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MODEL BUILDING AND THE SOCIAL ACCOUNTS: A SURVEY

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I. INTRODUCTION

In my papers [1, 2] to the two earlier conferences of this Association I was concerned, in the main, with some of the conceptual problems involved in setting up the social accounts; with problems of defining and classifying transactions and of arranging them in a convenient form. On this occasion I propose to examine some of the problems that arise in using the social accounts. Many of these uses involve an extension of social accounting, in the sense of measuring variables, into the field of model building which, from an applied point of view, requires both the formulation of relationships between the variables, and any others that may be relevant, and the estimation of the parameters in these relationships.

The construction of models of the economic process as a whole is, without doubt, one of the most ambitious tasks on which an economist can be engaged and the problems involved have been studied from many different points of view. Much work on model building has been undertaken as a theoretical exercise, that is as an attempt to see what kind of relationships are needed to produce variations of the kind which more or less casual empiricism suggests take place in the actual world. Many of the papers in this field are concerned with the behaviour of systems of linear equations and these indeed may be of considerable utility for certain purposes. It is frequently supposed, however, that a system of equations which is to represent the economic process should be capable of generating stable oscillations but, as has frequently been recognised [3, 4, 5, 6], this is only possible in the case of linear systems if, given the initial conditions, the coefficients take very special values. Accordingly it would appear that if maintained oscillations are to arise from the structure of the model itself and not as a consequence of influences not brought within the model such as sunspots [7], an exogenous cycle in capital formation [8, 9] or a series of random impulses [10], then non-linearity must enter in an essential way into the model with all the attendant complications of analysis [11].

Other investigations have been concerned with the statistical problem of estimating the parameters in systems of relationships. Here attention has been drawn to the importance from the standpoint of the classical method of least squares, of the assumption that in any equation the determining variables be jointly distributed independently of the disturbance term. Apart from the implications it may have for the choice of an estimating procedure, consideration of this assumption leads to the notion of identifiability which is important if the coefficients estimated are to be identifiable with the parameters in the theoretical model [12, 13, 14, 15].

Many studies have also been undertaken in which a model is constructed as a basis for prediction. The essential point here is that relationships expressing behaviour or technology can usefully be added to those of a definitional character arising from the accounts themselves so long as they serve to introduce information about the economic process. Thus if a satisfactory formulation of the consumption function with reasonably well estimated parameters can be introduced, this may be extremely valuable even if it is necessary to treat capital formation as exogenous because a satisfactory formulation of the relationship between capital formation and product cannot be made. The objective in this kind of work is to introduce information and to avoid the introduction of misinformation as can easily be done if undue insistence is placed on closed systems of a kind which can readily be manipulated. Here it is necessary to proceed in close contact with the facts as is amply illustrated by the history of attempts to formulate the consumption function in a satisfactory way.

The applied model building activity just described is directed towards the formulation of a set of relationships which will describe the economic process, or certain features of it, in a realistic way. The model is used in working out the implications of certain changes and any decisions which are taken as a consequence of these findings form the subject of a separate exercise. It is, however, possible to introduce certain policy objectives into the model, to treat some of the economic variables as adjustable parameters and to calculate the values of these parameters which are needed to satisfy the policy objectives. Work on these lines leads to what Frisch has termed 'decision models' [16, 17] and, generally, to linear programming or activity analysis [18, 19, 20] in which a criterion for an optimum solution is explicitly introduced.

II. THE SUBJECT MATTER OF THE PAPER

This paper is intended as an outline survey of developments in the construction of models of the economic process with particular emphasis on the empirical testing of relationships. The point of departure, set out in the following section, is a twofold classification of models based on the type of consolidation of the social accounts which provides the principal variables of the model. From a theoretical point of view it might not be supposed that this would be a particularly illuminating criterion of classification but in fact it serves very well to distinguish between the two principal lines of development which have been followed up in the last twenty years. The first type concentrates on the determinants of final demand without much regard to the details of the commodity composition of production. This kind of model may be associated primarily with the name of Keynes, though fiscal policy models and growth models of the Harrod-Domar types may, from my point of view, also be placed in this category. The second type concentrates on the technological relationships within the sphere of production. This kind of model is associated primarily with the name of Leontief and embraces the input-output model which he has made famous and also its normative correlative, activity analysis. This is followed by a brief discussion in section IV of the distinction between static and dynamic models. The following sections, V through XV, are taken up with a discussion of particular models or parts of models. In section V a simple national budgeting model of the type that has been used in the United Kingdom is set out. This model involves only accounting relationships. If estimates of the future values of the variables which it contains are to be narrowed down information about behaviour or technology must be introduced. Final demand models, the first type just mentioned, involve information on the determinants of final demand. Accordingly section VI indicates the developments that have taken place in the formulation of the consumption function and in the measurement of its parameters. Section VII contains some brief remarks on the measurement of future levels of government current purchases and section VIII is devoted to developments in the measurement of foreign trade elasticities. Section IX describes various methods proposed to introduce capital formation into the system of relationships and the experience gained from the various relationships proposed.

These sections complete what I have to say on the components of final demand models and section X is devoted to various complete models of this kind. Section XI deals with fiscal policy models and especially that of Kaldor which illustrates the way in which a model may be set up to show directly the implications of various policies.

In section XII the simple static input-output model is described and this is followed by some references in section XIII to input-output tables that have been constructed and the work that has been done on the inversion of the matrices of production coefficients that can be derived from them. Section XIV contains a brief description of the proposals that have been made for a dynamic input-output model and section XV is concerned with activity analysis and its relationship to inter-industry models and to ordinary economic theory. Some conclusions that seem to emerge from this survey are given in section XVI and the concluding section, XVII, is devoted to references.

III. MODELS AND THE CONSOLIDATION OF THE SOCIAL ACCOUNTS

Since my subject has many aspects it seems desirable at an early stage to lay down certain broad distinctions as an aid to a systematic arrangement of topics. For present purposes it is convenient to consider models in terms of the alternative consolidations of the social accounts with which they are associated. If we think of the entries, receivables and payables, in these accounts as the elements in the row and column pairs of a square matrix, then we can introduce two extreme forms of simplification by alternative consolidations. One of these would be to consolidate all accounts relating to the same form of economic activity and so obtain single accounts for aggregate production, consumption and adding to wealth and also, for an open economy, for external transactions. The second type of consolidation would be to consolidate all accounts relating to the same institutional transactor and so obtain a single account for each branch of activity, including the activity of converting consumers' goods into factor services.

This distinction reflects the fact that, in applied work at any rate, it is necessary to concentrate on certain aspects of the economic process to the exclusion of others, and corresponds to the distinction between elementary final demand models and elementary input-output models. The first type of model is concerned primarily with questions of final demand and income generation and does not raise the question of production possibilities in commodity terms. In the simplest form it is the basis of the Keynesian theory of income determination which depends on two accounting identities, namely that income, Y, equals consumption, C, plus capital formation, I, and that capital formation equals saving, S, on a consumption function relating consumption to income and on a level of capital formation assumed to be determined exogenously. These four variables, Y, C, I and S, are the ones which appear in the first type of consolidation of a simple, closed economy.

The second type of model concentrates on the interrelationships in the sphere of production since, basically, each type of transactor is regarded as in control of a transformation process which converts its input into its output. If all non-production accounts are consolidated together and labelled 'final demand' and if this final demand is assumed to be determined exogenously then there results a simple form of open-ended input-output system.

Thus it may be said [21] that final demand models result from a concentration on the entries in non-production accounts without explicit consideration of the production implications of different levels of effective demand whereas input-output models spell out the technological relationships between industries without explicit consideration of the determination of the level of final demand.

Simple models of either type can be formulated if it is assumed that the expenditures by any account depend, in a simple way, on, and only on, that account's total revenue. Such models I have elsewhere termed 'simple transaction models' [22]. The properties of these models have also been set out by Solow [23]. Reference may be made at this point to the works of Goodwin [24, 25] and Chipman [26] on the matrix multiplier.

