Review of Income and Wealth
Series 64, Number 3, September 2018
DOI: 10.1111/roiw.12300

THE MEASUREMENT OF CAPITAL: RETRIEVING INITIAL VALUES FROM PANEL DATA

BY XI CHEN* and TATIANA PLOTNIKOVA

ANEC/STATEC

Abstract

A common problem with micro-level analysis is that capital stock data is missing. Typically, a feasible measure of capital is calculated by accumulating investment flows from an initial value of the capital stock. As the time dimension of most disaggregated data is rather short, the choice of this initial value can have significant effects on the resulting capital estimates. Most empirical studies impute the initial value using a single arbitrary proxy. In this paper, we propose a panel data framework that assigns weighting coefficients to multiple proxy variables. We conduct a series of Monte Carlo experiments to test the performance of the proposed method and apply the method to a U.S. manufacturing dataset. The results suggest that our method improves the approximation of the capital stock and thus in turn reduces the bias in the production function estimation.

JEL Codes: E22, C18, C80

Keywords: capital stock measurement, production function estimation, Monte Carlo simulation, non-linear regression

1. INTRODUCTION

Capital measurement is an essential component of economic research. Although the notion of capital input appears frequently in the fields of growth accounting and production analysis, the questions of what the capital input is and how to measure it still remain (Hicks, 1974). The difficulty of capital measurement is due to the ambiguity in the theoretical conceptualization and the lack of data for empirical investigation. The specific issues of capital measurement include: the evaluation of capital efficiency, retirement, and user cost (Jorgenson, 1963; Hulten, 1990; Triplett, 1998); the aggregation of heterogeneous capital (Diewert, 1980); and the relationship between capital stocks and capital services (Berndt and Fuss, 1986; Inklaar, 2010).[1] While these issues are widely addressed

Note: We would like to thank Bertrand Koebel, Jacques Mairesse, Eric Bartelsman, and two anonymous referees for very helpful comments. An earlier version of this paper was presented at the ICOAE 2014, NAPW 2014, and IAAE 2015 conferences. We would like to thank the participants of these conferences for useful feedback. This research has been financially supported by the Luxembourg National Research Fund (FNR) and co-founded under the Marie-Curie Actions of the European Commission (FP7-COFUND).

*Correspondence to: Xi Chen, ANEC/STATEC, B.P. 304, L-2013, Luxembourg (xi.chen@statec.etat.lu).

[1]A summary of these issues can be found in OECD (2009). Becker and Haltiwanger (2006) also provide a comprehensive description of the sources and methodology of capital construction on the micro and macro level for the U.S. data.

in the literature, the problem of missing initial capital has not received much attention. This paper is aimed at improving initial capital estimates for micro-level analysis. We first evaluate the implications of missing initial capital in the context of production analysis, and then propose a panel data framework that distributes the aggregate initial capital stocks across production units. The proposed method is applicable to data with a short time dimension and is free from the ad hoc assumptions of traditional methods.

In empirical production analysis, researchers study the process that combines different inputs to produce outputs. Typically, the main ingredients of the analysis are input variables, such as labor, capital, and materials. While some inputs, such as labor and materials, are often available in firms' records, the data for capital are generally missing. For instance, the labor service that embodies the labor input can be measured as total hours worked by the labor force. The direct analogy of labor service for physical capital is the capital service measured as total hours worked by machines. Unfortunately, the latter information is not available in most production datasets. Therefore, researchers first assume that the capital service is proportional to the productive capital stock, which in theory can be measured. Then, the productive capital stock is calculated by applying the perpetual inventory method (henceforth, PIM). Following this method, the current capital stock is calculated as the weighted sum of an initial capital stock and subsequent investment flows. Besides physical capital, the PIM is also applied to construct various types of stock variables, such as intangible capital (Cummins, 2005; Corrado *et al.*, 2009; Fukao *et al.*, 2009).

One implementation problem of the PIM is that the initial value of the capital stock is unobserved. The implications of this problem depend on (i) the availability of the investment data and (ii) the objective of the study. First, when long series of investment flows are available, the initial condition of the PIM can be set sufficiently far back in the past such that the initial capital stock is relatively small with respect to the sum of subsequent investment flows. In this case, the problem of the missing initial value may play a rather unimportant role. However, a production dataset that has long series of investments is rare in practice. At the aggregate level, only a few countries have historical data with several decades of investment records. At the disaggregated level, investment data are often limited to a few observation points. Second, if the research question is to understand the dynamic pattern of capital in a pure time series framework (where there is only one statistical unit), the problem of the missing initial value may have a limited impact. However, when cross-sectional variation is added to the econometric exercise (where there are several statistical units with different starting points), such as in a panel data regression, the distribution of the initial capital stock will have a significant influence on the estimation results.

In this paper, we focus on the missing initial capital stock in the PIM framework and the consequences of the missing initial value for the production analysis of disaggregated data. The common practice of approximating the initial capital stock in micro-level studies relies on proxy variables. In the case of physical capital, the most frequent approach is to initialize the PIM using the book value of fixed tangible assets (Olley and Pakes, 1996; Pavcnik, 2002; Levinsohn and Petrin, 2003; Foster *et al.*, 2016); other authors propose to use a production-related variable, such as labor demand, intermediate materials, energy consumption, or purchased services, as the proxy variable for the initial capital stock (Martin, 2002; Gilhooly, 2009). While some proxies, such as the book value, are used more often than others, there is no empirical evidence to conclude that any one of them is superior to the others. Thus, we propose a generalized method that avoids the arbitrary choice of a single proxy. In this framework, the approximation of the initial capital stock is based on multiple proxy variables, instead of a single variable as in the traditional approaches. A set of weighting coefficients is estimated and attributed to the corresponding proxy variables. These coefficients represent the importance of each proxy variable in the approximation. Thus, this method is data-driven rather than based on ad hoc assumptions.[2]

The Monte Carlo simulation is used to assess the performance of the proposed method. We find that our method is superior to the traditional approaches in two aspects. First, the estimates of the capital stock using the proposed method are more correlated with the true capital stock than those obtained from the single-proxy approach. Second, the estimated output elasticity with respect to capital in the production function is less biased when the proposed method is used to approximate the capital input. Besides the simulation study, we also apply the method to a U.S. manufacturing industries dataset and reach similar conclusions. Although the examples we use in the paper may seem to refer mainly to the physical capital, our method can also be applied to deal with the problem of a missing initial value of other quasi-fixed inputs, such as intangible capital.

The remainder of this paper is organized as follows: we first present the problem of missing initial capital in Section 2. In Section 3, we briefly review the existing approaches that deal with the missing initial value. Then, we propose a generalized method. The results of empirical studies based on the simulated data are reported in Section 4. The results based on the real-world data are reported in Section 5. Section 6 concludes.

2. The Initial Value Problem

Since investment flows are the main source of information on capital, a substantial part of the literature focuses on the question of how to convert investment flows into productive capital stocks using the PIM. The specification of this calculation depends on the initial value, the choice of age–efficiency profiles, the retirement pattern of different assets, and the specification of the aggregation function across assets (OECD, 2009). In this paper, we focus solely on the problem of the missing initial value. Considering that this problem is not widely discussed in the literature, we start by formulating it in the PIM framework. Then, in order to convince the reader of the importance of the issue, we determine the potential bias of the production function estimation due to the mis-measurement of the initial capital stock.

[2]The three types of information that are used in the initial capital approximation are: firm accounting data, production data, and other indirect information on capital, which includes, for example, insurance records, property records, and share valuation. While in this paper we consider only the two first categories, other types of information can be easily incorporated into the method.

