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THE `TROUBLING TRADEOFFS' PARADOX AND A RESOLUTION

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Ravallion (2012a) argues that the Human Development Index (*HDI*) embeds questionable tradeoffs between the dimensions used to compute the index. To alleviate these problems he proposes the adoption of one of the indices developed by Chakravarty (2003). In this paper I identify the following paradox: while the Chakravarty indices clearly exhibit more sensible tradeoffs than the *HDI*, the *HDI* produces more sensible rankings than the Chakravarty indices. To solve the paradox I identify the axioms behind each methodology responsible for the unintuitive tradeoffs and rankings and illustrate how to develop an index with these questionable axioms removed. This approach can result in methodologies that exhibit more intuitive tradeoffs by design, as it seeks inputs from the public as to what those tradeoffs ought to be, and produces rankings that are more in line with what the *HDI* wishes to measure: human development and capabilities, as conceptualized by Sen (1985).

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1. INTRODUCTION

In 2010 the United Nations Development Program (UNDP) unveiled a new methodology for computing the Human Development Index (*HDI*). Given the significance the *HDI* has as the flagship indicator of multidimensional development worldwide, this new methodological change received a fair amount of attention in the news media and in academic and policy circles. Most prominently, Martin Ravallion, former Director of the Development Research Group at the World Bank, discussed the new methodology at length in two of his papers: "Mashup Indices of Development" and "Troubling Tradeoffs in the Human Development Index" (Ravallion, 2012a and 2012b; see also Fleurbaey and Blachet 2013). This attention is warranted: if the *HDI* is to be used properly in policy circles worldwide it is important that we all strive to comprehend the rationale behind those methodological changes.

The *HDI* is a multidimensional index and therefore exhibits different combinations of dimensional achievements that lead to the same level of "human development" as computed by such index. These are the so-called implicit tradeoffs

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embedded in a particular methodology. The main criticism that Ravallion (2012a) raised about the new *HDI* is that it exhibits questionable tradeoffs between life expectancy and per capita income, and between educational attainment and per capita income. The problem is that the tradeoffs seem to be too high for some countries and too low for others. For example, the marginal rate of substitution between life expectancy and income in the *2010 HDI* for the ten countries with the highest per capita income is, on average, about *240 times larger* than the same calculation done for the 10 countries with the lowest per capita income, whereas the marginal rate of substitution between educational attainment and income for the ten countries with the highest per capita income is, on average, about *170 times larger* than the same calculation done for the ten countries with the lowest per capita income. A striking feature of these differences is that they are even more extreme than those exhibited by the distribution of world per capita income.[1] These extraordinary differences in the tradeoffs across rich and poor countries become a problem if they are used as a guide for resource allocation, as they imply that the poorest countries should only be willing to spend about *one half of one percent* of what the richest countries would spend for the same gain in health or education capabilities. These implications are viewed as extreme and undesirable by many in academic and policy circles (Easterly, 2010; Ravallion, 2012a).

To soften these tradeoffs Ravallion (2012a) proposes the adoption of one of the indices developed by Chakravarty (2003) and argues that these indices do better in the sense of exhibiting far more reasonable tradeoffs than the *2010 HDI*. For example, the marginal rate of substitution between life expectancy and income in the "*r=0.5*" Chakravarty index for the ten countries with the highest per capita income is, on average, about *nine times larger* than the same calculation done for the ten countries with the lowest per capita income, whereas the marginal rate of substitution between educational attainment and income for the ten countries with the highest per capita income is, on average, about *eight times larger* than the same calculation done for the ten countries with the lowest per capita income. While differences across rich and poor countries persist, these differences are, on average, *between one and two orders of magnitude* smaller than the differences these rates of substitution exhibit in the *2010 HDI* and indeed a *full order of magnitude* smaller than the differences in the world distribution of per capita income across countries. They are, therefore, far more palatable.

While comparing methodologies in terms of their implied rates of substitution among core dimensions is key, it is just as important to take a look at the implied rankings that they produce. In this respect, the Chakravarty index adopted by Ravallion and the *2010 HDI* methodology agree considerably in terms of how they rank countries[2] but, when they differ, the *2010 HDI* appears to produce more intuitive rankings. Consider, for example, the four countries that drop

[1]The per capita income for the ten richest countries in the 2010 UNDP sample is, on average 100 times larger than the same calculation done for the ten countries with the lowest per capita income.

[2]The Spearman and Kendall rank correlation coefficients between these rankings are, respectively, 0.94 and 0.99.

out of the list of top ten countries with the highest level of human development as we move from the *2010 HDI* methodology to the Chakravarty index. These countries are New Zealand, Ireland, Sweden and Germany. These countries all belong to the top quintile of the distributions of life expectancy, educational attainment and income across countries and are replaced, in the Chakravarty ranking, by Qatar, Luxembourg, Hong Kong and Singapore. The countries in this second set do much worse in either health or education (or both) than the countries in the first set,[3] but they are all top ten countries in terms of income. That is enough for the Chakravarty index to rank these countries among the top ten countries in terms of human development despite their low and uneven development in the other dimensions.

That one of the methodologies produces more intuitive tradeoffs while the other produces more intuitive rankings is somewhat of a paradox. After all, there is a very close conceptual connection between the tradeoffs embedded in any index and the implied rankings. The main contribution in this paper is that I explain the source of the discrepancies by appealing to the normative principles embedded in these different methodologies, as these methodologies interact with the data. This examination reveals that the logarithmic transformation of income that follows from one of the assumptions embedded in the *2010 HDI* is about five times more important than its multiplicative structure in explaining the dramatic variations in the tradeoffs between life expectancy and income across countries.[4] On the other hand, the additive structure that follows from one of the assumptions embedded in the Chakravarty indices makes it possible for a country to rank well overall based solely on the country's very high income, and regardless of how poorly the country does in the other dimensions. The advantage of performing this kind of diagnostic is that it gives us hints as to which normative principles are responsible for the unintuitive tradeoffs and which are responsible for the unintuitive rankings, therefore opening the door to an approach that adopts perspectives from both methodologies.

In an attempt to illustrate what such approach can look like Zambrano (2015) develops a hybrid of sorts between the UNDP perspective and that espoused by Ravallion and in the present paper I compare these three methodologies, in terms of tradeoffs and rankings, from an empirical standpoint. The hybrid methodology produces intuitive tradeoffs by design, as it seeks inputs from the public and the decision-makers as to what those tradeoffs ought to be in the determination of the index. At the same time, because it removes the possibility of having a country rank highly solely because of its achievements in only one of the dimensions, it produces country rankings that are less correlated with income, and more in line with what the *HDI* is intended to measure: human development and capabilities, as conceptualized by Sen (1985).

The structure of the rest of this paper is the following. Section 2 succinctly explains what the *HDI* intends to measure and provides a rationale for the 2010

[3]Regarding educational attainment alone Luxembourg, Hong Kong and Singapore rank 48th, 43rd, 56th respectively in the 2010 UNDP sample (which has 169 countries). Qatar in turn ranks 47th and 97th in both life expectancy and in educational attainment in the same sample.

[4]This is shown in Section 3 of the paper.

formulation. This section is fundamental, as what the *HDI* is intended to measure is often misunderstood in academic and policy circles. Section 3 briefly discusses the Chakravarty indices, their properties, and compares them to the *2010 HDI*. Section 4 studies the rankings implied by three different Chakravarty indices, and compares them to the *2010 HDI* ranking. Section 5 reviews the hybrid methodology discussed above and investigates its normative and practical properties. Section 6 concludes.