Consider a system of four social accounts which consists of two production accounts, one for agriculture and one for all other branches of economic activity, one appropriation account (which may be assumed to relate simply to households) and one capital transactions account. Suppose further that the entries in this system are as follows

		Production Accounts		Appropriation Account (Households)	Capital Transactions Account	Σ
		Agriculture	All other			
Production Accounts	Agriculture .	—	100	250	—	350
	All other .	70	—	30	50	150
Appropriation Account (Households) . . .		280	50	—	—	330
Capital Transactions Account		—	—	50	—	50
Σ		350	150	330	50	

The final demand variables are obtained by a consolidation of the first two accounts, that is of the production accounts, which yields

	Production Account	Appropriation Account	Capital Transactions Account	Σ
Production Account . .	—	280	50	330
Appropriation Account .	330	—	—	330
Capital Transactions Account . . .	—	50	—	50
Σ	330	330	50	

The inter-industry variables are obtained on the other hand by a consolidation of the last two accounts, that is of the non-production accounts, which yields

		Production Accounts		Non-production Accounts	Σ
		Agriculture	All Other		
Production Accounts	Agriculture .	—	100	250	350
	All other .	70	—	80	150
Non-production accounts .		280	50	—	330
Σ		350	150	330	

Let **B** denote a 2×2 behaviour matrix, associated with the first consolidation, the elements of which are marginal propensities to spend, that is they show the sums transferred from current account j to current account i which are associated with a unit of income of account j. Let **r** be a vector of receipts into current accounts. The elements of **r** are composed of receipts from current accounts and receipts from capital accounts. The former are clearly equal to **B r** and the latter may be denoted by **k**. Thus

$$\begin{aligned} \mathbf{r} &= \mathbf{B}\,\mathbf{r} + \mathbf{k} \\ &= (\mathbf{I} - \mathbf{B})^{-1}\,\mathbf{k} \end{aligned} \tag{1}$$

Since, in this example,

$$\mathbf{k} = \begin{bmatrix} 50 \\ 0 \end{bmatrix} \tag{2}$$

and

$$\mathbf{B} = \begin{bmatrix} 0 & 28/33 \\ 1 & 0 \end{bmatrix} \tag{3}$$

it follows that

$$\mathbf{r} = \begin{bmatrix} 33/5 & 28/5 \\ 33/5 & 33/5 \end{bmatrix} \begin{bmatrix} 50 \\ 0 \end{bmatrix} = \begin{bmatrix} 330 \\ 330 \end{bmatrix} \tag{4}$$

This model exhibits the revenue of the current accounts as a matrix multiplier times a vector of capital outlays which are assumed to be exogenous. Thus, for example, income, 330, can be seen as the ordinary investment multiplier $1/(1-28/33)$ times investment, 50.

The open-ended simple transaction model associated with the inter-industry variables works out in a precisely similar way. Let **A** denote a 2×2 technical matrix the elements of which show the sums transferred from production account s to production account r which are associated with a unit of revenue of account s, that is the input from r into s which produces a unit of output in s. Let **q** be a vector of the total revenue, or product in money terms, of each industry. The elements of **q** are composed partly of intermediate products given by the vector **A q** and partly of final products given by a vector which may be denoted by e. Thus

$$\begin{aligned} \mathbf{q} &= \mathbf{A}\,\mathbf{q} + \mathbf{e} \\ &= (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{e} \end{aligned} \tag{5}$$

Since in this example

$$\mathbf{e}=\begin{bmatrix}250\\80\end{bmatrix} \tag{6}$$

and

$$\mathbf{A}=\begin{bmatrix}0 & 2/3\\1/5 & 0\end{bmatrix} \tag{7}$$

it follows that

$$\mathbf{q}=\begin{bmatrix}15/13 & 10/13\\3/13 & 15/13\end{bmatrix}\begin{bmatrix}250\\80\end{bmatrix}=\begin{bmatrix}350\\150\end{bmatrix} \tag{8}$$

This model exhibits the revenues of the production accounts as a matrix multiplier times a vector of final purchases which in the open-ended model are assumed to be exogenous.

Thus, in the final demand model, current receipts are made to depend on capital outlays while in the inter-industry model production receipts (total outputs) are made to depend on final purchases.

These examples are obviously highly simplified and it might be thought that the distinction drawn would inevitably disappear if a more realistic type of model were made the starting point of the discussion. In fact, however, it will be found that much of the work done in this general area can be classified along the lines suggested. This is not altogether surprising since a model which represented every aspect of the economic process in considerable detail would, even if it could be constructed at all, be unnecessarily cumbersome for specific purposes and therefore hardly worth-while if only specific types of application were intended. Thus, for example, if the object of the model were to indicate the magnitude of alternative fiscal changes intended to overcome a trade depression it would probably be of only secondary interest to examine in detail the commodity implications of changes in the level of effective demand; it would usually be reasonable to assume that a higher level of final demand could in fact be supplied. This assumption would not, however, necessarily be reasonable in all circumstances. If the depression had been very severe considerable changes might have taken place in industrial capacity with the consequence that even relatively small changes in supply with the existing structure of relative prices could not be taken for granted. A similar result might follow if the injection of final demand were heavily concentrated on particular commodities or processes and in any event considerations of supply would again come to the fore as total product approached the capacity of the system. In all such cases the aim of model building should be to select as far as possible the important variables and relationships and to introduce them, somehow or other, into the picture even if this cannot be done with great precision or elegance.

IV. STATIC AND DYNAMIC MODELS

The relationships in a static model connect variables belonging to the same time period whereas in a dynamic model this is not always the case and one or more of the relationships connect variables relating to different time periods. Many economic variables are clearly influenced by preceding events and in a sense, therefore, dynamic models, containing differences or derivatives of various orders, are superior to static models in representing the economic process. It does not follow from this that static models are not useful nor even that they may not be more useful than the dynamic models that can be constructed in a given state of knowledge. In the first place it will frequently be the case that there is a static model to the values in which the variables in a dynamic model will converge. In such a case all that will be lost by adopting the static formulation will be a knowledge of the transient states through which the system will pass before a steady state is reached. Second, and this is perhaps the more important from a practical point of view, it may be that those relationships in the system which are capable of a satisfactory numerical representation are in fact very little affected by past influences and that the variables which are determined by relationships in which past influences are strong must, for lack of information, be treated as exogenous. In such a case a partial, static model which, as far as it goes, gives a good representation of the actual world combined with separate estimates of the exogenous variables in the relevant periods is likely to prove more serviceable than a 'complete' dynamic model in which some of the relationships are not in fact reliable.

V. A NATIONAL BUDGETING MODEL

At this point I propose to take up the discussion of various types of model. I shall start with final demand models and consider in the first place a partial model sometimes used in estimating the transactions in the national accounts for the coming year. The only relationships introduced into this model to restrict the estimates are the accounting identities of the system. These are made use of in several different ways. In the first place some of them may be used to obtain residual estimates of certain flows. In a closed system of n articulated accounts there are (n−1) independent identities permitting at most this number of residual estimates. If full use is not made of these relationships for residual estimation then there will be at least one transaction that is estimated both directly and residually. A comparison of these estimates may be used as an indicator of the inflationary or deflationary pressure to be expected in the system in the coming year. Any policy decisions intended to remove such pressure imply changes in some of the variables of the system. Thus something can be said about the way in which a balance is sought and in the light of this an attempt may be made to set out the balanced system intended as a consequence of policy changes.

The model just described can be illustrated by the steps taken in the United Kingdom in the years since the war to draw up the annual national accounts projections which in balanced form appeared in a number of Economic Surveys [27]. The following account is based on a paper given by Jackson [28] at an earlier conference of this Association. It is intended to give a picture of the typical procedure adopted over a particular period and there is no reason to suppose that precisely this method is still in use.

The various steps can be seen from Diagram 1 below in which the five nodes represent the five accounts in the system and the directed branches represent the flows. The procedure is to estimate directly most of the flows into and out of the various accounts and to obtain others as residuals. As will be seen the resulting system of estimates is overdetermined; in fact the number of residuals is one less than the number of independent accounting relationships.

As regards the direct estimates themselves it should be recognized that national budgets cannot be drawn up by an individual statistician working by himself. Account has to be taken of the various plans and possibilities that are developing and this means that many individuals and committees have to be consulted about the various estimates. This consultation takes time and it is often unavoidable that later information can be used for some but not all of the estimates. This results in a certain amount of inconsistency which in practice cannot be avoided.

The estimates thus embody information derived from plans of all kinds which are being decided as the calculations proceed. By the time they are completed many decisions will already have been taken and these decisions cannot be directly influenced but only subsequently modified by the work. The decisions which, as a rule, have not been taken by the time the national budget is complete lie mainly in the sphere of fiscal policy and, in the United Kingdom at any rate, it is mainly in the 'financial' as opposed to the 'real' sphere that national budgeting may have an immediate influence.

Before the work is begun it is necessary to decide on the level of prices in terms of which the calculations are to be made. In general the level ruling at the end of the preceding year is adopted. However, in calculating the income from direct taxes an attempt is made to take a realistic view of the probable level of taxable income in money terms.

In the following brief account, the whole operation is divided into seven stages. These are indicated by the numbers against the flows in Diagram 1. Direct estimates are represented by solid lines and residual estimates are represented by dotted lines.

The seven steps in the calculation are as follows:

(1) The first step is to estimate the items in the account with the rest of the world. Exports are estimated first despite the fact that this implies an assumption about imports which cannot be taken into account. Estimates are then made of factor income from abroad (net) and of positive commitments for investment abroad. Account is then taken of the possible level of grants from abroad and of any additional foreign investments that it is thought can be financed. Imports are then obtained as the residual item in this account.

(2) The second step is to estimate the gross domestic product at factor cost. This calculation is based on assumptions about the level of employment, the industrial distribution of the labour

force and the productivity of labour so far as 'wage-containing' product is concerned. To this is added an estimate of the income derived from dwelling houses. The whole of this flow is shown in the diagram as paid into private consumption by production.

(3) A number of transfers between private and public authority consumption are estimated next. The first of these are social security and similar transfer payments and the interest on the national debt which are transfers from the public authority sector to the private sector. Second, an estimate is made of direct taxes of all kinds which are transfers in the opposite direction. Finally, since the whole of the gross domestic product was paid into private consumption at stage (2), that part which in fact accrues to public authorities is estimated and is shown as a flow from private to public authority consumption.

