the review of income and wealth

Review of Income and Wealth
Series 60, Number 2, June 2014
DOI: 10.1111/roiw.12000

NON-LINEAR PRICING AND PRICE INDEXES: EVIDENCE AND IMPLICATIONS FROM SCANNER DATA

BY KEVIN J. FOX*

School of Economics and CAER, University of New South Wales

AND

DANIEL MELSER

Moody's Economy.com, Sydney, Australia

Non-linear pricing, the fact that prices do not necessarily change in proportion to size, is a ubiquitous phenomenon. However, it has been neither particularly well understood nor well measured. Non-linear pricing is of practical importance for statistical agencies who, in constructing price indexes, are often required to compare the relative price of a product-variety of two different sizes. It is usually assumed that prices change one-for-one with package and pack size (e.g. a 1-liter cola costs half as much as a 2-liter bottle). We question the wisdom of such an assumption and outline a model to flexibly estimate the price-size function. Applying our model to a large U.S. scanner dataset for carbonated beverages, at a disaggregated level, we find very significant discounts for larger-sized products. This highlights the need to pursue methods such as those advocated in this paper.

JEL Codes: C43, C50, D00, E31

Keywords: hedonic regression, local regression, non-linear pricing, quantity discount

1. INTRODUCTION

In a modern economy there is a dizzying array of consumer-products. These products are invariably available in a wide range of flavors, colors, brands, grades as well as package sizes—from mini-cans, bottles, boxes, or bags to extra large and family sizes—and pack sizes—singles, doubles, half-dozens, dozens, and mega-packs. This proliferation in the available product varieties has posed problems for the construction of price indexes on a number of fronts. In particular, as the product varieties change and evolve, so must the indexes which purport to measure fluctuations in their prices. Comparing items on a quality-adjusted basis has become a major challenge for statistical agencies.

This quality adjustment problem, of comparing two different items, is most often encountered in high-tech sectors where new and improved products replace older outdated varieties. However, it is also apparent in the more mundane

Note: We are appreciative of the financial support of the Australian Research Council (LP0667655) in undertaking this research, and the very constructive comments we received from two anonymous referees and participants at the UNSW Economic Measurement Group Workshop. Any remaining errors are our own.

*Correspondence to: Kevin J. Fox, School of Economics and CAER, University of New South Wales, Sydney, NSW 2052, Australia (k.fox@unsw.edu.au).

product categories stocked by supermarkets and drug stores; for example, Greenlees and McClelland (2011) examined the quality adjustment problem for food categories priced in the U.S. Consumer Price Index (CPI), such as breakfast sausage. While much of the academic literature on quality change and price indexes has focused on the introduction of new goods (e.g. Diewert, 1998; Hausman, 2003; Pakes, 2003; Bils, 2009), one of the most common quality adjustment problems faced by statistical agencies is in comparing the prices of the same good sold in different sizes; for example, if the price of a 2-liter bottle of cola is collected in one period, but in the following period a 1-liter price quote is available. How should these prices be compared and hence inflation estimated in such a case? Should the price of the former be halved? This raises the question of what the market relationship is between products of different size.

There is general interest, at both a theoretical and practical level, about the relationship between product price, package, and pack size. The ubiquitousness of lower unit-prices for larger-sized products has spawned a large literature. Recent work in this area, such as Cohen (2008) and McManus (2007), has focused upon estimating a structural model of consumer and producer interaction. Much of the previous literature has explored the existence of size discounts through the prism of price discrimination. Our goal is somewhat different. We propose to focus on the reduced form of the hedonic function, as is common in the price index literature, and to empirically document the price-size nexus. This provides a contribution to understanding size discounts but also outlines a novel and flexible approach to estimating the hedonic function which can be actively pursued by statistical agencies in improving the accuracy of their size adjustments and hence of their price indexes.

Our empirical focus is on the carbonated beverages product category. We use a large, highly granular, U.S. scanner dataset from Information Resources Inc. (IRI) to examine the relationship between price, package, and pack size. This dataset covers 50 markets (metropolitan areas) and the 24 months from the beginning of 2005 to the end of 2006. Carbonated beverages are interesting for a number of reasons. They are sold in a variety of package sizes, such as small 8-oz and 12-oz cans all the way up to large 2-liter and 3-liter bottles, while also being available in different pack sizes, such as 6-packs, dozens, and the like. This is evident in our data. Table 1 shows the breakdown of the value of sales, as a proportion of total sales, by various pack and package sizes. Total sales over the two-year period amounted to \$159.5 million. As can be seen, much of the sales are focused around the single 2-liter and 20-oz packages. However, larger pack sizes, such as half-dozens and dozens, were also very widely purchased. This distinction between package and pack size, which is clear with regard to carbonated beverages, has not been made clear in the literature thus far. It is an important one for statistical agencies, however, as the adjustments required along these two size dimensions may differ. The carbonated beverages product category is also interesting because it is a relatively significant expenditure item for consumers. It constitutes around one-third of 1 percent of the U.S. CPI.[1] Because carbonated drinks is an important

[1]The U.S. city average weights for December 2009 give the carbonated drinks expenditure category a weight of 0.294 percent for the CPI for All Urban Consumers and 0.380 percent for the CPI for Urban Wage Earners and Clerical Workers.