2.1. *The PIM Framework*

For simplicity, we only consider the case with a single capital asset. Nevertheless, the proposed method can be easily applied to the case of multiple assets. The empirical application of the multiple assets model will depend on the availability of corresponding disaggregated investment data. We use two classical assumptions to frame the study. First, the productive service of an individual capital asset is assumed to be proportional to the corresponding productive stock.[3] Second, the capital accumulation follows a geometric age–efficiency profile. While other types of age–efficiency profile are proposed in the literature, the geometric profile is the most commonly used one, because of its great simplicity.[4] Given these assumptions, the current capital stock (K_{it}^*) is the sum of an initial capital stock (K_{i0}^*) and investment flows ($I_{it}, I_{it-1}, .., I_{i1}$):

$$K_{it}^* = I_{it} + (1-\delta)I_{it-1} + \ldots + (1-\delta)^{t-1} I_{i1} + (1-\delta)^t K_{i0}^*, \tag{1}$$

where $i=1, \ldots, N$ indexes statistical units; $t=1, \ldots, T$ indexes time, and δ denotes the depreciation rate. The initial value of capital stock is often referred to as the benchmark capital, which is not directly observed in the data.

If investment series are available for a very long time span and the depreciation rate of the capital asset is positive, the measurement error of the initial capital may become rather insignificant over time. However, the time span of most micro-level data is short, not to mention the fact that some statistical units in such data are not observed over several sequential years.

Several types of approximation methods for the initial capital stock are proposed in the literature (see Section 3.1). However, the approximation may be subject to errors. Thus, in the following subsection we evaluate the bias of the estimated technology parameters due to the approximation error. In particular, we focus on the output elasticity with respect to capital.

2.2. *Bias in the Production Function Estimation*

We rewrite equation (1):

$$K_{it}^* = SI_{it} + (1-\delta)^t K_{i0}^*, \tag{2}$$

where $SI_{it} \equiv I_{it} + (1-\delta)I_{it-1} + \ldots + (1-\delta)^{t-1} I_{i1}$ is the sum of accumulated investment flows in the period t. In practice, K_{i0}^* is unobserved and therefore approximated by K_{i0}, which is subject to a multiplicative error:

$$K_{i0} = K_{i0}^* \eta_{i0}, \tag{3}$$

where η_{i0} is the classical measurement error with the expected value of one. This term represents the difference between the approximated initial capital stock and

[3]The focus of this analysis is on the individual asset: thus, the proportionality is at the asset level. For the case of multiple assets, Jorgenson (1963) and Jorgenson and Griliches (1967) have proposed measurements of aggregate capital service that take asset heterogeneity into account.

[4]The distinction between different profiles has been largely discussed in the literature (Hulten, 1990).

its true value. To illustrate the consequence of mis-measured initial capital, we examine the potential bias of a Cobb–Douglas production function estimation:

$$\log Y_{it}=\beta_l \log L_{it}+\beta_k \log K_{it}^{*}+\xi_{it}, \tag{4}$$

where Y_{it} is the value-added output, L_{it} is the labor input, and ξ_{it} is an i.i.d. error term. The parameters β_l and β_k are the output elasticity parameters with respect to labor and capital, respectively. Since its true value, K_{i0}^{*}, is not observed, the measurement of capital input K_{it} is generated using the PIM with the approximated initial value, K_{i0}.

We focus on the bias of the estimated β_k due to the measurement error η_{i0}. The regression equation (4) can be rewritten as

$$\log Y_{it}=\beta_l \log L_{it}+\beta_k \log K_{it}+\beta_k(\log K_{it}^{*}-\log K_{it})+\xi_{it}. \tag{5}$$

Regressing $\log Y_{it}$ on the observed variables $\log L_{it}$ and $\log K_{it}$ will yield a biased estimator of β_k because of the omitted term $\beta_k(\log K_{it}^{*}-\log K_{it})$. For a given period of regression (t), the cross-sectional estimation of β_k is

$$\hat{\beta}_k=\beta_k\left(1+\frac{\sigma_{kk^*}-\sigma_k^2}{\sigma_k^2}\right), \tag{6}$$

where σ_{kk^*} denotes the covariance between $\log K_{it}$ and $\log K_{it}^{*}$; σ_k^2 is the variance of $\log K_{it}$. In the best-case scenario where K_{it}^{*} and K_{it} are identical, the estimator of β_k is unbiased. In contrast, when K_{it}^{*} and K_{it} are weakly correlated, this estimator is biased downwards.

Now, we relate this bias to the initial measurement error in equation (3). Equation (6) shows that the bias is due to the difference between $\log K_{it}^{*}$ and $\log K_{it}$. Using the PIM, we can rewrite this difference in terms of the initial approximation error, η_{i0}:

$$\log K_{it}^{*}-\log K_{it}=\log\left(1+(1-\delta)^t\frac{K_{i0}^{*}}{SI_{it}}\right)-\log\left(1+(1-\delta)^t\frac{K_{i0}^{*}\eta_{i0}}{SI_{it}}\right). \tag{7}$$

The relationship between the measurement error in the period t and the error in the initial period is highly non-linear. Thus, there is no simple tractable expression that relates the estimation bias to η_{i0}. However, for illustration purposes, we consider a special case in which the period of regression t is relatively far from the initial period. Then, the depreciated initial capital $(1-\delta)^t K_{i0}^{*}$ is relatively small compared to the weighted sum of investments SI_{it}. In this case, we can use a first-order Taylor expansion to linearize equation (7):

$$\log K_{it}^{*}-\log K_{it}\simeq(1-\delta)^t D_{it}, \tag{8}$$

where the term $D_{it}\equiv K_{i0}^{*}(1-\eta_{i0})/SI_{it}$ can be viewed as the relative magnitude of the approximation error. Using equation (8), we arrive at the following expression:

$$\hat{\beta}_k = \beta_k \left(1 + \frac{(1-\delta)^t \sigma_{kd}}{\sigma_k^2} \right), \tag{9}$$

where σ_{kd} denotes the covariance between $\log K_{it}$ and D_{it}. Similar to equation (6), the estimator $\hat{\beta}_k$ is downward biased and depends on three factors: (i) the time span between the period of regression and the initial period; (ii) the depreciation rate δ; and (iii) the relative magnitude of the approximation error and its correlation with the regressor $\log K_{it}$. In a given dataset, the two first points are beyond the control of researchers. We can only influence the last point. Thus, the next section presents a generalized method that can reduce the approximation error. We also note that the expression of bias given in equation (9) is obtained from a Taylor expansion by assuming that $|(1-\delta)^t K_{i0}^* / SI_{it}|$ is smaller than one.[5] When the ratio $|(1-\delta)^t K_{i0}^* / SI_{it}|$ is large, the Taylor linearization cannot be applied. In this case, numerical methods should be used to calculate the bias, as is done in Section 4.

For illustration, we evaluate the implication of missing initial capital in the context of estimating a production function. However, the choice of the initial capital values also affects more direct applications of capital measurements. For example, the capital–output ratio is often used in development economics to explain the growth rate (i.e. the Harrod–Domar model). This indicator may not only be used at the macro level but also at the sectoral level within an economy. In the latter case, the initial value is of great importance in cross-unit comparisons.

3. The Treatment of the Missing Initial Capital

A range of methods have been used to approximate the initial value of capital stock. The ideas behind these "traditional" approaches can be categorized into three classes: (i) the direct use of book values; (ii) the use of production-related variables; and (iii) the use of assumptions on past investment flows. In this section, we give a brief review of these approaches. Then, we propose a generalized framework that combines them.