(4) The next stage is to estimate all provisions for depreciation and private saving, that is the saving of businesses and households. This is one of the most difficult parts of the whole calculation, especially as far as the estimation of household saving is concerned. The whole flow is shown as going from private saving to adding to wealth.

With the estimation of this item all the flows at the private consumption account have been calculated except for private (household) consumption. This flow can accordingly be obtained as a residual. On some occasions it has been found more practicable to estimate private consumption directly and to obtain private saving plus provisions for depreciation as a residual.

(5) When the value of private consumption is known, it is possible, by considering its probable composition, to estimate indirect tax revenues and also the level of subsidies needed to stabilize a certain set of retail prices. Hence net indirect taxes, shown as a flow from the production account to the public authority account, can be estimated.

(6) Public authority consumption (current expenditure on goods and services) is the next flow to be considered. This is obtained from the parliamentary estimates for the central government and from estimates made for local authorities. When this is done, public authority saving (surplus or deficit) can be estimated as a residual.

At this point it is of interest to consider the network formed by the residuals in the system of estimates. It can be seen from

Diagram 1

The Stages in the Construction of the National Budget of the United Kingdom

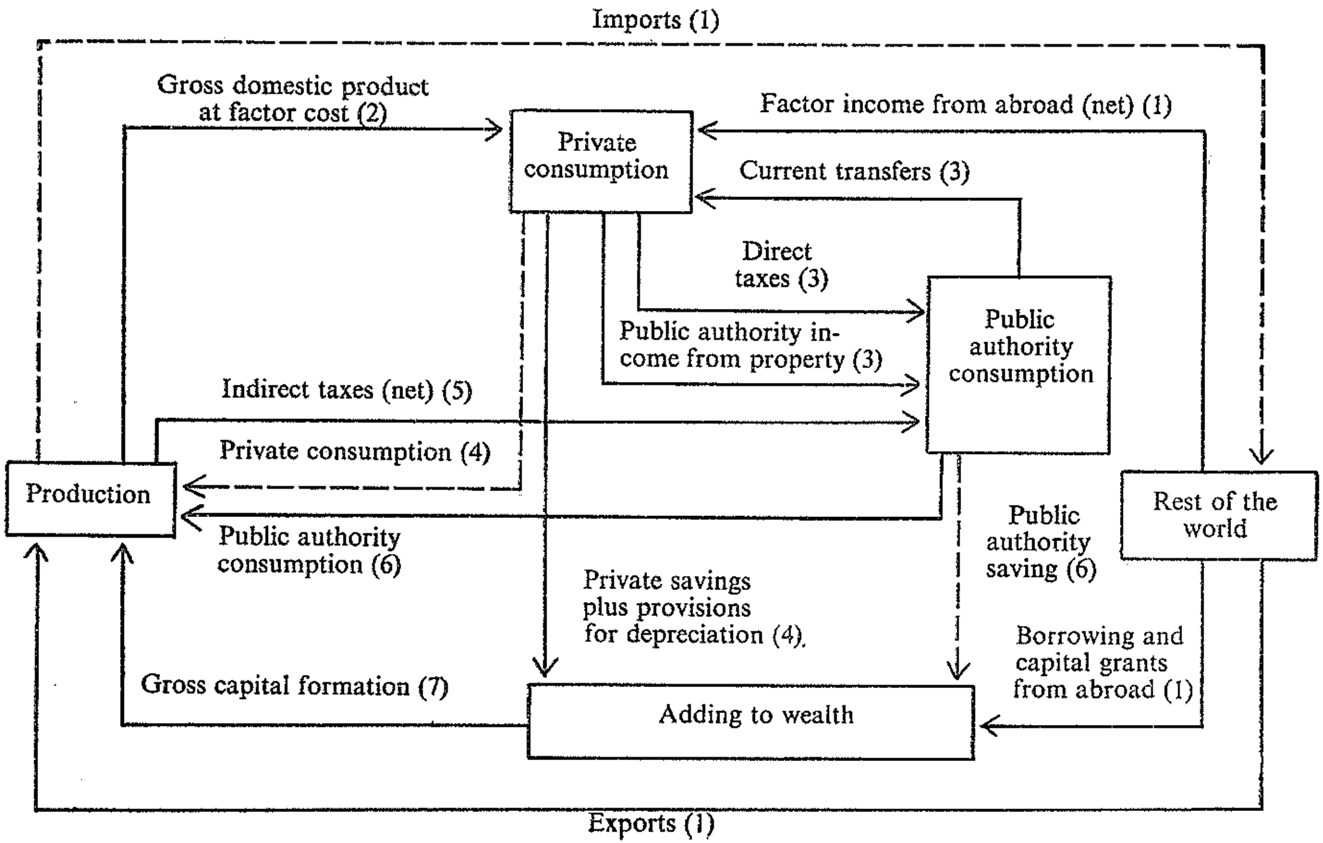


Diagram 1 that only three out of the four independent accounting identities have been used so far and that the network of residuals is not connected but falls into two parts. No mention has so far been made of gross capital formation and the diagram shows that this flow, which goes from adding to wealth to production, could be estimated as a residual at this stage.

(7) The final stage is to make a direct estimate of gross capital formation by reference to the plans for capital expenditure in different parts of the economy. The set of estimates is now overdetermined since one of the accounting relationships has not been used. Unless, therefore, the direct estimate of gross capital formation happens to be equal to the residual estimate, mentioned under (6) above, the flows at production and at adding to wealth will not balance. In practice a gap of considerable magnitude is frequently revealed by these calculations. If the direct estimate of gross capital formation exceeds the indirect estimate of it, that is to say the direct estimate of the supply of finance at adding to wealth, then, according to the estimates, the plans for spending are in excess of the supply of goods available. The situation is therefore an inflationary one and, if the supply of goods cannot be increased, policy must be directed to reducing demand by reducing either current or capital expenditure. Conversely, in the opposite situation, policy must be directed to expanding demand if the level of employment assumed in the calculation of the gross domestic product is in fact to be reached.

It will be seen that the procedure just described is highly flexible. Nothing more is required than that certain accounting identities will, in fact, be satisfied. That this should be so is necessary by definition. But if more restrictions can be introduced there is nothing to prevent this being done. In practice it appears that further restrictions, especially knowledge about the responses of consumers, are much needed and that a failure to introduce them means that the results obtained from the model, though consistent with the restrictions imposed, are often at some variance with events. To some extent this is to be expected on account of the individual features of any one period which could hardly be represented in a model. But on the whole it seems plausible that much of the gap between predictions and events is due to the fact that behaviouristic restrictions are not adequately formulated and taken into account. This, accordingly, is the next subject to examine.

Before an attempt is made to do this, it may be noted that the whole procedure just described may be of considerable value even if the picture which it gives of the shape of things to come is numerically rather wide of the mark. It certainly has the merit of showing to those whose minds are directed to certain parts of the picture how their intentions and objectives are related to those of others and to the productive potential of the system. In this way some mistakes at least may be avoided. It may well be the case that really disastrous policies would sometimes be attempted if there were no means of gaining even a rough picture of aggregate demand and supply.

VI. INFORMATION ABOUT CONSUMERS' BEHAVIOUR

If further information is to be introduced into the model just described, an obvious subject for further investigation is the behaviour of final buyers. While these buyers cannot act in a manner which is inconsistent with the accounting identities, their actions are not fully determined by these and in fact certain responses to the situation in which they find themselves are more likely than others. To suppose otherwise implies that nothing is known about the relevant types of economic behaviour and this, almost certainly, is to err on the side of pessimism. At all events, in order to get any further an attempt must be made to appraise what we do know in this matter.

The usually accepted point of departure is to concentrate on the responses of private consumers; that is to consider what is known empirically about the consumption function. It has been well known for some time [29] that a simple equation relating consumption to income gives a good fit to time series over certain periods and this is perhaps not surprising since consumption forms so large a part of total product. At the same time the fit is not perfect and so a disturbance term should be added to the right-hand side of the consumption equation to indicate the influence of omitted variables and other imperfections in the formulation.

If, however, a simple linear relationship is used to connect data on income and consumption (or saving) which cover a long period, say back to 1870, and also the periods before and after the second world war, two problems emerge. First there appears to be a substantial difference between adjustments to income when it is at an abnormally high or low level, as occurs over the trade cycle, and adjustments to long-term average income levels which may be regarded as reflecting normal growth [30, 31, 32]. Second there is evidence of an upward shift in the pre-war consumption function in the years since the end of the second world war.

Suggestions for meeting these difficulties have been made by Modigliani [33] and Duesenberry [34]. The essential feature of these proposals is that the highest level of income so far experienced should appear as an additional variable in the consumption equation. The effect of this is to make the short-term consumption function shift upwards through time so that, as income rises over time, the observed relationship between consumption and income is more nearly one of proportionality. The consumption function thus becomes an irreversible relationship in which the past history of living standards achieved exercises a certain influence on future levels of consumption. A further modification introduced by these writers is to concentrate on the explanation of the average propensity to consume rather than on total consumption. The object of this is to reduce the danger of spurious effects resulting from the common trends of C and Y.