2. Foundations

The Human Development Index is an index that aims to track the *capabilities* available to the individuals in a society. The term *capabilities* refers to the opportunities a person has to exercise his or her "freedom to attain different kinds of alternative lives between which a person can choose" (Sen, 2008, p. 23).[5]

2.1. *Capabilities and Human Development*

An extensive literature on capabilities exists, stemming from the work by Amartya Sen in 1985. I will not survey it here, but it is important to recall that the Human Development Index, since it was launched in 1990, has always been an attempt to measure capabilities.[6] From the 1990 HDR: The *HDI* is "an index that captures the three essential components of human life (…) longevity and knowledge refer to the formation of human capabilities, and income is a proxy measure for the choices people have in putting their capabilities to use."[7]

The basic building block in the capability approach is the idea of a *functioning*. *"A functioning is an achievement of a person: what she manages to do or be (…) The capability of a person is a derived notion. It reflects the various combinations of functionings he or she can achieve"* (Sen, 2003b, p. 5).

To measure capabilities in practice we need to think of which human functionings we wish to keep track of in a country, and how we want to think about how those functionings transform into capabilities.[8] "Some functionings are very elementary, such as being adequately nourished, being in good health, and so on, and these may be strongly valued by all, for obvious reasons. Others may be more complex, but still widely valued, such as achieving self-respect, or being socially integrated" (Sen, 2008, p. 24).

In choosing which dimensions of human functioning to track for the construction of a capabilities index the UNDP must balance being broad in scope with being able to gather data of sufficient quality for a large number of countries

[5]This Section follows Sections 2–4 in Zambrano (2014).

[6]For an account of the collaboration between Mahbud ul Haq and Amartya Sen that led to the publication of the first Human Development Report in 1990 see UNDP (2010b), and Sen (2003a pp. vii–xiii).

[7]UNDP (1990, p. 14). This is one of many references that can be given on the matter. See, e.g., many of the references in the reader compiled by Fukuda-Parr and Kumar in 2003.

[8]This is to some extent as in the approach of "household production functions" developed by Becker (1976) and Lancaster (1966), but the capabilities approach goes well beyond that in the inclusion of functionings that cannot be easily seen as detached objects that the person or the household happen to "own" or "produce." See Sen (1989, p. 104).

on a frequent (annual) basis. Which functionings are essential to monitor can also evolve over time, and vary both across and within countries. From the outset, the *HDI* keeps track of people's abilities (i) to live a long and healthy life, (ii) to have access to knowledge and (iii) to have command over the resources that would allow them to participate in community life and to make the choices that would permit them to live a full, meaningful life.

The first two, life and education, are elementary and intrinsically valuable functionings. Command over resources, however, has a different status in the capabilities approach, as "commodity ownership is rarely sought for itself, since commodities are means to other ends (. . .) For example, having food helps us to be nourished, to enjoy eating, to entertain friends, and so forth" (Sen, 2008, p. 24). One is interested in keeping track of the set of functionings that become available to the individual thanks to commodity ownership, not necessarily in keeping track of commodity ownership for its own sake.

2.2. *Computing the Human Development Index*

Until 2009 the *HDI* was calculated as the arithmetic mean of suitably normalized values for life expectancy, educational attainment, and income but ever since the publication of the first Human Development Report in 1990 many researchers have pondered whether this was the best way to calculate the *HDI*. In spite of the improvement that the old *HDI* offered relative to the mere comparison of per capita GNP values, the old *HDI* was subject to a number of well-grounded criticisms.

First, many relevant aspects of human development are missing in the computation of the *HDI*, such as social integration, freedom or sustainability. Second, it is not clear that the variables chosen to approximate health, education and material wellbeing are necessarily the most sensible ones. Third, it does not take into account the distribution of achievements, focusing only on aggregate statistics about the dimensional achievements for each country. Fourth, its additive structure has undesirable implications regarding the substitutability between dimensions.[9] Fifth, it lacks normative (axiomatic) justification, which makes it difficult to interpret.[10] See Herrero, Martínez and Villar (2010) for details.

These are all important problems, and the 2010 reformulation of the *HDI* addresses only the ones arising from the additive structure of the old *HDI* according to which a country could be deemed to have a high level of human development even when its health, education or income levels remain at critically low (subsistence) levels.

2.3. *The 2010 Formula*

Consider a collection of countries, each characterized by aggregate level of achievements, or human functionings, in health, h, education, e, and income, y.

[9]A problem discussed in a more general setting in Section 4 below.

[10]Although such axiomatization is now available for the additive HDI. See Chakravarty (2011a) and Zambrano (2014).

Let *H, E, Y* denote the corresponding spaces to which they belong. In what follows assume that *H* and *E* are closed intervals $[h^o,h^*]$ and $[e^o,e^*]$. *Y*, on the other hand, is best thought of as $[y^o,\infty)$, although in practice one also picks an upper bound for *y*, called y^* and thus $Y=[y^o,y^*]$. Those lower bounds (possibly zero) on the values of *h, e* and *y* represent normative values below which subsistence is not believed to be possible. The upper bounds represent the highest level any society has been known to achieve in those dimensions. The upper bounds can be as high as normatively desired. Let $\Omega = H\times E\times Y$. A *capabilities index* is a non-constant, continuous real-valued mapping $C:\Omega\to\mathbb{R}$. Associated with each capabilities index *C* there are three *partial capability measurement functions* $C_h:H\to[0,1]$, $C_e:E\to[0,1]$ and $C_y:Y\to[0,1]$ and a *capabilities aggregator* $I^C:[0,1]^3\to\mathbb{R}$ such that, by composition,

$$C(h,e,y)\equiv I^C\big(C_h(h),C_e(e),C_y(y)\big).$$

This representation emphasizes that identifying the principles we would want our capabilities index to satisfy has precise implications about what kinds of "normalizations" of the data are to be employed.

The 2010 Human Development Index is given by the following formula:

$$I\big(C_h,C_e,C_y\big)=C_h^{1/3}\cdot C_e^{1/3}\cdot C_y^{1/3} \tag{1}$$

where $C_h=\frac{h-h^o}{h^*-h^o}$, $C_e=\frac{e-e^o}{e^*-e^o}$ and $C_y=\frac{\log y-\log y^o}{\log y^*-\log y^o}$.

The old HDI is defined in a similar way, with expression (1) above replaced by

$$I\big(C_h,C_e,C_y\big)=\frac{C_h+C_e+C_y}{3} \tag{2}$$

2.4. *Properties*

Consider the following principles one may want a capabilities index to satisfy:

1. Monotonicity: For each (h,e,y), (h',e',y') in Ω, with $h>h'$, $e>e'$ and $y>y'$, we have $C(h,e,y)>C(h',e',y')$.
   The first assumption establishes that the capabilities index increases when all of the arguments increase simultaneously.
2. Independence: Let (h,e,y), (h',e',y') be in Ω with $h,h'>h_o$; $e,e'>e_o$ and $y,y'>y_o$. Then
   - $C(h,e,y)\geq C(h,e',y')$ implies $C(h',e,y)\geq C(h',e',y')$,
   - $C(h,e,y)\geq C(h',e,y')$ implies $C(h,e',y)\geq C(h',e',y')$ and
   - $C(h,e,y)\geq C(h',e',y)$ implies $C(h,e,y')\geq C(h',e',y')$.