3.1. *The Traditional Approaches*

Many datasets contain book values of capital assets, which can be used to approximate the initial capital. A large number of empirical studies use this approach, including Olley and Pakes (1996) for the Longitudinal Research Database, Liu (1993), Pavcnik (2002), and Levinsohn and Petrin (2003) for Chilean data, and Foster *et al.* (2016) for the Annual Survey of Manufactures data, among others. While many authors find the book value a reliable proxy, the main concern about this approach is that the book value does not necessarily capture the productive capital stock but, rather, reflects firms' accounting practices for fiscal purposes, such as the accelerated depreciation of assets. In some cases, using book values as a proxy for productive capital may be more

[5]For example, if the depreciated initial capital $(1-\delta)^t K_{i0}^*$ represents 20 percent of recently added investment, SI_{it}, the first-order approximation gives $\log(1.2) \simeq 0.2$, which is 9 percent off the true value of 0.182.

problematic if accounting practices differ among statistical units (e.g. different industries or regions), possibly due to varying capital composition or different accounting regulations.

An alternative approach is to use proxies that may be strongly correlated with productive capital stocks, such as worked hours, intermediate materials, or energy consumption. The idea is to use these proxies for allocating an aggregate capital stock among production units. The aggregate value of capital can be either obtained from an additional source or estimated. For instance, Martin (2002) uses the average material demand over the total industry demand as proxy (henceforth, this is referred to as the shares of material). The underlying assumption is that the share of capital is proportional to the share of the proxy. The main issue with this approach is the arbitrary choice of the proxy, which may result in quite different estimates of capital stocks.

The last category includes the methods that have emerged from macro-level studies, which assume that the economy is in the long-term equilibrium.[6] These methods can be applied to micro-level data if their assumptions are considered realistic. They rely on the available time series of economic variables, such as output and investment, and an assumption on the growth rate of the latter. For example, following the seminal work of Harberger (1978), in a firm-level study, Hall and Mairesse (1995) assume that past investment flows grow at a constant growth rate, g. Using this assumption together with the PIM, we can obtain a simple approximation of initial capital: $K_{i0}=I_{i1}/(g+\delta)$.[7] Besides the assumption on the past investment pattern, the implementation of this method requires a guess about the value of past investment growth. Given the common problem of lumpy and short investment data, the estimation of g is very difficult, if not impossible.

3.2. *A Generalized Approach*

In the previous subsection, we presented three types of methods that have been used in the literature to deal with the missing initial capital problem. Two approaches rely on additional proxies; that is, book values or production-related variables. The last approach is based on the additional assumption of the past investment pattern. The implementation question faced by researchers is the choice among different methods, which are likely to produce diverging results. In this subsection, our aim is to develop a panel data framework that uses different sources of information simultaneously, and therefore is free from arbitrary choice.

We begin by describing the traditional approaches based on a single proxy. Formally, given a proxy variable, for example firms' energy consumption (E_{i0}), the initial capital stock can be calculated as

[6]For the application of this approach to macro data and comparing the steady-state approach to an assumed constant capital–output ratio, see Feenstra *et al.* (2015). Nehru and Dhareshwar (1993) provide an overview of several approaches to solving the problem of the initial capital stock for aggregated data.

[7]An extended version that takes the age of firms into account is proposed in Raknerud *et al.* (2007).

$$K_{i0} = \frac{E_{i0}}{E^{j}_{.0}} K^{j}_{.0}, \tag{10}$$

where $E^{j}_{.0} \equiv \sum_i E_{i0}$ is the (observed) total value of the proxy variable within a group of production units j. $K^{j}_{.0} \equiv \sum_i K_{i0}$ is the total value of the capital stock in the group j. This method redistributes the aggregate capital stock across disaggregated units according to the share of energy consumption. We can interpret the direct use of book values (B_{i0}) in the same manner as in equation (10):

$$K_{i0} = \frac{B_{i0}}{B^{j}_{.0}} K^{j}_{.0}. \tag{11}$$

Note that if the aggregate values are equal, $B^{j}_{.0} = K^{j}_{.0}$, we obtain $K_{i0} = B_{i0}$, which is equivalent to approximating the initial capital directly with the book value.

The traditional approaches described in equations (10) and (11) rely on a single proxy, by assuming that the chosen proxy is informative. A more general method is to jointly use multiple proxies. Suppose that there are R shares of proxies $Z_{i0} = (Z^1_{i0}, Z^2_{i0}, \ldots, Z^R_{i0})$. A natural extension of the traditional methods is

$$K_{i0} = (Z^1_{i0})^{\alpha_1} \cdot (Z^2_{i0})^{\alpha_2} \ \ldots \ (Z^R_{i0})^{\alpha_R} \cdot K^{j}_{.0} \cdot \varepsilon_{i0} = \prod_r^R (Z^r_{i0})^{\alpha_r} \cdot K^{j}_{.0} \cdot \varepsilon_{i0}, \tag{12}$$

with $\alpha \equiv (\alpha_1, \alpha_2, \ldots, \alpha_R) \geq 0$ representing the weighting coefficients of each corresponding proxy, and $\sum_r^R \alpha_r = 1$. A higher coefficient α_r assigns higher importance to the corresponding proxy in the construction of the initial capital stock. The individual deviation from the average approximation is captured by ε_{i0}, which is assumed to be an i.i.d. error term with $E[\varepsilon_{i0} | Z_{i0}, K^{j}_{.0}] = 1$. Equation (12) extends the single-proxy approach in two aspects. First, this setting allows us to use multiple proxies with weighting coefficients. Second, the error term allows for imperfect approximation. For example, suppose that we have two proxy variables, book values and energy consumption: the generalized framework can then be written as

$$K_{i0} = \left(\frac{B_{i0}}{B^{j}_{.0}} \right)^{\alpha_1} \cdot \left(\frac{E_{i0}}{E^{j}_{.0}} \right)^{\alpha_2} \cdot K^{j}_{.0} \cdot \varepsilon_{i0}.$$

When the variance of ε_{i0} is zero, the generalized framework coincides with equation (11) by setting $\alpha_1 = 1$ and $\alpha_2 = 0$, where only the book value is considered. The generalized framework is reduced to equation (10) when $\alpha_1 = 0$ and $\alpha_2 = 1$, where only the energy consumption is used.

The implementation of both the traditional and the generalized approaches requires the knowledge of the total sample capital stock for computing $K^{j}_{.0}$. Depending on the type of data at hand, different estimation strategies can be applied to obtain its value. In the ideal case, where the micro-level dataset has the full coverage of the economy, the official figures provided by national statistical offices on the aggregate capital can be directly used as the total sample value. However, a more common type of micro-level data are those collected as a part of national or sectoral survey programs, and covering only a part of the economy. Thus, one needs to estimate the capital shares of observed units in the whole economy using, for example, observations of turnover and employment. Since in this case the aggregate capital is estimated, approximation errors may affect the aggregation value, but they do not influence the distribution of capital across production units and cross-unit comparisons. At the firm level, datasets may contain the book values of capital stock. Therefore, one can assume that the total sample value is equivalent to the sum of the book values and use the generalized approach to estimate the initial capital value. In this case, the main reason for deviating from the traditional approach based on individual book values is that the generalized approach allows us to include additional proxies in order to improve the estimation results.

