

LIFE CYCLE PRICE TRENDS AND PRODUCT REPLACEMENT: IMPLICATIONS FOR THE MEASUREMENT OF INFLATION

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The paper explores the extent to which products follow systematic pricing patterns over their life cycle and the impact this has on the measurement of inflation. Using a large U.S. scanner data set on supermarket products and applying flexible regression methods, we find that on average prices decline as items age. This life cycle price change is often attributed to quality difference in the construction of CPI as items are replaced due to disappearance or during sample rotations. This introduces a systematic bias in the measurement of inflation. For our data we find that the life cycle bias leads to the underestimation of inflation by around 0.30 percentage points each year for the products examined.

JEL Codes: C43, D22, E31

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1. INTRODUCTION

One of the most distinctive features of modern economies is the extraordinary array of product varieties, brands, sizes, colors, editions, and flavors. But not every new product variety is destined to be a best seller, and product churn, along with choice, has also characterized this new economy. The rapid turnover in product varieties has been driven by technology in some areas—such as electronics, where old models are superseded by faster, smaller, and better models—but is also unequivocally apparent in less dynamic product categories. This process of product birth, evolution, maturity, and death is termed the product life cycle.

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This paper builds upon a modestly-sized literature addressing issues around the product life cycle. We focus on two issues in particular: first, identifying price trends as products mature; and second, examining the implications of these life cycle price trends on matched-model price indexes as disappearing items are replaced with new items.

Much of the interest in life cycle pricing is focused on the price dynamics as products enter and exit the market. For example, retailers adopting a “price skimming” approach may sell their new products at high prices to take advantage of the novelty factor, or they may sell them at low prices as suggested by the “market penetration” hypothesis (Kotler and Armstrong, 2011). Goods may exit the market at rock-bottom prices in order to clear shelf space for new items, or at relatively high prices in order to cater for market segments exhibiting strong preferences for old brands. Additionally, the prices of the product may systematically change over time for reasons related to technological advances, cost reduction, competition, firms’ pricing strategies, and many other reasons internal to the industry which are conceptually distinct from what is conventionally meant by quality differences. Hence, in a constant-utility cost-of-living index, these price changes over the product life cycle should be included in the measurement of inflation.¹

However, the life cycle price change is treated asymmetrically by statistical agencies in the construction of some matched-model price indexes. When a product is carried over from the current period to the next, the life cycle price difference is included, and correctly so, in the price index. But when a product disappears or is rotated out of the index, and is replaced by a new product, the life cycle price difference between the old and new products is often attributed, and this time wrongly, to a quality difference and hence is removed from the index. We argue that this systematically biases the measurement of inflation.

To see this consider a simple example. Suppose we have two items, J and K , and the price of the two items differs by 10 percent in the period when both prices are observed. We suppose that 30 percent of the price difference between the items is due to quality difference and the remaining 70 percent is due to the life cycle price difference. We suppose that these items live for three periods; their prices are constant in the first two periods and fall in the final period of their lives. This is illustrated in Figure 1a. The problem is that the 10 percent relative price difference between items J and K in period 3, as K replaces J in the sample, is wholly attributed to quality difference. As a result, in period 3 the price for K would be adjusted down multiplicatively by the relative price difference of 10 percent and then this adjusted price m (at 90) would be fed into the index. This is shown by the dashed line in Figure 1b. However, it is clear that the correctly adjusted price is m' (at 97) and the quality adjustment is overdone by $m'm$ percentage points. The

¹The life cycle price trends may arise due to inter-temporal price discrimination where monopolists charge different prices over time in order to maximize profits (Stokey, 1979). This result has been extended by others who have found that it may be optimal to discriminate price across time if consumers are less patient than sellers or if different types of consumers can be separated by charging a premium when selling to those who have the strongest desire for the new variety (Landsberger and Meilijson, 1985; Varian, 1989; Koh, 2006). However, these results apply primarily to durable goods.

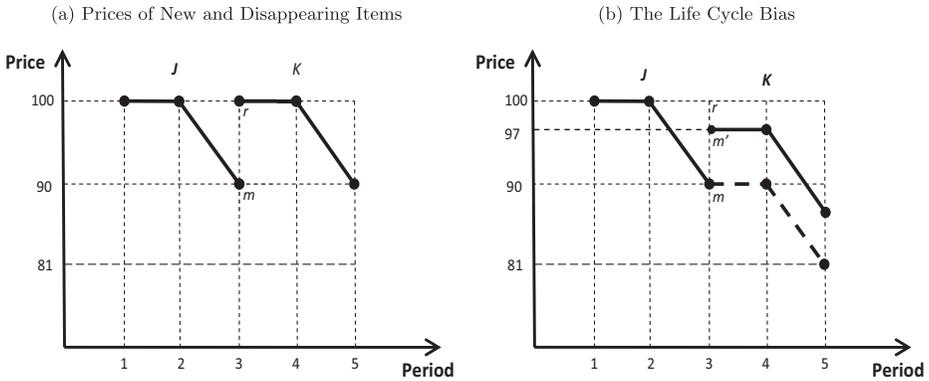


Figure 1. Quality Adjustment Using the Overlap Price Method

distance $m'm$, as we define, is the life cycle bias and in this case leads to an index which is biased downward by 7 percentage points.²

While the possibility of systematic life cycle effects on price indexes has been recognized in the measurement literature, there has been little quantification of this phenomenon. There are a few notable exceptions. Berndt *et al.* (1996), while investigating the effects of patent expiration and the entry of generic producers on the price of prescription drugs, find that the entry prices of new generic products tend to be lower than the prices of their patented antecedents. They conclude that the non-priced quality changes in the new generic products and how these new products are linked to their patented antecedents are potential sources of bias in official indexes. Berndt *et al.* (2003) find that prices for the established branded varieties tend to rise after patents expire as a select group of consumers with a strong preference for the branded variety remain willing to pay a premium for it. In other work, de Haan (2004) outlined a hedonic regression model which allowed for the systematic effects of entering and exiting varieties, though he did not proceed to estimate such a model. Silver and Heravi (2005), in a hedonic regression framework, show that indexes estimated only on matched products are biased because of sample degradation and systematic differences in the quality-adjusted prices of new, old and continuing items.

Given the paucity of empirical evidence, there has been speculation about the likely path of prices as products age. For example, a passage from the ILO (2004) CPI Manual argues that:

It may be that the prices of old items being dropped are relatively low and the prices of new ones relatively high, and such differences in price remain even after quality differences have been taken into account (Silver & Heravi, 2002).

²This method, which the ILO (2004) terms the overlap price method, is applied when the prices of both the new and disappearing items are available in the same period. When an overlap is not available, a common procedure is to extrapolate the price of the disappearing item forward to artificially create an overlap and attribute the price difference to quality change. This leads to just the same sort of error as the one that occurs in the overlap pricing method.

For strategic reasons, firms may wish to dump old models, perhaps to make way for the introduction of new models priced relatively high. (ILO, 2004, p. 100)

Some other studies have supported this view—that new goods are relatively highly priced and old goods are cheaper even after adjusting for quality difference (see Silver and Heravi, 2001; Schultze and Mackie, 2002, p. 162; Pakes, 2003; Triplett, 2006, p. 30). However, there remains little work in the literature quantifying such a phenomenon and particularly how this life cycle price difference influences the measurement of inflation. The studies, such as Boskin *et al.* (1997) and Lebow and Rudd (2003), which provide estimates of the overall bias in the CPI, did not explicitly take account of life cycle bias. In contrast, a great deal of attention has been given to identifying and quantifying quality change bias in the CPI (see, for example, Aizcorbe *et al.*, 2003; Pakes, 2003; Aizcorbe and Pho, 2005; Hill and Melsner, 2008; Bils, 2009; Broda and Weinstein, 2010; Erickson and Pakes, 2011; Greenlees and McClelland, 2011; Fox and Melsner, 2014).

Using data on 29 supermarket product categories included in the Dominick's scanner data set, we find that the life cycle component of price change is indeed significant. While there is variation in the size and direction of these life cycle pricing effects, after controlling for other inflationary factors that cause prices to change over time, we find that prices fall over their life for around two-thirds of our product categories. The average price change over the life of an item across the 29 supermarket products is a fall of around 3.93 percent. Given that the average age of items in our data is 22.6 months, this implies that prices on average fall by around 2.09 percent per year due to ageing. We estimate that ignoring this life cycle price difference at replacement leads to an annual downward bias of around 0.30 percentage points in the measurement of inflation for these products. Interestingly, the life cycle bias affects the measurement of inflation in the direction opposite to the quality change and substitution biases.

The paper is organized as follows. Section 2 describes our data and how the life cycle variables are identified. Section 3 introduces the regression models that are estimated in order to separate life cycle pricing patterns from the myriad of other effects that determine the price of a product. Section 4 discusses the regression results and the estimated life cycle functions. Section 5 examines the implications of the estimated life cycle price trends for matched-model price indexes. Finally, Section 6 provides a summary of the findings and draws some conclusions.