The second assumption states that if the capabilities set of society (h,e,y) is at least as large as that of society (h',e',y), then this relation holds for all common

values of income, y. Similarly for health and for education, with respect to the corresponding other two variables.

3. Subsistence: For all h, h' in H, e, e' in E and y, y' in Y
   - $C(h,e,y) \geq C(h',e',y_o)$
   - $C(h,e,y) \geq C(h',e_o,y')$ and
   - $C(h,e,y) \geq C(h_o,e',y')$.

The third assumption states that there are no trade-offs between any of the dimensions of achievement when the members of society are at their worst levels in any of the dimensions. Notice that the assumption places no inequality restrictions between h and h', e and e' in E or y and y'. The interpretation is that, even if, say, $h' > h$, and $e' > e$ it is enough for income to be at the `subsistence level' for society (h,e,y) to be deemed as having a higher level of human development than society (h',e',y_o).[11]

4. Partial Capabilities Growth: For h, h' in H, e, e' in E and y,y' in Y and "feasible" values for Δh, Δe and $\lambda \in \mathbb{R}_+$ (such that $h+\Delta h$, $h'+\Delta h$ are in H, $e+\Delta e$, $e'+\Delta e$ are in E, and $\lambda \cdot y$ and $\lambda \cdot y\prime$ are in Y)
   - $C_h(h+\Delta h)-C_h(h)=C_h(h\prime+\Delta h)-C_h(h\prime)$,
   - $C_e(e+\Delta e)-C_e(e)=C_e(e\prime+\Delta e)-C_e(e\prime)$ and
   - $C_y(\lambda \cdot y)-C_y(y)=C_y(\lambda \cdot y\prime)-C_y(y\prime)$.

According to the fourth assumption different functionings contribute to capabilities enhancement differently. Starting from h, the amount of change in health achievements required to produce an increase in *health capabilities* of a certain size is independent of h (and similarly for education, regarding *education capabilities*, and starting from e). Income changes contribute to capabilities in a different way: starting from y, the amount of income change required to produce an increase in *income capabilities* of a certain size is proportional to y.

5. Scale: Let (h,e,y) be such that $C_h(h)=C_e(e)=C_y(y)=c$. Then $C(h,e,y)=c$.
   The fifth assumption states that if the partial capabilities measures all take the same value, the capabilities index takes that value as well. This is a normalization condition.
6. Aggregation Symmetry: Let (C_h, C_e, C_y) be a vector of partial health, education and income capabilities and let $\pi(C_h, C_e, C_y)$ be a permutation of (C_h, C_e, C_y). Then $I(C_h, C_e, C_y) = I(\pi(C_h, C_e, C_y))$.
   The sixth assumption states that health, education and income *capabilities* contribute equally to the aggregate.
   The *2010 HDI* is the only index that satisfies Monotonicity, Subsistence, Independence, Scale, Aggregation Symmetry and Partial Capabilities Growth. The proof is in Zambrano (2014) which in turn is based on Herrero *et al*. (2010 a,b).

[11]Similar situations arise when it is health or education the variables that are at their "subsistence levels."

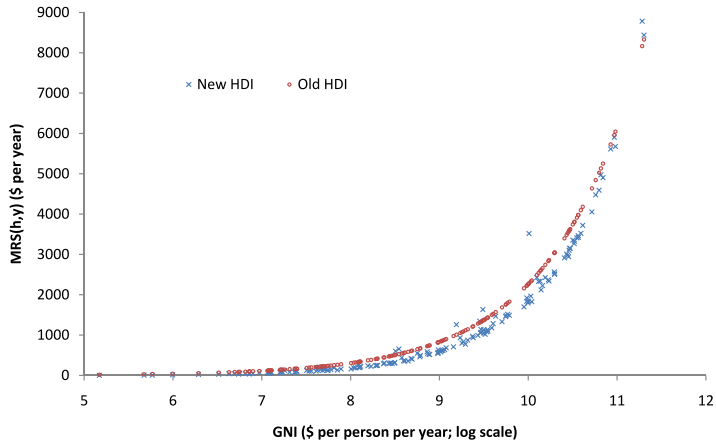

Figure 1. The tradeoff between life expectancy and income
Source: Calculations by the author based on UNDP (2010) data.

3. Tradeoffs Between Core Dimensions

In a sequence of papers Ravallion (1997, 2012a, 2012b) has stressed the importance of having a vigorous public debate about the tradeoffs implicit in the *HDI*. According to Ravallion, while the UNDP has been transparent about explaining that the *HDI* gives equal weights to the normalized values of the core dimensions, it has been less forthcoming in making it clear that different normalizing functions alter the weights of the core variables in the index, thereby altering the implied *HDI* rankings. All of this, of course, can be inferred by careful study of the publicly available methodologies employed by the UNDP to calculate the *HDI*, but doing this properly turns out to be a somewhat difficult task.

Addressing Ravallion's concerns is best done when one keeps close track of the historical links of the *HDI* with Sen's capability approach, and when one strives to understand the tradeoffs embedded in the *HDI* by appealing to the normative principles that it satisfies, as explained in Section 2. Next I show a detailed example of how this can be done.

3.1. *The Tradeoff Between Life Expectancy and Income in the* HDI

The marginal rate of substitution (MRS) between life expectancy and income in the *HDI*, given by $\frac{\partial HDI}{\partial h} / \frac{\partial HDI}{\partial y}$, establishes the rate at which one can substitute life expectancy for income and keep the capabilities index constant. Figure 1 below reproduces Figure 5 in Ravallion (2012a), which shows estimates of such rates for both the old and the new *HDI* formulations using the data employed in the 2010 Human Development Report.[12]

From a cursory examination of this picture it is clear that (i) the change in functional form does lower the tradeoff between life expectancy and income for

[12]I use the 2010 data and a common set of values for h^o,h^*, e^o,e^*, y^o *and* y^* to facilitate the comparison between methodologies as in Ravallion (2012a).

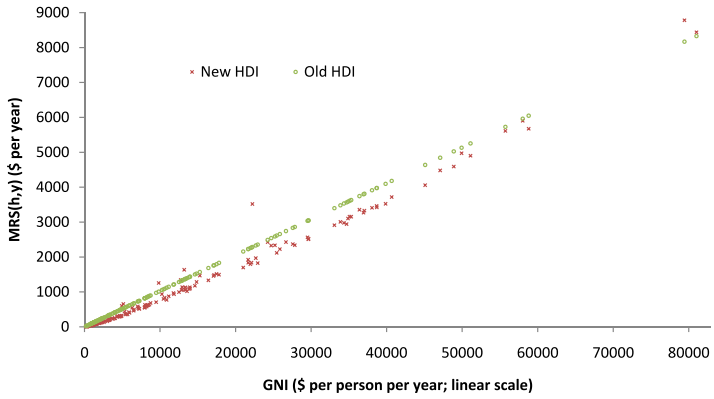

Figure 2. The tradeoff between life expectancy and income revisited
Source: Calculations by the author based on UNDP (2010) data.

all but a handful of countries, and (ii) the change in functional form cannot possibly explain the differences that this tradeoff exhibits across rich and poor countries, as these differences are marked for both functional forms. To wit: the average distance between the old and new levels for this tradeoff between life expectancy and income is 227 dollars per person across countries, which is merely 2.73 percent of the gap between the lowest and highest value for MRS under the old functional form and 2.59 percent of the gap between the lowest and highest value for MRS under the new functional form.