3.3. *The Estimation of the Weighting Coefficients*

In practice, the generalized framework given in equation (12) is useful only if the weighting coefficients are known or can be identified from the data. We could set the weighting coefficients according to some ad hoc assumptions. For example, set $\alpha_1 = \alpha_2 = \ldots = \alpha_R = 1/R$ by assuming that proxies contribute equally to the share of capital stock. Alternatively, we propose to estimate the weighting coefficients based on an optimality criterion. For this purpose, we need an additional assumption that the weighting coefficients in equation (12) are stable in the sample:

$$K_{is} = \prod_{r}^{R} (Z_{is}^{r})^{\alpha_r} \cdot K_{.s}^{j} \cdot \varepsilon_{is}, \tag{13}$$

where ε_{is} is an i.i.d. error term with $E[\varepsilon_{is} | Z_{is}, K_{.s}^{j}] = 1$, $\forall s \in [0, t]$.

Given that the inflows and outflows of capital stocks are fully characterized by the PIM, we have the following relationship:

$$SI_{it,s} = K_{it} - (1-\delta)^{t-s} K_{is}, \tag{14}$$

where $SI_{it,s} \equiv I_{it} + (1-\delta) I_{it-1} + \ldots + (1-\delta)^{t-s-1} I_{is+1}$ is the sum of the accumulated investment flows between the period t and s.[8] Combining equations (13) and (14), we obtain an empirical model that allows us to estimate α:

$$SI_{it,s} = \prod_{r}^{R} (Z_{it}^{r})^{\alpha_r} \cdot K_{.t}^{j} \cdot \varepsilon_{it} - (1-\delta)^{t-s} \prod_{r}^{R} (Z_{is}^{r})^{\alpha_r} \cdot K_{.s}^{j} \cdot \varepsilon_{is}.$$

The estimation model can be rewritten in the additive form:

[8]For instance, when $s = t-1$, $SI_{it,s} = I_{it}$, and when $s = 0$, $SI_{it,s} = SI_{it} = I_{it} + (1-\delta) I_{it-1} + \ldots + (1-\delta)^{t-1} I_{i1}$.

$$SI_{it,s}=\prod_{r}^{R}(Z_{it}^{r})^{\alpha_r}\cdot K_{.t}^{j}-(1-\delta)^{t-s}\prod_{r}^{R}(Z_{is}^{r})^{\alpha_r}\cdot K_{.s}^{j}+e_{it,s}, \tag{15}$$

where the composite error term $e_{it,s}$ is defined as

$$e_{it,s}\equiv\prod_{r}^{R}(Z_{it}^{r})^{\alpha_r}\cdot K_{.t}^{j}\cdot(\varepsilon_{it}-1)-(1-\delta)^{t-s}\prod_{r}^{R}(Z_{is}^{r})^{\alpha_r}\cdot K_{.s}^{j}\cdot(\varepsilon_{is}-1). \tag{16}$$

Thus, the estimated weighting coefficients ($\hat{\alpha}$) can be obtained using a non-linear least squares (NLS) estimator, which minimizes the sum of squared composite residuals given in equation (16).[9] Given the estimated weighing coefficients, the distribution of the initial capital stock can be retrieved from equation (12). The remaining capital stocks are calculated using the PIM with an estimate of the depreciation rate. Note that in our regression model in equations (15) and (16), the additive error term $e_{it,s}$ is heteroskedastic. Therefore, the standard errors of NLS estimates should be calculated using a heteroskedastic consistent variance estimator. Similar to linear models, weighted NLS and feasible generalized NLS may provide efficiency gains in this case. In addition, a decomposition of the estimated variance of $e_{it,s}$ can provide information on the quality of the approximation.

In practice, the implementation of the proposed method depends on (i) the choice of proxy variables; (ii) the choice of t and s in equation (14); and (iii) the assumption on the investment pattern. First, the previous literature suggests a number of proxy variables, such as book value, labor, materials, and energy, among others. According to the production theory, the capital stock is a quasi-fixed input and cannot be adjusted every period, unlike more flexible inputs, such as materials, energy, and labor. In order to use the variable inputs to approximate a quasi-fixed input, we can average the proxies over an arbitrary period. For instance, we can use the moving average of variable inputs. Depending on the number of periods in the dataset, we need to determine a reasonable number of periods in order to construct moving averages.

The second implementation issue is the specification of equation (14). For example, we can set $s=t-1$; then equation (14) becomes $I_{it}=K_{it}-(1-\delta)K_{it-1}$, for $t=1,\ldots,T$. In this case, we use every period of the sample to estimate the weighting coefficients. Alternatively, we may only use either the two first periods of the sample with $I_{i1}=K_{i1}-(1-\delta)K_{i0}$, or the first and the last period with $SI_{iT}=K_{iT}-(1-\delta)^{T}K_{i0}$. The sole requirement is that at least two periods of observations are available in the dataset. The flexibility of equation (14) is an advantage when dealing with disaggregated data, in which the investment variable is not always continuous over time and has many zeros. In these cases, summing up an arbitrary length $(t-s)$ of investment series could facilitate the estimation of α. The choice of this length depends on the availability of data.

[9]The essential consistency condition of this NLS estimator is as follows: $E[e_{it,s}|Z_{it},K_{.t}^{j},Z_{is},K_{.s}^{j}]=0$. Given the stochastic specification of error terms, the consistency condition is satisfied because $\prod_{r}^{R}(Z_{it}^{r})^{\alpha_r}\cdot K_{.t}^{j}\cdot E[\varepsilon_{it}-1|Z_{it},K_{.t}^{j}]-(1-\delta)^{t-s}\prod_{r}^{R}(Z_{is}^{r})^{\alpha_r}\cdot K_{.s}^{j}\cdot E[\varepsilon_{is}-1|Z_{is},K_{.s}^{j}]=0$.

Third, the generalized method can be also implemented together with an additional assumption on past investments to simplify the estimation procedure. Under the classical assumption that past investments grew at a constant rate, g (Harberger, 1978; Hall and Mairesse, 1995), we have $I_{i1}=(g+\delta)K_{i0}$. Substituting equation (12) into this equation yields

$$(17) \qquad I_{i1}=(g+\delta)\prod_{r}^{R}(Z_{i0}^{r})^{\alpha_r}\cdot K_{.0}^{j}\cdot\varepsilon_{i0},$$

and the corresponding model expressed in logarithmic terms is a simple linear regression model:

$$(18) \qquad \log(I_{i1}/K_{.0}^{j})=\log(g+\delta)+\sum_{r}^{R}\alpha_r\log Z_{i0}^{r}+\log\varepsilon_{i0}.$$

Although g and δ cannot be separately identified from the intercept term, the advantage of this approach is that an estimation of the past investment growth rate is not required. The weighting coefficients α can be consistently estimated by regressing $\log(I_{i1}/K_{.0}^{j})$ on $\log Z_{i0}^{r}$.

4. Monte Carlo Experiments

Since capital stocks are not directly observed, the proposed method is tested in this section on an artificial dataset. Monte Carlo experiments are used to illustrate the performance of the proposed method and to study the bias of the production function estimation. The generated dataset includes value-added output, capital stock, investment, and three proxy variables, (X^1, X^2, and X^3). We first use output, investment, and proxy variables (i.e. the series usually available to econometricians), to retrieve capital stocks and to estimate a production function. Then, the estimates based on the approximated capital are compared to their true values.