Modigliani proposes the equation

$$\frac{C_r}{Y_r}=\alpha+\beta\left(\frac{Y_r-Y_{or}}{Y_r}\right) \tag{9}$$

or

$$C_r=(\alpha+\beta)Y_r-\beta Y_{or} \tag{10}$$

where the variables are on a per capita basis, the suffix r denotes deflated values and the suffix o denotes the highest level of Y_r previously attained.

Duesenberry's relationship takes the form

$$\frac{C_r}{Y_r}=\alpha+\beta\left(\frac{Y_r}{Y_{or}}\right) \tag{11}$$

or

$$C_r=\alpha Y_r+\beta\left(\frac{Y_r^2}{Y_{or}}\right) \tag{12}$$

Numerical results for the United States based on these and other forms are conveniently presented by Davis [35] and, in more detail, by Ferber [36]. Davis also introduces a further variant of his own by substituting the highest consumption level previously achieved, C_{or} say, for Y_{or}. This is justified by reference to the fact that the standard of living which may be expected to influence future consumption is represented better by the past peak of real consumption than by the past peak of real income.

Considerable success was achieved by both Modigliani and Duesenberry in predicting post-war consumption levels on the basis of pre-war data. Davis estimates the parameters in various relationships from data relating to the years 1929–40 and obtains rather strikingly good estimates of consumption, especially from his own variant of the relationship, for the years 1946–50.

Further modifications on somewhat similar lines together with some interesting calculations have been presented recently by Brown [37]. This writer also estimates separately the change in consumption associated with changes in wage and non-wage components of income.

More recently a new and important contribution has been made by Modigliani and Brumberg. The results of their investigations are at the moment in course of publication [38, 39] and I am able to give a brief account of them through the kindness of the authors. Their concern is with the integration of individual and aggregate behaviour and with the reconciliation of observations from cross-section studies and from time series. Their line of attack on the problem is to consider the amount which an individual has available for consumption over the remainder of his lifetime expressed in terms of his current income and assets and the sum of his expected future income over the remainder of his earning span. Given assumptions about the individual's remaining earning span and his retirement span and supposing further that he proposes to spend on consumption all that is available to him in given proportions over his remaining years of life, then his current consumption can be expressed in terms of his age, expected future life, current assets, and current and expected future income. Given assumptions about the age composition of the population the individual functions can be aggregated to give an expression for total consumption. This leads to an expression in which the determining variables are total current income, total expected income and total assets. If a relationship can be accepted which expresses expected income in terms of current and past incomes *then the equation for total consumption can be further simplified* and expressed in terms of current and past incomes and current assets. This formulation has the advantage of relating individual and community behaviour and of specifying the rôle of assets as well as income in the determination of consumption.

The relationships considered so far contain aggregate consumption as a single variable. It may not always be appropriate to do this. In the first place there is ample evidence [40] that price substitution effects are a significant element in the explanation of the demand for individual commodities. It may be that such effects mainly influence the composition rather than the level of consumption but this is not necessarily so. Klein has shown [41] that in the simple case in which the deflated expenditure on each commodity is a linear function of real income and relative prices, that is if

$$(C_i/p_i)=\alpha_i+\beta_i(Y/p)+\gamma_i(p_i/p), \tag{13}$$

in which C_i denotes the expenditure on commodity i, p_i denotes the price of commodity i and p denotes the average level of retail prices, then, with consistent measures of price and consumption aggregates, *relative prices should only be omitted* from the aggregate consumption function if

$$\Sigma_i \gamma_i p_i=\lambda p \tag{14}$$

In the case investigated by Klein, which relates to durable and non-durable goods and to services in the United States over the years 1919–39, it in fact appears that the ratio λ is fairly stable.

A more important fact, in all probability, is the existence, in some economies on a considerable scale, of durable consumers' goods the demand for which can hardly be analysed in terms of income and prices without reference to the stocks of such goods in the hands of consumers. The demand for these goods is *postponable and also capable of reaching* saturation. If their supply is reduced for any considerable time, the demand for renewals will accumulate in a way which can hardly occur in the case of perishable commodities. Further, as the stock of any one such good is built up, future demand will tend to relate more and more to requirements for renewals as opposed to further net additions to the stock. It may well be that these kinds of effect largely cancel out in the aggregate for past periods but it is not clear that this need always be the case and a separate treatment of durable consumers' goods therefore seems desirable if long-term predictions are attempted.

VII. INFORMATION ABOUT GOVERNMENT BEHAVIOUR

The usual practice is to treat government current expenditure as an exogenous variable. In the case of short-term predictions this seems to be a reasonable procedure since in many cases it will probably be difficult to produce better estimates of next year's expenditure than those contained in the estimates submitted to parliaments or other governing bodies. Such information at least provides a starting point, though examination of the detailed figures may suggest at least the direction of important modifications.

For longer periods ahead this direct source of information is not available. A detailed examination of past records may, however, reveal many commitments of a more or less fixed kind and also the existence of certain trends in the development of services especially where these are performed in partial independence by numerous local authorities. By such means it may be possible to narrow the area of sheer guesswork about the future to those parts of government expenditure which are largely independent of known social policies and tendencies.

VIII. INFORMATION ABOUT FOREIGN TRADE DEMANDS

In principle it would always be possible to set up a simple transaction model for the whole world, suitably divided into areas, and to use this for an approximate analysis of the particular country under investigation. The relevant information can hardly be said to exist, however, at the present time and, moreover, a rather more elaborate model would almost certainly be necessary for this purpose since relative prices are important in this context. In these circumstances it seems best to study the demand for the country's imports and exports, each subdivided in what appears to be the most appropriate manner, by means of comparatively simple relationships.

A large amount of econometric work has been done on foreign trade elasticities. Reference may be made to Tinbergen's paper on the measurement of price substitution elasticities [42] and his subsequent discussion with Polak [43] and to the extensive calculations of Tse Chun Chang which are conveniently brought together in [44]. In brief it may be said that these results suggest that while elastic substitution effects are found for some commodities, particularly for staple commodities sold on well-organized markets, the substitution effects for the total imports or exports for many countries are extremely low. If correct, these conclusions would have important implications for foreign trade policy.

These results were criticized by Orcutt in a paper [45] which contains a useful bibliography of earlier empirical analyses. He points out the importance of common shifts in supply and demand schedules in yielding estimates of substitution elasticities that are too low and also the effects of grouping commodities and of concentrating on short-run effects which may not represent the full response to price changes in the long run. More recently a number of further studies have been made by means of methods other than the straightforward analysis of annual time series by the method of least squares.

An interesting attempt was made by Morgan and Corlett to apply simultaneous equation methods in this field [46] but the results obtained show very large margins of error and can hardly be regarded as successful. The elasticity of substitution between British and American exports of manufactures has been investigated by MacDougall [47, 48] on the basis of extensive cross-section studies using individual commodities. He converts the measures gained in this way to yield an estimate of the substitution elasticity between the two groups of exports and derives a value some ten times the estimate given in [44].

More recently an important contribution has been made by Harberger [49] in relation to the demand of the United States for imports. He returns to the single-equation least squares analysis of annual time series but with a difference which is likely to be important far outside this particular field of application. As already mentioned in the introduction, it is necessary for the successful application of the method of least squares to be able to assume, among other things, that the determining variables are jointly distributed independently of the disturbance in the regression equation. This suggests that the possible causes for shifts in the demand equation should be carefully studied from the standpoint of their possible association with the determining variables. From these considerations he shows that in the case under investigation the least squares estimate will provide a lower limit to the absolute value of the estimates. He then discusses various assumptions that seem reasonable in the light of theory and the observations which would help to set an upper limit. He concludes that in the cases studied this upper limit is anywhere from five to ten times the value of the least squares estimate and thus corroborates the general findings of MacDougall on the basis of an improved application of the methods the results of which were criticized by MacDougall. In my view, Harberger's method stands in need of further consideration and, if possible, of a more rigorous formalization. It does, however, illustrate very forcibly the importance, in this particular application, of one of the crucial assumptions of estimation by the method of least squares and indicates the need for an economic analysis of the disturbance term, an element in the estimation procedure which so often is treated in a purely mechanical way, in its relationship to the determining variables.

IX. RELATIONSHIPS BETWEEN CAPITAL FORMATION AND PRODUCT

All components of final demand other than capital formation have now been considered. This item presents very considerable difficulties and has not been the subject of such extensive econometric work as have the consumption function and foreign trade elasticities. From the standpoint of short-term forecasting it may, at the present time, be best to treat this variable as exogenous and to attempt to determine its immediate future values by sample surveys of investment intentions. From a more general point of view, on the other hand, this method is hardly satisfactory and an attempt must be made to bring capital formation into the system of explicit relationships. The usual theoretical proposals for doing this will now be briefly examined. The first is the acceleration principle and its developments, the flexible accelerator and the capacity principle, and the second derives from the recognition that capital formation will add to the future capacity of the system.

The notion of the acceleration principle appeared in a paper [50] by J. M. Clark which was published during the first world war. It was not, however, until many years later that attempts were made to test this principle, in the first instance by Kuznets [51] and by Tinbergen [52, 53, 54]. A further empirical study was published at the end of the second world war by Manne [55], who paid special attention to the effects of excess capacity, and the subject was further investigated by Chenery [56], who compared the predictive ability of the acceleration principle and a modification of it which he termed the capacity principle. This last may be compared with Goodwin's flexible accelerator [57].