2. DATA AND EXTRACTING LIFE CYCLE VARIABLES

This paper investigates pricing patterns over the product life cycle using a large scanner data set for supermarket products sold at the *Dominick's Finer Foods* chain of food stores in and around the Chicago area.³ We study all 29 product categories included in the data. There are 96 stores with prices for hundreds of items for each product category recorded at a weekly frequency from September

³The data set is available at <http://research.chicagogsb.edu/marketing/databases/dominicks/index.aspx>.

TABLE 1
CONSTRUCTING THE LIFE CYCLE VARIABLES

Items	Period (valid observation [O], missing obs. [-])												Observed [†]	
	1	2	3	4	5	6	7	8	9	10	11	12	a_{it}	d_{it}
A: Birth & death unknown	O	O	O	O	O	O	O	O	O	O	O	O	X	X
B: Death unknown	-	-	-	-	O	O	O	O	O	O	O	O	✓	X
C: Birth unknown	O	O	O	O	O	O	O	O	O	-	-	-	X	✓
D: Birth & death known	-	-	-	O	O	O	O	O	-	-	-	-	✓	✓

[†] a_{it} and d_{it} refer to age and reverse age, respectively.

1989 to May 1997—a period of almost eight years (though not all the products are available for the entire sample).⁴ We aggregate the data across stores to monthly unit values, as monthly is the most common calculation frequency for CPIs globally. The long coverage of the data set—the longest publicly available scanner data set known to the authors—provides the richness required to extract and estimate the age-related price effects.

While “product life cycle” is a relatively familiar term in marketing and economics, there has been little explicit specification of exactly how it is characterized. A natural approach is to identify the state of the life cycle with reference to an item’s *age*—the time since a product first appeared on the market. While age is indeed a key characteristic of an item’s life cycle, what is also relevant is the number of periods from the current period until an item disappears from the market. We refer to this as *reverse age*. Note the identity: *age* + *reverse age* = *length of life*. Therefore, we need only two of these three variables in order to fully define the stage of an item’s life cycle. The reverse age is potentially important because if there are specific price dynamics associated with the end of life, such as run-out sales near product death, then this can be linked with reverse age. In the data set, items with different length of life appear and disappear throughout time. This implies that in a given time there are a range of items at different points in their life having different values for the age and reverse age variables. Examining both age and reverse age implies a symmetric and balanced treatment of the product life cycle and gives us the ability to model price dynamism throughout the life cycle.⁵

The extraction of the life cycle variables from the data is not straightforward. Table 1 illustrates the problem faced for different types of items. For items of type *A*, which are available in the market in both the first and the last period of the sample, we are unable to identify either their birth or death. Consequently, we cannot determine the age and reverse age of these items. The *B* items were available from the beginning of the sample period and, therefore, their first appearance in the market is unknown. However, since these items disappeared before the end of the sample, we can identify the death of these items. The situation for the *C* items is just the opposite, where the age can be determined but reverse age cannot. Finally, in the case of the *D* items both the age and reverse age can be determined.

⁴An item is identified by a unique Universal Product Code (UPC), referring to a unique bundle of characteristics, e.g. “a 14 ounce can of Coca Cola available in a 6-pack.”

⁵See Erickson and Pakes (2011) for a discussion on the relationship between product prices and exit decisions.

TABLE 2
LIFE CYCLE STATISTICS

Products	Expenditure Share across Products (%) ¹	Months of Data	Number of Items	Percentage of Items in Type: [*]				Average Expenditure Share (%) of Items in Type:				D-type Items				Life Span (months)									
				A		B		C		D		A		B			C		D		No. of Obs.	% Disappear by (years)		Median	Std.
				A	B	C	D	A	B	C	D	A	B	C	D		2	4	6**						
Analgiesics	1.4	100	587	16.9	29.8	21.6	31.7	36.9	34.9	11.6	16.6	4.973	46.2	86.0	98.4	27.5	17.5								
Bath Soap	0.1	69	491	6.5	44.8	16.1	32.6	21.9	49.0	12.1	17.1	2,521	65.6	98.5	—	18.5	11.3								
Beer	3.7	78	680	6.2	29.9	31.9	32.1	7.8	11.1	53.3	27.8	4,214	63.8	95.0	100.0	18.5	14.2								
Bottled Juices	3.6	100	465	11.6	25.0	16.1	47.3	27.9	30.5	17.1	24.6	4,946	69.1	88.6	98.2	18.0	17.7								
Cereals	10.3	100	430	21.2	28.8	21.9	28.1	40.5	31.1	15.8	12.5	2,448	59.5	97.5	98.2	19.0	17.9								
Cheeses	8.8	100	615	19.4	31.9	19.2	29.6	46.6	27.5	11.9	14.0	3,691	66.5	90.7	98.9	18.0	16.2								
Cigarettes	1.8	100	700	12.1	63.6	3.7	20.6	16.6	46.3	35.4	1.6	2,599	55.6	82.2	95.8	20.5	21.0								
Cookies	4.6	100	1,038	10.3	22.1	27.7	40.0	27.4	29.9	18.6	24.1	8,761	53.5	89.6	98.6	22.0	16.6								
Crackers	1.3	100	300	18.0	28.0	27.3	26.7	35.6	41.4	8.9	14.1	1,522	66.3	96.5	98.8	19.0	13.2								
Canned Soup	3.3	100	414	19.3	17.9	33.1	29.7	29.6	19.8	25.3	25.3	4,329	30.9	73.2	98.4	35.0	18.7								
Dish Detergent	1.7	100	264	13.3	27.7	22.4	36.7	19.6	25.7	29.7	25.0	2,395	56.7	84.5	99.0	20.0	19.6								
Front-end-candies	1.4	100	436	14.7	24.5	26.2	34.6	38.2	29.4	14.1	18.3	3,644	54.3	90.7	99.3	22.0	16.7								
Frozen Dinners	1.5	65	231	13.9	27.3	24.2	34.6	26.6	38.3	16.4	18.8	1,634	73.8	90.0	—	20.0	13.4								
Frozen Entrees	5.3	100	820	10.2	31.2	21.6	37.0	22.1	37.3	17.9	22.6	6,909	61.7	89.8	97.7	21.0	17.1								
Frozen Juices	2.6	100	166	28.9	23.5	13.86	33.7	59.2	16.5	10.1	14.2	1,334	53.6	85.7	96.4	23.0	20.8								
Fabric Softeners	1.7	100	298	7.1	36.9	20.8	35.2	8.2	33.9	31.8	26.0	2,814	52.3	79.1	98.1	23.0	18.3								
Grooming Products	1.6	69	1,225	15.4	38.7	19.4	26.5	33.0	48.5	8.8	9.8	4,369	64.0	96.3	—	18.0	12.5								
Laundry Detergents	5.5	100	540	1.5	24.1	25.2	49.3	0.7	29.9	29.4	40.0	7,133	54.9	82.3	97.7	23.0	19.3								
Oatmeal	1.1	78	92	35.9	20.7	21.7	21.7	63.5	20.6	8.8	7.2	497	50.0	100.0	—	24.5	10.9								
Paper Towels	2.4	100	136	7.4	23.5	14.7	54.4	15.6	35.6	21.5	27.4	1,623	43.2	63.0	98.7	29.0	24.2								
Refrigerated Juices	5.7	100	207	14.0	37.2	15.0	33.8	41.1	35.2	21.5	12.5	1,647	55.0	86.4	94.3	20.0	19.9								
Soft Drinks	18.0	100	1,470	12.8	42.2	13.3	31.7	33.3	47.2	9.9	9.6	9,527	54.1	89.7	98.7	23.0	16.5								
Shampoos	1.7	69	2,571	12.1	31.12	29.5	27.2	27.0	40.9	19.6	12.5	10,186	50.9	97.0	—	24.0	12.2								
Snack Crackers	2.6	100	381	13.9	28.9	21.3	36.0	33.1	33.4	13.5	20.0	3,264	54.7	87.0	96.4	21.0	18.7								
Soaps	1.5	70	306	15.7	42.8	19.6	19.6	33.4	39.3	13.1	14.2	1,458	51.0	95.5	—	24.5	13.2								
Toothbrushes	0.4	100	438	5.9	29.7	29.5	34.9	12.3	40.4	28.6	18.6	3,232	58.2	88.5	96.1	20.0	19.8								
Canned Tuna	1.9	100	260	17.3	34.6	30.8	17.3	40.4	35.3	18.2	6.1	1,461	36.0	78.9	95.6	35.0	20.7								
Toothpastes	1.1	100	567	7.9	36.0	22.2	33.9	21.0	44.3	13.4	21.3	4,382	48.4	92.2	96.9	25.0	16.4								
Bathroom Tissues	3.4	100	114	14.0	35.1	15.8	35.1	28.6	33.6	17.6	20.3	1,014	52.5	92.1	100	23.5	15.0								
Average All Items [‡]	100.0	93.0	16,242	13.9	31.6	21.7	32.8	29.2	34.0	18.8	18.0	108,527	55.3	88.2	97.8	22.6	16.9								

Notes: ¹Share of each product in the total expenditure on all product categories. ²See Table 1 for definitions of different types of items. ³—implies “not applicable” because the length of sample is smaller. ⁴The figures for “Expenditure Share across Products (%)”, “Number of Items”, and “No. of Obs.” are summations while the others are averages across products.

