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PRODUCTIVITY ASPECTS OF ACCOUNTS DEFLATION: DATA FOR IRELAND

By R. C. Geary

IN constant-price series as ordinarily understood, volume is envisaged as measured in quantum units of the ordinary kind, for the computing formula is a weighted mean of quantities expressed in number, weight, etc. For certain purposes it may be useful to measure volume by units of factor (labour and capital) input. This is how productivity (output per unit of factor input) enters the picture. What items in the accounts can be expressed in terms of input units having regard to utility and practicability?

There can be little doubt about the utility of expressing gross domestic product in input terms. Clearly the nation has gained if, since the base period, product in real terms has advanced per unit of factor input, whether this advance has resulted in increased leisure or increased production for home consumption, or for export whereby the nation can enjoy increased imports. Or, from a different point of view, productivity analysis can be regarded as explaining a rise in real product in the current compared with the base period. It would therefore seem a useful exercise to analyse current real product in its components (*a*) factor input and (*b*) productivity increase, since the base period. The national productivity index would follow immediately from this calculation as the quotient of real domestic product by factor input, equal to unity, of course, in the base year.

The exchange implications of increased productivity also seem worth investigation. If for the same factor input as in the base year the nation achieves an increased volume of exports (and thereby increased imports in exchange) in the current year, one might expect the emergence of a 'gain from productivity' term in the relevant real account. The same consideration applies to sectors within the economy.

Increased productivity can occur in one or both of two ways (i) a *structural* change within the economy whereby factor inputs shift proportionately from economic branches of lower productivity to those of higher productivity, e.g. from agriculture (in most countries) to non-agriculture and (ii) increased

technological productivity in one or more sectors. Assuming the availability of the data mentioned below, these constituents can be estimated in a very simple manner. Factor output accrues from the application of labour and capital, so that it seems natural to estimate current factor input at base-year rates as the sum product of current employee hours by base-year rate per hour and current volume of capital by base-year rate of return on capital. In the case of unincorporated enterprises, return on capital must be computed on profits after allowing for adequate remuneration of working proprietors and family members. Let volume of gross domestic product (GDP) be $Q(= P'$ in the Introduction) and let H and D be hours worked and capital invested (at base year values) in the current year. Let superscripts $(^0)$ indicate actual base-year values. Then

$$Q^0 = Q_h{}^0 + Q_d{}^0, \tag{1}$$

where $Q_h{}^0$ and $Q_d{}^0$ are respectively value of employee compensation and profits and rents (before depreciation) in the base year. *Expected product* Q_1 at base year rate of input of labour and capital is estimated as

$$Q_1 = \frac{Q_h{}^0}{H^0} \cdot H + \frac{Q_d{}^0}{D^0} \cdot D. \tag{2}$$

Then

$$Q \equiv Q_1 + (Q - Q_1) \tag{3}$$

where Q_1 (= expected product) is the current factor input constituent and $(Q - Q_1)$ is the current productivity constituent. The latter obviously has the character of an 'increase', since it is zero in the base year. The productivity index is Q/Q_1 (= unity in the base year).

To analyse the total productivity constituent into its structural and technological components, suppose that the economy consists of n sectors,[1] each homogeneous in general characteristics. Then expected output can be estimated as

$$Q_2 = \sum_{i=1}^{n} \left(\frac{Q_{ih}{}^0}{H_i{}^0} \cdot H_i + \frac{Q_{id}{}^0}{D_i{}^0} \cdot D_i \right) \tag{4}$$

[1] The more numerous the sectors the better, best of all the individual establishment as the 'sector' for the purpose of the calculation.

where symbols with subscript i indicate that they relate to the ith sector. In every case $\Sigma_i X_i$, of course, equals X, where X is any of the symbols.

