

EXTREME VALUES, MEANS, AND INEQUALITY MEASUREMENT

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We examine some ordinal measures of inequality that are familiar from the literature. These measures have a quite simple structure in that their values are determined by combinations of specific summary statistics such as the extreme values and the arithmetic mean of a distribution. In spite of their common appearance, there seem to be no axiomatizations available so far, and this paper is intended to fill that gap. In particular, we consider the absolute and relative variants of the range, the max-mean and the mean-min orderings, and quantile-based measures. In addition, we provide some empirical observations that are intended to illustrate that, although these orderings are straightforward to define, some of them display a surprisingly high correlation with alternative (more complex) measures.

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1. INTRODUCTION

The measurement of income inequality has been an active field of investigation for over a century, and early classical contributions include those of Lorenz (1905), Gini (1912), Pigou (1912), and Dalton (1920). While much of the literature focuses on a relative notion of inequality (i.e. on scale-invariant measures), absolute indices (which are translation-invariant) are examined as well. Centrist or intermediate measures that represent compromises between the relative and the absolute approach are discussed in Kolm (1976a,b), Pfingsten (1986), and Bossert and Pfingsten (1990). The normative approach connects inequality to welfare and can be traced back to Kolm (1969), Atkinson (1970), and Sen (1973) in the case

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of relative measures, and to Kolm (1969) and Blackorby and Donaldson (1980) if an absolute notion of inequality is adopted. Ethical measures of inequality in an ordinal setting are analyzed by Blackorby and Donaldson (1984), Ebert (1987), and Dutta and Esteban (1992).

In this paper, we follow an ordinal approach to inequality measurement and, therefore, focus on inequality orderings. Our main results provide characterizations of some simple measures of inequality that are familiar from the literature. The first of these are range-based measures that perform inequality comparisons by means of the difference between maximal and minimal income in the absolute case, and the ratio of the maximum and the minimum in a relative setting. The max-mean orderings use the difference and the ratio of the maximum and the arithmetic mean, and the mean-min measures employ the arithmetic mean and the minimal income. In addition, we examine inequality orderings that focus on the income gaps (in the absolute case) or the income shares (for relative measures) of the top or bottom quantile of an income distribution. All of these inequality orderings satisfy three standard axioms, namely, S-convexity, continuity, and replication invariance. However, as far as we are aware, they have not been axiomatized yet.

The primary motivation of our analysis is rooted in the observation that many of the measures discussed here are well known and well established in the literature. Despite this fact, there are no characterizations available so far, and it seems to us that this gap ought to be filled. With this objective in mind, clearly the properties we employ in our axiomatizations cannot but reflect the nature of these indices. As a consequence, whatever perceived shortcomings there are in the comparisons according to these measures are inevitably mirrored in the corresponding recommendations of (some of) the axioms.

Clearly, the measures discussed here are rather coarse because of their limited use of income distribution statistics and, therefore, we do not mean to advocate their use over all competing suggestions. Nevertheless, as discussed by Leigh (2009, p. 162) in the context of justifying the use of the top income shares, when some data are absent or reliable estimates of the entire income distribution are not available, they can serve as a useful proxy for measuring inequality. In particular, in light of the interdependence between different parts of the income distribution resulting from economic activities, they could be a useful and easy-to-use tool for drawing inferences about overall inequality from limited data; see Atkinson (2007, pp. 19–25) and Atkinson *et al.* (2011, pp. 7–12) for discussions regarding top income shares. Alvaredo (2011) examines connections between the Gini coefficient and top income shares from a theoretical perspective. Therefore, we think that it is worthwhile to provide axiomatic characterizations of those inequality orderings. Our results also clarify under which circumstances we may safely rely on the proxies provided by our orderings. While some of the axioms we employ may appear to have somewhat controversial recommendations, they mirror the coarse nature of the underlying inequality orderings. The analysis carried out in this paper suggests that, in the presence of data limitations, the relatively coarse measures characterized here are capable of providing quite close approximations.

Among the orderings we consider, the range-based inequality orderings that compare the distance between (or the ratio of) the maximal and the minimal income do not utilize the average income. In this sense, these inequality orderings are

coarser than the others. To present axiomatic characterizations of these inequality orderings, we employ some suitably adapted axioms that appeared in the literature on ranking sets of outcomes under complete uncertainty. These properties, reformulated in the context of income inequality measurement, are concerned with how we should rank income distributions when the information on the realized income levels in the distributions is reliable but that on their frequency distribution is not. Our characterizations of the other inequality orderings, on the contrary, rely on properties regarding the composition of progressive and regressive transfers in addition to standard axioms. The results are established in a coherent and systematic manner by showing how subsets of the axioms employed successively restrict the informational basis that can be utilized in measuring inequality.

As is the case for much of the literature on the measurement of income inequality, we allow the population (and the population size) to vary. However, all the distributions we consider are finite. Moreover, even if a finite distribution is interpreted as a sample from an underlying larger distribution, this underlying distribution is also assumed to be finite (although it may be arbitrarily large). As a consequence, we do not have to be concerned with potential issues that may arise in the context of finite samples from *infinite* distributions.

In addition to presenting their axiomatic characterizations, it is important to empirically examine the usefulness of these inequality orderings. In analogy with Leigh's (2007) study of the relative performance of top income shares in comparison with other inequality measures, we provide an empirical analysis of the correlation between the range-based and quantile-based orderings and some classical indices including the Gini coefficient. We find that there is some surprisingly significant agreement when considering the movements of the measures and more commonly employed inequality orderings.

In the following section, we introduce our basic notation and definitions. The range-based measures, the max-mean orderings, the mean-min ordinal indices, and the quantile shares and gaps are characterized in Section 3. In each case, axiomatizations of both the requisite absolute ordering and its relative counterpart are provided. Section 4 contains our empirical study, and Section 5 concludes. Proofs of the results in Section 3 are provided in an online appendix, where the independence of the axioms used in our characterizations is also established.

2. NOTATION AND DEFINITIONS

2.1. Range-Based and Related Inequality Orderings

Let \mathbb{N} be the set of positive integers. The sets of all real numbers, all non-negative real numbers, and all positive real numbers are denoted by \mathbb{R} , \mathbb{R}_+ , and \mathbb{R}_{++} . For $n \in \mathbb{N}$, let $\mathbf{1}^n$ denote the n -dimensional vector consisting of n ones and, for all $i \in \{1, \dots, n\}$, e^i is the i th unit vector in \mathbb{R}^n . For simplicity, we suppress the dependence of this unit vector on n ; the dimension of e^i will always be apparent from the context. For all $n \in \mathbb{N}$ and for all $x \in \mathbb{R}^n$, the arithmetic mean of x is denoted by $\mu(x)$; that is, $\mu(x) = \sum_{i=1}^n x_i/n$.

We distinguish two domains that are relevant in this paper. In the context of absolute inequality orderings, incomes may take on any real value and, analogously,

relative inequality orderings are restricted to positive incomes. Thus, we define the (variable-population) domains $D = \cup_{n \in \mathbb{N}} \Omega^n$, where $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$. A vector $x \in D$ is interpreted as an income distribution.

An *inequality ordering* is an ordering $R \in D^2$, and we write xRy for $(x, y) \in R$. Thus, the expression xRy means that the income inequality in x is at least as high as the inequality in y . The asymmetric part of R is P , and the symmetric part of R is I .

We begin our axiomatic analysis with two properties that require very little discussion. The majority of approaches to the measurement of income inequality can be classified as being either *absolute* or *relative* in nature. Notable exceptions are the centrist measures examined by Kolm (1976a,b); see also Pfingsten (1986) and Bossert and Pfingsten (1990) for a notion of inequality that is intermediate between the absolute and relative extremes.

An absolute inequality ordering is invariant to equal absolute changes of all incomes. That is, it is required to satisfy the axiom of translation invariance.

Translation Invariance

For all $n \in \mathbb{N}$, for all $x \in \mathbb{R}^n$, and for all $\delta \in \mathbb{R}$,

$$(x + \delta \mathbf{1}^n)Ix.$$

Analogously, a relative inequality ordering is invariant to changes in the scaling of all incomes by a common positive factor.

Scale Invariance

For all $n \in \mathbb{N}$, for all $x \in \mathbb{R}_{++}^n$, and for all $\lambda \in \mathbb{R}_{++}$,

$$\lambda xIx.$$

The first two orderings that we consider in this paper are the *absolute range* R_{xn}^a associated with $\Omega = \mathbb{R}$ and the *relative range* R_{xn}^r with the domain generated by $\Omega = \mathbb{R}_{++}$, defined as follows. For all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}^n$, and for all $y \in \mathbb{R}^m$, we let

$$xR_{xn}^a y \Leftrightarrow \max\{x_1, \dots, x_n\} - \min\{x_1, \dots, x_n\} \geq \max\{y_1, \dots, y_m\} - \min\{y_1, \dots, y_m\}.$$

Cowell (2011, p. 155) refers to a representation of this ordering as the *range*. The measure that is obtained by dividing R_{xn}^a by the mean income $\mu(x)$ (which requires the domain to be restricted to \mathbb{R}_{++}) is what he labels the *standardized range*. The latter also appears in Sen (1973, p. 24).