This can also be viewed in a more transparent way in a chart that plots the "unlogged" GNI against the old and new values for the tradeoff between life expectancy and income, shown below as Figure 2. The chart reveals the familiar fact that the tradeoff between life expectancy and income depended linearly on income in the case of the old *HDI* and the fact that this has effectively not changed much in the case of the 2010 *HDI*, as the relationship between the magnitude of this tradeoff and income, while non-linear in theory, is virtually linear in practice.

What explains the similarities between both series in Figure 2? It is that both the old and the new *HDI* transform income into capabilities at the same (decreasing) rate: *they both satisfy Partial Capabilities Growth*. Partial Capabilities Growth implies a normalization for income that, for both the additive and multiplicative formulations, has the following implication: as income approaches its "subsistence level" in the sample it becomes very difficult for a country to move up in the rankings even if it improves dramatically on the other dimensions at the cost of a very small drop in income. That this was solely due to this normalization in the case of the old *HDI* follows from the fact that this feature about the tradeoffs is not implied by the old functional form absent this normalization.

More is true, in fact. The marginal rate of substitution between health and income for each country can be decomposed in the following way:

$$MRS^{M,l} \equiv MRS^{A,a} + \left(MRS^{M,a} - MRS^{A,a}\right) + \left(MRS^{M,l} - MRS^{M,a}\right), \tag{3}$$

where $MRS^{M,l}$ is the *MRS* for the multiplicative *HDI* with the logarithmic normalization of income (the 2010 formulation), $MRS^{M,a}$ is the MRS for the multiplicative *HDI* with an affine normalization of income (such as the one used for health or education) and $MRS^{A,a}$ is the *MRS* for the additive *HDI* with an affine normalization of income. One can call the first term in parenthesis in expression (3) the *geometric mean effect*, as it measures the changes in *MRS* solely due to a move from an arithmetic to a geometric mean, while keeping the income unlogged. One can call the second term in parenthesis in expression (3) the *logged-income effect*, as it measures the changes in *MRS* solely due to a move from an affine to a logarithmic normalization of income, while keeping the geometric mean as the aggregation method.

Noticing that this is a full decomposition, and that $MRS^{A,a}$ does not vary across countries, one can ask the following question: what contributes more to the variations of $MRS^{M,l}$ across countries: the geometric mean effect, or the logged income effect? A variance decomposition exercise yields the answer: 83 percent of the variations in the *MRS* across countries are due to the logged-income effect and only 17 percent of the variations are due to the geometric mean effect.[13] In other words, it is the logarithmic transformation of income (which follows from Partial Capabilities Growth), much more than the adoption of a multiplicative aggregation method (which follows from Subsistence), that which is responsible for the troubling tradeoffs present in the *HDI*.[14]

4. Alternative Formulations

Reasonable people can disagree about the magnitude of the tradeoffs between the core dimensions and, once one has identified a formula for the *HDI* by identifying the principles one wishes to the formula to satisfy, *in terms of the core dimensions*, those disagreements can be investigated by considering how reasonable those principles are in the design of a capabilities index. As discussed above, the *2010 HDI* can be fully characterized in terms of Monotonicity, Subsistence, Independence, Scale, Aggregation Symmetry and Partial Capabilities Growth. Any, or all, of those can be rejected on a priori, normative grounds.[15]

Having said this, Monotonicity, Subsistence, Independence, Aggregation Symmetry, and even Scale and Partial Capabilities Growth, have strong, intuitive appeal. Proposing any alternative index would imply rejecting one or more of these principles.

[13]The contribution of the geometric mean effect to the variations in $MRS^{M,l}$ across countries is calculated as $\frac{Cov\left(MRS^{M,l},MRS^{M,a}-MRS^{A,a}\right)}{Var(MRS^{M,l})}$. The contribution of the logged-income effect is, in turn, calculated as $\frac{Cov\left(MRS^{M,l},MRS^{M,l}-MRS^{M,a}\right)}{Var(MRS^{M,l})}$.

[14]That the old *HDI* already exhibited problematic tradeoffs was, indeed, a point first made in Ravallion (1997).

[15]To be sure, an index that can be characterized in terms of principles such as the ones mentioned above is not superior to one which hasn't. It is just more transparent, and most likely easier to understand, which is of course important if the index is to be employed correctly for the purposes for which it is being designed.

Ravallion, for example, brings forth as an alternative the class of normalized indices developed by Satya Chakravarty. Such indices, $I_r^C : [0,1]^3 \to \mathbb{R}$, take the form

$$I_r^C = \frac{1}{3} C_h{}^r + \frac{1}{3} C_e{}^r + \frac{1}{3} C_y{}^r \quad (0 < \mathrm{r} < 1)$$

and satisfy assumptions regarding Symmetry, Normalization, Dimensional Concavity[16] and Consistency in Aggregation.[17] Importantly, I_r satisfies these assumptions but is not characterized by them: there are other indices that satisfy those assumptions as well (Chakravarty, 2003 and 2011b). Ravallion proposed using a normalized index from this class together with the affine normalizations $C_x = \frac{x - x^o}{x^* - x^o}$, $x = h, e, y$;[18] thus generating the family of development indices

$$C_r(h,e,y) = \frac{1}{3}\left(\frac{h-h^o}{h^*-h^o}\right)^r + \frac{1}{3}\left(\frac{e-e^o}{e^*-e^o}\right)^r + \frac{1}{3}\left(\frac{y-y^o}{y^*-y^o}\right)^r.$$

While these indices satisfy Monotonicity and Independence none of them satisfy Subsistence[19] which has the implication that the citizens in a country could presumably have a rich set of "possible livings" available to them (a big capabilities set) even as their health, education or income remains at critically low (subsistence) levels. This does not seem tenable.[20]

On the other hand, Consistency in Aggregation brings back a problem that had been previously identified in the old *HDI* by Herrero, Martínez and Villar (2010a), and that the *2010 HDI* had solved: that changes in the historically observed maxima for *h*, *e* and *y amounts to modifying the weights with which those variables enter the index*. It is not at all clear why, from a normative standpoint, an increase of, say, *h**, should lessen the importance that *h* has in the computation of the *HDI*. This is, however, the case under the Chakravarty indices discussed above.

4.1. *Implied Rankings*

Ravallion pointed out that it is ultimately up to the users of the new *HDI* to judge for themselves if they accept its tradeoffs. Most users will relate to these tradeoffs not by evaluating ratios of derivatives as economists do but instead by evaluating the rankings implied by those tradeoffs and deciding if these make

[16]He calls this "Lower Gain in Indicator at Higher Levels of Attainment Difference."

[17]Essentially, an addititive separability assumption.

[18]Not quite the normalizations Chakravarty used for the empirical illustration of his indices. Chakravarty understood that income deserves a special treatment in the capabilities approach, and he used the piece-wise linear concavifications of income that were being used at the time of his writing by the UNDP for the calculation of the old, additive, *HDI* (prior to the UNDP's adoption of the logarithmic normalization). See Rodríguez (2010).