For the calculation of Q_2 the economic structure is the current structure, so that $(Q - Q_2)$ estimates the technological component in the total increase in productivity $(Q - Q_1)$. The technological component arises through employees being better trained or working harder, by better management, by improved machinery and working conditions, etc. The present approach affords no means of estimating the separate contributions of these different causes. Finally,

$$(5) \qquad Q \equiv Q_1 + (Q - Q_2) + (Q_2 - Q_1),$$

where the three terms on the right side represent respectively: (a) factor input at base-year rates; (b) increase since base year in productivity due to technology; (c) increase in productivity due to structural change in the economy.

In formulas (2) and (4) the concept of capital, valued at base-year prices, intervenes. These statistics are available in very few countries. It is likely, however, that, to a degree of accuracy required for the computation of Q_1, many countries could make sufficiently reliable estimates. If capital is estimated for this purpose by one of the accumulation methods, it probably matters little (at any rate in the short term) whether annual capital formation is defined as 'net' (i.e. of capital consumption), 'gross' (i.e. before deduction of capital consumption), or 'gross-gross' (i.e. including, in addition, repairs and maintenance). In fact, the last term in formula (2) involves the ratio D/D^0, which may be much the same on all definitions.

To discuss the implications of productivity in the exchange aspect it will suffice to consider the external account. The method involves estimation of the GDP[1] content of exports in base and current year, both at base-year prices and then proceeding with productivity analysis exactly as in the case of *total* GDP already dealt with. Specific requirements in respect of exports of goods and services at base year prices or rates are therefore as follows:

[1] In this outline external factor services are assumed to be nil, for simplicity of exposition, so that GDP = GNP. No considerable difficulty would be experienced in introducing them, however.

For base year—
Import content (M_E^0)
Employee hours (H_E^0) and remuneration (Q_{hE}^0)
Capital invested (D_E^0) and profits etc. (Q_{dE}^0)

For current year—
Import content (M_E)
Employee hours (H_E)
Capital invested (D_E)

The subscript E is inserted to indicate relationship to exports. These quantities are derived from input–output tables for base and current years by the method described and illustrated in the appendix. Then for the current years e, the real GDP of E, is defined as

$$e = E - M_E \tag{6}$$

and e_1, the factor input, or expected output of current factors at base-year rates, as

$$e_1 = (Q_{hE}^0 H_E / H_E^0) + (Q_{dE}^0 D_E / D_E^0). \tag{7}$$

Finally

$$e \equiv e_1 + (e - e_1) \tag{8}$$

where $(e - e_1)$ is the productivity gain in respect of exports.

In the Introduction the exchange account in *base-year prices* was found to be of the form

$$E + T = M + N, \tag{9}$$

where N is the real surplus estimated from the current excess (positive or negative) of exports over imports by price deflation according to the rules suggested. T is the balancing item, the trading gain. (These letters are primed in the Introduction, but no confusion is likely to arise by using unprimed symbols here.) Subtracting M_E from both sides of (9), we find

$$e + T = m + N \tag{10}$$

where $m = M - M_E$. From the factor input viewpoint the exchange equation might be written

$$e_1 + t = m + N \tag{11}$$

which may be regarded as defining a new term t. From (10) and (11)

$$t = (e - e_1) + T, \tag{12}$$

so that t encompasses not only the trading gain T, due to different trends in import and export prices between base and current years, but also the export productivity gain.

Unless and until the system of balancing accounts in price-deflated terms establishes itself, there does not seem to be much point in considering further, at this stage, the problem of establishing accounts in units of factor input. It may be interesting to observe, however, how it was necessary to have regard to both input–output analysis and real capital estimation in connection with the study of productivity changes. It may also be worth remarking that it does not appear necessary to express imports in terms of factor (external, of course) content, an exercise which would probably be impracticable.

The viewpoint taken is that the essence of international exchange is the barter of the factor input of the nation for imported goods and services in the customary units (weight, number, etc.). For intersectoral studies within the nation, it might, on the other hand, be useful to have the exchange entirely in input terms as a guide to public policy, which might be concerned with directing factors from sectors of lower to those of higher productivity.