TABLE 1
Value of Sales by Package and Pack Size (% of total)

Package Size	Pack Size 1	4	6	12	18	24	Total
8 oz/237 ml	0.1013	0.0406	1.4549	0.0666	0.0174	0.0011	1.6819
12 oz/355 ml	0.6125	0.3548	2.7882	48.5913	0.0471	8.6243	61.0182
14 oz/414 ml	0.1094	0.0000	0.0032	0.0184	0.0023	–	0.1333
16 oz/473 ml	0.0134	0.0223	0.6107	0.0009	–	–	0.6473
20 oz/591 ml	6.1494	0.0070	3.1745	0.2166	–	0.0000	9.5475
24 oz/709 ml	0.0796	–	3.9681	0.0040	–	–	4.0517
33.8 oz/1 l	2.0529	0.0000	–	0.0263	–	–	2.0792
50.7 oz/1.5 l	0.4553	–	–	–	–	–	0.4553
67.6 oz/2 l	19.6175	0.0032	–	–	–	–	19.6207
101.4 oz/3 l	0.7650	–	–	–	–	–	0.7650
Total	29.9563	0.4279	11.9996	48.9241	0.0668	8.6254	100.0000

Note: The values for pack and package size indicate intervals with respect to the previous value. For example for package size the intervals are; 0 oz to 8 oz, greater than 8 oz and less than 12 oz, etc.

expenditure category it is often the focus of competitive pricing strategies amongst supermarkets, such as being a loss leader (Lal and Matutes, 1994; Huang and Lopez, 2009).

In the following section we outline the problems faced by statistical agencies in quality-adjusting for changes in pack and package sizes. Section 3 outlines some of the theory around size discounts and formulates a flexible hedonic model for estimating these effects. In Section 4 we apply these methods to the IRI scanner dataset on carbonated beverages and outline the results. We conclude by discussing some of the implications of these results for the construction of price indexes.

2. Price Indexes and Size Adjustments

The standard approach used by statistical agencies when comparing the same product of different size is to make the prices comparable by multiplying one of them by the appropriate size ratio (see for example, Armknecht and Maitland-Smith, 1999; ILO, 2004; Triplett, 2004). There are at least three situations where this approach may be applied.

First, often the pricing specifications adopted by statistical agencies will stipulate that a particular package size be collected. For example, the specification may say that a 2-kg bag of potatoes should be recorded. When this is not available it is often the case that another size is recorded and the price is automatically size-scaled, by either the price collector or the index calculation computer system, so that it represents the desired specification size.

Second, a comparability problem will often arise when there is a change in package size for a particular product. For example, a manufacturer may decide to increase the size of a bottle of cola from 2 liters to 2.25 liters. If the price of the 2-liter variety was originally collected then this poses a comparability problem with the price in the later period. A common approach adopted by statistical agencies is to scale the price of the new item by the difference in package sizes—in our example the scale factor would be 0.8889 = 2/2.25. This adjusted price is then directly compared with the price for the 2-liter product in the earlier period in estimating inflation.

Finally, retailers sometimes undertake promotional offers such as “buy-one-get-one-free” (BOGOF). Suppose that a price for a single bottle of juice was collected in some previous period while in the current period the retailer introduced a BOGOF-offer at the same price. There are two approaches that are sometimes adopted by statistical agencies to this problem. The first is to note that it still costs the same amount to buy a bottle of juice so the price has not changed for someone who wants just a single bottle. While this is true it neglects the fact that the consumer has gained utility from the additional unit of the product which is essentially treated as being free. A second, more common, approach in this case is for the statistical agency to undertake the comparison on a price-per-unit basis. Because two units are received in the current period, the price in this period is divided by two and compared with the base price.

Each of these examples illustrates the way in which changes in package size and pack size can impact upon price indexes. Implicit in statistical agencies’ adjustments for changes in pack and package size is the assumption of a linear relationship between these factors and price. As noted by Triplett (2004), it is certainly not clear that this is a valid assumption:

> Considering that the relation between size and price is seldom linear, it is a bit surprising that statistical agencies use predominantly the simple linear form of package size adjustment . . . (Triplett, 2004, p. 20)

If price-per-unit varies, then assuming a linear relationship will give incorrect, and potentially biased, measures of price inflation if there are systematic changes in size over time. This raises the important question of how this problem should be dealt with in the construction of official price indexes.

The ILO manual on consumer price indexes (ILO, 2004) echoes Triplett (2004) and cautions against the mechanical application of size-based quality adjustment:

> It is generally a considerable oversimplification to assume that the quality of a product changes in proportion to the size of some single physical characteristic. For example, most consumers are very unlikely to rate a refrigerator that has three times the capacity of a smaller one as being worth three times the price of the latter. Nevertheless it is clearly possible to make some adjustment to the price of a new quality or different size to make it more comparable with the price of an old quality. There is considerable scope for the judicious, or common sense, application of relatively straightforward quality adjustments of this kind. (ILO, 2004, p. 29)

While this is certainly sound advice, it is remains somewhat unclear as to the sort of “common sense” adjustment rules which could or should be applied in practice.

Given the current practices adopted by statistical agencies, the difficulties outlined in the ILO CPI manual in making linear size adjustments, and the fact that very little empirical research has been undertaken in this area, there is clearly scope for further investigation. Of primary interest is determining the relationship between price and size. We proceed to do exactly this in our empirical application below but first discuss the related literature and econometric models of the relationship between price, pack, and package size.