The relative counterpart of the absolute range is the relative range R_{xn}^r , defined by

$$xR_{xn}^r y \Leftrightarrow \frac{\max\{x_1, \dots, x_n\}}{\min\{x_1, \dots, x_n\}} \geq \frac{\max\{y_1, \dots, y_m\}}{\min\{y_1, \dots, y_m\}}$$

for all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}_{++}^n$, and for all $y \in \mathbb{R}_{++}^m$.

The *absolute max-mean inequality ordering* $R_{x\mu}^a$ is defined by letting, for all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}^n$, and for all $y \in \mathbb{R}^m$,

$$xR_{x\mu}^a y \Leftrightarrow \max\{x_1, \dots, x_n\} - \mu(x) \geq \max\{y_1, \dots, y_m\} - \mu(y).$$

The scale-invariant counterpart of $R_{x\mu}^a$ is the *relative max-mean inequality ordering* $R_{x\mu}^r$, defined as

$$xR_{x\mu}^r y \Leftrightarrow \frac{\max\{x_1, \dots, x_n\}}{\mu(x)} \geq \frac{\max\{y_1, \dots, y_m\}}{\mu(y)}$$

for all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}_{++}^n$, and for all $y \in \mathbb{R}_{++}^m$.

The *absolute mean-min inequality ordering* $R_{\mu n}^a$ is given by

$$xR_{\mu n}^a y \Leftrightarrow \mu(x) - \min\{x_1, \dots, x_n\} \geq \mu(y) - \min\{y_1, \dots, y_m\}$$

for all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}^n$, and for all $y \in \mathbb{R}^m$. Chakravarty (2010, p. 34) refers to a representation of this ordering as the *absolute maximin index* because of its link to the maximin social welfare function.

Finally, the *relative mean-min inequality ordering* $R_{\mu n}^r$ is obtained by defining, for all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}_{++}^n$, and for all $y \in \mathbb{R}_{++}^m$,

$$xR_{\mu n}^r y \Leftrightarrow \frac{\mu(x)}{\min\{x_1, \dots, x_n\}} \geq \frac{\mu(y)}{\min\{y_1, \dots, y_m\}}$$

or, equivalently,

$$xR_{\mu n}^r y \Leftrightarrow \frac{\min\{x_1, \dots, x_n\}}{\mu(x)} \leq \frac{\min\{y_1, \dots, y_m\}}{\mu(y)}.$$

Therefore, according to $R_{\mu n}^r$, inequality increases if and only if the ratio of the minimum income to the mean income decreases. In analogy to the absolute case, Chakravarty (2010, p. 24) uses the term *relative maximin index* for a representation of $R_{\mu n}^r$.

2.2. Quantile-Based Inequality Orderings

To discuss the inequality orderings that are based on top and bottom income shares and gaps, we need to employ a slightly modified framework. Let $q \in \mathbb{N}$ with $q \geq 3$. The set D of income distributions considered now is defined by $D = \cup_{n \in \mathbb{N}} \Omega^{nq}$, where $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$. This modification guarantees that q equal-sized groups of individuals in an income distribution are well defined. Note that, for any $n \in \mathbb{N}$ and for any $x \in \Omega^{nq}$, there exists a unique permutation π_x of $\{1, \dots, nq\}$ such that $x_{(\cdot)} = (x_{\pi_x(1)}, \dots, x_{\pi_x(nq)})$ is a non-decreasing rearrangement of x and, for all $i, j \in \{1, \dots, nq\}$ with $i < j$, if $x_{\pi_x(i)} = x_{\pi_x(j)}$ then $\pi_x(i) < \pi_x(j)$. That is, $\pi_x^{-1}(i)$ is interpreted as the income rank of individual i from the bottom in x , where ties of income levels are broken with respect to individual names represented by numbers. For any $n \in \mathbb{N}$, for any $x \in \Omega^{nq}$, and for any $\ell \in \{1, \dots, q\}$, we define $G_\ell(x)$ by

$$G_\ell(x) = \{i \in \{1, \dots, nq\} | (\ell - 1)n + 1 \leq \pi_x^{-1}(i) \leq \ell n\},$$

that is, $G_\ell(x)$ is the group of individuals in the $\ell^{th}q$ -quantile in x . In this paper, the $\ell^{th}q$ -quantile of income distribution x represents the ℓ^{th} worse-off group of individuals according to the income ranking π_x^{-1} , rather than the ℓ^{th} cut-off point. Therefore, if $q = 10$, $G_1(x)$ is the group of individuals in the bottom decile and $G_{10}(x)$ is that in the top decile. For all $n \in \mathbb{N}$, for all $x \in \mathbb{R}_{++}^{nq}$, and for all $\ell \in \{1, \dots, q\}$, we write $\mu_\ell(x)$ as the mean income of the $\ell^{th}q$ -quantile of x , that is, $\mu_\ell(x) = \sum_{i \in G_\ell(x)} x_i/n$.

According to the modification of the domain of an inequality ordering, we say that an inequality ordering R on D is absolute if it satisfies the translation invariance axiom reformulated as follows.

Translation Invariance

For all $n \in \mathbb{N}$, for all $x \in \mathbb{R}^{nq}$, and for all $\delta \in \mathbb{R}$,

$$(x + \delta \mathbf{1}^{nq})Ix.$$

Analogously, an inequality ordering R is said to be relative if it satisfies the following reformulation of the scale-invariance property.

Scale Invariance

For all $n \in \mathbb{N}$, for all $x \in \mathbb{R}_{++}^{nq}$, and for all $\lambda \in \mathbb{R}_{++}$,

$$\lambda xIx.$$

We define the *top income gap inequality ordering* R_t^a by letting, for all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}^{nq}$, and for all $y \in \mathbb{R}^{mq}$,

$$xR_t^a y \Leftrightarrow \mu_q(x) - \mu(x) \geq \mu_q(y) - \mu(y).$$

The scale-invariant analogue of R_t^a is the (relative) *top income share inequality ordering* R_t^r , defined as follows. For all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}_{++}^{nq}$, and for all $y \in \mathbb{R}_{++}^{mq}$,

$$xR_t^r y \Leftrightarrow \frac{\sum_{i \in G_q(x)} x_i}{\sum_{i=1}^{nq} x_i} \geq \frac{\sum_{i \in G_q(y)} y_i}{\sum_{i=1}^{mq} y_i}.$$

Since the pioneering work by Piketty (2001), top income shares have been widely employed in the literature on the empirical analysis of inequality in the long run; see, for instance, Atkinson *et al.* (2011) and Leigh (2009). Note that, since $\sum_{i \in G_q(x)} x_i / \sum_{i=1}^{nq} x_i = \mu_q(x) / (q\mu(x))$, an ordinally equivalent representation of R_t^r is given by

$$xR_t^r y \Leftrightarrow \frac{\mu_q(x)}{\mu(x)} \geq \frac{\mu_q(y)}{\mu(y)}.$$

The *bottom income gap inequality ordering* R_b^a is given by letting, for all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}^{nq}$, and for all $y \in \mathbb{R}^{mq}$,

$$xR_b^a y \Leftrightarrow \mu(x) - \mu_1(x) \geq \mu(y) - \mu_1(y).$$

Finally, we define a relative analogue of the bottom income gap inequality ordering. The *bottom income share inequality ordering* is the inequality ordering R_b^r defined as follows. For all $n, m \in \mathbb{N}$, for all $x \in \mathbb{R}_{++}^{nq}$, and for all $y \in \mathbb{R}_{++}^{mq}$,

$$xR_b^r y \Leftrightarrow \frac{\sum_{i \in G_1(x)} x_i}{\sum_{i=1}^{nq} x_i} \leq \frac{\sum_{i \in G_1(y)} y_i}{\sum_{i=1}^{mq} y_i}.$$

Analogously to the top income share inequality ordering, an ordinally equivalent representation of R_b^r is given by

$$xR_b^r y \Leftrightarrow \frac{\mu(x)}{\mu_1(x)} \geq \frac{\mu(y)}{\mu_1(y)}.$$

3. CHARACTERIZATIONS

The use of translation invariance is restricted to absolute inequality orderings, whereas scale invariance is employed in the relative case, and both of these properties are employed throughout this section to distinguish these two notions of inequality. All other axioms can be defined for both options, that is, for $\Omega = \mathbb{R}$ and for $\Omega = \mathbb{R}_{++}$. Each of the following subsections addresses one type of ordering considered in this paper.

The first set of results on range-based measures borrows, to a large extent, from the literature on ranking sets under uncertainty by adapting some of the axioms that appear in this area to our framework. The remaining subsections rely on more traditional properties that are familiar from the theory of social index numbers. The general pattern that emerges is that, for each category of the remaining inequality orderings, all but one axiom (or, in the case of quantiles, two axioms) is well established, and the measures are set apart by the additional property (or properties). In particular, the characterizations of the inequality orderings that are based on the maximum and the mean rely on properties such as the well-known principle of progressive transfers, continuity, and replication invariance as standard requirements; the additional axiom is a principle that prescribes trade-offs between specific conflicting progressive and regressive transfers to be resolved in a consistent way in different situations. A parallel approach is applied in the case of the mean and the minimum, with the difference that the resolution of the aforementioned trade-off proceeds in a different direction. Finally, for the measures that are based on quantiles, we again formulate suitably adapted principles that resolves trade-offs in specific ways. In addition, a neutrality property that ensures an additive structure within quantiles is employed. Intuitively, a second additional property is necessitated in this case because some within-quantile structure needs to be established, a requirement that is not present for the other categories of indices.