These are the items which appear in, and disappear from, the market within the coverage of the sample period and, therefore, their whole life cycle is observed. The inability to calculate age and/or reverse age for all items led us to run different versions of our empirical model based upon different combinations of the *B*, *C*, and *D* items. This is discussed in more detail in the next two sections.

There are two further data issues to be addressed. First, we chose 3 months as the minimum life length and all products with a shorter life-span were dropped. Second, because an item may be temporarily out of stock, or it may not sell and hence would not be observed in our data, we must be careful in defining when an item appears and disappears relative to the start and end of our data set. We allowed for a buffer of 3 months at the start and end of our data where no products were assumed to appear or disappear. We also considered a minimum life length and buffer of 6 months, but this did not alter the main thrust of the results, particularly in terms of their implications for quality adjustment and the measurement of inflation.

Our data are summarized in Table 2. Around 13.9 percent of items are of the *A* type, that is, items where both age and reverse age cannot be identified. On the other hand, 32.8 percent of items are *D*-type items, that is, the items whose both age and reverse age can be identified. Of these *D*-type items we find that more than 88.2 percent of them disappear within a period of 4 years, which is much lower than the sample coverage of 8 years and explains why there are a relatively small number of long-lived items (97.8 percent of items disappear within 6 years). However, the long-lived items account for a larger share of expenditure, with 29.2 percent of the monthly expenditure for all products on average. The average monthly expenditure shares for the *B* and *C* items is 52.8 percent, it is 18.0 percent for *D* items. We find that the average of the median life span of the 29 products is 22.6 months. However, there is considerable dispersion in the distribution of product length of life, both within and across product categories. The median life of the products ranges from 18 months (bottled juices, cheeses, and grooming products) to 35 months (canned soup and canned tuna). The wide variation in the age profile of items allows us to obtain a comprehensive picture of the age effect on price movements.

3. MODELLING LIFE CYCLE PRICE TRENDS

Using the data sets outlined in the previous section we proceed to estimate the age effect on prices for our products. We specify different models depending on whether the data contain the age and/or reverse age of the items. The models take the natural logarithm of prices as the dependant variable and control for the cross-sectional and time-series variation in the prices of items using fixed effects. We include dummy variables for items to control for cross-sectional price differences. These dummy variables reflect the difference in price for the quality-related features of the products. For example, in the case of cigarettes, the item-dummies will reflect the difference in price related to factors such as packet size, nicotine content, packaging, and so forth. We include dummy variables for time periods to control for product-wide temporal variations in price. This gives a model which is

the temporal variant of the widely used country-product dummy (CPD) method due originally to Summers (1973). Here, however, we have added a life cycle function.⁶

More formally let us define dummy variables for varieties (z_{nj}) such that $z_{nj} = 1$ when $n = j$ for item n and zero otherwise, and for time periods (x_{ts}) such that $x_{ts} = 1$ when $t = s$ for time t and zero otherwise, and leave the life cycle function, $f(a_{nt}, d_{nt})$, general at this stage. This gives us our basic model, with a mean zero error term (e_{nt}) appended:

$$(1) \quad \ln(p_{nt}) = \sum_{j=1}^N \beta_j z_{nj} + \sum_{s=1}^T \delta_s x_{ts} + f(a_{nt}, d_{nt}) + e_{nt}, \quad n = 1, \dots, N; t = 1, \dots, T.$$

With regard to $f(a_{nt}, d_{nt})$, let us begin by supposing that it takes the following form:

$$(2) \quad f(a_{nt}, d_{nt}) = \alpha I_a \log(a_{nt}) + \gamma I_d \log(d_{nt}).$$

Here a_{nt} and d_{nt} denote the age and reverse age of item n in period t , I_a (I_d) denotes the indicator function taking the value of 1 if the model includes a_{nt} (d_{nt}). Note that though the time dummy and age are linearly dependent, there is no such dependence problem in equation (1) because $f(a_{nt}, d_{nt})$ is specified in a non-linear (logarithmic) form. The life cycle function has the interpretation of age and reverse age acting as depreciation/appreciation factors upon price.⁷

However, the log-linear functional form places a great deal of a priori structure on a potentially very complex empirical relationship. A flexible alternative is to model $f(a_{nt}, d_{nt})$ non-parametrically, inserting dummy variables for each unique value of a_{nt} and d_{nt} . However, this is not fully identified and will place too few restrictions on the function, meaning that the results are likely to be driven by sampling variability rather than the underlying data generating process. It seems reasonable to impose some continuity restrictions on the life cycle function, as pricing effects are likely to change relatively slowly. For example, the price of a good of age 5 is likely to be more similar—after controlling for other factors—to the price of a good of age 4 and 6 than, say, age 25.

A natural approach, given these imperatives, is a smoothing spline regression with individual functions for age and reverse age. Here the functions themselves are left completely general except that we penalize for rapid changes in their curvature. Consider the penalized smoothing problem shown below where we specify a spline function for each of the age variables:

$$(3) \quad \min_{\beta, \delta, f_a, f_d} \sum_{n=1}^N \sum_{t=1}^T \left[\log(p_{nt}) - \sum_{j=1}^N \beta_j z_{nj} - \sum_{s=2}^T \delta_s x_{ts} - f_a[\log(a_{nt})] - f_d[\log(d_{nt})] \right]^2 + \lambda_a \int [f_a''(v)]^2 dv + \lambda_b \int [f_d''(v)]^2 dv.$$

⁶The CPD or its variants are widely used in different contexts. See, for example, Prasada Rao (2004), Diewert (2005), and de Haan and Krsinich (2014).

⁷The papers on price indexes which include age within the hedonic regression framework include Berndt *et al.* (1995), Cole *et al.* (1986), and Lee *et al.* (2005).

The first objective of the optimization is fidelity to the data. In addition, we add a penalty for rapid changes in the curvature of the functions reflected in the integral over the squared second derivative of f_a and f_d . The smoothing parameters, λ_a and λ_b , represent the relative weights that are given to fidelity and smoothness. As $\lambda_a, \lambda_b \rightarrow \infty$ the selected functions will have no second-order curvature. This implies that the estimators are linear, that is, $f_a(a_{nt}) \rightarrow \alpha \log(a_{nt})$ and $f_d(d_{nt}) \rightarrow \gamma \log(d_{nt})$. It can also be seen that the spline smoothing model nests the non-parametric dummy variable approach as $\lambda_a, \lambda_b \rightarrow 0$. The smoothing parameters themselves are chosen using the generalized cross validation approach (GCV) of Craven and Wahba (1979). This is similar to the standard cross validation approach but tends to be more robust to outliers.

4. MODEL RESULTS AND LIFE CYCLE PRICE TRENDS

Our a priori preference lies with the models where both age and reverse age are included, requiring the use of D -type items only. We denote these models by $D(a, d)$ and $D(a_s, d_s)$ for log-linear and spline models, respectively. Turning to the log-linear model first, Table 3 shows the estimated coefficients of age ($\hat{\alpha}$) and reverse age ($\hat{\gamma}$) of equation (2) for our 29 products. Here $\hat{\alpha}$ is negative for 22 products (14 are significant at the 5 percent significance level) and positive for 7 products (2 are significant), and $\hat{\gamma}$ is positive for 17 products (12 are significant) and negative for 12 products (6 are significant). At least one of the coefficients between $\hat{\alpha}$ and $\hat{\gamma}$ is significant for 25 products (the products where both the coefficients are insignificant are bath soaps, cheeses, cookies, and soaps). The average of $\hat{\alpha}$ is -0.0071 which corresponds to a monthly change in prices of approximately -0.71 percent (-1.24 percent for the significant coefficients) and the average of the $\hat{\gamma}$ is 0.25 percent (0.42 percent for the significant coefficients) (see Table 4). The F -test for the joint significance of a_{nt} and d_{nt} is significant for 21 product categories, providing support to the hypothesis that both age and reverse age are important determinants of price.

In the $D(a, d)$ model, a negative age coefficient combined with a positive reverse age coefficient implies falling prices over the life span (this holds for 13 products). On the contrary, a positive age coefficient in combination with a negative reverse age coefficient implies a rising pricing pattern over the life cycle (holds for 3 products). In the cases where coefficients have the same sign, positive or negative, the resulting pricing pattern depends on the relative magnitude of the coefficients and the stage and length of the life of products. The results of the $D(a, d)$ models show that for most products prices fall as items age, indicating that the price skimming strategy is the dominant strategy followed by firms. The magnitude of the life cycle price effects is discussed more later.