The foregoing is merely an outline of an approach to the productivity problem. As to its practicability, it may suffice to say that it postulates the existence at base-year prices of estimates of wealth classified by branches of economic activity and input–output tables, in addition, of course, to such statistics as employee hours worked. In practice, with a single input–output table for some recent year, it should not be an insuperable task to adjust it to give current estimates for the matrix, for the marginal totals and the final bill of goods in the different categories (government consumption, private consumption, capital formation, exports) in the different branches of economic activity. Having by matrix-inversion established the (a) import, (b) employee remuneration, and (c) other factor content per unit of final bill of goods, each of the three ratios for each branch of activity could be distributed proportionately into the categories, government, consumption, etc. A few countries

would even now appear to be in a position to make the calculations envisaged over an interval of, say, five years as a single span. In fact, productivity changes in short periods are small, and their effects might be masked in errors of estimation of other elements in year to year analyses, apart from the unavailability of annual input–output tables.

Three of the five accounts contemplated in the Introduction are displayed in Table I for Ireland at constant (1954) prices in the years 1938, 1950–57.[1] Obviously there would be no difficulty about supplying the two additional accounts if these were required. It may be well to explain how the table was constructed, in particular to identify the items which are residuals. Gross domestic product (GDP) at market prices (item 1.8) was found as items 1.4 + 1.5 + 1.6 − 1.7 separately computed. This was carried to 1.3. Item 1.2 was then estimated at what indirect taxes less subsidies would have been in the years indicated at the specific rates obtaining in base year 1954; and the difference between 1.3 and 1.2 gave 1.1 GDP at factor cost.

In turn, GDP at factor cost is the sum of 2.5 and 2.6, the two items representing respectively the Q_1 and $(Q - Q_1)$ in formula (3) above. Item 2.7 is found by deflating the corresponding current value by the non-factor import price index, as in the case of the external surplus, item 3.6. The trading gain, item 2.8, in Account 2 comes from the external account 3 (item 3.3). GNP at factor cost, the sum of items 2.5, 2.6, 2.7, and 2.8 is carried to item 2.4. Item 2.2 is employee increment deflated by a suitable consumer price index and item 2.1 the same current value deflated by a wage/earnings index. Item 2.3 (gross property income) is a residual. Direct deflation of this latter item is scarcely conceivable. Anyway, profits in current prices have essentially the character of a residual: they are what is left when current expenses have been met.

The three accounts contain in effect two items each deflated in two different ways, namely GDP at factor cost ((i) 2.5 and (ii) sum of 2.5 and 2.6 = 1.1) and employee compensation ((i) 2.1 and (ii) sum of 2.1 and 2.2). The device of using 'increments' has been resorted to in order to make the accounts additive. Obviously if the consumers' account (4 in the Introduction)

[1] Extracted or computed from data for Ireland in the UN *Yearbook of National Accounts Statistics*, latest issue 1958, and in the Irish Central Statistics Office publications *Irish Statistical Survey*, *Trend of Employment and Unemployment*, and the *Irish Statistical Abstract*.

TABLE I

Ireland, 1938, 1950–57: Three Accounts at Constant (1954) Prices

(£ million)