3.1. Range Inequality Orderings

Our first axiom in this subsection requires that the inequality ordering R is *anonymous*, paying no attention to the names of the individuals. Clearly, this is a fundamental equity property. We acknowledge that, according to some views, a distinction could be made between individuals on the basis of compensatory notions. However, we are confident that anonymity constitutes a normatively extremely appealing principle. This is especially the case if it is assumed (as is done here) that the individual incomes represent the basic information on which inequality judgments are to be founded.

Anonymity

For all $n \in \mathbb{N}$ and for all $x, y \in \Omega^n$, if x is a permutation of y , then xIy .

In addition to anonymity, the results of this subsection make use of properties that involve the comparison of income distributions of different dimensions. The first of these is straightforward. *Equality indifference* requires that all equal distributions are equally unequal, independent of the number of people involved. As is the case for anonymity, the intuitive appeal of this condition is immediate.

Equality Indifference

For all $n, m \in \mathbb{N}$ and for all $\alpha, \beta \in \Omega^1$,

$$\alpha \mathbf{1}^n I \beta \mathbf{1}^m.$$

The first part of the following expansion-dominance axiom is borrowed from the literature on ranking sets of outcomes in the presence of complete uncertainty; see, for instance, Kannai and Peleg (1984) and Bossert and Slinko (2006). In contrast to that literature, we must allow for incomes being equal within a distribution and, moreover, the role played by lowest incomes is different from that played by worst elements in a set of possible outcomes. Thus, our formulation differs from that in the literature on ranking sets. The second part of the property reflects the coarse nature of the inequality orderings discussed here by requiring that adding individuals with incomes between the extremes of a distribution does not increase inequality.

Expansion Dominance

(i) For all $n, m \in \mathbb{N}$, for all $x \in \Omega^n$, and for all $y \in \Omega^m$, if $y_1 = \dots = y_m > \max\{x_1, \dots, x_n\}$, then

$$(y, x)Px.$$

(ii) For all $n \in \mathbb{N}$, for all $x \in \Omega^n$, and for all $\alpha \in [\min\{x_1, \dots, x_n\}, \max\{x_1, \dots, x_n\}]$,

$$xR(x, \alpha).$$

Part (i) of the above expansion-dominance axiom is based on the observation that if an income distribution is expanded by adding any number of individuals with a common income level that is above the highest in the original distribution, the resulting larger distribution should display a higher level of inequality. Again, this is intuitively plausible because the new distribution increases maximal income without changing the distribution among those who are present prior to the expansion. Part (ii) clearly is more controversial because it reflects a feature of the range-based measures—namely, that they are insensitive with respect to expansions of a distribution that leave the extreme values unchanged.

Another modification of a requirement from the literature on choice under complete uncertainty is the following conditional version of an independence property. Again, the axiom differs from the corresponding condition for set rankings because of the different interpretation—primarily because equal income levels within a distribution must be accommodated.

Conditional Independence

For all $n, m \in \mathbb{N}$, for all $x \in \Omega^n$, for all $y \in \Omega^m$, and for all $\alpha \in \Omega^1$, if xPy , $\min\{x_1, \dots, x_n\} = \min\{y_1, \dots, y_m\}$, $\alpha \geq \max\{x_1, \dots, x_n\}$, and $\alpha > \max\{y_1, \dots, y_m\}$, then

$$(x, \alpha)R(y, \alpha).$$

Conditional independence is a robustness condition. Starting with two distributions x and y (not necessarily of the same population size), if x is considered more unequal than y , then the addition of an individual whose income exceeds the maximal income in y and is at least as high as the maximal income in x should not overturn this strict relation.

Our first observation shows that the conjunction of the four axioms of this subsection implies that an income distribution x of any dimension must be as unequal as the distribution that is composed of the maximal and the minimal values of x . See, for instance, Kannai and Peleg's (1984, p. 174) Lemma and Bossert and Slinko (2006, pp. 108–09) Theorem 1 for analogous results in the context of set rankings.

Theorem 1 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$. If R satisfies anonymity, equality indifference, expansion dominance, and conditional independence, then, for all $n \in \mathbb{N}$ and for all $x \in \Omega^n$,

$$xI(\max\{x_1, \dots, x_n\}, \min\{x_1, \dots, x_n\}).$$

The following theorem characterizes all inequality orderings that satisfy the axioms defined in this subsection. It turns out that these measures can be expressed by means of an ordering defined on the pairs of maximal and minimal incomes. Thus, only the extreme values may be utilized as a consequence of the axioms and, moreover, the ordering of the pairs must be increasing in the maximal income.

Theorem 2 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$. R satisfies anonymity, equality indifference, expansion dominance, and conditional independence if and only if there exists an ordering \succeq (with asymmetric and symmetric parts $>$ and \sim) on $S = \{(\alpha, \beta) \in \Omega^2 \mid \alpha \geq \beta\}$ such that

- (i) for all $n, m \in \mathbb{N}$, for all $x \in \Omega^n$, and for all $y \in \Omega^m$,
 $xRy \Leftrightarrow (\max\{x_1, \dots, x_n\}, \min\{x_1, \dots, x_n\}) \succeq (\max\{y_1, \dots, y_m\}, \min\{y_1, \dots, y_m\})$;
- (ii) $(\alpha, \alpha) \sim (\beta, \beta)$ for all $\alpha, \beta \in \Omega^1$;
- (iii) \succeq is increasing in its first argument.

We now state the two main results of this subsection. Adding translation invariance to the axioms of Theorem 2 characterizes the absolute range, whereas the relative range is obtained if scale invariance is used in the place of translation invariance.

Theorem 3 Let $\Omega = \mathbb{R}$. R satisfies anonymity, equality indifference, expansion dominance, conditional independence, and translation invariance if and only if $R = R_{xn}^a$.

As a remark aside, note that Theorems 1–3 remain true if $\Omega = \mathbb{R}$ is replaced with $\Omega = \mathbb{R}_+$; this is apparent from inspecting their proofs.

Theorem 4 Let $\Omega = \mathbb{R}_{++}$. R satisfies anonymity, equality indifference, expansion dominance, conditional independence, and scale invariance if and only if $R = R_{xn}^r$.

3.2. Max-Mean Inequality Orderings

We characterize the absolute and relative max-mean inequality orderings using four axioms in addition to translation invariance and scale invariance, respectively.

The first of these can be considered the cornerstone of inequality measurement. To introduce it, we require the definition of a *doubly stochastic* matrix. For any $n \in \mathbb{N}$, an $n \times n$ matrix is doubly stochastic if all its elements are nonnegative and its rows and columns sum to one. Given $n \in \mathbb{N}$ and $x \in \Omega^n$, multiplying x by an $n \times n$ doubly stochastic matrix B yields an income distribution $Bx \in \Omega^n$ that has the same total income and is a smoothing of x in the sense that each component is a convex combination of x . Indeed, it is known that for any rank-ordered distribution $x \in \Omega^n$, Bx can be obtained by a finite sequence of progressive transfers (Hardy *et al.*, 1934; Marshall and Olkin, 1979). The property of *Schur-convexity* (or *S-convexity*, for short) asserts that such a smoothing of an income distribution does not increase inequality.

S-Convexity

For all $n \in \mathbb{N}$, for all $x \in \Omega^n$, and for all $n \times n$ doubly stochastic matrices B , $xR(Bx)$.

The property of S-convexity is equivalent to the conjunction of anonymity and the well-known *Pigou-Dalton transfer principle* (Pigou, 1912; Dalton, 1920). Clearly, S-convexity is uncontroversial because the axiom captures the very notion of inequality measurement: if incomes move closer together, inequality cannot increase.

Our next property, *continuity*, requires that small changes in incomes do not lead to large changes in inequality. Thus, the axiom ensures that income measurement is robust in the sense that a small measurement error does not lead to an excessive change in the assessment performed by an inequality ordering. This is another standard requirement commonly imposed on inequality orderings and other (ordinal) social indicators.

Continuity

For all $n \in \mathbb{N}$ and for all $x \in \Omega^n$, $\{y \in \Omega^n | yRx\}$ and $\{y \in \Omega^n | xRy\}$ are closed in Ω^n .

Replication invariance, which first appeared in Dalton (1920) under the name of the *principle of proportionate additions to persons*, requires that inequality be invariant under any k -fold replica of an income distribution.

Replication Invariance

For all $n, k \in \mathbb{N}$ and for all $x \in \Omega^n$, $xI(\underbrace{x, \dots, x}_{k \text{ times}})$.

Replication invariance ensures that an averaging view is adopted when comparing distributions of different dimension; this is an immediate consequence of the requirement that a replicated distribution must be as unequal as its original.