3. Modeling Price, Pack, and Package Size

In developing a model of the price–size relationship it is important to have an idea of the likely drivers of non-linear pricing. These will be important in determining the structure of the model, and the degree of flexibility that is required in isolating the function of interest. The literature on non-linear pricing has stressed price discrimination as a primary cause of such pricing policies. The first paper to rigorously address the issue was Spence (1976) but the literature has grown considerably since this time. If the producer has some sort of pricing power, and consumers differ, then it may be that a non-linear pricing schedule will separate the consumers and lead to higher firm profits. Maskin and Riley (1984) showed, for a simple utility function and consumers with heterogenous preferences, that the optimal pricing function for a monopolistic firm will have declining unit costs.

Size discounts may also arise from differences in the marginal costs of production. Clements (2006) emphasized, amongst other factors, that the cost of the packaging may be lower for larger-sized products. Consider the example of bottled drinks. A 2-liter bottle has twice the volume of a 1-liter bottle but the surface area of the package is only 40–50 percent larger. Yet the cost of packaging is unlikely to be big enough to have a major influence upon price. Other areas where there may be cost economies of size are in retailing, transport, advertising, and storage. It is likely that the costs of selling an item—processing it at the checkout—as well as transporting, storing, and stocking the shelves with it, will be almost independent of pack or package size, implying declining unit costs.

If we take the consumer perspective, rather than that of the firm, we note that there may be greater storage or transport costs from purchasing a larger package or pack size. Hence consumers may need to be compensated for this by paying a lower price per unit. Furthermore, there may also be a greater risk of wastage if larger package sizes (and to a lesser extent pack sizes) are purchased. For example, if a consumer purchases a large bottle of carbonated soft drink they may only consume part of the contents in the first sitting. There is a risk that the remainder will spoil. This would lower the value of larger package sizes to consumers. Yet contrary to this intuition, Gertsner and Hess (1986) have argued that consumers will save shopping time, and could have lower overall transport costs, by purchasing in bulk. Hence they argue that it is possible that we may see premiums, rather than discounts, for larger products.

Hong *et al.* (2002) considered a model with two types of consumers, price-sensitive "comparison shoppers," and "captives" who buy from a fixed store. They allowed for consumer inventories so that the "shoppers" can stock up during sales. They found that the level of consumer inventories leads to state-dependent price dispersion, with prices and quantities displaying negative serial correlation. This inventory-building framework can be used to think about the choice of both package and pack size by consumers as they build inventories during sales. Recent work by McManus (2007) and Cohen (2008), focusing upon coffee and paper towels, respectively, has taken a unified approach and specified and estimated a structural model of both consumption and production. Consumers make discrete choices over different products of various sizes, and producers sell at some markup over marginal costs. This approach, and that of Hong *et al.* (2002), helps to understand the structural factors generating non-linear pricing regimes but goes much further, and makes many more assumptions, than is required for price index construction. What is needed by statistical agencies is some estimate of the relative prices of products of different package sizes so that prices can be appropriately adjusted if necessary.

3.1. *A Flexible Hedonic Model of the Price–Size Function*

We propose to examine the question of non-linear pricing within a hedonic framework. A hedonic pricing model can be justified on a consumer-basis as, under certain conditions, representing consumer preferences (see Diewert, 2003a). Alternatively it can be motivated from a producer perspective as reflecting the marginal costs of production as well as markups (Pakes, 2003). More generally, as Rosen (1974) noted in his seminal work on the subject, the hedonic function is a reduced form that is likely to reflect all aspects of the strategic interaction between buyers and sellers in a market; incorporating the effects of cost, technology, preferences, and market power. From a price index perspective this is desirable. Our objective is not necessarily to estimate either producer technology or consumer preferences but to provide a simpler, more basic, representation of the budget constraint which prevailed at a specific time and/or place and which can be used in producing a quality-adjusted price index. We propose a flexible non-parametric hedonic regression method which represents the relationships of interest succinctly but with a suitable degree of fidelity to the data.

In the empirical investigation which follows, our highly granular scanner dataset enables us to control for the attributes of each product and isolate the impact of pack and package size on price. In particular, we have data on prices p_{imt} for varieties $i = 1, 2, \ldots, I$, each representing a specific barcode, over markets $m = 1, 2, \ldots, M$ and time periods $t = 1, 2, \ldots, T$. The attributes which are likely to determine the price of each item are its pack (x_i^A) and package size (x_i^B)—the primary objects of interest—the market and time period in which it was sold, and the particular features of the product such as brand and flavor.

First, to these latter price-drivers. The way in which consumers discriminate amongst different varieties is potentially quite complex given the myriad of brands and flavors on the market. But some care needs to be taken in controlling for these quality differences in order to isolate the effects of size on price. Our approach is to control for these quality differences using a large number of dummy variables for different brands and flavors. Define a brand dummy as $b_{i\iota} = 1$ when product i is from brand $\iota = 1, 2, \ldots, L$ and zero otherwise, and a flavor dummy $f_{i\tau}$ similarly. Brand and flavor represent quite detailed characteristics of the product. For example we may have a brand "Pepsi" which comes in the flavors "Cola," "Vanilla Cola," "Lime Cola," "Cola with Lemon Twist," "Berry Cola" and so forth. Each of these components helps to control in a very specific way for observed price heterogeneity. However, in our hedonic model we also allow for possible interactions between brand and flavor. This may be important if the marginal price of flavor, e.g. "Lime Cola," is valued differently by consumers across brands, such as "Pepsi" or "Coca Cola." In addition to these two factors we also include a dummy variable for each supermarket chain, c_{imtv} = when product i in time t and market m is sold in chain $v = 1, 2, \ldots, V$ and zero otherwise. This is to allow for the possibility of different levels of service and shopping quality across retailers. These three factors are included in our hedonic model as dummy variables and we represent their combined effect by the dummy function $d(b_{i\iota}, f_{i\tau}, c_{imtv})$. This function helps to control very tightly for any price effects due to the nature of the product.