Account and item	1938	1950	1951	1952	1953	1954	1955	1956	1957
1. Domestic Product Account									
1.1 GDP at factor cost	340·3	396·4	394·9	417·6	426·4	434·8	439·0	436·8	447
1.2. Indirect taxes *less* subsidies	58·9	63·6	70·5	65·7	65·3	64·2	66·7	63·5	62
1.3. GDP at market prices	399·2	460·0	465·4	483·3	491·7	499·0	505·7	500·3	509
1.4. Consumption expenditure	378·9	450·6	459·9	441·9	453·3	466·3	486·5	472·3	455
1.5. Gross capital formation	49·6	73·4	79·2	67·0	76·9	67·7	80·5	61·3	61
1.6. Non-factor exports	100·9	130·8	133·1	148·3	154·2	154·2	148·6	148·1	168
1.7. *Less* non-factor imports	−130·2	−194·8	−206·8	−173·9	−192·7	−189·2	−209·9	−181·4	−175
1.8. Expenditure on GDP at market prices	399·2	460·0	465·4	483·3	491·7	499·0	505·7	500·3	509
2. Gross National Income Account									
2.1. Employee input	203·1	223·8	222·7	220·4	221·0	224·1	220·4	217·7	215
2.2. Employee increment	−21·8	−12·8	−10·9	−12·0	−5·7	—	6·6	14·1	10
2.3. Gross property income	185·8	202·4	188·0	217·1	232·0	229·6	228·4	214·1	226
2.4. Gross national income at factor cost	367·1	413·4	399·8	425·5	447·3	453·7	455·4	445·9	451
2.5. Factor input into GDP at factor cost	432·2	443·2	440·8	436·5	431·9	434·8	435·5	430·3	426
2.6. Productivity increment	−91·9	−46·8	−45·9	−18·9	−5·5	—	3·5	6·5	21
2.7. Net factor income exports	29·0	18·3	17·0	16·6	18·1	18·9	16·9	16·5	17
2.8. Trading gain	−2·2	−1·3	−12·1	−8·7	2·8	—	−0·5	−7·4	−13
2.9. GNP at factor cost	367·1	413·4	399·8	425·5	447·3	453·7	455·4	445·9	451
3. External Account									
3.1. Non-factor exports	100·9	130·8	133·1	148·3	154·2	154·2	148·6	148·1	168
3.2. Net factor income exports	29·0	18·3	17·0	16·6	18·1	18·9	16·9	16·5	17
3.3. Trading gain	−2·2	−1·3	−12·1	−8·7	2·8	—	−0·5	−7·4	−13
3.4. Receipts from abroad	127·7	147·8	138·0	156·2	175·1	173·1	165·0	157·2	172
3.5. Non-factor imports	130·2	194·8	206·8	173·9	192·7	189·2	209·9	181·4	175
3.6. Surplus	−2·5	−47·0	−68·8	−17·7	−17·6	−16·1	−44·9	−24·2	−3
3.7. Disposal of receipts from abroad	127·7	147·8	138·0	156·2	175·1	173·1	165·0	157·2	172

had been required, savings could be treated in an analogous manner the two deflating price indexes being (i) a composite 'consumers'' price index (though business savings might give some conceptual trouble) and (ii) the price index for gross capital formation. The fact that some items can be deflated in more than one way is not a deterrent to the compilation of accounts at constant prices. On the contrary, these accounts are a convenient way for juxtaposing the separately deflated items, and the juxtaposition is economically revealing.

The estimates for some of the items in the table, notably items 1.2 Indirect taxes less subsidies (and therefore the residual item 1.1 GDP at factor cost), 2.1 Employee input and 2.5 Factor input into GDP (and therefore the residual item 2.6 Productivity increment) could certainly be improved. It should be possible to estimate these items fairly accurately from available official records unfortunately inaccessible to the writer. It is hoped that others in a position to do so may be inspired or provoked to correct these figures. As regards items 2.1 and 2.5, it was necessary to rely mainly on labour-force statistics of persons at work (instead of hours worked, though official statistics show that there was not much change in hours worked per week during the period 1938–57). As regards productivity, no comprehensive statistics of stock of tangible capital are yet available for Ireland. I have felt emboldened to make the productivity calculation (using very speculative capital data in conjunction with labour force statistics) because the capital constituent is only a small fraction of total GDP factor input: in the United States the ratio capital/labour in input is currently only 1/4[1] and the ratio is almost certainly much smaller for Ireland, so that input is dominated by the labour constituent (in the wider sense of including adequate remuneration of working proprietors and family members). The figures (unforced) for the doubtful items do not look too bad.