Replication invariance in conjunction with translation invariance (if $\Omega = \mathbb{R}$) or scale invariance (in the case $\Omega = \mathbb{R}_{++}$) implies equality indifference. To see that this is the case, let $\Omega = \mathbb{R}$, $n, m \in \mathbb{N}$, and $\alpha, \beta \in \Omega^1$. Translation invariance implies $\alpha \mathbf{1}^n I \beta \mathbf{1}^n$. By replication invariance, we obtain $\beta \mathbf{1}^n I \beta \mathbf{1}^{nm}$ and $\beta \mathbf{1}^{nm} I \beta \mathbf{1}^m$. Since R is transitive, it follows that $\alpha \mathbf{1}^n I \beta \mathbf{1}^m$. Analogously, it can be verified that replication invariance and scale invariance together imply equality indifference if $\Omega = \mathbb{R}_{++}$.

The only axiom of this subsection that is not entirely standard is the following *composite transfer principle for top income*. It prescribes certain consequences of a composition of rank-preserving progressive and regressive transfers involving three income recipients. Consider three individuals i, j , and n . Suppose that n is the best-off in the entire population and i is worse off than j . The axiom asserts that a composition of a progressive transfer from j to i and a regressive transfer from j to n increases inequality as long as the relative ranking of all individuals involved is preserved. This axiom strengthens an idea embodied in the *joint transfer axiom* in Sen (1974). A crucial feature of the axiom (and of the inequality orderings that satisfy it) is that a trade-off between two conflicting transfers (one regressive, one progressive) must always be resolved in the same direction.

Composite Transfer Principle for Top Income

For all $n \in \mathbb{N}$ and for all $x, y \in \Omega^n$ with $x_k \leq x_{k+1}$ and $y_k \leq y_{k+1}$ for all $k \in \{1, \dots, n-1\}$, if there exist $i, j \in \{1, \dots, n-1\}$ with $i < j$ and $\delta, \epsilon \in \mathbb{R}_{++}$ such that $x - y = \delta(e^i - e^j) + \epsilon(e^n - e^j)$, then xPy .

The following theorem provides a preliminary result that is analogous to Theorem 1 of the previous section.

Theorem 5 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$ and suppose that R satisfies S-convexity, continuity, replication invariance, and the composite transfer principle for top income. For all $n, m \in \mathbb{N}$, for all $x \in \Omega^n$, and for all $y \in \Omega^m$, if $\max\{x_1, \dots, x_n\} = \max\{y_1, \dots, y_m\}$ and $\mu(x) = \mu(y)$, then xIy .

Parallel to Theorem 2, the following result characterizes all inequality orderings that satisfy the axioms introduced in this subsection. As the theorem shows, these orderings only utilize the maximum and average incomes and are increasing in the maximum income.

Theorem 6 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$. R satisfies S-convexity, continuity, replication invariance, and the composite transfer principle for top income if and only if there exists a continuous ordering \succsim on $S = \{(\alpha, \beta) \in \Omega^2 \mid \alpha \geq \beta\}$ such that

(i) for all $n, m \in \mathbb{N}$, for all $x \in \Omega^n$, and for all $y \in \Omega^m$,

$$xRy \Leftrightarrow (\max\{x_1, \dots, x_n\}, \mu(x)) \succsim (\max\{y_1, \dots, y_m\}, \mu(y));$$

(ii) \succsim is increasing in its first argument.

The subsection is concluded with characterizations of the absolute and relative max-mean inequality orderings.

Theorem 7 Let $\Omega = \mathbb{R}$. R satisfies S-convexity, continuity, replication invariance, the composite transfer principle for top income, and translation invariance if and only if $R = R_{x\mu}^a$.

Theorem 8 Let $\Omega = \mathbb{R}_{++}$. R satisfies S-convexity, continuity, replication invariance, the composite transfer principle for top income, and scale invariance if and only if $R = R_{x\mu}^r$.

3.3. Mean-Min Inequality Orderings

We characterize the absolute and relative mean-min inequality orderings using an axiom dual to the composite transfer principle for top income, which we call the *composite transfer principle for bottom income*. Consider again three individuals i, j , and 1. Now suppose that j is better-off than i and 1 is the worst-off in the entire population. The composite transfer principle for bottom income asserts that a composition of a progressive transfer from i to 1 and a regressive transfer from i to j decreases inequality as long as the ranking of all individuals is preserved. This axiom is similar to the *transfer sensitivity* axiom in Shorrocks and Foster (1987); see also Kamaga (2018) and Bossert and Kamaga (2020). In the context of welfare measurement, the property employed by these authors is implied

by the conjunction of the strong Pareto principle and the well-known axiom of *Hammond equity*; see Hammond (1979, p. 1132).

Composite Transfer Principle for Bottom Income

For all $n \in \mathbb{N}$ and for all $x, y \in \Omega^n$ with $x_k \leq x_{k+1}$ and $y_k \leq y_{k+1}$ for all $k \in \{1, \dots, n-1\}$, if there exist $i, j \in \{2, \dots, n\}$ with $i < j$ and $\delta, \varepsilon \in \mathbb{R}_{++}$ such that $x - y = \delta(e^1 - e^i) + \varepsilon(e^j - e^i)$, then yPx .

In analogy to the previous subsections, we begin with a preliminary result. This is followed by a characterization of all inequality orderings that satisfy the axioms of the previous subsection when the composite transfer principle for top income is replaced with the corresponding principle for bottom income.

Theorem 9 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$ and suppose that R satisfies S-convexity, continuity, replication invariance, and the composite transfer principle for bottom income. For all $n, m \in \mathbb{N}$, for all $x \in \Omega^n$, and for all $y \in \Omega^m$, if $\min\{x_1, \dots, x_n\} = \min\{y_1, \dots, y_m\}$ and $\mu(x) = \mu(y)$, then xIy .

Note that, unlike Theorems 1–3, the proof of Theorem 9 does not apply if $\Omega = \mathbb{R}$ is replaced with $\Omega = \mathbb{R}_+$; this is the case because Step 1 of its proof presented in the appendix cannot be established on this alternative domain. For that reason, we allow for negative income values in the absolute case.

The following theorem axiomatizes the class of continuous inequality orderings that only utilize the mean and minimum incomes and are decreasing in the minimum income.

Theorem 10 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$. R satisfies S-convexity, continuity, replication invariance, and the composite transfer principle for bottom income if and only if there exists a continuous ordering \succsim on $S = \{(\alpha, \beta) \in \Omega^2 \mid \alpha \geq \beta\}$ such that

- (i) for all $n, m \in \mathbb{N}$, for all $x \in \Omega^n$, and for all $y \in \Omega^m$,

$$xRy \Leftrightarrow (\mu(x), \min\{x_1, \dots, x_n\}) \succsim (\mu(y), \min\{y_1, \dots, y_m\});$$
- (ii) \succsim is decreasing in its second argument.

Finally, we characterize the absolute and relative mean-min inequality orderings.

Theorem 11 Let $\Omega = \mathbb{R}$. R satisfies S-convexity, continuity, replication invariance, the composite transfer principle for bottom income, and translation invariance if and only if $R = R_{\mu m}^a$.

Theorem 12 Let $\Omega = \mathbb{R}_{++}$. R satisfies S-convexity, continuity, replication invariance, the composite transfer principle for bottom income, and scale invariance if and only if $R = R_{\mu m}^r$.

3.4. Top Income Gaps and Shares

We begin by presenting the restatements of S-convexity, continuity, and replication invariance defined on the requisite domain; as is the case for the properties introduced in Section 2, this needs to be done because we focus on quantiles in this subsection.

S-Convexity*

For all $n \in \mathbb{N}$, for all $x \in \Omega^{nq}$, and for all $nq \times nq$ doubly stochastic matrices B , $xR(Bx)$.

Continuity*

For all $n \in \mathbb{N}$ and for all $x \in \Omega^{nq}$, $\{y \in \Omega^{nq} | yRx\}$ and $\{y \in \Omega^{nq} | xRy\}$ are closed in Ω^{nq} .

Replication invariance*

For all $n, k \in \mathbb{N}$ and for all $x \in \Omega^{nq}$, $xI(\underbrace{x, \dots, x}_{k \text{ times}})$.

There are now two new axioms that play a crucial role in identifying the indices considered in this subsection. The first of these, *transfer neutrality within quantiles*, requires that inequality be invariant with respect to a transfer within a quantile as long as the individuals involved remain in the same quantile. This is an inequality-measurement analogue of the *incremental-equity* property introduced by Blackorby *et al.* (2002) in the context of welfare measurement. Parallel to Blackorby *et al.* (2002) characterization of utilitarianism, the axiom is primarily responsible for the linearity inherent in criteria that depend on arithmetic means—in our case, the means of the quantiles. As alluded to earlier, the reason why an axiom of this nature is required in this subsection but not earlier is the necessity to address within-quantile issues.

Transfer Neutrality within Quantiles

For all $n \in \mathbb{N}$ and for all $x, y \in \Omega^{nq}$, if $G_\ell(x) = G_\ell(y)$ for all $\ell \in \{1, \dots, q\}$ and there exist $\ell' \in \{1, \dots, q\}$ and $i, j \in G_{\ell'}(x)$ such that $x_i - y_i = y_j - x_j$ and $x_k = y_k$ for all $k \in \{1, \dots, nq\} \setminus \{i, j\}$, then xIy .

The following theorem characterizes the class of inequality orderings that satisfy the four axioms presented earlier. As the theorem shows, this class consists of all continuous and S-convex orderings that only utilize the mean incomes of the quantiles.