4.1. *The Design of the Experiment*

In this experiment, we assume that there are one thousand units in production for five periods (a balanced panel). The initial allocation of the capital stock, K_{i0}^{*}, is drawn exogenously from a log-normal distribution with a mean of 2 and a standard deviation of 1; that is, $K_{i0}^{*}\sim\log N(2,1)$. Assuming a fixed depreciation rate of $\delta=8$ percent, the capital formation in period t is given by equation (2). We consider a simple linear investment rule:

$$(19) \qquad I_{it+1}=1.5\,K_{it}^{*0.2}\,\varepsilon_{it}^{I},$$

where $\varepsilon_{it}^{I}\sim\log N(0,0.5)$ is an exogenous shock on the investment decision. Proxy variables are generated as follows:

$$X_{it}^{1} = 5\,K_{it}^{*0.3}\,\varepsilon_{it}^{1};\ X_{it}^{2} = 10\,K_{it}^{*0.8}\,\varepsilon_{it}^{2};\ X_{it}^{3} = 10\,K_{it}^{*0.8}\,\varepsilon_{it}^{3}, \tag{20}$$

where $\varepsilon_{it}^{1} \sim \log N(0,1)$, $\varepsilon_{it}^{2} \sim \log N(0,0.5)$, and $\varepsilon_{it}^{3} \sim \log N(0,0.5)$ represent exogenous shocks.

In order to keep the simulation as well as the estimation simple, we consider a Cobb–Douglas production function with an error term $u_{it} \sim \log N(0,1)$ and technology parameters $\beta_k = 0.4$ and $\beta_x = 0.6$:

$$Y_{it} = K_{it}^{*\beta_k}\,X_{it}^{1\beta_x}\,u_{it}, \tag{21}$$

where X_{it}^{1} appears in the production function, and X_{it}^{2} and X_{it}^{3} do not directly contribute to the production. Since the output is not affected by technical change or any other unobserved terms, the production function estimation in this Monte Carlo study does not suffer from endogeneity problems such as in Olley and Pakes (1996). This restriction allows us to focus only on the missing initial capital problem.

Six capital approximations are considered in this Monte Carlo experiment. In the most general case, the aggregate capital (total sample value, $K_{.0} = \sum_{i}^{N} K_{i0}$) is distributed across disaggregated units according to the following equation:

$$K_{i0} = \left(\frac{X_{i0}^{1}}{X_{.0}^{1}}\right)^{\alpha_1} \cdot \left(\frac{X_{i0}^{2}}{X_{.0}^{2}}\right)^{\alpha_2} \cdot \left(\frac{X_{i0}^{3}}{X_{.0}^{3}}\right)^{\alpha_3} \cdot K_{.0} \cdot \varepsilon_{i0}. \tag{22}$$

The measurements of the capital stock generated by the single-proxy approaches are denoted by K^1, K^2, and K^3. The superscript indicates the selected proxy; for instance, K^1 is based on the share of X^1 by assuming $\alpha_1 = 1$, $\alpha_2 = 0$, and $\alpha_3 = 0$. In addition, in the case of single-proxy approaches, we impose $\varepsilon_{i0} = 1$ for all $i = 1, \ldots, N$. The measurements of the capital stock based on the generalized approach with multiple proxies are denoted by K^{12}, K^{23}, and K^{123}. Two of them, K^{12} and K^{23}, use two proxies, while K^{123} uses all available proxies.[10] The weighing coefficients (α) in the generalized approach are obtained by estimating equation (15) with $s = t-1$.

The Monte Carlo experiment proceeds in the following steps. It is repeated $M = 200$ times with different seeds of a random number generator (i.e. seed = $12345 + m$ for $m = 1, \ldots, M$). For each replication, the statistics of interest (in Steps 4 and 5 below) are stored, and the results of these experiments are reported as averages over M replications.

- Step 1: Obtain the true initial values for the sample of N production units.
- Step 2: Generate the variables of interest from equations (2), (19), (20), and (21).
- Step 3: Estimate different initial values of capital, and use the PIM to generate the remaining values for T periods with fixed depreciation.

[10]One combination, K^{13}, is not reported in this section because its results are very similar to those of K^{12}.

TABLE 1
THE AVERAGE CORRELATION MATRIX OF THE SIMULATED DATA

	Y	X^1	X^2	X^3	K^*	I
Y	1	0.387	0.175	0.177	0.233	0.066
X^1		1	0.118	0.120	0.154	0.050
X^2			1	0.533	0.723	0.187
X^3				1	0.724	0.188
K^*					1	0.241
I						1

- Step 4: Compare the different estimates of capital in terms of their correlation with the true values.
- Step 5: Estimate the technology parameters in equation (21) and evaluate the estimation bias.

4.2. *Results*

The data are generated in such a way that X^2 and X^3 are highly correlated with the true capital, whereas X^1 is less correlated (see Table 1). Thus, our expectation is that the traditional approaches based on the proxy X^2 or X^3 will give good approximations of the initial capital stock. In practice, however, we may not always choose the best proxy, since the correlation between capital and proxy is unknown. The generalized method considers multiple proxies and attributes a weighting coefficient to each of them. In this way, the generalized method is not only free from the ad hoc choice of proxy, but it also uses more information for the approximation. In Table 2, we compare the different approaches in terms of the correlation with the true capital.

TABLE 2
THE AVERAGE CORRELATION OF DIFFERENT CAPITAL MEASURES WITH THE TRUE CAPITAL STOCK

(a) Fixed depreciation rate						
t	$\log K^1$	$\log K^2$	$\log K^3$	$\log K^{12}$	$\log K^{23}$	$\log K^{123}$
0	0.287	0.846	0.847	0.821	0.914	0.903
1	0.372	0.868	0.868	0.849	0.926	0.918
2	0.426	0.880	0.880	0.866	0.934	0.927
3	0.469	0.889	0.890	0.879	0.939	0.933
4	0.506	0.897	0.898	0.890	0.944	0.939
(b) Varying depreciation rate						
t	$\log K^1$	$\log K^2$	$\log K^3$	$\log K^{12}$	$\log K^{23}$	$\log K^{123}$
1	0.287	0.848	0.850	0.822	0.915	0.903
2	0.371	0.864	0.866	0.845	0.922	0.913
3	0.418	0.860	0.862	0.847	0.913	0.906
4	0.447	0.840	0.841	0.831	0.886	0.881
5	0.461	0.804	0.805	0.800	0.846	0.843

The correlation coefficients in Table 2 (a) rely on the assumption that the depreciation rate is known and fixed (8 percent in our data-generating process). Table 2 (b) shows the results when the depreciation rate is varying across units. In the latter case, we modify the data-generating process such that δ in equation (2) becomes a normally distributed random variable with a mean of 0.08 and a standard deviation of 0.1. From Table 2 (a), we see that the generalized method with two strong proxies $\log K^{23}$ in the initial period $t = 0$ is the best approximation (91.4 percent), followed by $\log K^{123}$ (90.3 percent). The measurements based on a single strong proxy, $\log K^2$ and $\log K^3$, are also highly correlated with the true capital stock. Their performance is even better than the multiple-proxy approach, $\log K^{12}$, where one strong and one weak proxy are used. $\log K^1$ is the approximation that is least correlated with the true values. The different methods converge as t increases, but the generalized method ($\log K^{23}$ and $\log K^{123}$) has a clear advantage for short panels. The correlation coefficients with varying depreciation are similar to those with fixed depreciation in the initial period. For the following periods, the correlation coefficients increase less rapidly and even decrease as t increases. This is due to the mis-specification of the depreciation rate in the PIM. However, the capital measurement based on the generalized method remains highly correlated with the true capital stock.