The intrusive items 2.6 and 2.8 can better be seen in their true perspective than the absolute figures by considering *changes* from year to year. GDP at factor cost indicates the production of the nation, GNP or national income the nation's welfare in so far as this is measurable by statistics. Table II shows how trend in economic welfare, as defined, has been influenced by

[1] *Basic Facts on Productivity Change*, by S. Fabricant. Occasional Paper 63, National Bureau of Economic Research, Inc., 1959.

changes in the productivity increment and the trading gain, by reference to national production during the period 1950–57.

TABLE II

Year-to-year Changes in Certain Items in Table I, 1950–57

(£ million, 1954)

Item	1950/51	1951/52	1952/53	1953/54	1954/55	1955/56	1956/57
1.1. GDP at factor cost . .	− 1·5	+22·7	+ 8·8	+8·4	+4·2	−2·2	+10
2.4. GNP at factor cost . .	−13·6	+25·7	+21·8	+6·4	+1·7	−9·5	+ 5
2.6. Productivity increment .	+ 0·9	+27·0	+13·4	+5·5	+3·5	+3·0	+14
2.8. Trading gain .	−10·8	+ 3·4	+11·5	−2·8	−0·5	−6·9	− 6

The first fact to note about Table II is that changes in the productivity increment and the trading gain are of the same order of magnitude as the changes in GDP and GNP. The productivity increment is, of course, an element in both, so that their trend must be influenced by productivity. The trading gain, on the other hand, is in GNP as defined but not in GDP. The magnitude of the changes in the trading gain makes it imperative to decide whether the trading gain is, in fact an element in economic welfare and if so (though this problem is of less importance) whether it is correctly measured. The relationship between changes in GDP and the productivity increment are evident from Table II: far less so is the relationship between GNP and productivity. In fact, the discrepancy between the changes in GDP and GNP are clearly explicable by changes in the trading gain (or loss) in all periods except perhaps 1954/55. Reference to Table I will show that the marked rise of £12 million (at 1954 prices) in GDP at factor cost between 1954 and 1957 has been more than wiped out by the unfavourable relative trend in the terms of trade. Another showing of Table I is the remarkable constancy in real terms of the employee (sum of items 2.1 and 2.2) and property (item 2.3) shares of gross national income at practically half and half throughout the whole period 1938–57.

On the showing of Tables I and II the popular view that personal compensation can change *pro rata* with productivity alone, without inflation, is untenable. In a country like Ireland with a

relatively large external trade – the ratio exports/GNP is about 2/5 – responsible groups should note that the trading gain must also be taken into account. Rational policy might in fact have regard to the sum of these two elements in claiming, or advising as to, the proper level of income distribution as the share of GNP, the national cake, to which each group is fairly entitled. Without begging any questions, the fact that, between 1950 and 1957, at 1954 prices, GDP at market prices rose by £49 million while consumption expenditure rose by only £4 million (items 1.3 and 1.4 of Table I, both official estimates) suggests that the generally unfavourable terms of trade experience must have been an important factor in accounting for the discrepancy. It is not, of course, the only factor.

In the rise of productivity in Ireland since 1938 the technological component ($Q - Q_2$) was far more important than the structural component ($Q_2 - Q_1$) (see formula (5)). For the calculation of Q_2 according to formula (4) only three sectors (i.e. $n = 3$) were distinguished namely: (i) agriculture, forestry, fishing; (ii) industry; (iii) services. Following are the figures:

TABLE III

Ireland, 1938, 1950–57—Technological and Structural Components in Productivity

(£ million, 1954)

Year	Factor input (Q_1)	Technological component ($Q - Q_2$)	Structural component ($Q_2 - Q_1$)	Factor output (Q) = GDP
1938	432·2	−84·0	−7·9	340·3
1950	443·2	−44·8	−2·0	396·4
1951	440·8	−44·6	−1·3	394·9
1952	436·5	−18·0	−0·9	417·6
1953	431·9	− 5·7	0·2	426·4
1954	434·8	—	—	434·8
1955	435·5	3·1	0·4	439·0
1956	430·3	5·9	0·6	436·8
1957	426	20	1	447

The regular change in the structural component was due to the marked and continuous decline during the whole period 1938–57 in the numbers in agriculture, with the lowest output per unit of factor input of the three sectors.