Theorem 13 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$. R satisfies S-convexity*, replication invariance*, continuity*, and transfer neutrality within quantiles if and only if there exists a continuous and S-convex ordering \succ^* on $S^* = \{z \in \Omega^q | z_\ell \leq z_{\ell+1} \text{ for all } \ell \in \{1, \dots, q-1\}\}$ such that, for all $n, m \in \mathbb{N}$, for all $x \in \Omega^{nq}$, and for all $y \in \Omega^{mq}$,

$$(1) \quad xRy \Leftrightarrow (\mu_1(x), \dots, \mu_q(x)) \succeq^* (\mu_1(y), \dots, \mu_q(y)).$$

The second new axiom we use to characterize the top income gap inequality ordering and its relative counterpart is the *composite transfer principle for top quantile*. This axiom parallels the composite transfer principle for top income, but the requirement is restricted to income distributions involving q individuals. Thus, it is logically weaker than the direct reformulation of the composite transfer principle for top income.

Composite transfer principle for top quantile

For all $x, y \in \Omega^q$ with $x_\ell \leq x_{\ell+1}$ and $y_\ell \leq y_{\ell+1}$ for all $\ell \in \{1, \dots, q-1\}$, if there exist $\delta, \varepsilon \in \mathbb{R}_{++}$ and $i, j \in \{1, \dots, q-1\}$ with $i < j$ such that $x - y = \delta(e^i - e^j) + \varepsilon(e^q - e^j)$, then xPy .

Adding the composite transfer principle for top quantile to the axioms of Theorem 13, we obtain the following preliminary result that is analogous to Theorem 5.

Theorem 14 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$ and suppose that R satisfies S -convexity*, continuity*, replication invariance*, transfer neutrality within quantiles, and the composite transfer principle for top quantile. For all $n, m \in \mathbb{N}$, for all $x \in \Omega^{nq}$, and for all $y \in \Omega^{mq}$, if $\mu_q(x) = \mu_q(y)$ and $\mu(x) = \mu(y)$, then xIy .

The following theorem characterizes all inequality orderings that satisfy the axioms introduced in this subsection. These inequality orderings only utilize the mean incomes of the top quantile and the entire population, and they are increasing in the mean income of the top quantile.

Theorem 15 1. Let $\Omega = \mathbb{R}$. R satisfies S -convexity*, continuity*, replication invariance*, transfer neutrality within quantiles, and the composite transfer principle for top quantile if and only if there exists a continuous ordering \succeq on $S = \{(\alpha, \beta) \in \Omega^2 \mid \alpha \geq \beta\}$ such that

- 2. (i) for all $n, m \in \mathbb{N}$, for all $x \in \Omega^{nq}$, and for all $y \in \Omega^{mq}$,

$$xRy \Leftrightarrow (\mu_q(x), \mu(x)) \succeq (\mu_q(y), \mu(y));$$

(ii) \succeq is increasing in its first argument.

- 3. Let $\Omega = \mathbb{R}_{++}$. R satisfies S -convexity*, continuity*, replication invariance*, transfer neutrality within quantiles, and the composite transfer principle for top quantile if and only if there exists a continuous ordering \succeq on $S = \{(\alpha, \beta) \in \Omega^2 \mid \alpha \geq \beta > \alpha/q\}$ such that

- 4. (i) for all $n, m \in \mathbb{N}$, for all $x \in \Omega^{nq}$, and for all $y \in \Omega^{mq}$,

$$xRy \Leftrightarrow (\mu_q(x), \mu(x)) \succeq (\mu_q(y), \mu(y));$$

(ii) \succsim is increasing in its first argument.

Again, adding translation invariance and scale invariance, respectively, to the axioms of Theorem 15, we obtain characterizations of the top income gap inequality ordering and the top income share inequality ordering.

Theorem 16 Let $\Omega = \mathbb{R}$. R satisfies S -convexity*, continuity*, replication invariance*, transfer neutrality within quantiles, the composite transfer principle for top quantile, and translation invariance* if and only if $R = R^a$.

Theorem 17 Let $\Omega = \mathbb{R}_{++}$. R satisfies S -convexity*, continuity*, replication invariance*, transfer neutrality within quantiles, the composite transfer principle for top quantile, and scale invariance* if and only if $R = R'_t$.

3.5. Bottom Income Gaps and Shares

We characterize the bottom income share inequality ordering and the mean-bottom inequality ordering using the *composite transfer principle for bottom quantile*, which is an axiom dual to the composite transfer principle for top quantile. The composite transfer principle for bottom quantile requires the same property as the composite transfer principle for bottom income, but the property applies only to income distributions for q persons.

Composite Transfer Principle for Bottom Quantile

For all $x, y \in \Omega^q$ with $x_\ell \leq x_{\ell+1}$ and $y_\ell \leq y_{\ell+1}$ for all $\ell \in \{1, \dots, q-1\}$, if there exist $\delta, \varepsilon \in \mathbb{R}_{++}$ and $i, j \in \{2, \dots, q\}$ with $i < j$ such that $x - y = \delta(e^1 - e^i) + \varepsilon(e^j - e^i)$, then yPx .

In analogy to the previous section, we characterize all inequality orderings that satisfy the axioms of the previous subsection when the composite transfer principle for top quantile is replaced with that for bottom quantile. We begin with the following preliminary result.

Theorem 18 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$ and suppose that R satisfies S -convexity*, continuity*, replication invariance*, transfer neutrality within quantiles, and the composite transfer principle for bottom quantile. For all $n, m \in \mathbb{N}$, for all $x \in \Omega^{nq}$, and for all $y \in \Omega^{mq}$, if $\mu_1(x) = \mu_1(y)$ and $\mu(x) = \mu(y)$, then xIy .

The following theorem forms the basis of our final two axiomatizations.

Theorem 19 Let $\Omega \in \{\mathbb{R}, \mathbb{R}_{++}\}$. R satisfies S -convexity*, continuity*, replication invariance*, transfer neutrality within quantiles, and the composite transfer principle for bottom quantile if and only if there exists a continuous ordering \succsim on $S = \{(\alpha, \beta) \in \Omega^2 \mid \alpha \geq \beta\}$ such that

- (i) for all $n, m \in \mathbb{N}$, for all $x \in \Omega^{nq}$, and for all $y \in \Omega^{mq}$,

$$xRy \Leftrightarrow (\mu(x), \mu_1(x)) \succsim (\mu(y), \mu_1(y));$$

(ii) \succsim is decreasing in its second argument.

Adding translation invariance and scale invariance, respectively, to the axioms of Theorem 19, we obtain characterizations of the bottom income gap inequality ordering and the bottom income share inequality ordering.

Theorem 20 Let $\Omega = \mathbb{R}$. R satisfies S -convexity*, continuity*, replication invariance*, transfer neutrality within quantiles, the composite transfer principle for bottom quantile, and translation invariance* if and only if $R = R_b^a$.

Theorem 21 Let $\Omega = \mathbb{R}_{++}$. R satisfies S -convexity*, continuity*, replication invariance*, transfer neutrality within quantiles, the composite transfer principle for bottom quantile, and scale invariance* if and only if $R = R_b^r$.

4. EMPIRICAL CONSIDERATIONS

The measures characterized in this paper are easily understood and computed. They can be considered somewhat coarse, and the purpose of this empirical section is to explore their linear correlation with more standard indices of inequality. We proceed by calculating some of the characterized indices and some standard indices (see below for details) for comparison purposes, employing a strategy that is inspired by Leigh (2007). In particular, we estimate the equations

$$(2) \quad SIneq_{i,t} = \alpha + \beta Ineq_{i,t} + \varepsilon_{i,t},$$

$$(3) \quad SIneq_{i,t} = \alpha + \beta Ineq_{i,t} + \gamma_i + \varepsilon_{i,t},$$

$$(4) \quad SIneq_{i,t} = \alpha + \beta Ineq_{i,t} + \gamma_i + \delta_t + \varepsilon_{i,t},$$

where $SIneq_{i,t}$ is one of several standard indices of inequality in country i in year t . These alternative measures are given by (1) the absolute Gini coefficient, the variance, and the Kolm index with parameter values of 10^{-4} and $5 \cdot 10^{-4}$ in the absolute case; and (2) the Gini coefficient and the Atkinson index with inequality-aversion parameter values of 0.5 and 1 for the relative measures. The variable $Ineq_{i,t}$ indicates one of the inequality measures characterized in this paper. Equation (3) also includes a country-specific term γ , allowing us to estimate the association between indices within countries, while (4) controls for the year fixed effect δ in addition, to include the effects of common macroeconomic shocks between countries in specific years. A fourth model that may be estimated adds covariates (e.g. GDP growth rates, unemployment, and fertility rates) that could mediate the association between inequality indices. In addition, nonlinear associations among the indices could be explored. We leave these extensions for future research.

We use all the waves of the Luxembourg Income Study (LIS) datasets that were available as of May 2019, retaining the countries for which at least 4 years for the period 1974–2016 are covered. This leaves us with 36 countries in total and

a global sample of 299 observations; for the countries retained, see Table A1 in the appendix. We follow the LIS rules for their provision of the key figures since we wish to be of guidance for researchers who decide to use the indices already available from LIS. In particular, in this specification, (1) the income measure is disposable household income equalized by means of the square root equivalence scale; (2) the unit of analysis is the individual; (3) incomes are bottom-coded at 1% of equalized mean income and top-coded at 10times mean income; (4) missing and zero incomes are excluded. As an alternative, to test the sensitivity of our results to the LIS top-coding rules, we also provide the results without top coding and include all the observations on the right tail of the income distribution. The incomes are expressed in 2011 constant US dollars.