The $D(a, d)$ results, however, may not be representative of all items as these models just use the D -type items (see Table 1 for the definition of different item types). In order to check whether the selection of items matters while estimating the age effects, we compare estimates of α and β obtained from different models using larger sets of data. First, we check whether the exclusion of B -type items makes any significant difference in the estimated age coefficients. For this purpose, we compare between $BD(a)$ and $D(a)$, where while both models include the age of

TABLE 3
REGRESSION RESULTS FOR THE LOG-LINEAR AND SPLINE MODELS

Products [†]	Estimated Coefficients Obtained from $D(a, d)$		F -stat.: Joint Significance of (a_{nt}, d_{nt}) in Models:		F -stat.:
	Age($\hat{\alpha}$)	Rev.Age($\hat{\gamma}$)	$D(a, d)$	$D(a_s, d_s)$	$D(a_s, d_s)$ vs $D(a, d)$
Analgesics	-0.0081***	0.0081***	24.95***	13.74***	9.91***
Bath Soap	0.0094	0.0011	1.15	11.09***	14.39***
Beer	0.0007	-0.0040**	2.79*	3.10***	3.20***
Bottled Juices	-0.0202***	0.0100***	47.07***	16.83***	6.64***
Cereals	-0.0249***	0.0158***	23.57***	8.60***	3.56***
Cheeses	-0.0015	0.0015	0.64	5.88***	7.62***
Cigarettes	0.0065***	-0.0041***	8.97***	7.22***	6.60***
Cookies	-0.0036	-0.0018	1.89	3.04***	3.43***
Crackers	-0.0115**	-0.0027	2.59*	5.37***	6.28***
Canned Soup	-0.0241***	0.0024***	40.25***	15.60***	7.27***
Dish Detergent	-0.0102***	-0.0018	5.02***	1.76*	0.68
Front-end-candies	-0.0116***	-0.0061***	9.78***	6.27***	5.08***
Frozen Dinners	-0.0021	0.0160***	4.22***	9.54***	11.26***
Frozen Entrees	-0.0185***	-0.0025	12.19***	8.13***	6.76***
Frozen Juices	-0.0154*	0.0060	2.48*	4.47***	5.12***
Fabric Softeners	-0.0094***	-0.0054***	5.85***	3.45***	2.64**
Grooming Products	0.0040	0.0012	0.82	2.53***	3.10***
Laundry Detergents	-0.0233***	0.0035*	41.82***	18.20***	10.22***
Oatmeal	-0.0276***	-0.0031	3.58**	4.31***	4.49***
Paper Towels	-0.0007	-0.0118**	5.53***	2.12**	0.98
Refrigerated Juices	-0.0144***	-0.0041	3.79**	2.26**	1.74
Soft Drinks	-0.0166***	0.0038***	25.24***	8.23***	2.55**
Shampoos	-0.0002	0.0095***	25.20***	11.83***	7.34***
Snack Crackers	-0.0007	0.0076***	8.93***	3.81***	2.10***
Soaps	0.0023	0.0025	0.31	0.80	0.97
Toothbrushes	0.0286***	0.0064**	18.69***	18.82***	18.64***
Canned Tuna	-0.0124**	0.0099***	6.16***	3.06***	2.01*
Toothpastes	-0.0055	0.0079***	7.86***	4.51***	3.38***
Bathroom Tissues	0.0058	-0.0164***	13.19***	12.17***	11.52***

Notes: *Significant at the 10% level, **5% level, ***1% level.

[†]The minimum adjusted R^2 of the regressions is 0.92.

items, the former includes both B - and D -type items and the latter includes only D -type items. The t -tests show that the differences in $\hat{\alpha}$ are not significantly different from zero for 19 products between the $BD(a)$ and $D(a)$ models. Similarly, we conduct sensitivity analysis to check whether the exclusion of C -type items makes any significant difference in the estimated reverse age coefficients. This is achieved by comparing the results obtained from $CD(d)$ and $D(d)$ models, where both models include the reverse age of items, and the former includes both C - and D -type items and the latter includes only D -type items. We find that the differences in the $\hat{\gamma}$ are not significantly different for 21 products between the $CD(d)$ and $D(d)$ models. Note that the $BD(a)$ and $CD(d)$ models include the maximum number of items where the age and reverse age are identified, respectively.

In the above set of comparisons, the data truncation allows us to compare models which include either the age or the reverse age of items. Therefore, we conduct a second set of sensitivity analysis where we compare these models with

TABLE 4
AVERAGES OF THE REGRESSION COEFFICIENTS OBTAINED FROM THE LOG-LINEAR MODELS*

Averages [†]	Age Coefficients ($\hat{\alpha}$) Obtained from the Model:			Reverse Age Coefficients ($\hat{\gamma}$) Obtained from the Model:		
	$BD(a)$	$D(a)$	$D(a, d)$	$CD(d)$	$D(d)$	$D(a, d)$
Simple: All coefficients	-0.0080 (0.0089)	-0.0086 (0.0113)	-0.0071 (0.0123)	0.0025 (0.0083)	0.0034 (0.0089)	0.0025 (0.0086)
Simple: Significant coefs	-0.0120 (0.0070)	-0.0154 (0.0095)	-0.0124 (0.0137)	0.0036 (0.0101)	0.0050 (0.0109)	0.0042 (0.0104)
Weighted: All coefs	-0.0098	-0.0127	-0.0117	0.0029	0.0047	0.0032

Notes: The figures in parentheses are the std deviation of the coefficients across products.

*For detailed results, see the online appendix.

[†]The second average is obtained from the products with significant coefficients (5% significance level) while the final weighted average uses the expenditure shares in column 2 of Table 2.

the model which includes both the age and reverse age variables. In particular, we conduct a *sign* test on the estimated age coefficients obtained from the $D(a)$ and $D(a, d)$ models and, similarly, on the estimated reverse age coefficients obtained from the $D(d)$ and $D(a, d)$ models (note that $D(a)$ and $D(d)$ models are common between the first and second sets of comparisons). If the signs remain the same, it would mean that the coefficients are stable and there is no systematic difference in the direction of the life cycle bias across different model specifications. Indeed, the tests show just this; that there are no significant differences in the paired $\hat{\alpha}$ and the paired $\hat{\gamma}$ coefficients. But the sign test has the disadvantage that it does not take into account the magnitude of the paired differences. It may happen that the magnitude in one direction is much higher than in the other direction, and in that case the sign test could give misleading results. Therefore, we conduct a Wilcoxon signed-rank test which takes into account both the direction and magnitude of the paired differences. The p -values of these tests are 0.46 and 0.37 for $\hat{\alpha}$ and $\hat{\gamma}$, respectively, thus indicating that these paired differences are random.⁸

The averages of the estimated coefficients across products are provided in Table 4. These averages show that the effects of life cycle variables on prices are similar across models and data sets. As a whole, the estimation of different models and the subsequent tests indicate that although there are differences for some products, the main thrust of the results in terms of the estimated life cycle price trends remains the same across different sets of items. The fact that the B -type items on average entered the market later than the D -type items, and the C -type items entered the market earlier than the D -type items, does not in general lead to different—after controlling for inflation—life cycle pricing patterns.

⁸The null hypothesis of the sign test states that for a random pair of measurement (x_i, y_i) , x_i and y_i are equally likely to be larger than the other. For our purpose, to conduct the test, we take the paired difference of the estimated coefficients, and replace each positive difference with a positive sign and each negative difference with a negative sign. When the null hypothesis of no difference is true, the sum of the positive signs is approximately equal to the sum of the negative signs. The test statistic W is binomially distributed, $W \sim \text{Bin}(n, 0.5)$, irrespective of the population F_x (see, for example, Garthwaite *et al.* 2002 for a description of the test).

However, note that none of the regressions above use the *A*-type items. For our purpose, *A*-type items are the least important set of items. This is because the *A*-type items do not appear in or disappear from the market within our sample, and therefore the question of forced substitution and the consequent life cycle bias occurring through the *A*-type items does not arise. The *A*-type items may, however, be replaced through the routine sample rotations followed by a few statistical agencies. But given that the *A*-type items account for a small portion of total items (13.9 percent in our data set), their contribution to the overall bias is expected to be small. This is in fact confirmed by the sensitivity analysis conducted in the next section.⁹

Having shown the robustness of the results obtained from the simple log-linear models, and using different data sets, we now move to the estimation of the spline models, $D(a_s, d_s)$. These spline models enable us to better capture the non-linearity of the age effect, if any, over the life cycle. The *F*-test of no life cycle price effect is rejected in this model for every product category except dish detergents and soaps (see Table 3). While both the log-linear and spline models are statistically significant for most products, *F*-tests support the more flexible spline model for 24 products. This leads us to use the results from the spline models in the next section to find the implications of the life cycle price changes on price indexes.

The life cycle pricing functions for the spline model corresponding to products living for 2 years are shown in Figure 2. The figure shows that the price falls for 18 products, rises for 8 products, and is unchanged or non-monotonic (U or inverse-U shaped) for 3 products. The $D(a, d)$ and $D(a_s, d_s)$ models exhibit life cycle price trends which are broadly similar for most products. The exceptions are the products where the curvature of the life cycle price trend obtained from the spline models is non-monotonic and changes its direction over the life cycle.