The various relationships can be set out in the following way. Let the capital formation of a period be denoted, as before, by I, the product of that period by Y and the capital stock at the beginning of the period by K. Then

$$I = \Delta K \tag{15}$$

where Δ denotes the operation of taking forward first differences. In its simplest form the acceleration principle is based on the assumption that the capital stock at the end of the period will need to be proportional to the product of the period, that is

$$EK = \alpha Y \tag{16}$$

where E is an operator such that $E^{\theta}x_t \equiv x_{t+\theta}$ and is related to Δ by the equation $\Delta \equiv E - 1$. If (16) is multiplied by E^{-1} and if a substitution for K is made from (16) into (15) there results

$$I = \alpha \Delta' Y \tag{17}$$

where Δ' denotes the operation of taking backward first differences. This operator is related to E and Δ by the equations $\Delta' \equiv E^{-1}\Delta \equiv 1 - E^{-1}$.

Equation (17) may be further complicated in various ways and the following treatment is essentially that given in [56]. First it may be desirable to allow for a further lag between the change in product and the actual emergence of the capital formation and at the same time to alter the factor of proportionality as given in (16). In this case (16) is replaced by

$$E^{\theta}K = \beta\alpha Y \tag{18}$$

whence

$$I=\beta\alpha E^{-\theta}\Delta Y \qquad (19)$$

It was in essentially this form that the acceleration principle was originally tested with results which, on the whole, were disappointing. Many reasons might be given for this but one which seems likely to be of particular importance in practice is that the formulation does not allow for fluctuations in capacity utilization, a phenomenon clearly observable in the actual world. To allow for this it may be assumed that with a lag, θ, and subject to some damping factor, β, due perhaps to uncertainties, capital formation will be undertaken to the extent of the difference between required capital stock given by αY and actual capital stock given by K. This leads to the equation

$$I=\beta E^{-\theta}(\alpha Y-K) \qquad (20)$$

which with $\beta=1$ and $\theta=0$ reduces to (17) but in other cases will give quite different results. A further refinement is to replace K by γK where γ denotes the optimum degree of use of the capital stock. If this is done (20) is replaced by

$$I=\beta E^{-\theta}(\alpha Y-\gamma K) \qquad (21)$$

A modified form of (19) and (21) are applied in [56] to six industries in the United States over the inter-war period. It was found that in the majority of cases the capacity principle gave a closer explanation of observed variations than did the acceleration principle. There were exceptions to this, however, which suggest that the two principles should be regarded as alternative types of explanation, the choice between which in any instance would depend largely on the position with regard to excess capacity.

Thus, as was the case with the consumption function, it appears that in this case also improved results can be obtained if the first simple formulations are replaced by more complicated, though still relatively simple, expressions. Also, as has been suggested by many writers in connection with foreign trade elasticities, it seems particularly important in this case to subdivide capital formation in making the analysis since the responses of different industries may vary and they may not all be capable of representation by the same type of expression.

The unsatisfactory results obtained from the application of a simple multiplier-accelerator model to the Unites States as a whole can be seen from the papers by Fisher [58, 59].

Reference may conveniently be made at this point to the methods of solving a system involving a non-linear accelerator, as given by Goodwin in [6], set out by Bothwell [60] and by Strotz, McAnulty and Naines [61]. The latter investigation employs electro-analogue methods and thus links up with the proposals by Tustin [62, 63, 64] to formulate economic models in terms of control system engineering.

The second proposal for introducing capital formation into the system of relationships involves an explicit connection between the capital formation in a given period and the subsequent increase in the capacity to produce. With a fixed propensity to save, the use of the additional capacity will require in each subsequent period an ever larger level of capital formation and hence further increases in capacity in the future. In its simplest form this hypothesis can be expressed by the relationship

$$\Delta Y = \kappa I \tag{22}$$

which states that the increase in potential product is proportional to capital formation. The elaboration of this relationship in the light of statistical information has not, so far as I know, been undertaken.

If either (17) or (22) is combined with a saving function and the identity $S \equiv I$, it provides the basis for a model of economic growth [65]. The use of (17) leads to the type of model that has been developed on the basis of Harrod's work [66, 67]. Thus if saving is assumed proportional to income and the acceleration principle takes its simplest form, the model consists of the three equations

$$S = \sigma Y \tag{23}$$

$$I = \alpha \Delta' Y \tag{24}$$

$$S \equiv I \tag{25}$$

whence

$$Y = \frac{\alpha}{\alpha - \sigma} E^{-1} Y \tag{26}$$

The use of (22) on the other hand leads to the type of model initiated by Domar [68, 69]. A simple version of this type can be expressed in the equations

$$S=\sigma Y \tag{27}$$

$$\Delta Y=\kappa I \tag{28}$$

$$S\equiv I \tag{29}$$

whence

$$Y=(1+\sigma\kappa)E^{-1}Y \tag{30}$$

Extensive theoretical developments of this kind of model are to be found in a number of papers by Domar [70, 71, 72], Fellner [73] and Eisner [74, 75]. It will be observed that if the accelerator as in (24) and the capital coefficient as in (28) are retained in the same model, the relationship

$$\kappa(a-\sigma)=1 \tag{31}$$

is implied.

X. COMPLETE MODELS OF THE FINAL DEMAND TYPE

The first studies to be mentioned in this section are the remarkable pioneer works in general econometric model building undertaken by Tinbergen before the second world war [76, 77, 78] and a methodological study of the same author [79]. These investigations gave rise to a considerable discussion of the various problems of model building in which, in particular, Keynes [80, 81], Tinbergen himself [82, 83], Koopmans [84] and Haavelmo [85] took part. These econometric studies of Tinbergen involved more component relationships that those discussed in the preceding sections since, in the study for the United States, for example, certain relationships concerned with price formation and with the money and capital markets were introduced. Another general model of the same period is the one constructed by Radice [86] for the British economy based on data for the inter-war period.

The next set of studies to be noticed relate to the United States in the transitional and post-transitional periods following the second world war. Estimates relating to the end of 1945 and the beginning of 1946 were presented at the time by Hagen and Kirkpatrick and subsequently published [87]. Calculations relating to 1950 or thereabouts were published by Smithies [88], Mosak [89] and the National Planning Association [90]. By contrast with Tinbergen's pre-war models, these were all comparatively simple. They were based essentially on projections of the capacity to produce of the American economy and on estimates of final demand. Government expenditures and capital formation were treated as exogenous but use was made of relationships between consumption and product. The general conclusion of these models was that full employment would be hard to achieve and maintain in the absence of special government measures.

The discrepancy between fact and forecast was immediately apparent in the case of the model relating to 1945–6. These results, and especially the treatment of the consumption function, were criticized by Woytinsky [91] whose views on this subject were subsequently elaborated [92]. An explanation of the bulge in consumers' expenditure on non-durable goods was put forward by Bassie [93] and a post-mortem on the transition predictions was conducted by Klein [41], by Hagen himself [94] and by Sapir [95]. A criticism of the consumption function used by Smithies was the point of departure for Modigliani's proposal [33] for a revised formulation of this relationship.

A return to more elaborate models was undertaken by Klein and subsequently published [96]. Government expenditures are treated as exogenous but demand equations are given for consumers' goods and for capital formation, subdivided into private producers' plant and equipment, housing (owner-occupied and rented shown separately) and inventories. Among further relationships introduced are demand equations for labour, for active cash balances and for idle balances. Use is made of the reduced form method of estimation and an interesting comparison is made with the results of applying the single-equation least squares method. On the whole the differences between the two methods are in this case rather small.

A detailed and interesting discussion of this model has been undertaken by Christ [97] who, among other things, compares its predictive ability with that of naive models in which the hypothesis is that next year's value of a variable will be equal to this year's plus a random element or to this plus a trend factor. Klein's comments on this discussion also appear in [97].

I believe that since the publication of [96] Klein has undertaken a further elaboration of his model but, as far as I know, the results have not yet been published.

Another model which relates to the American economy and is based on quarterly data over the inter-war period is due to

Colin Clark [98]. Recently, another projection which carries forward the earlier study [90] has been undertaken at the National Planning Association by Colm [99].

XI. A FISCAL POLICY MODEL

The models described in the preceding section permit a set of endogenous variables to be expressed in terms of a set of exogenous variables. Thus, for example, the simplest model of income determination might be expressed in the form

$$Y=C+I \tag{32}$$

$$I=S \tag{33}$$

$$S=\sigma Y \tag{34}$$

If I is given a specific value then the associated values of Y, C and S can be calculated. On the other hand if a desired value is assigned to Y then the value of I required for that value of Y, given of course the validity of the model, can be determined.

A model which was set up to indicate the implications of various types of fiscal policy in bringing about full employment was devised by Kaldor for the United Kingdom [100]. This is essentially a static final demand model in which the principal relationships refer to tax payments and saving out of different forms of income. The system is closed by various assumptions about pairs of the four fiscal magnitudes, the rates of direct and indirect taxation, government expenditure on goods and services and government net lending. This model has been formulated algebraically by Jackson and myself [101]. An examination of this type of model, equally applicable to models for the transition and post-transition periods in the United States, referred to in the last section, has been made by Hart [102].