[19]Including the old *HDI*, which is the limiting case of I_r^C as r approaches 1.

[20]The indices C_r do not satisfy Partial Capabilities Growth either, but I myself do not deem this a problem, as I explain in Section 5 below. Nor do they satisfy Scale, but that is arguably an issue of lesser importance.

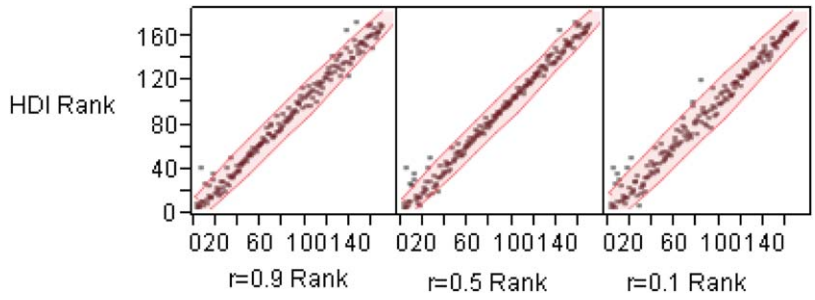

Figure 3. Rank scatterplot
Source: Calculations by the author based on UNDP (2010) data.

sense to them. Those kinds of evaluations are legitimate, and they can also help researchers understand if they got the tradeoffs right.

To hone in the intuition that comes from exercises of this sort, let's perform some comparison of the rankings arising from four different formulas intended to capture different degrees of substitutability between health, education and income: C_r indices for $r=.1$, $r=.5$ and $r=.9$, with the normalizations suggested by Ravallion, and the 2010 *HDI*. The results are reported in Table A in the Appendix.[21] The first thing to notice about these rankings is that there is a very strong linear association between them, as can be inferred from Figure 3.[22]

The next thing to notice is that there are nevertheless a number of countries for which the different rankings differ markedly. Consider, for example, the collection of 12 countries outside of the 95 percent density ellipses highlighted in Figure 3.[23] All but one of these countries (New Zealand) rank much better according to at least two of the Chakravarty indices than according to the 2010 *HDI*. What's the reason?

4.1.1. *The (sometimes) useful business of ranking core dimensions*

Table 1 reveals the answer. It contains the rankings of these 12 countries according to the four indices, and also according to the health, education and income variables alone.[24]

Figure 4 depicts some of that information as well, and groups the countries in terms of some common features they share according to this data.

Luxembourg, for example, ranks in the top 10 according to the Chakravarty indices even though it ranks 24th in life expectancy and 48th in educational

[21]Zambrano (2014) performs a similar comparison, between the old and the *2010 HDI*.

[22]The Spearman's rank correlation coefficients between the *2010 HDI* ranking and these three Chakravarty rankings are all greater than 0.985. The Kendall's rank correlation coefficients between the *2010 HDI* ranking and these three Chakravarty rankings are all greater than 0.911.

[23]These ellipses were computed in JMP Pro 10. Other ways of singling out the list of countries for which these rankings differ significantly produce very similar results.

[24]In producing the educational attainment rankings one can use an additive or a multiplicative aggregation function (the one I chose) for the education variables "mean years of schooling" and "expected years of schooling." The rankings that arise are from either aggregation method are so similar that the arguments that follow yields exactly the same conclusions either way.

Table 1

The Rankings Of Selected Countries According To Assorted Criteria

	2010 HDI	$C_{0.9}$	$C_{0.5}$	$C_{0.1}$	Life exp.	Education	Income
Hong Kong	21	12	9	10	2	43	10
Luxembourg	24	9	7	6	24	48	6
Singapore	27	16	10	9	14	56	8
United Arab Emirates	32	17	11	7	37	79	4
Brunei Darussalam	37	29	23	17	38	83	7
Qatar	38	5	4	3	47	97	2
Kuwait	47	32	26	19	35	107	5
Botswana	98	110	94	76	145	78	60
Equatorial Guinea	117	123	113	83	155	131	39
New Zealand	3	8	17	27	15	1	33
Liberia	162	136	143	163	133	133	167
Zimbabwe	169	145	156	169	166	113	169

Source: Calculations by the author based on UNDP (2010) data.

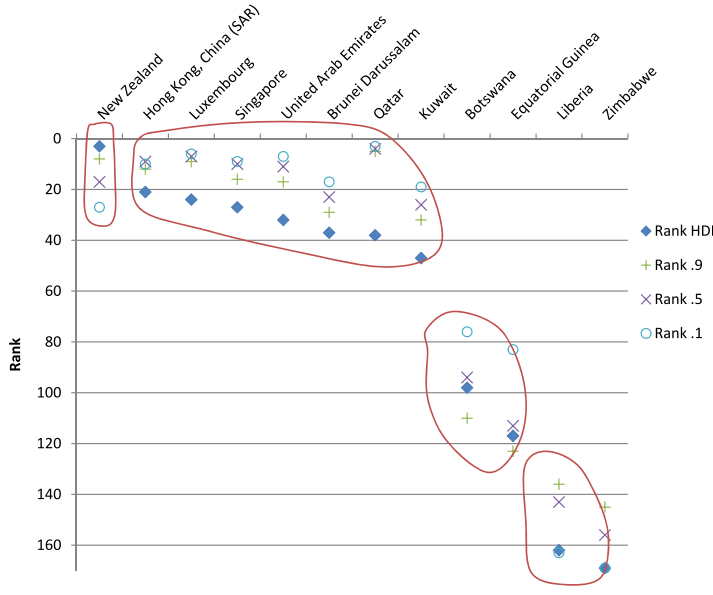

Figure 4. Countries for which the *2010 HDI* and the Chakravarti indices differ significantly
Source: Calculations by the author based on UNDP (2010) data.

attainment. Qatar, in turn, ranks in the top five according to the Chakravarty indices even though it ranks 47th in life expectancy and 97th in educational attainment. Similar points can be made about the top 20 position of Hong Kong, Singapore and the United Arab Emirates according to the Chakravarty indices despite their rankings in terms of educational attainments being outside of the top 40 and about the top 32 position in terms of the Chakravarty indices[25] for

[25]In the most conservative of cases.

Brunei and Kuwait despite their life expectancy and educational attainment rankings being outside of the top 32.[26] What all these countries have in common is that *they are all top ten countries in terms of income per capita* and that is just enough for all three of the Chakravarty indices to rank these countries pretty high up[27] despite their low, or uneven, development in the other dimensions.[28] This is something that the *2010 HDI* tends to avoid.

Table 1 also features Liberia and Zimbabwe, which are on the other end of the income spectrum. Not only are they comparatively low, they are also very close to the normatively determined minimum level for income of $163 per capita per year. As a consequence of this, the *2010 HDI* would have these countries stay at the bottom of the ranking even though Liberia has comparatively better numbers for life expectancy and education (ranking 133 in each) than income (rank of 167), and Zimbabwe has even better numbers for education (with a ranking of 113) versus income (rank of 169). $C_{0.1}$ would agree with the *2010 HDI* in keeping the ranking of these countries low whereas the $C_{0.5}$ and $C_{0.9}$ indices would allow these countries to gain between 13 and 26 positions in the ranking thanks to their better position in the other dimensions in spite of their critically low income levels.