While the life cycle pricing effects are statistically significant, they are also of a magnitude that is economically meaningful. Since $D(a, d)$ and $D(a_s, d_s)$ models in general perform better than the other models, we look at the magnitudes of the age effect obtained from these models. The results are given in Table 5. Because of the inclusion of reverse age in the model, the age effect becomes conditional on the length of life of items. That is, once we fix the length of life, the age effect can be obtained from the life cycle price difference between two points in the life of items. For example, using the age function in equation (2), the difference in the log of prices at two ages is: $\log(p_n^*/p_n) = \alpha[\log(a_{nt}^*/a_{nt})] + \gamma[\log(d_{nt}^*/d_{nt})]$, where the asterisk denotes the later stage of life. Take, for example, an analgesic item, which lived for 12 months. We want to obtain an estimate of the life cycle price difference from the item's birth to its death. Substituting $\alpha = -0.0081$ and $\gamma = 0.0081$ (see Table 3 for the estimates of the age effects) and setting $(a_{nt}, d_{nt}) = (1, 12)$ and $(a_{nt}^*, d_{nt}^*) = (12, 1)$, we obtain $\log(p_n^*/p_n) = -0.0403$. This means that the price of this analgesic item, which lives for a year, falls by approximately 4.03 percent over

⁹The *A*-type items have larger expenditure shares compared to their proportions of the total items in the data. However, it is typical for statistical agencies to use a symmetric weighted index rather than an expenditure share weighted index at such a low level of item definition.

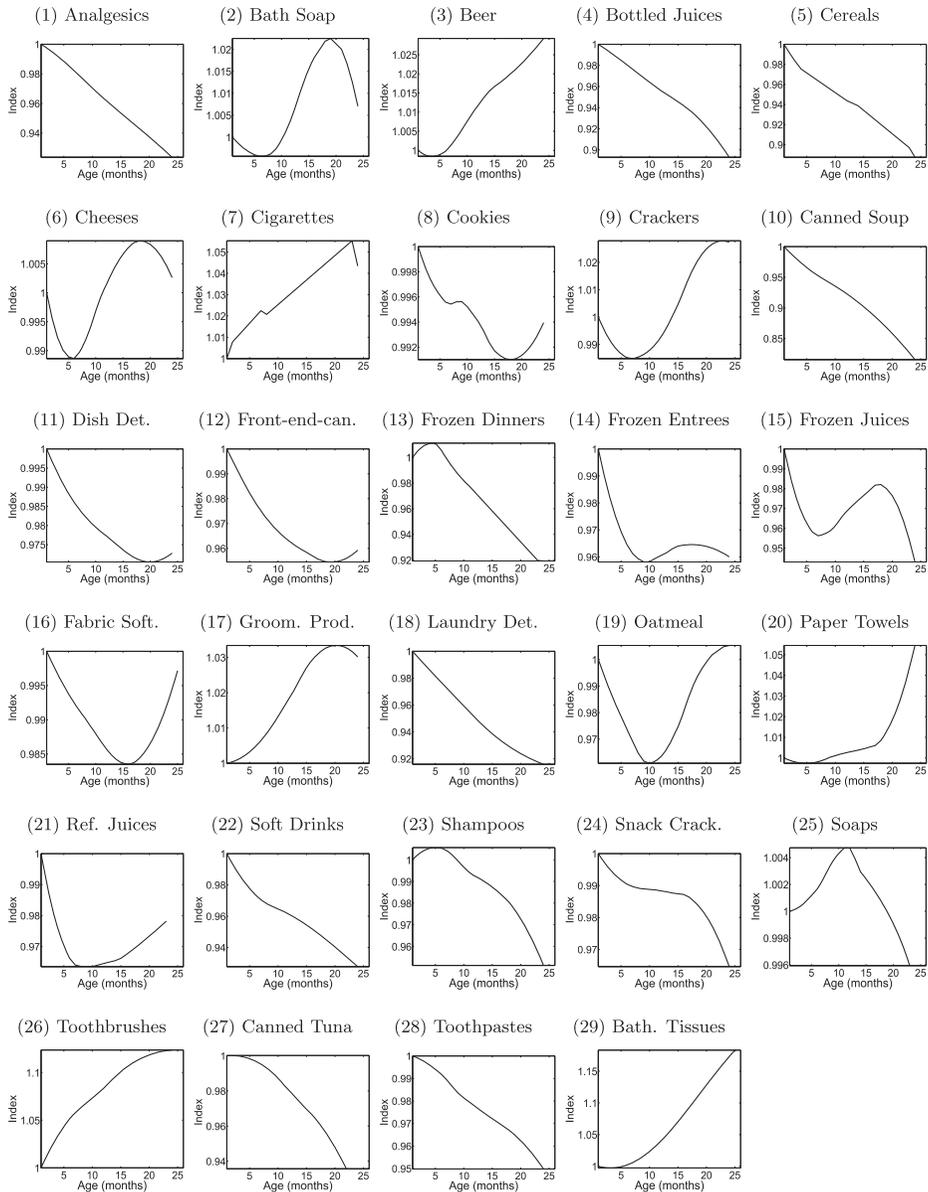


Figure 2. Life Cycle Price Trends (for a life span of 2 years for each product)

the course of its life. This fall in price takes place after controlling for price variations across different items in the product category (through the use of item dummies) and product-wide temporal variations in price (through the use of time dummies). Table 5 shows that the largest rise is seen for bathroom tissues while the largest fall is for canned soup.

TABLE 5
SUMMARY OF PRICE TRENDS OVER THE LIFE CYCLE (%)

Products	Log-Linear Model			Spline Model		
	Average Life Cycle Price Change of Items with Length of Life Equal to:			Average Life Cycle Price Change of Items with Length of Life Equal to:		
	1 year	3 years	Median [†]	1 year	3 years	Median [†]
Analgesics	-4.03	-5.81	-5.37	-4.81	-9.30	-8.24
Bath Soap	2.06	2.97	2.42	1.86	-2.58	1.98
Beer	1.17	1.68	1.37	1.53	3.48	2.45
Bottled Juices	-7.50	-10.82	-8.73	-7.80	-12.21	-9.54
Cereals	-10.11	-14.58	-11.98	-11.52	-9.87	-11.38
Cheeses	-0.75	-1.08	-0.87	-2.08	1.41	-0.95
Cigarettes	2.63	3.80	3.20	3.42	5.01	4.55
Cookies	-0.45	-0.65	-0.56	0.04	-0.66	-4.18
Crackers	-2.19	-3.15	-2.59	-0.81	3.48	1.71
Canned Soup	-11.95	-17.24	-17.10	-13.60	-20.04	-19.89
Dish Detergent	-2.09	-3.01	-2.52	-1.43	-3.21	-2.40
Front-end-candies	-1.37	-1.97	-1.70	-0.74	-6.03	-3.37
Frozen Dinners	-4.50	-6.49	-5.42	-7.41	-10.89	-8.07
Frozen Entrees	-3.98	-5.73	-4.87	-4.67	-4.29	-3.94
Frozen Juices	-5.32	-7.67	-6.71	-9.30	-6.12	-5.92
Fabric Softeners	-0.99	-1.43	-1.25	0.42	-0.72	-0.10
Grooming Products	0.70	1.00	0.81	0.76	3.84	2.18
Laundry Detergents	-6.66	-9.60	-8.40	-4.87	-9.63	-8.26
Oatmeal	-6.09	-8.78	-7.84	-0.37	1.47	0.73
Paper Towels	2.76	3.98	3.74	5.41	6.00	5.90
Refrigerated Juices	-2.56	-3.69	-3.09	-3.44	-2.37	-2.48
Soft Drinks	-5.07	-7.31	-6.40	-5.35	-11.25	-5.10
Shampoos	-2.41	-3.48	-3.08	-3.80	-4.44	-4.88
Snack Crackers	-2.06	-2.97	-2.53	-3.75	-6.24	-2.07
Soaps	-0.05	-0.07	-0.06	-0.34	-0.93	-0.23
Toothbrushes	5.52	7.96	6.65	5.85	19.38	10.30
Canned Tuna	-5.54	-7.99	-7.93	-6.00	-6.60	-6.50
Toothpastes	-3.33	-4.80	-4.31	-3.44	-5.91	-5.21
Bathroom Tissues	5.52	7.96	7.01	8.17	22.05	18.14
Averages: [‡]						
Simple	-2.37	-3.41	-3.04	-2.35	-2.32	-2.23
Weighted	-3.71	-5.35	-4.63	-3.99	-4.66	-3.93

Notes: [†]Median life length of each product is provided in Table 2 with the average being 22.6 months.

[‡]Weights for the weighted average are the expenditure shares of the products obtained using the full data set.

The magnitude of the age effects from the spline models can be obtained in a similar way. However, in the case of spline functions, unlike the log-linear functions, because of the curvature, the same age difference at two different stages of life may not produce the same life cycle price difference. Take, for example, the case of cheeses, where while the first six months of life produces a positive age effect, the next six months produces a negative age effect. Table 5 shows the magnitude of age effects for the products with a length of life of 1 year, 3 years, and the median life span of the product (median life span is provided in Table 2). For the spline models these age effects are obtained by taking the life cycle price

difference between the beginning and end of a product's life. The table shows that, on a weighted average basis, the fall in price due to life cycle pricing is 3.93 percent for all products living their respective median length of life. This corresponds to an annual fall in price of 2.09 percent given that the average of the median life span of products is 22.6 months.