XII. A SIMPLE INTER-INDUSTRY MODEL

Final demand models, various aspects of which have been described in the preceding sections, have never received a single formulation generally accepted by all investigators. While, as suggested in section III, they may all be said to derive from attempts to explain the interactions of certain aggregates of transactions obtained from a particular consolidation of the social accounts, the pioneer work of Tinbergen in this field was undertaken expressly to test the various relevant theories as summarized at the time by Haberler [103]. Such models naturally contain certain general features in common but they also contain many special features which vary from one writer to another. Moreover, in their development, much has been gained by the concentration on particular relationships taken out of the system as a whole.

In contrast to this situation the view of the economic process which forms the basis of inter-industry models is extremely simple in form. The formulation is due to Leontief [104, 105] and some of its special features have been elegantly presented by Goodwin [106]. It is extensively discussed in [18] and various aspects of it have been examined by Cameron [107, 108] and by Klein [109].

In this model an economic system is regarded as a system of transformation processes each engaged in converting inputs into output. In the closed version of the model households are no exception since they are to be regarded as converting consumption goods into factor services. These services form a single scarce factor which limits the scale on which the system can operate.

The model contains a quantity circuit and a price circuit which can be considered independently. On the quantity side, the inputs into the process which produces factor services (final demand) are proportional to the amount of these services in use. In order to produce the necessary inputs which comprise final demand, materials and other commodities are needed in certain fixed proportions. In this way output levels for all processes other than the provision of factor services are determined and the circuit is closed since these output levels identically demand in the aggregate the quantity of factor services which gave rise to them in the first place. Thus let

ν denote a scalar, the total quantity of supposedly homogenous factor services in use;

$\mathbf{n}$ denote a vector, the elements of which are the quantities of factor services in use in each process (other than households which use none of these services);

e denote a vector, the elements of which are the quantities of each final good demanded, that is the levels of input of each commodity into the process (households) engaged in producing factor services;

q denote a vector, the elements of which are the quantities of each commodity (other than factor services) produced;

M denote a matrix, the elements of which are the quantities of each commodity flowing from its producer to its users (other than households).

Then the accounting structure of the system may be written in the form

$$\mathbf{W}=\left[\begin{array}{c:c} \mathbf{M} & \mathbf{e} \\ \hdashline \mathbf{n}' & 0 \end{array}\right] \tag{35}$$

and ν, $\mathbf{n}$, $\mathbf{e}$ and $\mathbf{q}$ are related by the equations

$$\nu=\mathbf{i}'\mathbf{n} \tag{36}$$

$$\mathbf{e}=\nu\mathbf{d} \tag{37}$$

$$\mathbf{q}=(\mathbf{I}-\mathbf{A})^{-1}\mathbf{e} \tag{38}$$

$$\mathbf{n}=\hat{\mathbf{q}}\mathbf{c}=\hat{\mathbf{c}}\mathbf{q} \tag{39}$$

where $\mathbf{i}$ and $\mathbf{I}$ denote respectively the unit vector and matrix, a prime denotes transposition, $\mathbf{c}$ and $\mathbf{d}$ denote vectors of proportionality, $\mathbf{A}$ denotes a matrix of commodity inputs per unit of output and a circumflex denotes a diagonal matrix with the elements of the associated vector in its leading diagonal.

Continued substitution for $\mathbf{n}$ from (39), $\mathbf{q}$ from (38) and $\mathbf{e}$ from (37) into (36) shows that

$$\begin{aligned}\nu&=\nu\mathbf{c}'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{d}\\&=\nu\end{aligned} \tag{40}$$

identically. For, by assumption, each account in (35) balances whence the expression $(\mathbf{I}-\mathbf{L})$, say, derived from the matrix of coefficients.

$$\mathbf{L}=\left[\begin{array}{c:c} \mathbf{A} & \mathbf{d} \\ \hdashline \mathbf{c}' & 0 \end{array}\right] \tag{41}$$

is singular and so, by a Cauchy expansion in terms of the elements of its last row and column,

$$\mathbf{c}' \operatorname{adj} (\mathbf{I}-\mathbf{A})\mathbf{d}=|\mathbf{I}-\mathbf{A}|, \tag{42}$$

where adj $(\mathbf{I}-\mathbf{A})$ denotes the adjoint of $(\mathbf{I}-\mathbf{A})$, whence

$$\mathbf{c}'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{d}\equiv 1 \tag{43}$$

The price circuit may be set out in a precisely similar way. Let

- λ denote a scalar, the rate of remuneration per unit of factor services;
- $\mathbf{f}$ denote a vector, the elements of which are the direct factor costs per unit of each commodity produced;
- $\mathbf{p}$ denote a vector, the elements of which are the prices of each commodity.

Then factor costs per unit are proportional to λ and prices are composed of commodity and factor inputs per unit of output. Hence

$$\mathbf{f}=\lambda\mathbf{c} \tag{44}$$

$$\mathbf{p}=(\mathbf{I}-\mathbf{A}')^{-1}\mathbf{f} \tag{45}$$

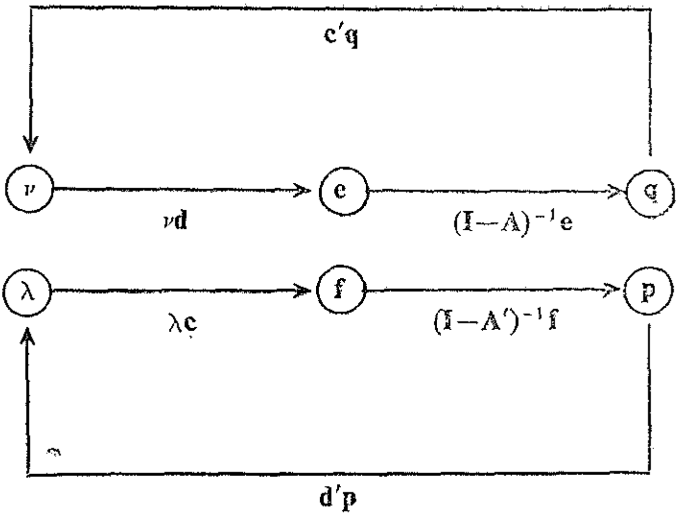
$$\lambda=\mathbf{d}'\mathbf{p} \tag{46}$$

The duality of these relationships, pointed out by Goodwin [106], can be seen from the following diagram in which the upper circuit relates to quantities and the lower circuit relates to unit values.

In this diagram the variables, scalars and vectors, are enclosed in circles and the relationships connecting them are shown against the branches by which the circles are joined. The condition that ensures that the quantity circuit will yield an identity in ν also ensures that the price circuit will yield an identity in λ.

Thus the system takes the form of a simple transaction model in which quantities and prices are treated separately. In its

Diagram 2



open-ended form either the bill of final goods, **e**, is assumed to be given whence the commodity implications, **q**, of meeting this demand can be worked out or the distribution of direct factor costs per unit of product, **f**, is given and this permits a calculation of prices per unit, **p**.

It will be seen that to achieve these results heroic assumptions have had to be made. Thus in all circumstances the bill of final goods has the same composition; it can vary only in size. If the elements of **e** are interpreted as consumers' goods it would seem reasonable to suppose that the demand for them depended not simply on ν, the scale of operations of the system, but also on **p** and λ. Indeed it is quite possible to replace (37) by a more orthodox set of final demand equations. But if nothing more is done the change will be unavailing since the price-cost theory represented in the model will ensure that relative prices cannot change. As has been demonstrated by Samuelson and by Georgescu-Roegen in [18] and clearly illustrated by Koopmans [110], this result must necessarily follow provided that (i) each process has only one output and (ii) there is only one scarce primary commodity, the homogenous factor services in this case, which is obtained from outside the system of productive processes and used by each of them. Thus the purpose intended in introducing ordinary demand equations for final goods would not be served unless at the same time a further change was made such as the introduction of more than one scarce factor between which substitution could be made. It does not appear to be an easy matter to do this in an elegant way in accord with the spirit of the rest of the model and investigation on these lines seems to lead quickly back to a general Walrasian system with its attendant complexity.

XIII. INFORMATION FOR INTER-INDUSTRY MODELS

For practical purposes it is usual to adopt quantity units, the £'s worth, such that all the prices are unity and all the quantities are values in the money of a particular period. Since the model is homogenous, involving as it does only factors of proportionality, the elements in matrix A which are the parameters of the open-ended model can be calculated from a single cross-section study of inter-industry flows. There are, of course, considerable conceptual as well as practical difficulties in doing this [111] but it has in fact been done for a number of countries.

The first input-output table was given by Leontief in [104]. It related to the United States in 1919. In [105: 1941] there was added to this a table for 1929. In these tables the productive system is divided into forty-one industries. In the early 1940's the Bureau of Labor Statistics in Washington became concerned with problems of employment adjustment after the cessation of munitions production and a small research unit under the direction of Leontief was established at Harvard. Between 1942 and 1944 a table was prepared for 1939. The economy was divided into ninety-six sectors and a table in which these are aggregated into forty-seven sectors is given in [105: 1951].