In sum, from the examination of Table 1 and Figure 4 above it follows that in the case in which not all three of the Chakravarty indices rank a country consistently above the *2010 HDI* rank it is not the case that there is a Chakravarty index that tracks the *2010 HDI* more closely than the others: in some cases the *2010 HDI* will tend to agree more with $C_{0.1}$, sometimes it will be much harsher than $C_{0.1}$ and yet tend to agree more with $C_{0.5}$, and sometimes $C_{0.1}$ and $C_{0.5}$ will be much harsher than the *2010 HDI* and the *2010 HDI* will tend to agree more with $C_{0.9}$.

Thus, whenever these Chakravarty indices and the *2010 HDI* disagree substantially, an intermediate value for r, like $r=0.5$, need not execute, in practice, a "compromise" between high and low degree of substitutability between the core dimensions in the same way that the *2010 HDI* executes such substitutability. This need not be so, as sometimes indices with very different properties map into a particular data set in exactly the same way. Hence the value of Figures 3 and 4: they visually tell the story of the extent to which these different approaches are the same and the extent to which they differ as they interact with real world human development data.

4.1.2. *More Tests*

Even after having seen all this, one should test the soundness of the *2010 HDI* and of the Chakravarty indices further, before discarding any of them, by asking the question: when the *2010 HDI* and the Chakravarty indices differ significantly, which index provides the ranking that accords most *with what these*

[26]With Brunei and Kuwait ranking 83rd and 107th in terms of educational attainment, respectively.

[27]A fact that follows from Chakravarty's Consistency in Aggregation assumption, which is responsible for the additive separability of his indices.

[28]This is similar to the case of Botswana and Equatorial Guinea, also in Table 1. Those countries also rank much better in terms of income than on anything else, and that drives $C_{0.1}$ to place them, respectively, 22 and 34 places higher in the ranking (relative to their position according to the *2010 HDI*).

indices of capabilities are intended to measure? Let's look at an example of how one can go about answering that question for the case of New Zealand, the only country in Table 1 which does significantly worse according to all three Chakravarty indices vs. the *2010 HDI*.

New Zealand is not in the top 20 in terms of income alone. With a level of income per capita that is about 3.5 times the median per capita income worldwide, is by no means a poor country. It does have, however, a top 20 position in both life expectancy and in educational attainment. In educational attainment it is actually at the top. These attainments, *and a high enough income*, are sufficient for the *2010 HDI* to rank New Zealand as a top five country in terms of human development. The Chakravarty indices, on the other hand, give New Zealand a position that is between five and 24 positions lower than the new HDI does (depending on the different values for r), placing New Zealand (in terms of the countries in Table 1), behind (i) Qatar (for $r=0.9$); (ii) Hong Kong, Luxembourg, Singapore and The United Arab Emirates (for $r=0.5$ or 0.9); and (iii) Brunei and Kuwait (for $r=0.1$ or 0.5 or 0.9). The inescapable question then becomes: If the Human Development Index is intended to measure the size of the set of *possible combinations of functionings* from which the citizens of these countries can freely choose so that they are able to live rich, meaningful lives, are the rankings implied by the Chakravarty measures for these countries (versus, say, those produced by the *2010 HDI*) warranted?

Ultimately, that is for each reader to decide, and the point I am trying to make here is that these are the type of questions the final users of the *HDI* are most likely to ask before they decide if they accept the tradeoffs embedded in a given index of human development.

5. A Hybrid Methodology

We learned in Section 3 that the logarithmic transformation of income embedded in the calculation of the *2010 HDI* is about five times more important than its multiplicative structure in explaining the dramatic variations in the tradeoffs between life expectancy and income across countries. We then learned in Section 4 that the additive structure of the Chakravarty indices makes it possible for very rich countries to rank high in terms of human development even if they do poorly on any or all of the other dimensions. Thus, if we wish to produce an index of development that simultaneously produces sensible tradeoffs *and rankings* we can achieve that by keeping the index multiplicative, but with a normalization of income different from the logarithmic one that can be parametrized in such a way that it will produce sensible tradeoffs by design.

An example of a methodology consistent with these principles is espoused in Zambrano (2015), who proposes an index that satisfies Monotonicity, Subsistence, Independence, Scale and Aggregation Symmetry, as with the *2010 HDI*, but that replaces Partial Capabilities Growth with Proportional Capabilities Growth.

4′. Proportional Capabilities Growth: For h, h' in H, e, e' in E and y, y' in Y and "feasible" values for the growth factors $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}_+$ (such that $\lambda_1 \cdot h$ and $\lambda_1 \cdot h\prime$ are in H, $\lambda_2 \cdot e$ and $\lambda_2 \cdot e'$ are in E, and $\lambda_3 \cdot y$ and $\lambda_3 \cdot y'$ are in Y)

- $C_h(\lambda_1 \cdot h) - C_h(\lambda_1 \cdot h') = \lambda_1 \cdot [C_h(h) - C_h(h')]$,
- $C_e(\lambda_2 \cdot e\ ) - C_e(\lambda_2 \cdot e') = \lambda_2 \cdot [C_e(e) - C_e(e')]$ and
- $C_y(\lambda_3 \cdot y) - C_y(\lambda_3 \cdot y') = \lambda_3{}^r \cdot \left[C_y(y) - C_y(y')\right]$ for some $r \in [0, 1]$.

This assumption establishes that if health achievements grow by a fixed proportion this causes an increase in the growth of *health capabilities* by the same proportion. Similarly for education, regarding *education capabilities*. On the other hand, if income grows by a fixed proportion this causes an increase by a fraction, *r*, of that proportion in the rate of growth of *income capabilities*, the fraction being the same for all income levels.[29]

Given such $r \in [0, 1]$, there is only one formula consistent with all these principles:

$$I\left(C_h, C_e, C_y\right) = C_h^{\,1/3} \cdot C_e^{\,1/3} \cdot C_y^{\,1/3},$$

where $C_h(h) = \frac{h - h^o}{h^* - h^o}$, $C_e(e) = \frac{e - e^o}{e^* - e^o}$, and $C_y(y) = \begin{cases} \dfrac{y^r - y^{or}}{y^{*r} - y^{or}} & r \in (0, 1] \\ \dfrac{\log y - \log y^o}{\log y^* - \log y^o} & r = 0 \end{cases}$.

The proof is in Zambrano (2015).[30]

Notice that as *r* approaches zero the development index described above approaches the official UNDP formulation for the *HDI* adopted in 2010, case in which the partial capabilities measurement function for income takes the value $C_y(y) = \frac{\log y - \log y^o}{\log y^* - \log y^o}$. The *2010 HDI* is therefore a special case from the family of indices presented above.

5.1. *The Identification of "r"*

The principles identified above define a family of development indices, parametrized by a scalar, *r*. I call any such index a "Polity *HDI*." The reason is that the parameter *r* can be determined by asking questions to the public or the policy makers about their points of view regarding how policies that affect different dimensions of well-being contribute to human development and then determining the value of *r* that is consistent with those points of view. For example, one can ask the public or decision-makers a question like the following:

> Consider country "A", with Life Expectancy (h) and Income Levels (y) equal to the median of those variables worldwide (about 73 years and 7500 PPP dollars per capita, respectively). What percent of such annual per capita income do you believe people in such country should be inclined to

[29]Technically, the requirement is for the first two difference functions identified in the axiom to be linear homogeneous and for the third difference function to be homogeneous of degree $r \in [0, 1]$.