In summary, the results provide statistically compelling and robust evidence—across models and data sets—for the existence of life cycle pricing effects. The results also show that both the physical age and the stage-of-life (reflected in reverse age) are important in adequately representing life cycle price movements. Moreover, allowing for the possibility of curvature in these effects, via spline functions, generally provides a better representation of the life cycle effect. A distinctive downward price trend was evident for around two-thirds of the products. This provides empirical support for the prevalence of the price skimming strategy—similar to that reported by Silver and Heravi (2005)—where producers and retailers take advantage of the novelty factor for new items to earn a premium upon introduction. While the downward sloping price trend is the dominant pricing pattern, the price trend is found to be upward sloping for some products. These products include: beer, cigarettes, crackers, grooming products, paper towels, toothbrushes, and bathroom tissues. This pricing pattern is likely to reflect the important part that taste and brand loyalty play in these particular markets. Here there is an apparent parallel with the results of Berndt *et al.* (2003) for prescription drugs following patent expiration. However, brand loyalty is likely to play a lesser role in the case of paper towels and bathroom tissues. Here it may be that the market penetration strategy prevails. Prices are set low at introduction to attract a large number of customers and gain the benefit of economies of scale.

5. IMPLICATIONS FOR PRICE INDEXES

5.1. *The Life Cycle Bias Formula*

In this section we focus on the impact of the life cycle pricing results for quality adjustment as items are introduced and removed from the index. We suppose that the object of estimation is the Jevons index for a product category. We selected a symmetric weighted index because the expenditure share information are not typically available at the low level of item definition such as ours. The Jevons index has good axiomatic properties and can also be justified on economic grounds (ILO, 2004), and hence is consistent with either a cost-of-living or cost-of-goods approach (Schultze and Mackie, 2002). In modifying our notation from earlier, we now use n to represent each component in the index, that is, each observation in the sample. In a given period this corresponds to a particular item, but as items disappear or are removed from the index the item which fills position n in the index may change. At any one time the sample is made up of the following index set of observations, $V = \{1, 2, \dots, N\}$. In comparing two periods, t and $t + 1$, these observations may be divided into two mutually exclusive sets: (a) those observations for which the same item is included, or matched, in both periods $n \in V_M$, and (b) those observations for which an item either disappears, $n \in V_D$, or is rotated out of the index, $n \in V_R$. When the item which corresponds to

observation n in time period t is different from that for period $t + 1$ we denote this by including an asterisk on the later-period price. This gives a price relative of the form $\frac{p_{nt+1}^*}{p_{nt}} = \frac{p(f_{nt+1}^*, z_n^*, x_{t+1})}{p(f_{nt}, z_n, x_t)}$, where f_{nt} denotes the point in the life cycle pricing, z_n the utility-determining quality characteristics, and x_t denotes pure inflationary factors. In this case, the price drivers—life cycle, quality, and pure inflationary factors—between the two items at position n are likely to take on different values. Hence an adjustment, κ_n , is required to ensure that the price for the new item is comparable to the old item.

Given the two sets of items we may decompose the index $P_{t,t+1}$ as follows:

$$(4) \quad P_{t,t+1}(\kappa) = \left[\prod_{n \in \mathcal{M}} \left(\frac{p_{nt+1}}{p_{nt}} \right)^{1/N} \right] \left[\prod_{n \in \mathcal{R} \cup \mathcal{D}} \left(\frac{p_{nt+1}^*/\kappa_n}{p_{nt}} \right)^{1/N} \right].$$

For those items which are being rotated into the index or disappear, and for which an overlap price exists, the quality adjustment is obtained from the ratio of prices in the common period, t . This is shown in the following, where we use equation (1) to illustrate the precise nature of the adjustment:

$$(5) \quad \hat{\kappa}_n = \frac{p_{nt}^*}{p_{nt}} = \frac{p(f_{nt}^*, z_n^*, x_t)}{p(f_{nt}, z_n, x_t)} = \frac{\exp(\beta_n^* + f(a_{nt}^*, d_{nt}^*) + \delta_t x_t)}{\exp(\beta_n + f(a_{nt}, d_{nt}) + \delta_t x_t)} \\ = \exp(\beta_n^* - \beta_n) \cdot \exp(f(a_{nt}^*, d_{nt}^*) - f(a_{nt}, d_{nt})).$$

The first term in $\hat{\kappa}_n$ expresses the quality difference between the new and disappearing items, where β^* and β are the estimates of the fixed effect corresponding to the new and disappearing item dummies obtained from equation (1). The new and disappearing items are likely to have different life cycle durations and be at different stages of their life, meaning that a_{nt}^* and d_{nt}^* differ from a_{nt} and d_{nt} , respectively. The second term provides an estimate of the life cycle price difference between the new and disappearing items. Thus the use of $\hat{\kappa}_n$ removes the life cycle price difference from the index.¹⁰

We now specify an alternative adjustment, which does not remove this important source of price change and where we compare the new and old prices only on the basis of their quality reflected in the z variable. By doing this we are instead including the life cycle price difference in the index. The adjustment is the following:

$$(6) \quad \tilde{\kappa}_n = \frac{p_{nt}^*}{p(f_{nt}^*, z_n, x_t)} = \frac{p(f_{nt}^*, z_n^*, x_t)}{p(f_{nt}^*, z_n, x_t)} = \exp(\beta_n^* - \beta_n).$$

Given $\hat{\kappa}_n$ and $\tilde{\kappa}_n$, it is easy to see that the life cycle bias in the quality adjustment for a particular item, denoted by Θ_n , is equal to:

¹⁰This requires making an assumption that $E(\exp(e_{nt})) = 1$, which is not the case. Possible corrections have been proposed by Kennedy (1981) and others. In our case, however, these corrections are small enough that they can be safely ignored.

$$(7) \quad \Theta_n = \frac{\hat{\kappa}_n}{\tilde{\kappa}_n} = \exp(f(a_{nt}^*, d_{nt}^*) - f(a_{nt}, d_{nt})).$$

Θ_n provides an estimate of distance $m'm$ shown in Figure 1b. Note that if the price skimming strategy is followed by firms (i.e., the entry price is higher than the exit price), then $f(a_{nt}^*, d_{nt}^*) - f(a_{nt}, d_{nt}) > 0$. Therefore, from equation (7), $\hat{\kappa}_n$ overestimates the quality difference between the new and disappearing items. On the other hand, if the market penetration strategy is followed (i.e., the entry price is lower than the exit price), then $f(a_{nt}^*, d_{nt}^*) - f(a_{nt}, d_{nt}) < 0$. In this case, $\hat{\kappa}_n$ underestimates the quality change between the new and old items.

Given that our target index is the Jevons index, we can find the average life cycle bias at replacement by taking the geometric mean of the bias incurred at each replacement:

$$(8) \quad \Theta = \left[\prod_{n \in V_R \cup V_D} \left(\frac{\hat{\kappa}_n}{\tilde{\kappa}_n} \right) \right]^{1/(N_R + N_D)} = \left[\prod_{n \in V_R \cup V_D} \Theta_n \right]^{1/(N_R + N_D)}.$$

Here N_R and N_D denote the number of rotated items and forced substitutions which are subjected to the life cycle bias, respectively. If, for example, Θ is 1.05, the quality adjustment at replacement is biased upward on average by 5 percentage points. The overall bias is equal to the average quality change bias (Θ) scaled by the weight of these items in the index, and is obtained as follows:

$$(9) \quad \Psi = [\Theta]^{(N_R + N_D)/N}.$$

Note that Θ and Ψ provide estimates of the life cycle bias on the quality adjustment. The life cycle bias in the measurement of inflation is obtained as follows:

$$(10) \quad \frac{P_{t,t+1}(\hat{\kappa})}{P_{t,t+1}(\tilde{\kappa})} = \left[\prod_{n \in V_R \cup V_D} \left(\frac{\tilde{\kappa}_n}{\hat{\kappa}_n} \right) \right]^{1/N} = \left[\prod_{n \in V_R \cup V_D} \left(\frac{1}{\Theta_n} \right) \right]^{1/N} = \left[\frac{1}{\Theta} \right]^{(N_R + N_D)/N} = \frac{1}{\Psi}.$$

This implies that life cycle bias will underestimate the measurement of inflation in our data, given that the estimated life cycle pricing pattern showed that the price skimming strategy is the dominant strategy followed by firms (see Figure 2 and Table 5).