Some use was made of this material from 1944 on by a small group at the Bureau of Labor Statistics [112, 113, 114, 115]. In the main these studies were concerned with the post-transitional problems of the American economy analysed by somewhat different methods from the investigations referred to in section X above.

In 1949 work was begun on a new and more detailed inter-industry study relating to 1947. A summary presentation in the form of a fifty-sector tabulation is given in [111]. The methods used in the various parts of this study are described in a series of papers submitted to the American conference on income and wealth and will eventually be published in [116]. Another paper, also given at this conference, is a useful review of input-output analysis both from the standpoint of economic theory and of its achievements in the sphere of prediction by Christ. On this occasion three important tables relating to approximately two hundred sectors were made available. These relate to (i) the inter-industry flow of goods and services by industry of origin and destination, (ii) direct purchase per million dollars of output and (iii) direct and indirect requirements per million dollars of final demand. A general explanation of these tables is given in [117]. The basic data were compiled in terms of five hundred sectors but it would hardly be practicable to present so large a mass of figures in tabular form.

The third table just referred to involves the inversion of a matrix of the order of 200×200. This formidable task was undertaken by the Univac at the Pentagon in a little over two days. The mathematical problems of matrix inversion and the errors to which such calculations are liable have been discussed in a recent paper by Dwyer and Waugh [118]. The bibliography attached to this contribution makes it unnecessary to consider the matter further here. It may be said, however, that provided one is willing to accept models which involve only the solution of linear equations, the derivation of a solution is well within the power of modern electronic computing equipment and such models are therefore determined by the available data rather than by difficulties of computation.

It may be mentioned that at the same conference a paper on the input-output analysis of the Puerto Rican Economy was presented by Gosfield.

The pioneering work of establishing an input-output table for the United Kingdom has been undertaken by Barna, who has given in [119, 120] a thirty-six industry table for 1935. Two small tables, relating to 1948 and 1950 and useful mainly for expository purposes, have been published by the Central Statistical Office [121, 122]. Late in 1952 a large-scale investigation relating to 1948 was started by the Board of Trade and the Department of Applied Economics in Cambridge.

In the Netherlands input-output tables on a twenty-six industry basis are available for 1938, for the years 1946 to 1950, for the second quarter of 1949 and for the fourth quarter of 1949 through the third quarter of 1951. These may be found in various official publications issued by the Central Bureau of Statistics [123, 124] and, for the years 1938 and 1946–8, in the National Accounts Study for the Netherlands issued by the O.E.E.C. [125]. For 1948, the information is presented on several bases of classification in [124]. An especially attractive feature of these tables is that they show the input-output table set within the wider framework of the social accounts.

Information on the experience of the Central Planning Bureau in the use of this material is given in a paper by Loeb [126] and a description of the model used at the time, 1950, by that institution is given by Sandee and Schouten [127]. This model makes use of inter-industry information but goes far beyond the simple technological relationships described in the preceding section.

For Denmark small input-output tables on a sixteen-industry basis in which current and capital uses are separately shown in detail are available for the years 1930 to 1939 in [128] and for 1946 in [129]. The information for 1946 together with a brief description of it is also available in [130].

For Norway information on a thirty-four industry basis has been published for 1948 in [131] and a description of input-output studies in Norway has been given by Aukrust [132].

Finally a most interesting study for Italy has recently been prepared and published by the Program Division of the M.S.A. Mission to Italy [133]. This contains a rectangular table relating to some two-hundred products and sixty industries. An attempt is made to test the accuracy of the model and it is used to predict the probable structure of the Italian economy in 1956 and as a basis for regional analysis.

XIV. DYNAMIC INTER-INDUSTRY MODELS

Little was said in section XII about the treatment of capital formation in inter-industry models. In the closed model originally proposed by Leontief there is only one account for each transactor and so current and capital purchases are unavoidably linked together. With the introduction of open-ended models this difficulty could be got round, by putting capital formation along with consumers' expenditures into the bill of final goods. This of course involves making capital formation exogenous. If this is to be avoided then some means must be found of relating capital purchases by a given industry to the output (total sales) of that industry which is more plausible than assumptions of proportionality.

Proposals for a dynamic inter-industry model have been made by Leontief [134] and a discussion of these is also given by Whitin [135]. These accounts are formulated in terms of differential equations but it seems equally appropriate to use difference equations for purposes of illustration and when this is done the proposals can readily be seen to be equivalent to a general introduction of the acceleration principle discussed in section IX above.

It is convenient at this point to refer back to (41). Each column of **L** contains the multipliers which relate the purchases of one account from each other account to its output (total sales). No time lags are involved, that is to say the purchases in period θ depend only on the output of period θ in every case. If the row sums of (35) are denoted by **w**, that is if

$$\mathbf{Wi}=\mathbf{w} \tag{47}$$

then the closed model can be expressed in the form

$$\mathbf{W}=\mathbf{L}\hat{\mathbf{w}} \tag{48}$$

or

$$(\mathbf{I}-\mathbf{L})\mathbf{w}=\mathbf{o} \tag{49}$$

Suppose now that purchases do not depend simply on outputs of the same period but also on the outputs of previous periods as well. This may be represented by replacing **L** by **LE** where the addition of the symbol for the lag operator indicates that whereas the elements of **L** are constants the elements of **LE** are polynomials in E, the values of the exponent being, of course, zero or negative. A general dynamic model can be represented by combining (47) with a modification of (48) in which **LE** replaces **L**. These relationships yield the equation

$$(\mathbf{I}-\mathbf{LE})\mathbf{w}=\mathbf{o} \tag{50}$$

whence

$$|\mathbf{I}-\mathbf{LE}|w=0, \tag{51}$$

where w is any element of **w**, is the general autoregressive equation of the system.

This formulation is too general to be of much value but a specific theory will be given by specifying the form of **LE**. Leontief proposes that it should take the form of a general linear function of the rate of change, or in finite difference terms

$$\mathbf{LE}=\mathbf{L}^*+\mathbf{L}^{**}\Delta' \tag{52}$$

Here the elements of $\mathbf{L}^*$ are the ratios of current purchases to output while the elements of $\mathbf{L}^{**}$ are the ratios of capital purchases to the change in output. This formulation is equivalent to adopting an all-pervading acceleration principle and is justified by similar considerations.

In the open-ended model it is necessary only to replace **A** by **AE** where

$$\mathbf{AE}=\mathbf{A}^*+\mathbf{A}^{**}\Delta' \tag{53}$$

In this case

$$\mathbf{q}=(\mathbf{A}^*+\mathbf{A}^{**}\Delta')\mathbf{q}+\mathbf{e} \tag{54}$$

whence

$$(\mathbf{I}-\mathbf{A}^*-\mathbf{A}^{**}\Delta')\mathbf{q}=\mathbf{e} \tag{55}$$

and so, as may be inferred from [134],

$$\mathbf{E}^\theta\mathbf{q}=\mathbf{C}\lambda^\theta+(\mathbf{I}-\mathbf{A}^*)^{-1}\mathbf{e} \tag{56}$$

where **C** is an nxn matrix of n^2 constants, n of which are independent, and

$$\lambda^\theta\equiv\begin{bmatrix}\lambda_1{}^\theta\\ \vdots\\ \lambda_n{}^\theta\end{bmatrix} \tag{57}$$

where $\lambda_1, \ldots, \lambda_n$ are the roots, supposedly distinct, of the determinantal equation

$$|(\mathbf{I}-\mathbf{A}^*-\mathbf{A}^{**})\lambda-\mathbf{A}^{**}|=0 \tag{58}$$

In [134] capital coefficients relating to fixed capital and to inventories are given for 1939. In view of the somewhat discouraging empirical findings with regard to the simple acceleration principle it seems likely that further modification in the proposed dynamic structure will be necessary.

Two recent papers by Holley [136, 137] describe respectively the dynamic theory of inter-industry relationships used by the U.S. Air Force and the actual empirical models which have been developed.

XV. ACTIVITY ANALYSIS

The inter-industry models described in sections XII and XIV aim purely at a description of certain features of the economic process. It is obvious that they are highly simplified but equally possible that they may prove practically useful despite their theoretical shortcomings. They are not, however, concerned with any problem of choice, with the best, in some sense, distribution of productive resources, since they show only the implications of a simplified version of an existing state of technology. Such a question as how best to organize production in order to meet the desires of final consumers is ruled out of court by the assumptions of the model.

In practice, however, there are frequently alternative ways of making things and the choice between these ways may be an important matter. If there is only one scarce non-produced factor which enters into all the processes then a given commodity can be supplied most effectively by concentrating on that process for producing it which requires, directly or indirectly, the smallest amount of the factor. If, however, there is more than one scarce factor or if there are processes which produce joint products, an efficient level of production cannot be reached simply from the above considerations. These propositions can readily be seen from the two commodity diagrams due to Koopmans [18, 110] and used also in a review by Chipman [138].