In terms of the temporal and spatial effects, we estimate a separate regression for each combination of time and market. This is because the preceding discussion, particularly with regard to the price discrimination motive for non-linear pricing, has emphasized just how much the price–size function may depend upon "local" factors. This allows in a flexible way for differences in prices across time and markets and also for changes in relative prices of different brands and flavors over these dimensions. Moreover, the stability of the size function across time and space is of some practical importance to statistical agencies in terms of identifying the extent to which common quality adjustment ratios can be used or whether individual markets and/or time periods require tailored adjustments. Given this, and denoting the general pack size and package size function in time t and market m as $s_{mt}(\ln x_i^A, \ln x_i^B)$, adding a random error term, e_{imt}, and hypothesizing a log-linear functional form, as advocated by Diewert (2003b), we have the following hedonic model:

$$(1) \qquad \ln p_{imt} = d_{mt}(b_{i\iota}, f_{i\tau}, c_{imtv}) + s_{mt}(\ln x_i^A, \ln x_i^B) + e_{imt},$$

for $i = 1, 2, \ldots, I$, $m = 1, 2, \ldots, M$, $t = 1, 2, \ldots, T$. We want to model the size function flexibly while also providing interpretable results. In particular we are especially interested in the price–size elasticity. That is the derivative of log price with respect to either log package size or log pack size:

$$(2) \qquad \frac{\partial \ln p_{imt}}{\partial \ln x_i^C} = \frac{\left(\dfrac{\partial p_{imt}}{p_{imt}}\right)}{\left(\dfrac{\partial x_i^C}{x_i^C}\right)} = \frac{\partial s_{mt}(\ln x_i^A, \ln x_i^B)}{\partial \ln x_i^C}, \quad C = A, B.$$

This elasticity is a particularly informative quantity because if the price–size effects are linear then the price–size elasticity will be equal to one. In the empirical section below our concern will be to examine this quantity across different markets and time periods and draw some conclusions about the extent of non-linear pricing.

A natural approach to modeling the size function is to simply include the log of package size and pack size linearly. While we will explore this model it has the drawback that it assumes that the elasticity between price and package and pack size shown in (2) is constant. Yet the relationship is potentially much more complex than this. For package size, for example, the elasticity is likely to vary both across different package sizes and also as pack size varies. A linear model cannot encompass these possibilities.

Our solution is to use a simple functional form but to estimate it "locally" by way of a local regression estimator (Cleveland, 1979). In particular we assume that $s_{mt}\left(\ln x_i^A, \ln x_i^B\right)$ is a linear function of its components but we estimate this function for each point in the product space. This helps to reveal the local behavior of the price–size surface. At each data point a weighted regression is estimated with the weights determined only by the size–distance from the reference point. Hence points which are neighboring to the point of interest in terms of pack and package size get the highest weights and further-away points get much lower weights. The exact weights used in our application are those of the tricube function suggested by Cleveland (1979). Here, if an observation indexed by *imt* is the reference observation then the weight for observation *jmt*, represented by $w_{jmt|imt}$, is proportional to $w_{jmt|imt} = \left(1 - \left(\frac{k_{jmt|imt}}{k^*_{imt}}\right)^3\right)^3$. Here $k_{jmt|imt}$ is the distance of the observation from the reference point and k^*_{itm} is the maximum such distance. The extension to two dimensions is straightforward following standardization.

4. An Empirical Examination

We apply our model to the carbonated beverages scanner dataset made available for academic research purposes by IRI (Bronnenberg *et al.*, 2008). The dataset is extremely rich and detailed. In our analysis we focus on data across 50 markets—essentially a mix of both large and small metropolitan areas[2]—across more than 2,000 stores from 116 chains in 2005 and 2006. The stores in the sample are anonymized due to confidentiality. However, we know from Bronnenberg *et al.* (2008) that they are the stores contained in IRI's national sample and only include chain stores. Independents are excluded. However, because chain stores usually dominate a market, the sample of stores is likely to be broadly representative of those operating in a region.

The data has unit prices for each Universal Product Code (UPC, i.e. barcode)—of which there are 5,304—by store at a weekly frequency. In terms of the product characteristics there are 457 brands and 454 flavors in total. We aggregate across weeks to create monthly price observations, and across stores to create chain average prices for each market. The standard calculation frequency for price index construction is monthly so this a natural time unit in our context. The aggregation across stores within a market, to chain-level average prices, provides us with a much more manageable dataset. These unit values are unlikely to be contaminated by any quality differences because the stores are members of the same chain and hence are likely to provide the same levels of service and other

[2]The markets are: Atlanta, Birmingham/Montgomery, Boston, Buffalo/Rochester, Charlotte, Chicago, Cleveland, Dallas, Des Moines, Detroit, Eau Claire, Grand Rapids, Green Bay, Harrisburg/Scranton, Hartford, Houston, Indianapolis, Kansas City, Knoxville, Los Angeles, Milwaukee, Minneapolis/St. Paul, Mississippi, New England, New Orleans, New York, Oklahoma City, Omaha, Peoria/Springfield, Philadelphia, Phoenix, Pittsfield, Portland, Providence, Raleigh/Durham, Richmond/Norfolk, Noanoke, Sacramento, Salt Lake City, San Diego, San Francisco, Seattle/Tacoma, South Carolina, Spokane, St. Louis, Syracuse, Toledo, Tulsa, Washington DC, West Texas/New Mexico.

such shopping amenities. The use of chain unit values may also reduce some of the "noise" in the data from sales cycles at individual stores, though Hwang *et al.* (2010) find a high degree of homogeneity in chain pricing patterns. While our aggregation leads to a dataset of manageable proportions it still yields a total of 2,528,533 observations spread over the 50 markets and 24 months. This gives an average of 2,107 observations per market-month.