All variables are standardized to Z-scores (i.e. to a mean of zero and a standard deviation of one) to facilitate comparisons of the estimated coefficients. As a result of this standardization, the slope β of the regression line in (2) is Pearson's correlation coefficient among the independent and dependent variables. This equivalence does not hold in the other two estimated models since these are multivariate regressions. Again, the reference value is one because an increase in one standard deviation of one index is associated with an increase in one standard deviation in the other. In the models of equation (3), we include a country-specific term γ . Equation (4) also controls for a year fixed effect δ . The β coefficient in (2), Pearson's correlation coefficient among the independent and dependent variables, could be considered a more informative measure to rely on to evaluate the reliability of the compared indices for the purpose of inter-country or longitudinal comparisons. Still, the other two models allow us to analyze the correlation within countries and consider the effects of common macroeconomic shocks.

Table 1 displays the results for the absolute inequality indices, and Table 2 contains those for the relative case following the LIS rules, while Tables 3 and 4 contain those without top-coding. Owing to the presence of high collinearity among the inequality indices (measured by a Variance Inflation Factor exceeding the reference value of 10 by a large margin), we cannot include all of them simultaneously in the regression. To avoid lengthy tables, we report the estimation results of pairs of the classical and our inequality measures in a single column. The classical measure we consider is indicated in the top row, and the inequality measures we characterize are listed in the first column. The equation numbers (2)–(4) in the top row indicate the three regression models without fixed effects, with country fixed effects, and with country and year fixed effects, respectively.

All coefficients are positive and significant in the LIS specification, while some coefficients lose significance without top/bottom coding and also in one case for the Atkinson index. Let us first focus on the discussion of the results with the full application of the LIS rules. We observe many correlation coefficients among the indices above 0.9, indicating that these indices are reliable proxies of each other. For the absolute case, the lowest observed correlation is never below 0.352 (between the Kolm index with a parameter value of 10^{-4} and the absolute mean-min indices in Table 1). The correlation coefficients for the relative measures range between 0.18 (observed between the Gini or the Atkinson index with a parameter value of 0.5 and the relative mean-min indices in Table 2) and 0.983 (between the Gini and the top income share indices).

TABLE 1
STANDARD ABSOLUTE INEQUALITY MEASURES AND OUR ABSOLUTE INEQUALITY MEASURES

Dependent variable	Absolute Gini				Variance				Kolm (parameter 10 ⁻⁴)				Kolm (parameter 5·10 ⁻⁴)			
	(2)	(3)	(4)	(3)	(4)	(2)	(3)	(4)	(3)	(4)	(2)	(3)	(4)	(2)	(3)	(4)
Absolute range	0.838*** (0.037)	0.606*** (0.038)	0.332*** (0.041)	0.809*** (0.055)	0.675*** (0.048)	0.429*** (0.049)	0.493*** (0.054)	0.555*** (0.062)	0.400*** (0.088)	0.869*** (0.025)	0.642*** (0.044)	0.406*** (0.057)				
Obs.	0.702 299	0.910 299	0.959 299	0.654 299	0.875 299	0.927 299	0.243 299	0.687 299	0.759 299	0.755 299	0.879 299	0.932 299				
Absolute max–mean	0.823*** (0.038)	0.577*** (0.038)	0.306*** (0.040)	0.789*** (0.056)	0.647*** (0.047)	0.402*** (0.048)	0.498*** (0.053)	0.530*** (0.061)	0.374*** (0.085)	0.856*** (0.026)	0.612*** (0.044)	0.375*** (0.055)				
Obs.	0.677 299	0.904 299	0.958 299	0.638 299	0.869 299	0.926 299	0.249 299	0.683 299	0.757 299	0.732 299	0.873 299	0.930 299				
Absolute mean–min	0.925*** (0.027)	1.167*** (0.026)	1.022*** (0.042)	0.829*** (0.049)	1.155*** (0.063)	0.965*** (0.086)	0.352*** (0.060)	1.031*** (0.089)	0.983*** (0.160)	0.917*** (0.026)	1.244*** (0.033)	1.211*** (0.060)				
Obs.	0.855 299	0.984 299	0.988 299	0.688 299	0.913 299	0.939 299	0.124 299	0.737 299	0.777 299	0.841 299	0.965 299	0.971 299				
Top 10% gap	0.991*** (0.008)	1.011*** (0.014)	0.930*** (0.024)	0.923*** (0.041)	1.004*** (0.058)	0.890*** (0.074)	0.420*** (0.051)	0.848*** (0.079)	0.724*** (0.145)	0.904*** (0.021)	1.027*** (0.037)	0.938*** (0.062)				
Obs.	0.982 299	0.990 299	0.993 299	0.851 299	0.921 299	0.945 299	0.176 299	0.726 299	0.765 299	0.817 299	0.950 299	0.958 299				
Bottom 10% gap	0.993*** (0.008)	1.060*** (0.012)	1.032*** (0.019)	0.923*** (0.042)	1.071*** (0.056)	1.030*** (0.070)	0.414*** (0.053)	0.880*** (0.080)	0.773*** (0.153)	0.926*** (0.019)	1.078*** (0.034)	1.045*** (0.063)				
Obs.	0.986 299	0.996 299	0.997 299	0.852 299	0.934 299	0.953 299	0.171 299	0.727 299	0.765 299	0.858 299	0.957 299	0.962 299				

Notes: Each column presents the results of the corresponding pairs of classical and our inequality measures according to the estimated equations (2) without fixed effects, (3) with country fixed effects, and (4) with country and year fixed effects, respectively. Robust standard errors in parentheses. ***, **, and * denote $p < 0.01$, $p < 0.05$, and $p < 0.1$, respectively. R-squared is in italic.

TABLE 2
STANDARD RELATIVE INEQUALITY MEASURES AND OUR RELATIVE INEQUALITY MEASURES

Dependent variables	Gini				Atkinson (parameter 0.5)				Atkinson (parameter 1)			
	(2)	(3)	(4)	(2)	(3)	(4)	(2)	(3)	(4)	(2)	(3)	(4)
Relative range	0.327*** (0.044) 0.107	0.123*** (0.019) 0.957	0.078*** (0.023) 0.967	0.335*** (0.042) 0.112	0.108*** (0.018) 0.954	0.066*** (0.022) 0.964	0.341*** (0.043) 0.116	0.112*** (0.021) 0.941	0.070*** (0.027) 0.953	0.341*** (0.043) 0.116	0.112*** (0.021) 0.941	0.070*** (0.027) 0.953
Obs.	299	299	299	299	299	299	299	299	299	299	299	299
Relative max–mean	0.334*** (0.050) 0.111	0.093*** (0.020) 0.954	0.051*** (0.023) 0.966	0.342*** (0.048) 0.117	0.078*** (0.018) 0.951	0.038* (0.022) 0.963	0.331*** (0.050) 0.110	0.070*** (0.021) 0.937	0.025 (0.026) 0.952	0.331*** (0.050) 0.110	0.070*** (0.021) 0.937	0.025 (0.026) 0.952
Obs.	299	299	299	299	299	299	299	299	299	299	299	299
Relative mean–min	0.180*** (0.025) 0.032	0.097*** (0.018) 0.954	0.059*** (0.018) 0.967	0.180*** (0.022) 0.032	0.090*** (0.016) 0.952	0.054*** (0.016) 0.964	0.208*** (0.022) 0.043	0.110*** (0.019) 0.941	0.073** (0.020) 0.954	0.208*** (0.022) 0.043	0.110*** (0.019) 0.941	0.073** (0.020) 0.954
Obs.	299	299	299	299	299	299	299	299	299	299	299	299
Top 10% share	0.983*** (0.010) 0.965	0.822*** (0.040) 0.981	0.763*** (0.039) 0.987	0.969*** (0.016) 0.939	0.814*** (0.045) 0.979	0.770*** (0.046) 0.984	0.965*** (0.018) 0.932	0.888*** (0.049) 0.972	0.857*** (0.056) 0.978	0.965*** (0.018) 0.932	0.888*** (0.049) 0.972	0.857*** (0.056) 0.978
Obs.	299	299	299	299	299	299	299	299	299	299	299	299
Bottom 10% share	0.892*** (0.118) 0.792	0.547*** (0.149) 0.971	0.471*** (0.141) 0.979	0.918*** (0.108) 0.843	0.620*** (0.144) 0.976	0.554*** (0.141) 0.982	0.927*** (0.099) 0.860	0.671*** (0.155) 0.968	0.615*** (0.156) 0.976	0.927*** (0.099) 0.860	0.671*** (0.155) 0.968	0.615*** (0.156) 0.976
Obs.	299	299	299	299	299	299	299	299	299	299	299	299

Notes. Each column presents the results of the corresponding pairs of classical and our inequality measures according to the estimated equations (2) without fixed effects, (3) with country fixed effects, and (4) with country and year fixed effects, respectively. Robust standard errors in parentheses. ***, **, and * denote $p < 0.01$, $p < 0.05$, and $p < 0.1$, respectively. R-squared is in italic.