Finally, it may be of interest to set down the productivity

indexes (i.e. Q/Q_1) for the economy as a whole and for the three sectors:

TABLE IV

Ireland 1938, 1950–57—Productivity and Real Wages/Earnings Indexes

(1954 as 100)

Year	Agriculture, forestry, fishing		Industry		Services	Total
	Productivity	Real wages	Productivity	Real earnings	Productivity	Productivity
1938	73	72	70	89	93	79
1950	85	86	87	94	96	89
1951	87	89	83	95	97	90
1952	97	90	92	95	98	96
1953	103	95	98	97	96	99
1954	100	100	100	100	100	100
1955	104	98	99	103	99	101
1956	106	108	96	106	102	102
1957	115	104	98	105	101	105

The continuous rise in aggregate productivity (last column Table IV), derived as the quotient of item 1.1 by item 2.5 in Table I, since 1954 is seen to be due to the rise in productivity in agriculture. In the two intervals of years 1938–50 and 1950–54 the productivity indexes for agriculture and industry were very similar, especially when allowance is made for the approximative character of the estimates. As might be expected, the rise in productivity of services is small over the whole period 1938–57.

As is customary, the productivity indexes are accompanied by real wages/earnings indexes, which must also be regarded as improvable in statistical quality: actually the figures represent the official indexes of weekly money earnings (transportable goods industries) or wages (agriculture), the former for the month of October and the latter for July deflated by the implicit private consumer-price index. Some doubt must therefore attach to changes between consecutive years, though the general showing may be valid enough. In agriculture, real wages followed productivity closely in the two intervals 1938–50 and 1950–54, and (if we regard the large increase in productivity between 1956 and 1957 as due to 1957 being a specially good year in this variable sector) the same may be regarded as true during the period

1954–57. For industry, rises in real earnings lagged behind the productivity increases in the intervals 1938–50 and 1950–54, but during 1954–57 industrial productivity flagged but real earnings rose.

Since no input–output table is yet available for Ireland – a deficiency in process of liquidation – it is not possible to apply the theory of productivity gain in external trade.

APPENDIX

EXPORT PRODUCTIVITY

Suppose that, for both base and current years, there is available an input–output table in the following highly consolidated form to which is appended ancillary data about employee hours (number) and capital invested:

Payments to \ Receipts from	Input of industry (1)	(2)	(3)	(4)	—	Final buyers: Consumers and capital (C)	Final buyers: Exports (E)	Total production, etc. (P)
Industry:								
(1)	—	A_{12}	A_{13}	A_{14}	—	C_1	E_1	P_1
(2)	A_{21}	—	A_{23}	A_{24}	—	C_2	E_2	P_2
(3)	A_{31}	A_{32}	—	A_{34}	—	C_3	E_3	P_3
(4)	A_{41}	A_{42}	A_{43}	—	—	C_4	E_4	P_4
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
Imports (M)	M_1	M_2	M_3	M_4	—	M'	—	M
Employee remuneration (Q_h)	Q_{h1}	Q_{h2}	Q_{h3}	Q_{h4}	—	—	—	Q_h
Profits, etc. (Q_d)	Q_{d1}	Q_{d2}	Q_{d3}	Q_{d4}	—	—	—	Q_d
Input (= production, etc.) (P)	P_1	P_2	P_3	P_4	—	C	E	—
Employee hours (H)	H_1	H_2	H_3	H_4	—			
Capital invested (D)	D_1	D_2	D_3	D_4	—			

It should be noted that in both tables (i.e. one for base and one for current year) all values are deemed expressed in base-year prices. Profits, etc., in the current-year table are residuals in each industry column, found by deducting inputs (including direct imports M_i) and employee remuneration (hours at base-year rates) from total production at base-year prices, derived as the row-wise sum of the entries in the table.