In order to determine whether brand and flavor dummy variables should be interacted in our hedonic specification, we estimated the local regression model both with and without interactions while including chain dummy variables. The R^2 when effects for brand and flavor are included separately is 0.8994. This rises to 0.9105 with interactions. Though introducing interactions leads to a large increase in the number of parameters, the rise in R^2 is highly statistically significant using an F-test so we include this interaction in our model. Likewise the inclusion of the chain dummy variables is also important. If we remove these, with brand and flavor interacted, the R^2 drops to 0.9041 which is a statistically significant change.

As noted, when we include the effects of size log-linearly we get a fixed elasticity for each of the size dimensions. For the non-parametric local regression approach we get a very flexible price–size surface. However, in implementing this estimation we are required to choose the smoothing level, in our case the proportion of observations which fall within the local regression window. The fact that the data are tightly centered around particular pack and package sizes, rather than scattered evenly across size–space, means that slight changes in location can lead to large changes in weights. After some experimentation the most stable and reliable estimates were obtained when we maximized the span and let the weighting function down-weight observations which were further away, in terms of size, and up-weight those observations which are closer. This leads to a degree of localization in the estimated size surface without introducing undue volatility and allows us to reliably impute size effects within sample, which is our primary objective. Comparing the fit of the linear size model to the local regression approach, with brand and flavor interacted and chain fixed effects, the R^2 falls to 0.9100. While this is a fairly modest decline the loss of explanatory power is statistically significant. This is because the local regression function can be estimated relatively cheaply, with the addition of only a small number of extra parameters. Given the statistical evidence we favor the local regression method over the linear approach.

The outcome of our estimation of the local regression model is a pricing surface in package size and pack size for each of the 1,200 ($= 50 \times 24$) market-months. As a way of summarizing these results we construct prices normalized on a single 1-liter bottle and averaged across all markets and time periods. The results are shown in Table 2, for certain pack and package sizes of interest, and are depicted in Figure 1 graphically.

These results indicate very significant discounts for size, particularly along the package size dimension. For example, an 8-oz bottle of soda costs 76.51 percent of that of a 1-liter bottle despite being less than a third of the size. Similarly, a 2-liter bottle of soda costs only 11.91 percent more than a 1-liter bottle on average. Along the pack size dimension there are also significant discounts available, though they are not quite as large as for package size. A 6-pack of the widely sold 12-oz package size costs, on average, only 2.19 times more than a single serving while a

TABLE 2

Price, Pack, and Package Size

Package Size	Pack Size: 1	4	6	12	18	24
8 oz/237 ml	0.7651	1.5942	1.9876	2.9234	3.6559	4.286
	[0.7642, 0.7662]	[1.5924, 1.5961]	[1.9849, 1.9903]	[2.9184, 2.9284]	[3.6490, 3.6628]	[4.2776, 4.2948]
12 oz/355 ml	0.8311	1.7551	2.1888	3.2092	3.9949	4.6807
	[0.8304, 0.8319]	[1.7536, 1.7566]	[2.1865, 2.1909]	[3.2051, 3.2139]	[3.9893, 4.0024]	[4.6739, 4.6910]
14 oz/414 ml	0.8568	1.8149	2.2611	3.3135	4.1226	*4.8324*
	[0.8562, 0.8575]	[1.8135, 1.8162]	[2.2590, 2.2633]	[3.3099, 3.3190]	[4.1173, 4.1312]	[*4.8252*, *4.8440*]
16 oz/473 ml	0.8789	1.8654	2.3210	3.4002	*4.2315*	*4.9629*
	[0.8783, 0.8795]	[1.8641, 1.8668]	[2.3190, 2.3236]	[3.3965, 3.4064]	[*4.2254*, *4.2409*]	[*4.9548*, *4.9756*]
20 oz/591 ml	0.9148	1.9458	2.4152	3.5382	*4.4103*	5.1796
	[0.9144, 0.9152]	[1.9446, 1.9477]	[2.4130, 2.4184]	[3.5336, 3.5453]	[*4.4030*, *4.4211*]	[5.1700, 5.1941]
24 oz/709 ml	0.9437	*2.0045*	2.4865	3.6454	*4.5542*	*5.3561*
	[0.9435, 0.9440]	[*2.0029*, *2.0067*]	[2.4838, 2.4902]	[3.6400, 3.6533]	[*4.5458*, *4.5661*]	[*5.3452*, *5.3720*]
33.8 oz/1 l	1	2.1013	*2.6090*	3.8374	*4.8223*	*5.6893*
	[1, 1]	[2.0990, 2.1040]	[*2.6054*, *2.6135*]	[3.8306, 3.8465]	[*4.8118*, *4.8361*]	[*5.6756*, *5.7078*]
50.7 oz/1.5 l	1.0688	*2.2104*	*2.7427*	*4.0597*	*5.1445*	*6.0949*
	[1.0684, 1.0692]	[*2.2074*, *2.2138*]	[*2.7382*, *2.7480*]	[*4.0509*, *4.071*]	[*5.1311*, *5.1617*]	[*6.0776*, 6.1179]
67.6 oz/2 l	1.1191	2.2904	*2.8391*	4.2238	*5.3837*	*6.3971*
	[1.1184, 1.1197]	[2.2869, 2.2946]	[*2.8338*, *2.8454*]	[*4.2135*, *4.2370*]	[*5.3681*, *5.4038*]	[*6.3770*, *6.4239*]
101.4 oz/3 l	1.1890	*2.4224*	*2.9997*	*4.4884*	*5.7553*	*6.8590*
	[1.1879, 1.1902]	[*2.4182*, *2.4278*]	[*2.9935*, *3.0074*]	[*4.4747*, *4.5021*]	[*5.7342*, *5.7759*]	[*6.8310*, *6.8869*]