TABLE 3
STANDARD ABSOLUTE INEQUALITY MEASURES AND OUR ABSOLUTE INEQUALITY MEASURES (WITHOUT TOP-CODING)

Dependent variable	Absolute Gini				Variance				Kolm (parameter 10 ⁻⁴)				Kolm (parameter 5·10 ⁻⁴)			
	(2)	(3)	(4)	(3)	(2)	(3)	(4)	(3)	(2)	(3)	(4)	(3)	(2)	(3)	(4)	
Absolute range	0.025 (0.035)	0.064*** (0.024)	0.015 (0.012)	0.562*** (0.084)	0.576*** (0.069)	0.561*** (0.067)	0.088** (0.044)	0.065*** (0.016)	0.003 (0.020)	0.067 (0.043)	0.063*** (0.019)	0.011 (0.012)				
Obs.	0.001 299	0.775 299	0.937 299	0.316 299	0.547 299	0.648 299	0.008 299	0.584 299	0.735 299	0.005 299	0.736 299	0.906 299				
Absolute max–mean	0.023 (0.034)	0.064*** (0.024)	0.015 (0.012)	0.561*** (0.084)	0.576*** (0.069)	0.561*** (0.067)	0.087** (0.044)	0.064*** (0.016)	0.003 (0.020)	0.066 (0.041)	0.062*** (0.019)	0.011 (0.011)				
Obs.	0.001 299	0.775 299	0.937 299	0.315 299	0.547 299	0.648 299	0.008 299	0.584 299	0.735 299	0.004 299	0.736 299	0.906 299				
Absolute mean–min	0.917*** (0.028)	1.179*** (0.028)	1.044*** (0.045)	0.467*** (0.050)	0.872*** (0.162)	0.829*** (0.203)	0.355*** (0.060)	1.025*** (0.088)	0.974*** (0.159)	0.917*** (0.026)	1.240*** (0.032)	1.207*** (0.059)				
Obs.	0.841 299	0.981 299	0.986 299	0.218 299	0.385 299	0.473 299	0.126 299	0.739 299	0.778 299	0.840 299	0.965 299	0.971 299				
Top 10% gap	0.981*** (0.013)	1.016*** (0.020)	0.904*** (0.034)	0.431*** (0.033)	0.584*** (0.100)	0.237 (0.187)	0.413*** (0.051)	0.852*** (0.080)	0.720*** (0.144)	0.901*** (0.021)	1.033*** (0.038)	0.929*** (0.064)				
Obs.	0.963 299	0.979 299	0.985 299	0.186 299	0.339 299	0.445 299	0.170 299	0.726 299	0.766 299	0.811 299	0.947 299	0.956 299				
Bottom 10% gap	0.990*** (0.009)	1.070*** (0.013)	1.053*** (0.021)	0.498*** (0.040)	0.812*** (0.143)	0.882*** (0.201)	0.420*** (0.052)	0.870*** (0.079)	0.756*** (0.149)	0.927*** (0.019)	1.070*** (0.034)	1.031*** (0.061)				
Obs.	0.981 299	0.995 299	0.996 299	0.248 299	0.399 299	0.483 299	0.176 299	0.728 299	0.766 299	0.859 299	0.957 299	0.962 299				

Notes: Each column presents the results of the corresponding pairs of classical and our inequality measures according to the estimated equations (2) without fixed effects, (3) with country fixed effects, and (4) with country and year fixed effects, respectively. Robust standard errors in parentheses. ***, **, and * denote $p < 0.01$, $p < 0.05$, and $p < 0.1$, respectively. R-squared is in italic.

TABLE 4
STANDARD RELATIVE INEQUALITY MEASURES AND OUR RELATIVE INEQUALITY MEASURES (WITHOUT TOP-CODING)

Dependent variables	Gini				Atkinson (parameter 0.5)				Atkinson (parameter 1)			
	(2)	(3)	(4)	(4)	(2)	(3)	(4)	(4)	(2)	(3)	(3)	(4)
Relative range	0.057 (0.116) 0.003 299	0.023* (0.013) 0.944 299	0.012 (0.012) 0.962 299	0.014 (0.012) 0.962 299	0.079 (0.127) 0.006 299	0.021 (0.015) 0.936 299	0.014 (0.015) 0.954 299	0.014 (0.015) 0.954 299	0.064 (0.118) 0.004 299	0.014 (0.013) 0.931 299	0.014 (0.013) 0.931 299	0.009 (0.014) 0.949 299
Relative max–mean	0.056 (0.115) 0.003 299	0.023* (0.013) 0.944 299	0.011 (0.012) 0.962 299	0.011 (0.012) 0.962 299	0.077 (0.125) 0.006 299	0.021 (0.015) 0.936 299	0.013 (0.015) 0.954 299	0.013 (0.012) 0.931 299	0.062 (0.116) 0.004 299	0.013 (0.012) 0.931 299	0.013 (0.012) 0.931 299	0.008 (0.014) 0.949 299
Relative mean–min	0.230*** (0.026) 0.053 299	0.100*** (0.018) 0.949 299	0.058*** (0.018) 0.963 299	0.058*** (0.018) 0.963 299	0.229*** (0.023) 0.052 299	0.089*** (0.016) 0.936 299	0.049*** (0.018) 0.955 299	0.049*** (0.018) 0.955 299	0.255*** (0.023) 0.065 299	0.112*** (0.018) 0.937 299	0.112*** (0.018) 0.937 299	0.072*** (0.075) 0.963 299
Top 10% share	0.948*** (0.022) 0.899 299	0.600*** (0.063) 0.962 299	0.529*** (0.064) 0.973 299	0.529*** (0.064) 0.973 299	0.909*** (0.031) 0.827 299	0.519*** (0.076) 0.949 299	0.455*** (0.079) 0.962 299	0.455*** (0.079) 0.962 299	0.927*** (0.027) 0.858 299	0.637*** (0.070) 0.951 299	0.637*** (0.070) 0.951 299	0.528*** (0.075) 0.963 299
Bottom 10% share	0.896*** (0.117) 0.803 299	0.564*** (0.146) 0.969 299	0.484*** (0.138) 0.978 299	0.484*** (0.138) 0.978 299	0.916*** (0.109) 0.839 299	0.659*** (0.147) 0.970 299	0.583*** (0.143) 0.977 299	0.583*** (0.143) 0.977 299	0.927*** (0.100) 0.860 299	0.673*** (0.152) 0.967 299	0.673*** (0.152) 0.967 299	0.610*** (0.152) 0.974 299

Notes. Each column presents the results of the corresponding pairs of classical and our inequality measures according to the estimated equations (2) without fixed effects, (3) with country fixed effects, and (4) with country and year fixed effects, respectively. Robust standard errors in parentheses. ***, **, and * denote $p < 0.01$, $p < 0.05$, and $p < 0.1$, respectively. R-squared is in italic.

The linear associations between the absolute indices are surprisingly high; see Table 1. Values very close to one are observed in all three models between all the absolute standard measures and the absolute mean-min, top 10% gap, and bottom 10% gap indices; the only exception is the correlation coefficient with the Kolm index with parameter value 10^{-4} , reported in the eighth column of the table. The results improve with the introduction of country and year fixed effects.

For the relative case in all models (Table 2), the value closest to one is observed for the top 10% income share index, followed by the bottom 10% income share index. The remaining indices do not perform that well, especially when year and country fixed effects are incorporated. As we wrote earlier, this might not be a concern if the purpose is to use these indices as proxies for international and inter-temporal comparisons. It is worth noting that the values of the coefficient of determination (R -squared) are always above 0.9 as soon as the country dummies are introduced in the model.

As expected, the full consideration of the highest incomes (see Tables 3 and 4) has an effect on the results, lowering the correlation coefficients between the standard measures and the coarser indices, apart from the two that exclude the maximum income from their definitions (the top 10% gap and share and the bottom 10% gap and share). The absolute and relative mean-min indices perform well, especially in the absolute case with the absolute Gini coefficient and the two versions of the Kolm index.

One explanation behind our results could be that the income distributions of the countries in our sample are broadly similar among themselves. To better explore this issue, we follow the suggestion of an anonymous referee, and replace the income distributions of the countries by randomly generating the distributions via the Stata package of van Kerm (2017). An alphabetical list of the LIS original countries by years is provided first, including 322 cases (of which we analyze 299 in our regressions). We simulate the first 110 as generalized beta of the second kind (GB2) distributions with the usual parameters a, b, p, q . The following 110 are generated as Singh-Maddala distributions with parameters a, b, q . For the remainder, we employ Dagum distributions with parameters a, b, p .

In our simulations, the parameters are random draws from uniform distributions, U , with different bounds. In particular, for a we use $U = (1.5, 7)$, for p the distribution is $U = (0.5, 25)$, and for q we employ $U = (0.6, 5)$. The bands for b differ between distributions and are $U = (90, 400)$ but for the Dagum were $U = (1, 7)$. We arbitrarily choose these bounds around those that are commonly found for income distributions (see, e.g. Chotikapanich *et al.*, 2018, for the case of the generalized beta of the second kind and the help file of the van Kerm, 2017, package). From each GB2 distribution, we then extract randomly incomes of 10,000 individuals and use these for the calculation of the inequality indices. The resulting income distributions are quite varied among themselves. To give a rough and concise idea of the main differences between the simulated distributions, we report some of the most well-known reference points: the Gini coefficient ranges between 0.04 and 0.91; the median and mean have values, respectively, between 1.21/1.44 and 3407.78/35069.55; the absolute range varies between 8.23 and $7.46e+07$.