Express each of the entries in each industry column per unit of production $P_j (j = 1, 2, \ldots)$ letting small letters represent these unitary quotients, so that $a_{ij} = A_{ij}/P_j$, $q_{di} = Q_{di}/P_i$, etc. There being n industries, let $\mathbf{a}$ represent the square $(n \times n)$ industry matrix $[a_{ij}]$, $\mathbf{r}$ any row vector $(1 \times n)$ and $\mathbf{x}_r$, the row vector to be determined from $\mathbf{a}$. Then set

$$\mathbf{r} + \mathbf{x}_r\mathbf{a} = \mathbf{x}_r\mathbf{I},$$

where $\mathbf{I}$ is the $(n \times n)$ unit matrix. Hence

$$\mathbf{x}_r = \mathbf{r}(\mathbf{I} - \mathbf{a})^{-1}$$

Once the matrix $(\mathbf{I} - \mathbf{a})$ is inverted [to give $(\mathbf{I} - \mathbf{a})^{-1}$] for both base and current years, the vector $\mathbf{r}$ will be taken in succession to represent for base-year direct imports into industry $\mathbf{m}^0$, employee hours $\mathbf{h}^0$ and remuneration $\mathbf{q}_h{}^0$, capital invested $\mathbf{d}^0$ and profits, etc., $\mathbf{q}_d{}^0$; and for current-year direct imports into industry $\mathbf{m}$, employee hours $\mathbf{h}$ and capital invested $\mathbf{d}$. There will result as many vectors $\mathbf{x}_r$, representing the sum of direct and indirect components per unit of production of each industry. Finally, with the usual rather heroic assumption (valid enough if the input–output tables are in considerable detail as to industry) that for each industry exports are typical in kind of total production, the entities specified in the text above are found as the scalar products $\mathbf{E}\mathbf{x}_r'$, $\mathbf{x}_r'$ being the transpose (i.e. column vector) of $\mathbf{x}_r$ and $\mathbf{E}$ the row vector of real exports.

The procedures outlined above will perhaps be clearer from the following example for two industries (i.e. $n = 2$).

All Values Deemed to be at Base-Year Prices or Rates

	(1)		(2)		*C*	*E*	*P*
Base Year							
(1)	—	—	40	(0·2)	30	30	100
(2)	20	(0·2)	—	—	40	140	200
M^0	50	(0·5)	60	(0·3)	60	—	170
$Q_h{}^0$	20	(0·2)	50	(0·25)	—	—	70
$Q_d{}^0$	10	(0·1)	50	(0·25)	—	—	60
P^0	100	(1·0)	200	(1·00)	130	170	—
H^0	100	(1·0)	220	(1·1)			
D^0	200	(2·0)	800	(4·0)			
Current Year							
(1)	—	—	52	(0·2)	35	63	150
(2)	45	(0·3)	—	—	50	165	260
M	75	(0·5)	65	(0·25)	70	—	210
Q_h	22·5	(0·15)	65	(0·25)	—	—	87·5
Q_d	7·5	(0·05)	78	(0·3)	—	—	85·5
P	150	(1·00)	260	(1·00)	155	228	—
H	112·5	(0·75)	208	(0·8)			
D	225	(1·5)	910	(3·5)			