Accordingly the inter-industry model may in principle be adapted to a situation involving choice by introducing more processes than products so that an economy would be represented in which there were alternative ways of making things. The question might then be asked how best to organize production, that is at what level to operate each activity given a certain supply of each non-produced factor and a system of preferences for final products. With only one scarce factor there would be a best way of making each commodity which alone would be employed for the purpose in all circumstances, but with more than one scarce factor the process adopted might change with a change in tastes and with joint production it might be best to derive the supply of some product from more than one process in proportions that could in principle be determined.

Questions of this kind can be reduced to the problem of minimizing a linear function of certain variables subject to a number of linear inequalities. It may be doubted whether, for the time being at any rate, methods of this kind will find practical application to questions of the organization of production in a whole economy, but they seem to offer practicable possibilities in the planning of production within a firm or even within a branch of activity. In the growing literature on the subject, applications will be found to: the routing of ships in one direction from one set of ports to another set of ports and the generalization of this to a continuous flow problem in which *the object is to minimize the total flow of empty shipping while* providing for a given flow of freight capacity between the ports of the system [18]; the mixing of nuts [20]; the blending of aviation gasoline [139]; certain problems of airline operation [140]. There is also scope for similar methods in planning activities which are not readily regulated by a price mechanism such as the internal operations of the army, navy and air force.

In concluding this section it may be useful to sketch the origins of activity analysis as indicated in the introduction to [18] and to relate it to ordinary economic theory. Briefly, it may be said that an economic system is designed to transform a set of primary factors into a set of commodities desired by the human agents in the system. The orders to the system may be supposed to be given by these human agents by means of a set of demand relationships in which each individual endeavours to maximize the utility to him of the goods he can obtain given the ruling prices and his income. These orders are received by the productive system which produces the goods demanded and the intermediate products necessary to produce these goods on the basis that each productive unit seeks to maximize its profit under conditions of perfect competition. The owners of the primary factors of production such as land and labour compete in supplying them to the productive system by offering them to the productive unit which offers the highest rate of reward. In static equilibrium the demand for and supply of each primary factor, intermediate product and final product will be equal and all intermediate products will be used up in the period in which they are produced. This system of relationships will determine the amounts of primary factors supplied, the amounts of final products demanded, the level of each kind of production and the relative prices of all factors and products.

A suitably elaborated description along these lines takes a central place in almost all systematic presentations of economics. It may be expressed in literary form as in Marshall's *Principles* or it may be set out in the form of a system of equations as in the works of Leon Walras.

In practice the amount of each primary factor supplied, each output level, each amount of final product demanded and each price must be non-negative and the question arises whether a unique solution exists to the system of relationships with these restrictions on the variables. It was shown about twenty years ago by Neisser [141] that in an economically plausible case no solution existed that was compatible with non-negative prices. Shortly afterwards Schlesinger suggested [142] a reformulation of the equilibrium conditions so that for each commodity either the amount supplied is equal to the amount demanded or the former exceeds the latter, in which case the price is zero. In this form the system was investigated by Wald [143, 144, 145] who proved the existence and uniqueness of a solution. More recently the existence of a solution to the Walrasian system as thus modified has been proved under very general conditions.

In 1937 von Neumann generalized this model of the economic process in certain respects [146, 147]. The most important of these was to introduce several methods of producing any given commodity so that the question of a choice of productive techniques could be investigated explicitly. Von Neumann's model is also in a limited sense dynamic since the whole system is subject to a constant rate of expansion. The rate achieved in the solution of the system is shown to be the maximum compatible with producers' technology and consumers' preferences.

A feature of these solutions is that the levels of production achieved lie on the boundary of the production possibilities of the economy. There is a sense therefore in characterizing the equilibrium level of production of such a system as efficient since if a little more of any one commodity were desired it would be necessary, with fixed amounts of the primary factors of production, to give up a part of the supply of one or more of the remaining commodities. The competitive solution, as it may be called, could perhaps be rejected on social grounds on account of the distribution of incomes to which it gave rise but it could not be rejected on economic grounds since, independently of prices, the system could not perform outside its boundary of production possibilities.

The formal description of the economic process and the attainment of equilibrium can be paralleled by a statement of actions to be followed by the participants in the process such that the competitive solution will be attained. Since each participant is attempting to maximize something (his profit, his satisfaction, or whatever it may be) it is clear that prices perform a central function in guiding the decisions of producers and consumers. Hence the importance of the price system or market mechanism in actual economic systems.

But suppose there were no price system; what guides would then exist for the participants in the economic system? This question was taken up many years ago by Barone [148]. He emphasized the idea that if production were in the charge of a government department, that department should, in the interests of efficiency, satisfy, in most of its operations, the same formal conditions as are satisfied in the model of the competitive economic process. To the charge that it would be impossible to make the calculations required for action in one central office, it was shown by a number of writers that this was not in fact necessary and that, for efficiency, it would be sufficient that certain simple rules should be followed by the directors of individual producing units.

These rules are needed as a means of decentralizing decisions but cases may arise where such decentralization is not practicable or is only practicable to a limited extent. In such cases there is no alternative to direct calculation. Since in most cases there are many commodities to be produced, many required as inputs in the processes of production and many processes to be chosen among and since these are not independent it is to be expected that an efficient solution will not be easy to find. If the cost of rapid calculation and the selection of the most efficient procedure could be neglected then the cost of each of the enormous number of different methods of obtaining the required outputs could be computed and the cheapest one could be selected. In fact, however, these costs are far from negligible and some more efficient procedure is in most cases essential.

It is with the solution of problems which require to be approached in this way that activity analysis is concerned. From a formal point of view it is possible to reduce programming problems to problems in game theory and *vice versa* [18]. In this way the present subject is linked with another rapidly developing branch of theory.

XVI. CONCLUSIONS

From this survey of developments in econometric model building certain conclusions may perhaps be drawn.

In the first place it is evident that considerable progress has been made in the last twenty years in a number of directions. Mistakes have been made all along the line either through inappropriate formulations of hypotheses, through inadequate factual information or through faulty methods of estimation. But in many cases these have been recognized for what they are and it has been possible to correct some of them.

In the second place improvements seem to come largely through the analysis of observations, through the relating of theories and facts by appropriate methods of estimation and inference. From the theory side it is not very difficult, once the basic ideas have been propounded, to play around with multipliers, accelerators, capital coefficients and such-like ingredients and produce new versions and special cases in bewildering profusion. It may be doubted whether our knowledge of the economic process can be much increased by such means. From the observational side, the history of economic research is full of attempts to make the facts do all the talking, a task which they are quite unable to perform. Among economists this was recognized long ago by Marshall and it is to be hoped that the time is not far distant when it will receive general assent. As regards estimation procedures, the important point seems to be to think in genuinely economic terms about the disturbances, which have to be introduced into all relationships other than those of identity, rather than to accept mechanically any one method of estimation which, whatever it may be, is bound to involve economic assumptions that cannot be verified in specific applications.

In the third place the nature of the variables which enter into the models I have described involves a particular responsibility on those whose work is to prepare the social accounts. It is to them in the first instance that model builders must look for their observations. As more experience in the formulation of particular relationships is gained through experimentation with alternative definitions and classifications it will be possible to make more use of the idea of observed regularities in establishing standard concepts. It is, of course, generally recognized that such a criterion would be useful as a supplement to the more formal and general criteria of national income taxonomy but in the past there has been relatively little work in discovering which of various possible concepts give rise to the greatest observed regularities. I have in mind such matters as the rather unsatisfactory treatment of durable consumers' goods and many items in the capital transactions account in most social accounts at the present time.

Finally, from the standpoint of getting useful results in the short-run a view must be taken of the variables to be treated as endogenous. The ultimate goal no doubt is a complete system. But at any one time and place the available versions of some of the relationships which would be needed for such a system may give such a poor correspondence with actuality that their introduction into the model does more harm than good. From the short-run point of view this means that attempts must be made to forecast the values of some at least partly endogenous variables like capital formation independently of the model. The development of surveys of investment intentions is an example of this technique. It is not possible to put into a model more information than is actually available about the relationships between its variables. Beyond this point any appearance of completeness is illusory. On the other hand it will frequently be possible to get information which is relevant but which cannot easily be worked into the model itself. Thus if an input-output model is used to review the implications of an expected change in final demand it is likely, if the change is at all large, that certain economies and substitutions will be made that are not reflected in the fixed technical coefficients of the model. A knowledge of such possibilities is clearly likely to be important if realistic conclusions are to be drawn.

Despite its length, I am aware that this survey is incomplete in many respects. I have said almost nothing about estimation procedures though this subject has taken a large place in the literature of the last ten years. It can, however, perhaps be regarded as a somewhat separate subject from the one which has been my main concern. I have not worked into the picture models which assign an important rôle to the money and banking system and to the relationships between financing variables in a system of separate capital transactions accounts. So far as I know rather little model building in this particular field has been done but the value of such studies as those of

Copeland [149] and Brill [150] for such purposes will be evident. There is also perhaps more in the old quantity theory of money than it has recently been customary to suppose, as is indicated by Friedman's attempt [151] at its rehabilitation.

More important than these omissions, however, is the fact that almost all my references are to books and papers written in the United States or in Britain. My concentration on these is due to the fact that I am better acquainted with this literature than with its counterpart originating in other parts of the world and to my own linguistic limitations. I shall be grateful to any reader who will help me to repair this defect.

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