The results reported in Table 2 are generally in line with our expectations. However, two interesting points are worth discussing. First, we note that $\log K^2$ is more correlated with the true capital stock than $\log K^{12}$. This suggests that the traditional approach based on a single strong proxy (X^2) outperforms the generalized method, which includes an additional weak proxy (X^1). The comparison between $\log K^{23}$ and $\log K^{123}$ shows a similar result. Second, we also note that $\log K^{23}$ is more correlated with the true capital stock than $\log K^2$ and $\log K^3$. In this case, where the two proxies in question are equally strong, the generalized method outperforms the single-proxy approaches.[11] Combining these observations, we can say that the multiple-proxy approach is the preferred one if the additional proxies actually generate relevant information for the capital approximation. However, if the additional proxies are weakly correlated with the true capital, adding these proxies yields few benefits and in some cases may even hinder the performance of capital approximation. In practice, without information on the true data-generating process, the correlation between proxies and the true capital stock is unknown. One advantage of the proposed approach is that it can reveal the quality of proxies through the estimated weighting coefficients. The upper panels of Tables 3 (a) and (b) report the estimated weighting coefficients in the generalized method, as well as the underlying assumptions in the single-proxy approaches. In the case of K^{12}, the proxy X^2 receives the largest weighting coefficient. In the case of K^{23} and K^{123}, both proxies X^2 and X^3 receive the largest weights. Finally, the estimation indicates that X^1 is the weakest proxy when it comes to approximating the capital stock. These findings show that the generalized method assigns the weights correctly to each proxy and is robust to the mis-specification of depreciation rates in the PIM.

[11]We also test for the case where two proxies are equally weak, and find that the generalized method still outperforms the single-proxy approaches.

TABLE 3

Average Estimates of Weighting Coefficients and Technology Parameters

(a) Fixed depreciation rate

	K^*	K^1	K^2	K^3	K^{12}	K^{23}	K^{123}
Weighting coefficients:							
α_1		1	0	0	0.278	0	0.167
					(0.019)		(0.015)
α_2		0	1	0	0.722	0.499	0.416
					(0.019)	(0.024)	(0.020)
α_3		0	0	1	0	0.501	0.417
						(0.024)	(0.021)
OLS regression at $t = 1$:							
β_k	0.400	0.259	0.370	0.369	0.388	0.393	0.404
	(0.027)	(0.028)	(0.028)	(0.028)	(0.030)	(0.029)	(0.030)
β_x	0.603	0.729	0.625	0.626	0.617	0.607	0.602
	(0.026)	(0.026)	(0.027)	(0.027)	(0.027)	(0.027)	(0.027)
OLS regression at $t = 4$:							
β_k	0.398	0.351	0.388	0.389	0.398	0.398	0.403
	(0.029)	(0.029)	(0.028)	(0.029)	(0.029)	(0.029)	(0.029)
β_x	0.602	0.648	0.610	0.609	0.605	0.602	0.600
	(0.030)	(0.030)	(0.030)	(0.030)	(0.030)	(0.030)	(0.030)

(b) Varying depreciation rate

	K^*	K^1	K^2	K^3	K^{12}	K^{23}	K^{123}
Weighting coefficients:							
α_1		1	0	0	0.280	0	0.170
					(0.016)		0.013
α_2		0	1	0	0.720	0.500	0.414
					(0.016)	(0.026)	(0.019)
α_3		0	0	1	0	0.500	0.415
						(0.026)	(0.020)
OLS regression at $t = 1$:							
β_k	0.401	0.259	0.370	0.371	0.387	0.394	0.404
	(0.029)	(0.028)	(0.028)	(0.030)	(0.030)	(0.030)	(0.031)
β_x	0.601	0.728	0.625	0.623	0.616	0.605	0.600
	(0.027)	(0.026)	(0.027)	(0.028)	(0.028)	(0.028)	(0.028)
OLS regression at $t = 4$:							
β_k	0.400	0.348	0.385	0.385	0.395	0.394	0.399
	(0.029)	(0.030)	(0.029)	(0.029)	(0.030)	(0.030)	(0.030)
β_x	0.599	0.652	0.613	0.613	0.608	0.606	0.604
	(0.031)	(0.032)	(0.032)	(0.032)	(0.032)	(0.032)	(0.032)

Next, we put the generated capital series to use for the purpose of estimating the production function given in equation (21). The bottom panels of Tables 3 (a) and (b) summarize the estimates of the technology parameters, β_x and β_k, based on the true capital stock (K^*) as well as its approximations. In this analysis, we consider two cross-sectional regressions at $t = 1$ and $t = 4$, to evaluate the magnitude and the persistence of estimation bias due to the initial approximation error. In the period following the initial capital approximation ($t = 1$), the errors are not absorbed in the PIM. Thus, the estimates based on K^1 suffer badly from a downward bias, as predicted in equation (6). This bias is less severe in the period $t = 4$, but still persists. At the same time, the estimates are significantly improved when K^{23} or K^{123} are used as measures of the capital input. Note that the addition of the weak proxy X^1 in K^{123} does not dramatically affect the estimation of the technology parameters, and the multiple-proxy approaches outperform the single-proxy approaches. These estimation results are stable with both fixed and varying depreciation rates. The main finding of this experiment is generally in line with the previous discussion and supports the use of our generalized method in production function estimation. First, the experiment shows that the approximation error of the initial capital measurement affects the production function estimation, and the bias persists over time. Second, it shows that when the initial capital is calculated using the multiple-proxy approach with optimal weighting coefficients, the estimation bias is negligible.

5. An Empirical Application

In this section, we test the performance of different initial capital approximations on a U.S. industry-level dataset. The industry-level data come from the NBER manufacturing productivity database, which essentially reflects the Census Bureau's Annual Survey of Manufactures. This database contains annual information on 462 manufacturing industries from 1958 to 2008, which cover the entire U.S. manufacturing sector at the six-digit NAICS level. The main variables are number of workers, total payroll, value of shipments, value added, end-of-year inventories, investment, and expenditure on energy and materials.

The industry-level capital stock is not directly measured in the Annual Survey of Manufactures. The capital series in the original NBER database is calculated using the PIM with stochastic service lives and beta decay (Bartelsman and Gray, 1996). This calculation requires two additional datasets: (i) the historical investment series that dates back to the year 1890 (collected by the Bureau of Economic Analysis); and (ii) the original Penn–Census–SRI data for the concordances between different industry classifications. Thus, we cannot replicate the capital calculation as in Bartelsman and Gray (1996). In the following empirical study, we consider an alternative approach. First, we regenerate the capital stock for the entire period of 1958–2008, using the PIM with a geometric age–efficiency profile and a fixed depreciation rate of 0.08. The starting values of this calculation are Bartelsman and Gray (1996)'s capital stocks in the year 1958. Then, we keep only the last ten years of the generated capital series (1999–2008). After 41 periods of the geometric PIM calculation with a positive depreciation rate, the initial choice of capital stock should become irrelevant in 1999. Thus, we can consider this generated capital stock as the true values of capital for the last ten periods. We denote this capital series as K^*. The objective of this exercise is to approximate K^*, using only the last ten periods of production-related variables.