We run our regressions on this simulated sample, without top/bottom coding, and report the results in Tables 5 and 6. The linear associations are even higher

TABLE 5
STANDARD ABSOLUTE INEQUALITY MEASURES AND OUR ABSOLUTE INEQUALITY MEASURES (SIMULATED SAMPLES)

Dependent variable	Absolute Gini				Variance				Kolm (parameter 10 ⁻⁴)				Kolm (parameter 5·10 ⁻⁴)				
	(2)	(3)	(4)	(3)	(2)	(3)	(4)	(3)	(2)	(3)	(4)	(2)	(3)	(4)	(2)	(3)	(4)
Absolute range	0.966*** (0.054)	0.954*** (0.058)	0.956*** (0.058)	0.964*** (0.109)	0.952*** (0.108)	0.951*** (0.111)	0.974*** (0.095)	0.964*** (0.095)	0.978*** (0.084)	0.964*** (0.097)	0.964*** (0.097)	0.978*** (0.084)	0.964*** (0.084)	0.964*** (0.084)	0.978*** (0.084)	0.964*** (0.084)	0.964*** (0.086)
Obs.	0.933 299	0.949 299	0.957 299	0.929 299	0.937 299	0.940 299	0.949 299	0.954 299	0.956 299	0.956 299	0.956 299	0.956 299	0.961 299	0.961 299	0.956 299	0.961 299	0.963 299
Absolute max–mean	0.966*** (0.055)	0.954*** (0.058)	0.956*** (0.059)	0.964*** (0.109)	0.952*** (0.108)	0.951*** (0.111)	0.974*** (0.095)	0.964*** (0.095)	0.977*** (0.084)	0.964*** (0.097)	0.964*** (0.097)	0.977*** (0.084)	0.969*** (0.084)	0.969*** (0.084)	0.977*** (0.084)	0.969*** (0.084)	0.969*** (0.086)
Obs.	0.932 299	0.949 299	0.956 299	0.929 299	0.937 299	0.940 299	0.948 299	0.954 299	0.955 299	0.956 299	0.956 299	0.955 299	0.960 299	0.960 299	0.955 299	0.960 299	0.963 299
Absolute mean–min	0.974*** (0.087)	1.021*** (0.057)	1.029*** (0.053)	0.880*** (0.210)	0.943*** (0.164)	0.960*** (0.157)	0.901*** (0.187)	0.967*** (0.141)	0.920*** (0.164)	0.984*** (0.133)	0.984*** (0.133)	0.920*** (0.164)	0.988*** (0.119)	0.988*** (0.119)	0.920*** (0.164)	0.988*** (0.119)	1.003*** (0.111)
Obs.	0.948 299	0.974 299	0.978 299	0.774 299	0.852 299	0.874 299	0.872 299	0.885 299	0.847 299	0.903 299	0.903 299	0.847 299	0.913 299	0.913 299	0.847 299	0.913 299	0.928 299
Top 10% gap	0.955*** (0.139)	0.977*** (0.118)	0.994*** (0.113)	0.829*** (0.274)	0.862*** (0.238)	0.890*** (0.232)	0.858*** (0.248)	0.894*** (0.212)	0.885*** (0.221)	0.922*** (0.204)	0.922*** (0.204)	0.885*** (0.221)	0.923*** (0.186)	0.923*** (0.186)	0.885*** (0.221)	0.923*** (0.186)	0.950*** (0.178)
Obs.	0.913 299	0.936 299	0.946 299	0.688 299	0.764 299	0.796 299	0.736 299	0.805 299	0.784 299	0.835 299	0.835 299	0.784 299	0.845 299	0.845 299	0.784 299	0.845 299	0.870 299
Bottom 10% gap	0.996*** (0.030)	1.005*** (0.023)	1.008*** (0.021)	0.927*** (0.143)	0.947*** (0.120)	0.957*** (0.115)	0.946*** (0.121)	0.968*** (0.098)	0.962*** (0.098)	0.978*** (0.092)	0.978*** (0.092)	0.962*** (0.098)	0.985*** (0.076)	0.985*** (0.076)	0.962*** (0.098)	0.985*** (0.076)	0.994*** (0.072)
Obs.	0.992 299	0.995 299	0.996 299	0.859 299	0.897 299	0.912 299	0.894 299	0.927 299	0.925 299	0.939 299	0.939 299	0.925 299	0.952 299	0.952 299	0.925 299	0.952 299	0.960 299

Notes. Each column presents the results of the corresponding pairs of classical and our inequality measures according to the estimated equations (2) without fixed effects, (3) with country fixed effects, and (4) with country and year fixed effects, respectively. Robust standard errors in parentheses. ***, **, and * denote $p < 0.01$, $p < 0.05$, and $p < 0.1$, respectively. R-squared is in italic.

TABLE 6
STANDARD RELATIVE INEQUALITY MEASURES AND OUR RELATIVE INEQUALITY MEASURES (SIMULATED SAMPLES)

Dependent variables	Gini				Atkinson (parameter 0.5)				Atkinson (parameter 1)			
	(2)	(3)	(4)	(4)	(2)	(3)	(3)	(4)	(2)	(3)	(3)	(4)
Relative range	0.383*** (0.121)	0.402*** (0.105)	0.408*** (0.109)	0.496*** (0.152)	0.523*** (0.131)	0.524*** (0.136)	0.476*** (0.139)	0.503*** (0.120)	0.507*** (0.125)			
Obs.	0.146 299	0.333 299	0.430 299	0.246 299	0.382 299	0.468 299	0.226 299	0.366 299	0.455 299			
Relative max–mean	0.684*** (0.107)	0.687*** (0.101)	0.725*** (0.127)	0.824*** (0.110)	0.829*** (0.111)	0.875*** (0.140)	0.766*** (0.113)	0.775*** (0.113)	0.820*** (0.142)			
Obs.	0.468 299	0.596 299	0.659 299	0.679 299	0.735 299	0.778 299	0.586 299	0.664 299	0.716 299			
Relative mean–min	0.418*** (0.160)	0.445*** (0.138)	0.450*** (0.144)	0.484*** (0.155)	0.522*** (0.139)	0.525*** (0.147)	0.498*** (0.170)	0.536*** (0.150)	0.537*** (0.157)			
Obs.	0.174 299	0.362 299	0.456 299	0.234 299	0.383 299	0.468 299	0.248 299	0.394 299	0.479 299			
Top 10% share	0.936*** (0.031)	0.951*** (0.035)	0.955*** (0.040)	0.801*** (0.052)	0.846*** (0.058)	0.856*** (0.066)	0.862*** (0.044)	0.902*** (0.049)	0.912*** (0.056)			
Obs.	0.877 299	0.902 299	0.913 299	0.642 299	0.719 299	0.751 299	0.743 299	0.794 299	0.818 299			
Bottom 10% share	0.901*** (0.102)	0.885*** (0.079)	0.866*** (0.081)	0.948*** (0.083)	0.952*** (0.067)	0.937*** (0.071)	0.956*** (0.083)	0.957*** (0.065)	0.941*** (0.067)			
Obs.	0.811 299	0.863 299	0.884 299	0.899 299	0.929 299	0.940 299	0.914 299	0.937 299	0.947 299			

Notes: Each column presents the results of the corresponding pairs of classical and our inequality measures according to the estimated equations (2) without fixed effects, (3) with country fixed effects, and (4) with country and year fixed effects, respectively. Robust standard errors in parentheses. ***, **, and * denote $p < 0.01$, $p < 0.05$, and $p < 0.1$, respectively. R-squared is in italic.

than before in all cases. For the absolute regression models, all coefficients are very close to the reference value of one. When we consider relative indices, the positive linear associations are considerably higher than in the LIS sample and always significant.

This simple simulation exercise leads us to conclude this empirical application offering some confidence to scholars who do not have access to individual and household micro-data: the crude indices they are able to compute are very good proxies of the ideal inequality measures.

5. CONCLUDING REMARKS

In this paper, we characterize some inequality measures that are based on simple summary statistics such as the minimum, the maximum, and the mean of an income distribution. Although most of these indices are well known, there do not appear to be any axiomatizations available. Our theoretical results are supplemented with an empirical analysis that is intended to show that there may be more to our contribution than merely filling a gap in the literature. Especially in the case of some absolute measures, it turns out that there are some strong correlations between these indices and inequality orderings that are of a more complex nature. This latter observation, along with Leigh's (2007) analysis, suggests that there is a surprisingly high level of agreement across indices when it comes to practical applications.

These findings are particularly reassuring for applications to countries and time periods where detailed information on the individual and household incomes is not available, such as those studies on the long-run and historical trends of inequality between and within countries.

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SUPPORTING INFORMATION

Additional supporting information may be found in the online version of this article at the publisher's web site:

Appendix A: Proofs of the Theorems

Appendix B: Independence of the Axioms