[30]It is noteworthy that in the function C_y just defined the concavification of income $f(x) = x^r$, $r \in (0, 1)$ is applied to the "raw" income achievements whereas in the Chakravarty formulation the same concavification is applied to the normalized income achievement $\frac{y - y^o}{y^* - y^o}$ so that the resulting function becomes $\left(\frac{y - y^o}{y^* - y^o}\right)^r$.

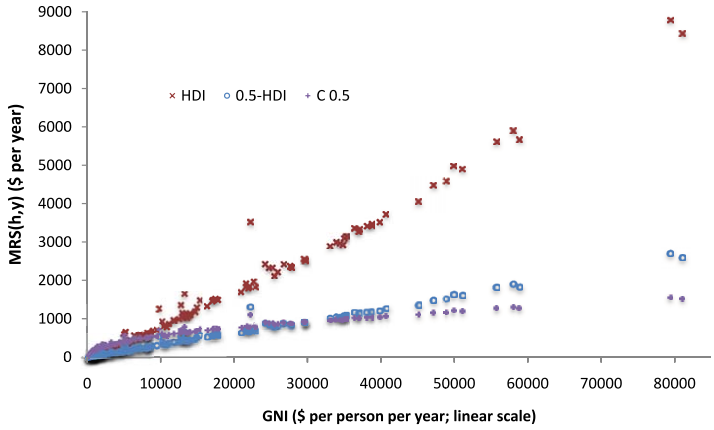

Figure 5. The tradeoff between life expectancy and income for three development indices
Source: Calculations by the author based on UNDP (2010) data.

sacrifice to gain a year in life expectancy and keep their level of human development constant?

Let M be the percentage of income that constitutes the answer to this question. It turns out that M uniquely determines r. For example, if $M \approx 3$ percent, then $r \approx 0.6$.[31]

5.2. *The Tradeoffs in Practice*

Figure 5 below is a depiction of how the marginal rates of substitution between health and income vary with income in the *2010 HDI*, in one of the Chakravarty indices adopted by Ravallion ($C_{0.5}$), and in a Polity *HDI* for *r=.5 (0.5-HDI)*. As expected, the tradeoffs between life expectancy and income are much more reasonable in the *0.5-HDI* than in the *2010 HDI*. In particular, they are much closer to the implied tradeoffs in the Chakravarty index than to those in the *2010 HDI*.

The near linearity of all these rates of substitution as a function of income can help us make this last point precise: The *2010 HDI* has marginal rates of substitution variations that are in the order of 10 percent of the income changes across countries, whereas the same calculation done for $C_{0.5}$ and *0.5-HDI* yield rates of substitution variations in the order of 2 percent and 3 percent of the income changes, respectively.[32]

[31]The value for r consistent with $M \approx 3$ percent satisfies the expression $M = \frac{1-\left(\frac{7500}{y_o}\right)^{-r}}{(73-h_o)}\frac{1}{r}$ for given values of $y_o = 100$ and $h_o = 20$, the ones used by the UNDP for the computation of the *HDI* in 2014. See Zambrano (2015) for details. That the parameters of a multidimensional development measurement function can be identified via tradeoff elicitation is also recognized in Chakravarty (2011a).

[32]These estimates come from regressing the marginal rates of substitution between health and income for all these three formulations as functions of income. In all three cases at least 90 percent of the variations in these rates of substitution across countries can be explained by per capita income variations, but the effect of income changes are, respectively, three and five times larger for the *2010 HDI* than for $C_{0.5}$ and *0.5-HDI*.

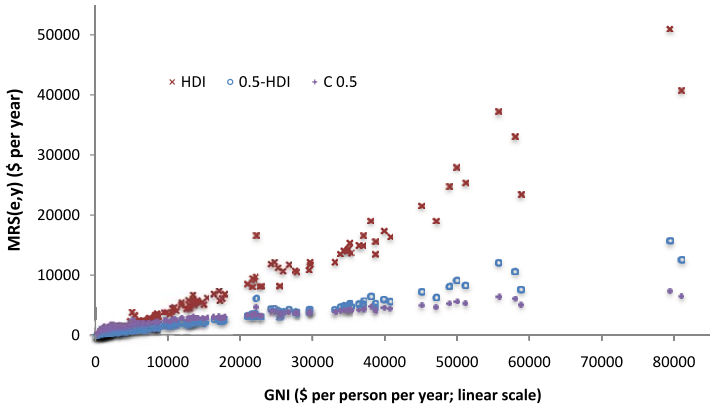

Figure 6. The tradeoff between education and income for three development indices
Source: Calculations by the author based on UNDP (2010) data.

Similarly, Figure 6 shows how the marginal rates of substitution between education and income vary with income in the same three development indices. Exactly the same pattern emerges: the tradeoffs between education and income are again much more reasonable in the *0.5-HDI* than in the *2010 HDI*, and they are much closer to the implied tradeoffs in the Chakravarty index than to those in the *2010 HDI*. The *2010 HDI* has marginal rates of substitution variations that are in the order of 50 percent of income changes across countries, whereas the same calculation done for $C_{0.5}$ and *0.5-HDI* yield rates of substitution variations that are in the order of 8 percent and 16 percent of income changes, respectively.[33]

Figures 5 and 6 thus reveal that the rates of substitution in all three indices are greatly affected by the vast income differences existing among rich and poor countries, and that those for $C_{0.5}$ and *0.5-HDI* are affected to a far less extent than those for *2010 HDI*.

5.3. *0.5-HDI* Rankings

The rank correlation coefficients between the *2010 HDI*, $C_{0.5}$ and *0.5-HDI* are very high.[34] These indices agree considerably in how they rank countries, as Figure 7 reveals.

The question remains: when they differ, which one produces more sensible rankings of human development and capabilities? We saw in Section 4.1.1. that the case can be made that the *2010 HDI* produces more intuitive rankings that the Chakravarty indices. Can the same be said about a comparison between the *2010 HDI* and the *0.5-HDI* rankings?

[33]These estimates come from regressing the marginal rates of substitution between education and income for all these three formulations as functions of income. In all three cases at least 91 percent of the variations in these rates of substitution across countries can be explained by per capita income variations, but the effect of income changes are, respectively, three and six times larger for the *2010 HDI* than for *0.5-HDI* and $C_{0.5}$.

[34]With Spearman rank correlation coefficients all greater than 0.99 and Kendall rank correlation coefficients all greater than 0.93.

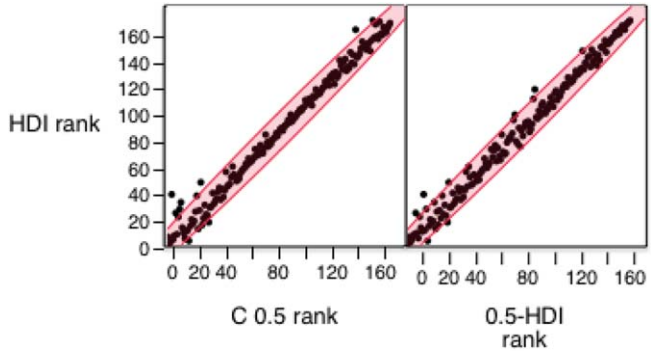

Figure 7. 2010 HDI, $C_{0.5}$ and *0.5-HDI* Rank scatterplot
Source: Calculations by the author based on UNDP (2010) data.