Notes: Italics indicates that a pack–package size combination was unavailable in our data. The numbers in square brackets represent 99% confidence intervals for the normalized price function.

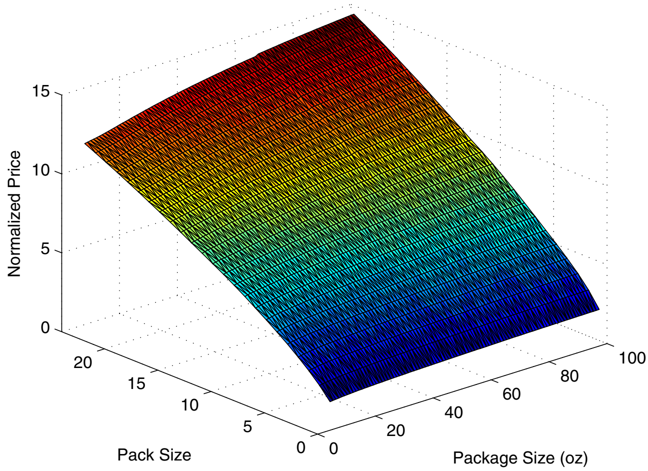

Figure 1. Price Surface (Average of All Time Periods and Markets)

24-pack costs just 4.68 times as much. Taken as a whole the results present very strong evidence that the package and pack size function is not linearly increasing, that is, prices do not rise proportionately with size. Furthermore, these effects are very accurately estimated. We constructed 99% confidence intervals for the relevant points on the size surface using a bootstrap approach. Here we randomly re-sampled residuals from the model, added these to the dependent variable, and re-estimated the models many times. This was computationally intensive yet we were able to undertake 200 such replications. The resulting confidence intervals are very tight. This is partly because of our large sample size but also due to the flexible nature of the model and the fact that we average over many markets and time periods. From a statistical agency perspective our estimated price–size function implies that linear price adjustment is likely to get the quality adjustment wrong, and significantly so.

In terms of implementing appropriate quality adjustment for changes in size, Table 2 potentially provides a way forward. The relative prices in this table give some indication of the appropriate scalar for linking products of different size into the index. However, an examination of the more disaggregated results is instructive.

First to package size. In Figure 2 we depict the price function over various package sizes, and its derivative, for a pack containing a single product for a selection of cities—Chicago, Indianapolis, and San Diego—in each of the 24 time periods. We are only interested in relative prices, so the price–size function is normalized so as to be one for a 1-liter bottle in each period. It is clear that there is significant variability in the function even within a given city across time. Indianapolis has the steepest package size function with an average elasticity of 0.2320, while in San Diego the average elasticity is just 0.0889. This points toward the particular competition dynamics in a city, and in a given time period, having a major role in determining the momentary size pricing function. Nevertheless, there are some features which do stand out. First, it is clear that the elasticity is not