A crude measure of the life cycle bias can be obtained from the estimated life cycle function in the following way. We showed that for the spline model the weighted average price fall across product categories was 3.93 percent for the items living their median life length (see Table 5). Now suppose that the average difference in the age-to-life ratio between the newly introduced and removed items is 0.50 (for example, for forced substitution it means that the age-to-life ratio of the new items is 0.50 and disappeared items is 1). This means that the life cycle price difference between the new and disappeared items is $3.93\% \cdot 0.50 = 1.97\% = \Theta$, which we argue is wrongly attributed to the quality difference. Let us suppose that for each month around 4 percent of sampled items are replaced through forced

substitutions and routine sample rotations. We further suppose that only one-fourth of these replaced items, that is, 1 percent of the sampled items, incur the life cycle bias. Then the annual bias at the product level is $1.97\% \cdot 0.01 \cdot 12 = 0.24\% = \Psi$. However, the magnitude of this estimate depends upon the age profile of old and replacement items as well as the particular shape of the life cycle function. We investigate a more precise estimation of this bias below.

5.2. *Simulation Evidence on Index Bias*

In order to obtain estimates of the life cycle bias in a particular product category, it is important to know what proportion of replacements in that product are undertaken using the overlap and imputation methods. Moulton and Moses (1997) provide information of their relative importance in the US CPI for 1983, 1984, and 1995 (see also Hulten, 1997). In 1995, the replacements undertaken through indirect methods—consisting of overlap and imputation methods which are the subject of our study—accounted for 24.1 percent of all forced substitutions (see Moulton and Moses, 1997, table 4). The corresponding percentage is 46.0 percent for the food and beverage items (many of the items in our data fall into this category). Using this information, we consider two scenarios with regard to the percentage of forced substitutions which we subject to the life cycle bias: $\Lambda = 40\%$ and 25% . We should note that the replaced items in any given period have a proportionately large impact on the reported inflation, accounting for around 50 percent of the overall price change in the U.S. in 1995 (Moulton and Moses, 1997, tables 5 and 6). Therefore, the life cycle bias, applicable to 40 or 25 percent of total replacements, can be quite significant.¹¹

Another way in which the life cycle bias may occur is during sample rotations. Sample rotations replace a certain proportion of items before they disappear to reflect the change in expenditure patterns in the market. The Bureau of Labor Statistics (BLS) in the U.S. reports that around 25 percent of the sample of price quotes are rotated each year (BLS, 2007, ch. 17). The sample rotation implicitly applies an aggregate version of the overlap method (ILO, 2004, ch. 7). Bils (2009) reports that at sample rotations, all the price differences are attributed to quality differences. Hence, if the life cycle price effect exists, it gets added to the quality effect. However, the sample rotation is not practised routinely by most statistical agencies. Therefore, we calculate the life cycle bias under scenarios with and without sample rotations. In particular, we consider three scenarios with annual sample rotation rates: $\Pi = 0\%$, 12% , and 24% (or equivalently, $\pi = 0\%$, 1% , and 2% per month). For the 12 and 24 percent of annual sample rotations, we consider two

¹¹When an item disappears, perhaps the most common practice is to select a similar item as a replacement. The life cycle bias is not incurred in the case of comparable replacements, because all the price difference between new and disappearing items is included in the measured inflation. When a comparable item is not found, agencies may use the direct quality adjustment method. Given that these methods are applied on a case-by-case basis, and judgments and consultations at various stages are involved, it is difficult to understand exactly how the price difference is allocated between the quality change and pure price change. Therefore, the life cycle bias originating from this method has not been considered in our analysis.

cases with respect to how we treat the A -type items. These are the items for which we do not have estimates of the life cycle pricing pattern. The base case is constructed with the assumption that A -type items do not incur the life cycle bias, thus providing a lower bound estimate of the potential bias. In the other case, we assume that the A -type items follow the same pricing pattern as the other items in a product category.

Note that when a sample rotation is introduced (or the rotation rate is increased), it subjects more items to the life cycle bias and, therefore, has the potential effect of increasing the bias at the overall index level. However, a larger rotation in general is expected to contain the age difference between the new and disappeared items over time and allow less items to reach their death while in the sample, implying that if the life cycle function exhibits a distinct upward or downward trend, the life cycle bias at replacement tends to reduce. Since sample rotation may affect the life cycle bias in both positive and negative ways, a priori it is not clear whether any particular pricing pattern or a more aggressive rotation strategy minimizes the bias.

In order to quantify the impact of life cycle bias we undertake a simulation which meshes together: (a) the initial age profile of the products in our data, (b) the empirically estimated life cycle functions, (c) the index which is the object of estimation, (d) assumptions about the proportion of replaced items incurring the life cycle bias, and (e) assumptions about the rate at which statistical agencies rotate samples. We undertake this simulation for each product though primarily focus on results averaged over all product categories.

We begin the simulation by drawing an initial random sample of N items from the empirical joint distribution of age and reverse age in our data for each product. Once the initial sample is constructed, these products are followed over time. In subsequent periods an item either continues to be available in the market, $n \in V_M$, disappears, $n \in V_D$, or is rotated out of the index, $n \in V_R$. We follow the actual items in the data so the rate of matched and disappearing items is determined empirically by that observed for a particular product category.¹²

When items disappear we introduce new items which we draw randomly from the initial sample of items. The new items are obviously at a different stage of their life to the items which have disappeared. More specifically, the new items are located at an earlier stage of their life than the disappeared items. This difference in the life cycle between the new and disappearing items introduces a price difference at the time of replacement. In addition, in each period we randomly remove items in proportion to the rotation rate from the sample of all items in that period, and similarly randomly introduce the same number of items which are drawn from the full sample of initial items. There will be an age difference between these two sets of items because these are drawn from samples

¹²It should be recognized that our purpose is to design a simulation which provides insight into the dynamics of the likely impact of life cycle pricing on price indexes rather than to reproduce the procedure followed by a particular statistical agency. The idea is to impose minimal restrictions and let the specifics of the age profile of the product in the data and the estimated life cycle function provide us with estimates of the life cycle bias. The design of our simulation, in some aspects, may differ from the replacement procedures followed by statistical agencies.

TABLE 6
QUALITY ADJUSTMENT BIAS DUE TO LIFE CYCLE PRICING (%)

Products	Scenarios*		Bias at Replacement (Θ) [†]			Overall Annual Bias (Ψ)			
	Λ	Π	40%, 25%	25%	25%	40%	25%	25%	25%
			0%	12%	24%	0%	0%	12%	24%
Analgesics			5.56	3.08	2.28	0.73	0.45	0.55	0.63
Bath Soap			1.24	0.77	0.58	0.20	0.13	0.16	0.18
Beer			-2.38	-1.38	-1.01	-0.46	-0.29	-0.32	-0.34
Bottled Juices			6.73	4.52	3.67	1.09	0.68	0.93	1.14
Cereals			7.53	3.58	2.46	0.88	0.55	0.59	0.64
Cheeses			-0.11	-0.31	-0.34	-0.02	-0.01	-0.06	-0.10
Cigarettes			-2.37	-0.61	-0.37	-0.11	-0.07	-0.08	-0.09
Cookies			-0.17	0.09	0.15	-0.03	-0.02	0.02	0.05
Crackers			-1.46	-0.87	-0.63	-0.26	-0.17	-0.18	-0.19
Canned Soup			13.56	7.56	5.53	1.79	1.12	1.32	1.48
Dish Detergent			0.43	0.73	0.77	0.06	0.04	0.13	0.22
Front-end-candies			0.54	1.15	1.27	0.08	0.05	0.22	0.37
Frozen Dinners			7.74	3.30	1.81	1.27	0.79	0.66	0.55
Frozen Entrees			1.03	1.23	1.22	0.15	0.10	0.25	0.37
Frozen Juices			5.77	3.65	2.89	0.57	0.35	0.53	0.66
Fabric Softeners			-1.15	0.08	0.37	-0.15	-0.09	0.02	0.11
Grooming Products			-1.09	-0.98	-0.90	-0.16	-0.10	-0.19	-0.26
Laundry Detergents			4.13	3.05	2.62	0.68	0.42	0.66	0.87
Oatmeal			-1.71	-0.80	-0.44	-0.24	-0.15	-0.13	-0.10
Paper Towels			-6.44	-2.93	-1.94	-0.82	-0.51	-0.54	-0.57
Refrigerated Juices			0.18	0.64	0.69	0.02	0.01	0.11	0.19
Soft Drinks			3.49	2.18	1.79	0.38	0.24	0.37	0.49
Shampoos			4.19	2.29	1.62	0.70	0.44	0.47	0.50
Snack Crackers			3.55	1.53	0.94	0.47	0.30	0.28	0.27
Soaps			1.23	0.44	0.23	0.14	0.09	0.07	0.06
Toothbrushes			-6.56	-5.72	-5.15	-0.98	-0.62	-1.16	-1.62
Canned Tuna			4.28	2.05	1.48	0.43	0.27	0.32	0.38
Toothpastes			4.57	2.34	1.66	0.64	0.40	0.45	0.50
Bathroom Tissues			-11.68	-6.52	-4.84	-1.58	-0.99	-1.20	-1.38
Averages [‡]									
Simple			1.40	0.83	0.64	0.19	0.12	0.15	0.17
Weighted			2.26	1.37	1.08	0.28	0.18	0.24	0.30

Notes: * Λ refers to the percentage of forced substitutions subjected to the life cycle bias and Π refers to the annual sample rotation rate.