Several variables in the NBER database can be used as proxy variables for approximating the initial capital stock. In particular, we select three variables: the deflated expenditure on materials (including energy), the deflated end-of-year inventories, and the stock of equipment. The two first variables come directly

TABLE 4
Correlations of Main Variables over the Period 1999–2008

	Capital	Investment	Materials	Inventory	Equipment
Capital	1	0.920	0.745	0.592	0.985
Investment		1	0.746	0.542	0.877
Materials			1	0.566	0.736
Inventory				1	0.580
Equipment					1

from the original survey data, and the third one is generated by Bartelsman and Gray (1996). As a component of the total capital stock, the stock of equipment is 99 percent correlated to the true capital series, which makes it the best proxy variable. Unfortunately, it is rarely the case that the stock of equipment is observed. Among the three variables, the second-best proxy is the materials, with 75 percent correlation to the true capital series, and the worst proxy is the end-of-year inventory, with 55 percent correlation to the true capital series (Table 4). Note that the three proxy variables are chosen for illustration purposes. The use of equipment as a proxy is an ideal case, but rarely feasible in practice. The materials are not as good as the stock of equipment, but more realistic. The use of inventory represents the case when we only have a weak proxy variable.

Based on the three proxy variables, there are several ways to approximate the initial capital values. The first category is the single-proxy approach: K^{M}, K^{Inv}, and K^{Eq} are generated according to the share of material, end-of-year inventory, and equipment, respectively. The second category is the generalized approach with two proxy variables: K^{M-Inv}, K^{M-Eq}, and K^{Inv-Eq}. The third one is the generalized approach with all proxy variables, $K^{M-Inv-Eq}$. The estimation procedure for the weighting coefficients is based on equation (15) with $s=t-1$ (similar to the one in the Monte Carlo simulation). Table 5 summaries the estimation results. In the four cases where multiple proxies are used, the estimates of the weighting coefficients are consistent with the observations in Table 4. The best proxy in terms of correlation with the true capital series, the stock of equipment, is attributed the highest weighting coefficient, while the worst proxy, the end-of-year inventory, is assigned the smallest coefficient.

Given the estimated weighting coefficients, we generate the capital measurement for the initial year, 1999, then apply the PIM for the remaining periods. Table 6 reports the correlations of different capital measures with the true capital

TABLE 5
Estimates of Weighting Coefficients

	K^{M-Inv}	K^{M-Eq}	K^{Inv-Eq}	$K^{M-Inv-Eq}$
α_{M}	0.726	0.045	–	0.039
	(0.080)	(0.026)		(0.026)
α_{Inv}	0.274	–	0.028	0.020
	(0.080)		(0.018)	(0.013)
α_{Eq}	–	0.955	0.972	0.942
		(0.026)	(0.018)	(0.029)

TABLE 6

The Correlation between Different Capital Measures and the True Capital Stock

t	K^{M}	K^{Inv}	K^{Eq}	K^{M-Inv}	K^{M-Eq}	K^{Inv-Eq}	$K^{M-Inv-Eq}$
1999	0.720	0.517	0.977	0.785	0.976	0.977	0.977
2000	0.776	0.608	0.982	0.844	0.982	0.983	0.982
2001	0.821	0.684	0.986	0.884	0.986	0.987	0.986
2002	0.849	0.729	0.988	0.904	0.988	0.989	0.989
2003	0.873	0.770	0.990	0.921	0.990	0.990	0.990
2004	0.894	0.808	0.992	0.935	0.992	0.992	0.992
2005	0.916	0.848	0.993	0.950	0.993	0.994	0.994
2006	0.932	0.879	0.995	0.960	0.995	0.995	0.995
2007	0.949	0.910	0.996	0.970	0.996	0.996	0.996
2008	0.963	0.936	0.997	0.979	0.997	0.997	0.997

stock. In the first year, among the single-proxy approaches, we find the expected ranking that K^{Eq} has the highest correlation with the true capital series, followed by K^{M}, and the least correlated measure K^{Inv}. The multiple-proxy approach with materials and the end-of-year inventory, K^{M-Inv}, performs better than the cases where only one of these proxies is used. When the stock of equipment is used, the multiple-proxy measures perform as well as the ideal case of K^{Eq}. Figure 1 shows that the correlations of different measures to the true capital converge to 1 over time.

Similar to the Monte Carlo experiments in Section 4, we compare different methods in the context of estimating a production function. Consider the value-added Cobb–Douglas production function, with capital and labor as input variables:

$$Y_{it}=K_{it}^{*\beta_K}L_{it}^{\beta_L}. \tag{23}$$

The two parameters of interest are the output elasticities β_K and β_L. The output variable is generated by setting β_K=0.4 and β_L=0.6. Then, we regress the

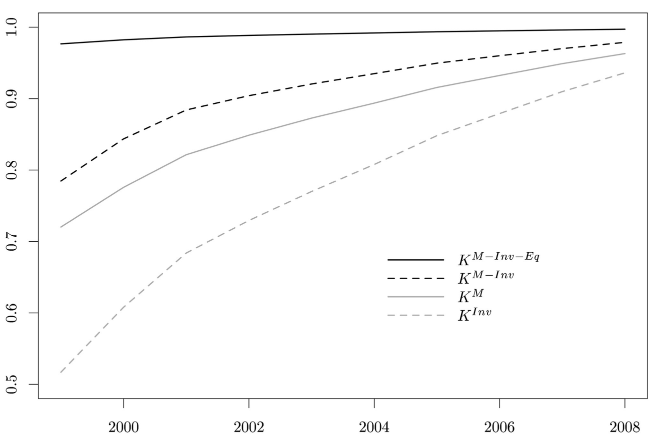

Figure 1. The evolution of the correlation coefficients between different capital measures and the true capital stock

TABLE 7
Estimation of the Production Function

	K^*	K^M	K^{Inv}	K^{Eq}	K^{M-Inv}	$K^{M-Inv-Eq}$
t= 1999						
β_K	0.400	0.388	0.358	0.382	0.389	0.386
		(0.007)	(0.008)	(0.002)	(0.007)	(0.002)
β_L	0.600	0.624	0.679	0.646	0.617	0.637
		(0.016)	(0.017)	(0.005)	(0.015)	(0.005)
t= 2003						
β_K	0.400	0.398	0.375	0.394	0.396	0.395
		(0.004)	(0.005)	(0.001)	(0.004)	(0.001)
β_L	0.600	0.599	0.644	0.619	0.603	0.615
		(0.010)	(0.012)	(0.003)	(0.010)	(0.003)
t= 2008						
β_K	0.400	0.396	0.381	0.400	0.393	0.400
		(0.003)	(0.003)	(0.001)	(0.003)	(0.001)
β_L	0.600	0.607	0.634	0.601	0.612	0.601
		(0.006)	(0.008)	(0.002)	(0.006)	(0.002)

Note: The estimated standard errors are reported in parentheses.

generated output variable on labor and different measures of capital using the OLS method. Table 7 summarizes the cross-sectional estimation results for t= 1999, 2003, and 2008. This econometric exercise produces a series of results that are very similar to those in the Monte Carlo experiments. First, the mis-measurement of the initial capital leads to a downward bias for the estimator of β_k and an upward bias for the estimator of β_L. Second, while its magnitude decreases over time, the bias persists when a weak proxy variable is used (see column K^{Inv} in Table 7). Third, the generalized method with multiple proxies performs as well as the case when the ideal proxy is used. For example, we can compare the results in column K^{Eq} (the ideal case) to those in column K^{M-Inv} (the case that combines two imperfect proxies). The two approaches yield similar estimates of β_K, which converge quickly to the true value of β_K.[12]

6. Conclusion

In this paper, we have proposed a simple approach to deal with the problem of the missing initial capital in the context of empirical production analysis. Our approach generalizes existing methods of initial capital approximation at the disaggregated level that usually rely on the arbitrary choice of a single proxy. In contrast, the generalized method incorporates multiple proxies and attributes a

[12]In this study, we also estimate equation (23) with the actual observed output. In this case, the input variables are likely to be correlated with error terms of the regression equation because of productivity shocks. Therefore, we estimate the production function using the control function approach (Olley and Pakes, 1996). The estimation results are similar to those reported in Table 7, and show that the generalized method can also reduce the bias of initial mis-measurement for more sophisticated production function estimators.

weighting coefficient to each of them. The estimates of the weights indirectly reflect the correlation between each proxy and the true capital stock, therefore capturing the quality of each proxy. In this way, the proposed method is also useful for selecting the most relevant proxies.