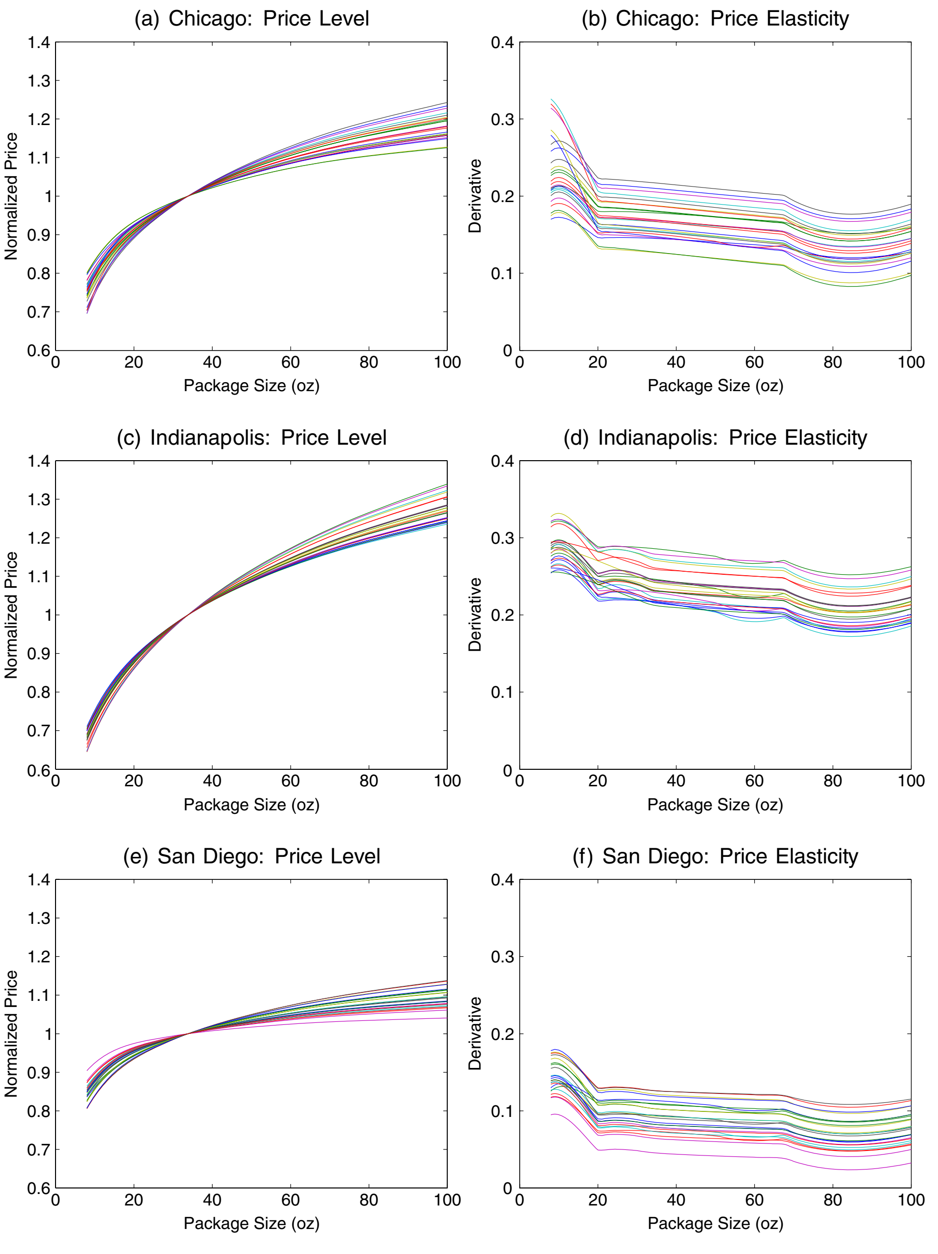

Figure 2. Package Size Function (Pack = 1)

constant. The advantage of the local regression method is that it can pick up the non-constancy in the elasticity and this is why it was preferred in the statistical tests. Second, the elasticity exhibits fairly consistent dynamics across package size. It usually starts off high, then falls quite rapidly before falling more slowly from around 20 oz.

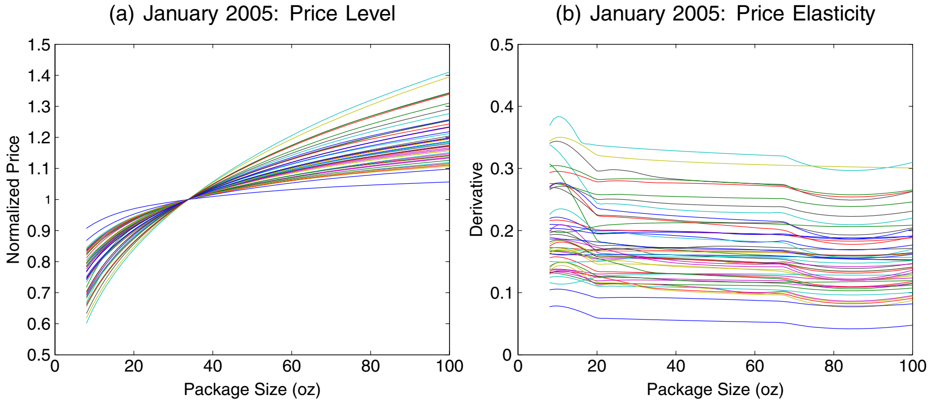

Figure 3. Package Size Function (Pack = 1)

An alternative way to examine the results is to hold time fixed and look across markets at the package size function. From this we can get an impression of the significant diversity in city pricing functions. The first month in our dataset, January 2005, is representative of our results. Figure 3 shows the nationwide diversity in pricing functions. It can be seen that in some markets there are portions of the package size pricing function which are almost horizontal with elasticities not much above zero. What is also apparent when comparing Figures 2 and 3 is that there is somewhat more heterogeneity across cities for a given time period than across time for a given city. This is potentially useful information for a statistical agency trying to replicate these results as it emphasizes that allowing flexibility across the spatial dimension is more important than across time.

We turn now to pack size and illustrate our results similarly. In Figure 4 we depict the pack size function across time for the markets of Atlanta, Boston and Los Angeles. In each of these figures we fix the package size at 12 oz. This was the most popular package size overall (see Table 1) and it was also available in every possible pack size. Figure 5 illustrates the diversity across markets by plotting the pack size function in January 2005 for all of the 50 markets.

Compared with package size the pricing of multipacks is closer to linear in that the elasticity is nearer one. However, it is still far from being equal to one. For our three cities, which are broadly representative, the average elasticity was 0.5783 for Atlanta, 0.5594 for Boston, and 0.4843 for Los Angeles. Unlike for package size, we never observed any non-monotonicity in our results for pack size. There is also less apparent heterogeneity in the pricing function for packs compared with packages. These results are likely to be the result of the greater scope that consumers have to “unbundle” multipacks compared with different package sizes. This may lead to a higher degree of substitution between different pack sizes than between package sizes and hence a higher degree of pricing homogeneity for the former. One particularly interesting feature of the pack size functions is the marked volatility in the elasticity around a 12-pack. Mostly there is a marked decline in the elasticity at this point followed by a rapid rise. This is particularly evident in Figure 5(f) for Los Angeles. This likely reflects the fact that 12-packs are