[†] Θ is the same for $(\Lambda, \Pi) = (40\%, 0\%)$ and $(25\%, 0\%)$.

[‡]Weights for the weighted average are the expenditure shares of the products obtained using the full data set.

of items of two different periods (the initial and current period). If the sample ages over time, then the removed items will, on average, be older than the new items. Whether, and to what extent, the sample ages over time depends on the initial age profile of the sample, the evolution of the age and reverse age distribution, and the rotation rate.

In our simulation we fix the sample size N at 10,000 for each product category and run it for 10 years with 1000 repetitions. The 10-year (120-month) run of the simulation is found to be sufficient for the behavior of the sample to stabilize—so that it was not influenced by the initially selected sample—and for the bias estimates to converge. The last 12 months of results for the simulation are averaged and are shown in Table 6. In particular we focus on the values of Θ and Ψ , the

average bias at replacement and total index bias, respectively. At the bottom of the table we provide simple and expenditure-weighted averages across product categories, though we limit our discussion to the weighted average result (the online Appendix includes figures showing how the simulation evolved over the 120-month period).

The average bias per replacement (Θ) and the overall bias in the index (Ψ) depend on the proportion of items which are subjected to the life cycle bias. With regard to the disappearance of items, on a weighted average basis, the disappearance rates for all products are 2.74, 2.66, and 2.60 percent for rotation rates of $\Pi = 0\%$, 12%, and 24%, respectively.¹³ This finding supports our earlier conjecture that a higher rotation rate leads to a lower disappearance rate. As discussed earlier, we subject only $\Lambda = 40\%$ and 25% of these disappeared items to incur the life cycle bias in each period.

Table 6 provides the estimates of Θ and Ψ obtained under four scenarios: $(\Lambda, \Pi) = (40\%, 0\%)$, $(25\%, 0\%)$, $(25\%, 12\%)$ and $(25\%, 24\%)$. With no sample rotation, Θ provides an estimate of the average life cycle bias per forced substitution. On a weighted average basis across products, the average life cycle bias per forced substitution—the same for Λ equal to 40 percent and 25 percent—is 2.26 percent. However, as we lower Λ from 40 percent to 25 percent, Ψ reduces from 0.28 to 0.18 percentage points. The size of Θ varies quite significantly across product categories and to a lesser extent across the sample rotation rate. For canned soup, with no sample rotation and where only 25 percent of forced substitutions incur the life cycle bias, the bias at replacement is 13.56 percent. This is the result of a very large fall in price over its life. Bathroom tissues, on the other hand, have a bias of -11.68 percent at replacement, given the strong upward trend in prices over their life. The results show, as expected, that the life cycle pricing pattern plays a significant role in determining the direction and the magnitude of the bias.

More sample rotations generally leads to the attenuation of the bias at replacement. These effects are often quite large. For paper towels, for example, with no rotation Θ is -6.44 percent but this falls to -1.94 percent with a 24 percent annual rotation rate. Sample rotation reduces Θ for 24 products. The five products for which rotation increases the bias are cheeses, dish detergents, front-end-candies, frozen entrees, and refrigerated juices. The life cycle bias is quite small for these products. Of these five products, the life cycle functions exhibit clear non-monotonicity for cheeses, frozen entrees, and refrigerated juices. If we aggregate over all products, we find that rotations do reduce the life cycle bias per replacement. On a weighted-average basis across products, Θ is reduced from 2.26 percent to 1.37 percent to 1.08 percent for annual rotation rates of 0, 12, and 24 percent, respectively.

With regard to Ψ , Table 6 shows that with more extensive use of rotation, Ψ decreases for four products, remains almost unchanged for two products, while it increases for the remaining 23 products. For many products, where rotations reduce Θ , the result reverses when looked at in terms of Ψ . For instance, in the case

¹³This is broadly consistent with Bilts *et al.* (2012), who report that the monthly average forced substitution rate was about 3 percent in the U.S. CPI Research Database they use for their study.

of analgesics, as we move from 0 to 24 percent rotation per year, Θ falls from 5.56 percent to 2.28 percent, but Ψ rises from 0.45 percent to 0.63 percent. On a weighted average basis the bias estimates are $\Psi = 0.18, 0.24$ and 0.30 percentage points for rotation rates of $\Pi = 0\%, 12\%$, and 24% , respectively.

Our simulations show that the life cycle bias is quite significant even when there is no sample rotation and when only 25 percent of the forced substitutions are subjected to the bias. The simulations show that there is a clear benefit of sample rotation because it reduces the life cycle bias per replacement. However, this benefit is not large enough to outweigh the accompanying cost of subjecting more items to the bias. This finding indicates that the life cycle bias is required to be dealt with at its cause and reducing the age difference between new and disappeared items through sample rotations may not make any significant improvement in the life cycle bias at the overall index level. However, it would be useful to see whether similar conclusions can be reached using other data sets.¹⁴

Note that these results are based on the assumption that no life cycle bias is incurred for the *A*-type items. If, on the other hand, the *A*-type items in a product category are assumed to follow the life cycle pricing pattern similar to the other items in that category, then life cycle bias increases by a small magnitude. For example, in the scenario $(\Lambda, \Pi) = (25\%, 12\%)$ and $(25\%, 24\%)$, Ψ for all products increases by 0.02 and 0.03 percentage points, respectively. This small impact of *A*-type items in the overall bias is expected because: (a) the forced substitutions, which do not subject *A*-type items to the bias, contribute more to the overall bias (at least two-thirds in our data) than the sample rotations; this is because the age difference between the new and disappearing items during replacements is, on average, larger at forced substitutions than at sample rotations; and (b) the *A* items account for a small portion of the total items (only 13.9 percent of the items of all products), so even within the sample rotation its contribution is small (see the online Appendix for detailed results).¹⁵

Broadly, these results illustrate that the life cycle bias incurred during replacements is quite large, ranging between 0.2 and 0.3 percentage points, and relatively robust to assumptions made regarding exactly how sampling is undertaken. This implies that statistical agencies should take more account of the stage of the life cycle of a product when undertaking quality adjustment. Perhaps the easiest approach would be to try to attenuate the bias by replacing items with new ones that are at a similar stage in the life cycle (ILO, 2004, ch. 7). Another alternative, which this paper has shown is possible to implement, is to estimate a hedonic pricing model which explicitly includes controls for the life cycle and use this in the quality adjustment process.

¹⁴These results should not be taken to mean that sample rotation is not a good idea. There are a range of other benefits of sample rotations, such as the fact that it means the sample will remain representative of the population of products. This benefit is not taken into account in our calculations. The argument made in this paper applies only to the particular type of bias that takes place due to the mix-up of life cycle price change and the quality change during replacements of old items with new items.

¹⁵An anonymous referee noted that the life cycle bias can originate from continuing item as well because of the divergence between the sample and population in terms of the life cycle characteristics of items. This source of bias can certainly be a focus of further research in the area.

6. CONCLUSION

The purpose of this paper has been two-fold. First, to shed light on the path of prices for commonly consumed supermarket products over their life cycle. Do life cycle price trends exist at all and are they of a sufficient magnitude to be economically meaningful? Second, to investigate the implications of these price-maturation effects for the estimation of price indexes. In particular, whether the failure to include these price changes in some matched-model indexes introduces any systematic errors into the measurement of inflation.

Using a large scanner data set, we answer both questions in the affirmative. We found statistically compelling and robust evidence—across models, products, and data sets—for the existence of the life cycle pricing effects. We also found these life cycle price trends to be economically significant. The dominant pricing pattern exhibits higher entry prices and lower exit prices of items for at least two-thirds of the products, providing support to the price skimming hypothesis. By taking the average of all products and after controlling for product-wide temporal variations in price, we found that prices fell by 2.09 percent per year due to ageing.

The implication of this finding is that index methods which ignore life cycle price differences during replacement of disappearing and removed items produce biased measures of price change. Given that prices tend to fall over the life of a product, the life cycle bias leads to the overestimation of quality change and consequently the underestimation of price change. We found that with only 25 percent of forced substitutions incurring the life cycle bias and a 24 percent annual sample rotation policy, the price change is on average underestimated by around 1.08 percent for replaced items. This bias is far from trivial. At the product level this bias corresponds to an underestimation of annual price change of 0.30 percentage points for the product categories examined.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article at the publisher's web-site:

Table A-1: Regression Results of the Log-Linear Models

Table A-2: Model Diagnostics

Table A-3: Overall Annual Bias (Ψ) due to Life Cycle Pricing (%) (A-type items are assumed to exhibit the same life cycle pricing pattern as the other items)

Figure A-1: Monthly Simulation Results Aggregated over all Products for Different Sample Rotations (Π) and fixed Λ at 25 percent—Aggregated by taking Simple Average

Figure A-2: Monthly Simulation Results Aggregated over all Products for Different Sample Rotations (Π) and fixed Λ at 25 percent—Aggregated by taking Expenditure Share Weighted Average