We conduct a series of experiments to show the impact of the initial capital approximation error and to test the performance of the new method. We find that the initial approximation error can cause severe bias in the production function estimation, especially for a dataset with a short time horizon. In this context, the proposed method is a promising tool because not only it relaxes the ad hoc assumptions of the traditional methods, but also yields a better approximation of the initial capital stock, thus reducing the potential bias in empirical analysis that comes from mis-measurement of the capital stock.

Based on the experiments conducted in this paper, we can offer a guideline for practitioners who would be interested in using the generalized method to select among different proxies. Suppose that we have one or several obvious candidates for the capital approximation as well as additional proxies. Researchers may wonder whether to use these additional proxies in capital approximation. To answer this question, we can examine the weighting coefficients of different proxies. Three potential cases may occur. First, weights are evenly assigned to all potential proxies. In this case, the additional proxies are as good as the preferred candidates. Thus, inclusion of the additional proxies is recommended. Second, the preferred proxy receives a sufficiently high weight, while one or several additional proxies receive low weights. Then, eliminating the weakest proxies may yield efficiency gains. In the third, and least favorable, case, one of our preferred proxies receives a low weight. In this situation, either we disregard this proxy or use the generalized method with all available proxies. The latter approach may still be preferred in some cases because, as our experiments show, the efficiency loss of including a weak proxy is limited.

There are several directions in which the proposed method can be further improved in the future. The generalized method is based on a set of assumptions that allow us to focus on the initial capital problem. Although these assumptions are standard in the literature, some of them could be relaxed depending on the availability of data or preliminary knowledge of capital accumulation. For example, the PIM could be modified to include the capital asset retirement (OECD, 2009), or to relax the assumption that productive capital services are proportional to capital stock (Müller, 2008). Furthermore, if detailed investments series on different capital assets are available, it would also be possible to apply the generalized method to each type of asset in order to account for their differences in terms of physical depreciation rate, lifetime, and the relevant proxy.

References

Bartelsman, E. and W. Gray, "The NBER Manufacturing Productivity Database," NBER Technical Working Paper No. 205, National Bureau of Economic Research, 1996.

Becker, R. A. and J. Haltiwanger, "Micro and Macro Data Integration: The Case of Capital," in D. W. Jorgenson, J. S. Landefeld, and W. D. Nordhaus (eds), *A New Architecture for the U.S. National Accounts*, The University of Chicago Press for the National Bureau of Economic Research, Chicago, 541–610, 2006.

Berndt, E. R. and M. A. Fuss, “Productivity Measurement with Adjustments for Variations in Capacity Utilization and Other Forms of Temporary Equilibrium,” *Journal of Econometrics*, 33, 7–29, 1986.
Corrado, C., C. Hulten, and D. Sichel, “Intangible Capital and U.S. Economic Growth,” *Review of Income and Wealth*, 55, 661–85, 2009.
Cummins, J. G., “A New Approach to the Valuation of Intangible Capital,” in C. Corrado, J. Haltiwanger, and D. Sichel (eds), *Measuring Capital in The New Economy*, The University of Chicago Press for the National Bureau of Economic Research, Chicago, 47–72, 2005.
Diewert, W. E., “Aggregation Problems in the Measurement of Capital,” in D. Usher (ed.), *The Measurement of Capital*, The University of Chicago Press for the National Bureau of Economic Research, Chicago, 433–538, 1980.
Feenstra, R. C., R. Inklaar, and M. P. Timmer, “The Next Generation of the Penn World Table,” *American Economic Review*, 105, 3150–82, 2015.
Foster, L., C. Grim, and J. Haltiwanger, “Reallocation in the Great Recession: Cleansing or Not?” *Journal of Labor Economics*, 34, S293–331, 2016.
Fukao, K., T. Miyagawa, K. Mukai, Y. Shinoda, and K. Tonogi, “Intangible Investment in Japan: Measurement and Contribution to Economic Growth,” *Review of Income and Wealth*, 55, 717–36, 2009.
Gilhooly, B., “Firm-Level Estimates of Capital Stock and Productivity,” *Economic and Labour Market Review*, 3, 36–41, 2009.
Hall, B. H. and J. Mairesse, “Exploring the Relationship between R&D and Productivity in French Manufacturing Firms,” *Journal of Econometrics*, 65, 263–93, 1995.
Harberger, A. C., “Perspectives on Capital and Technology in Less-Developed Countries,” in M. J. Artis and A. R. Nobay (eds), *Contemporary Economic Analysis*, Croom Helm, London, 15–40, 1978.
Hicks, J. R., “Capital Controversies: Ancient and Modern,” *American Economic Review*, 64, 307–16, 1974.
Hulten, C. R., “The Measurement of Capital,” in E. R. Berndt and J. E. Triplett (eds), *Fifty Years of Economic Measurement: The Jubilee of the Conference on Research in Income and Wealth*, The University of Chicago Press for the National Bureau of Economic Research, Chicago, 119–58, 1990.
Inklaar, R., “The Sensitivity of Capital Services Measurement: Measure All Assets and the Cost of Capital,” *Review of Income and Wealth*, 56, 389–412, 2010.
Jorgenson, D. W., “Capital Theory and Investment Behavior,” *American Economic Review*, 53, 247–59, 1963.
Jorgenson, D. W. and Z. Griliches, “The Explanation of Productivity Change,” *Review of Economic Studies*, 34, 249–83, 1967.
Levinsohn, J. and A. Petrin, “Estimating Production Functions Using Inputs to Control for Unobservables,” *Review of Economic Studies*, 70, 317–41, 2003.
Liu, L., “Entry–Exit, Learning, and Productivity Change Evidence from Chile,” *Journal of Development Economics*, 42, 217–42, 1993.
Martin, R., “Building the Capital Stock,” CeRiBA Working paper, The Centre for Research into Business Activity (CeRiBA), 2002.
Müller, S., “Capital Stock Approximation Using Firm Level Panel Data,” Discussion Paper 38, Bavarian Graduate Program in Economics (BGPE), 2008.
Nehru, V. and A. Dhareshwar, “A New Database on Physical Capital Stock: Sources, Methodology and Results,” *Revista de análisis económico*, 8, 37–59, 1993.
OECD, *Measuring Capital; OECD Manual*, 2nd edn, The Organisation for Economic Co-operation and Development, Paris, 2009.
Olley, G. S. and A. Pakes, “The Dynamics of Productivity in the Telecommunications Equipment Industry,” *Econometrica*, 64, 1263–97, 1996.
Pavcnik, N., “Trade Liberalization, Exit, and Productivity Improvement: Evidence from Chilean Plants,” *Review of Economic Studies*, 69, 245–76, 2002.
Raknerud, A., D. Rènningen, and T. Skjerpen, “A Method for Improved Capital Measurement by Combining Accounts and Firm Investment Data,” *Review of Income and Wealth*, 53, 397–421, 2007.
Triplett, J. E., “A Dictionary of Usage for Capital Measurement Issues,” presented at the *Second Meeting of the Canberra Group on Capital Stock Statistics*, 1998.