The units in which H^0 and H are expressed are immaterial, since finally only the ratios H^0/H will be involved. The matrices $(\mathbf{I} - \mathbf{a})$ and $(\mathbf{I} - \mathbf{a})^{-1}$ are as follows:

$$(\mathbf{I} - \mathbf{a}) \qquad\qquad (\mathbf{I} - \mathbf{a})^{-1}$$

$$\begin{array}{cc|cc} \text{Base} & & \text{Current} & \\ 1 & -0{\cdot}2 & 1 & -0{\cdot}2 \\ -0{\cdot}2 & 1 & -0{\cdot}3 & 1 \end{array} \quad \left| \quad \begin{array}{cc|cc} \text{Base} & & \text{Current} & \\ 1 & +0{\cdot}2 & 1 & +0{\cdot}2 \\ +0{\cdot}2 & 1 & +0{\cdot}3 & 1 \end{array} \right.$$

$$\left(\begin{array}{cc} 1 & -0{\cdot}2 \\ -0{\cdot}2 & 1 \end{array}\right); \left(\begin{array}{cc} 1 & -0{\cdot}2 \\ -0{\cdot}3 & 1 \end{array}\right) \quad \left| \quad \frac{1}{0{\cdot}96}\left(\begin{array}{cc} 1 & +0{\cdot}2 \\ +0{\cdot}2 & 1 \end{array}\right); \frac{1}{0{\cdot}94}\left(\begin{array}{cc} 1 & +0{\cdot}2 \\ +0{\cdot}3 & 1 \end{array}\right) \right.$$

Note: The values of the determinant $\mathbf{I} - \mathbf{a}$ are respectively 0·96 and 0·94 for base and current years.

$\mathbf{r}$	$\mathbf{r}(\mathbf{I} - \mathbf{a})^{-1} = \mathbf{x}_r$		$\mathbf{E}\mathbf{x}_r'$
	Base year		
			[$\mathbf{E} = (30,140)$]
$\mathbf{m}^0 = (0{\cdot}5, 0{\cdot}3)$	0·583	0·417	75·8
$\mathbf{q}_h{}^0 = (0{\cdot}2, 0{\cdot}25)$	0·260	0·302	50·1
$\mathbf{q}_d{}^0 = (0{\cdot}1, 0{\cdot}25)$	0·156	0·281	44·1
$\mathbf{h}^0 = (1{\cdot}0, 1{\cdot}1)$	1·271	1·354	227·7
$\mathbf{d}^0 = (2{\cdot}0, 4{\cdot}0)$	2·917	4·583	729·2
	Current year		
			[$\mathbf{E} = (63,165)$]
$\mathbf{m} = (0{\cdot}5, 0{\cdot}25)$	0·612	0·372	100·0
$\mathbf{q}_h = (0{\cdot}15, 0{\cdot}25)$	0·239	0·298	64·2
$\mathbf{q}_d = (0{\cdot}05, 0{\cdot}3)$	0·149	0·330	63·8
$\mathbf{h} = (0{\cdot}75, 0{\cdot}8)$	1·053	1·011	233·1
$\mathbf{d} = (1{\cdot}5, 3{\cdot}5)$	2·713	4·043	837·9

Hence expected GDP, or factor input e_1 in exports, is given by

$$e_1 = \frac{50{\cdot}1 \times 233{\cdot}1}{227{\cdot}7} + \frac{44{\cdot}1 \times 837{\cdot}9}{729{\cdot}2} = 102{\cdot}0$$

and actual GDP in exports E is

$$e = 228 - 100{\cdot}0 = 128{\cdot}0$$

The export productivity gain is accordingly 128·0 − 102·0 = 26·0. It is worth observing that actual GDP (in both base and current years), computed as the difference between gross exports and import content (170 − 75·8 = 94·2 in base year and 128·0 in current year), exactly equals the GDP regarded as the sum of the estimated factor inputs (50·1 + 44·1 = 94·2 in base year and 64·2 + 63·8 = 128·0 in current year). This property of input–output operation is quite general. What the operation does is to construct a production account for exports.