Table 2
The Rankings Of Selected Countries According To 0.5-Hdi

Hong Kong	10
Luxembourg	7
Singapore	15
United Arab Emirates	21
Brunei Darussalam	27
Qatar	13
Kuwait	32
Botswana	82
Equatorial Guinea	97
New Zealand	16
Liberia	165
Zimbabwe	169

Source: Calculations by the author based on UNDP (2010) data.

Consider again the countries in Table 1. Those were the few countries for which the *2010 HDI* and the Chakravarty Indices substantially differed. Table 2 complements Table 1 in that it presents rankings for these same countries according to the *0.5-HDI*. Table A in the Appendix presents the full *0.5-HDI* ranking.

First, consider New Zealand, which ranks as a top ten country according to the *2010 HDI*. The effect of income on human development is not as heavily discounted in *0.5-HDI* as it is in the *2010 HDI*, especially for medium-to-high income countries.[35] As a consequence, two of the top ten countries in terms of income that were not in the top 10 in the *2010 HDI* ranking are now top 10 countries according to the *0.5-HDI* (Luxembourg and Hong Kong) and four of the top ten countries in the *2010 HDI* ranking, who also happen to be high-income countries, improve their position within the top 10 according to *0.5-HDI*. All this causes New Zealand to place outside of the top ten in the *0.5-HDI* ranking. It nevertheless remains a top 20 country according to this index (ranks 16th). With

[35]Again, this is due to the adoption of a less concave normalization of income in *0.5-HDI* than in the *2010 HDI*, which uses the logarithmic normalization.

its ranking in terms of health, education and income of 15, 1 and 33, respectively, this is perhaps as it should be.[36]

Now consider the last two countries in Tables 1 and 2. For these two countries the *0.5-HDI* ranking is very similar to the one produced by *2010 HDI*. The reason is the following: Income for these two countries is so close to the subsistence level that more favorable rankings in life expectancy and education are not enough to compensate for this fact.[37]

Finally, let's look at the first nine countries in Tables 1 and 2. These countries rank better according to *0.5-HDI* than according to the *2010 HDI*. This is, again, because the effect of income on human development is not as heavily discounted in *0.5-HDI* as it is in the *2010 HDI*. This is also true with $C_{0.5}$, except that $C_{0.5}$ arguably goes too far in the other direction and allows more countries to place in the top 20 solely on the basis of a top 10 ranking in terms of income than the *0.5-HDI* does. Among those countries in Tables 1 and 2 this happens in the $C_{0.5}$ ranking for Luxembourg, the United Arab Emirates and Qatar, whereas this happens in the *0.5-HDI* rankings only in the case of Luxembourg.

6. Conclusions

The first goal of this paper is to document a curious fact: that among two competing indices for tracking the evolution of human development, the official UNDP methodology for the calculation of the *HDI*, and an alternative proposed by Ravallion (2012a), one produces more sensible country rankings and the other produces more sensible tradeoffs between the core dimensions of achievement employed in the computation of the indices. This is puzzling because there is a very close theoretical connection between rankings and tradeoffs as one can say that one conceptually determines the other, or vice versa. Why does this happen? The second (and main) goal of this paper is to explain why this is so.

On the one hand, the proposal by Ravallion, which is based on work by Chakravarty (2003), has the attractive feature that it has a free parameter, which can be calibrated so that the development index has tradeoffs that the public deems acceptable. On the other hand, it is an additive index, and treats all core variables symmetrically. These last two facts, in combination with the vast disparities in income across countries make it so that many countries will rank highly in terms of human development solely on the basis of their high per capita income, with little regard to how they perform in the other dimensions of well-being. This is viewed as unacceptable by many and it is, in essence, the reason why the Ravallion proposal produces unintuitive rankings.

The UNDP methodology, on the other hand, handles income very differently relative to how it treats health and education, in line with what the *HDI* is intended to measure: human development and capabilities. It is also not an additive index. These last two facts, when combined, rule out the kinds of unintuitive

[36]One can reach a similar conclusion under the $C_{0.5}$ ranking, although $C_{0.5}$ ranks the United Arab Emirates and Qatar over New Zealand, solely on the basis of their very high income, whereas *0.5-HDI* does so for Qatar alone.

[37]The *0.5-HDI* therefore gives them rankings that are very close to those given by the *2010 HDI* and that are between 13 and 22 spots lower in the development ranking than $C_{0.5}$.

rankings one observes not just in the indices considered by Ravallion ($r=.5$ and $r=.25$) but in any Chakravarty index (that is, for any value of r), as long as "raw" income is treated symmetrically with the other variables. But the UNDP drastically overshoots its correction of the importance of income for human development by adopting a logarithmic transformation as its way of "handling income differently." The implication is that, as a function of income, the *HDI* is steeper than any polynomial function of income at the subsistence income level and flatter than any polynomial function of income as income grows. This is, in essence, the main reason why the UNDP methodology produces unintuitive tradeoffs. Many blame the multiplicative structure of the *HDI* as the main culprit for the unintuitive tradeoffs, but this is not true. I have shown in Section 3.1. that 83 percent of the variations in the rates of substitution between health and income across countries can be attributed to the logarithmic normalization of income, rather than to the index's multiplicative structure.

The third goal of this paper is to provide an example of how one can address the problems identified above via the development of a hybrid methodology that adopts perspectives from both camps. On the one hand, it is multiplicative, therefore making it difficult for the world distribution of income to solely determine the human development rankings. On the other hand, it has a free parameter that can be calibrated to produce sensible tradeoffs by design. How the free parameter enters the scene must be handled with care, that is, with close regards to what the *HDI* is intended to measure. Because the functionings of being healthy and educated are *ends in themselves*, health and education capabilities grow with a change in health and education achievements in direct proportion to the absolute change in these achievements. Income, however, contributes to capabilities only instrumentally, that is, indirectly, and to a degree where differences of opinion can arise. Hence the (parametric) postulate: if income grows by a fixed proportion this causes an increase by a fraction, r, of that proportion in the rate of growth of *income capabilities*, the fraction being the same for all income levels. This formulation contains the *2010 HDI* as a special case ($r=0$).

While statisticians often prefer formulas without "free parameters," whether this is desirable or not actually depends on the purpose of the measurement exercise that is to take place. Leaving room in the methodology for input from the public to be factored in when assessing and monitoring human development addresses what is arguably Ravallion's main critique about most multidimensional indices of development, namely, that for the most part the discussions in policy circles often obscure the relevant tradeoffs between the core variables that are the ultimate drivers of the resulting rankings. This has the highly undesirable implication that it becomes difficult to understand what view of economic development one is implicitly adopting by accepting a particular ranking. I have shown in this paper an example of a methodology that addresses Ravallion's concerns, and that is broadly in line with the UNDP's reliance on Sen's capability approach. The methodology also happens to work rather well in producing both sensible rankings and tradeoffs, as documented in Sub-sections 5.2.−5.3.

While this does illustrate the strengths of this methodology, doing so has not been the main point of this paper. Rather, it has been to document how one can trace back the unsatisfactory implications, in terms of rankings and tradeoffs, of

any methodology to the particular normative principles responsible for the implications, and how one can use these findings in the development of more sensible methodologies.

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Supporting Information

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Table A: Rankings according to the *2010 HDI*, some Chakravarty Rankings, and the 0.5-HDI