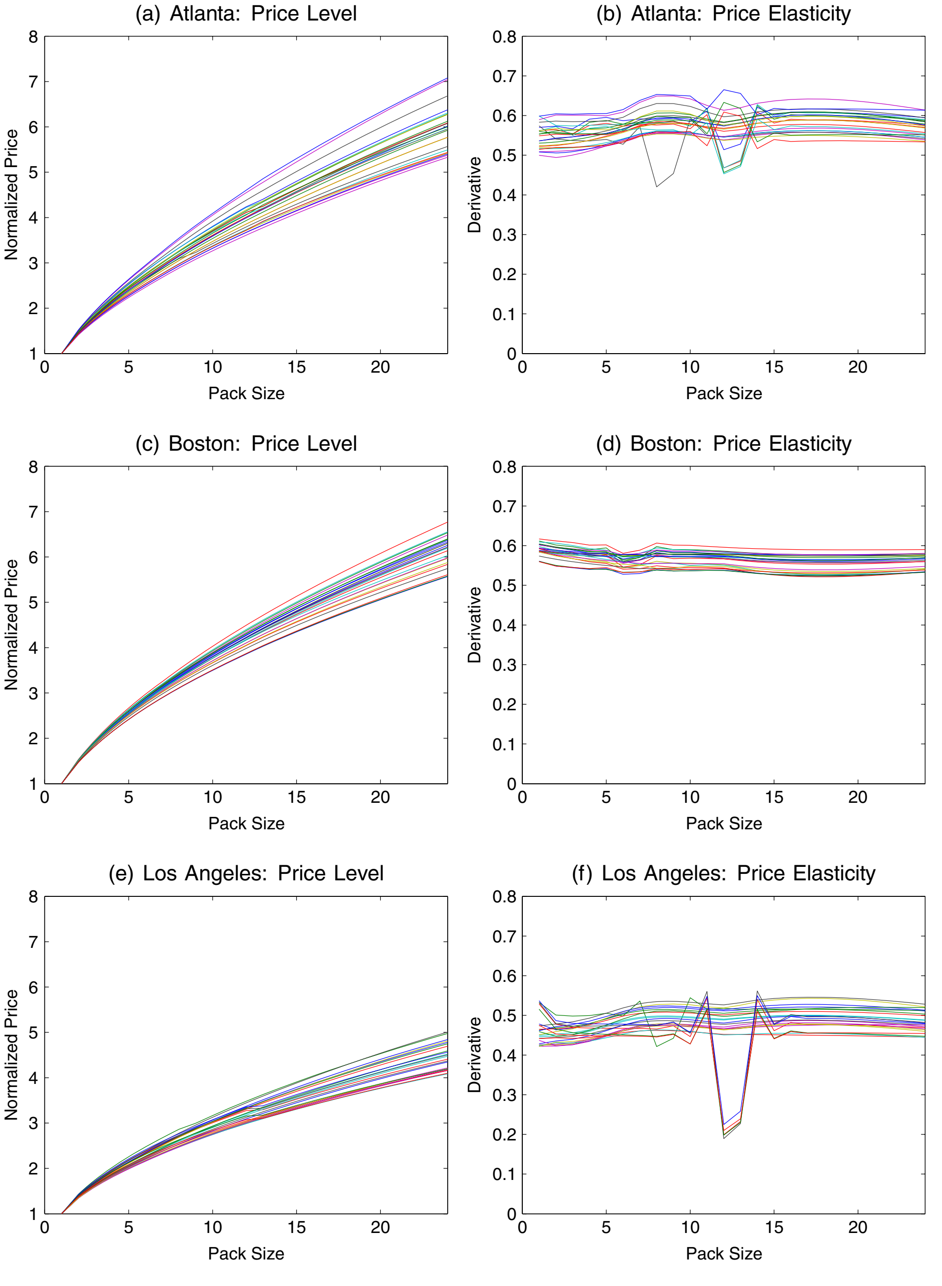

Figure 4. Pack Size Function (Package = 12 oz)

particularly prone to be put on sale or used as a loss leader, leading to a flattening in the size function at this point.

What does all this mean for price indexes? It is possible to quantify the impact of the use of linear quality adjustment methods compared with the appropriate

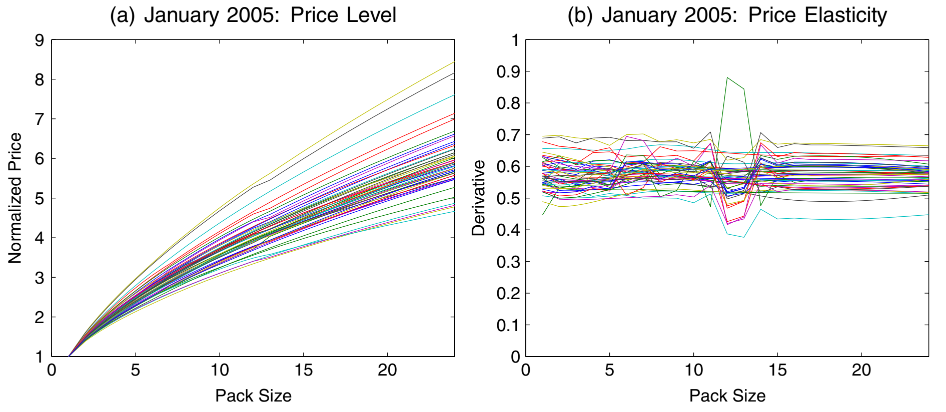

Figure 5. Pack Size Function (Package = 12 oz)

adjustment from our hedonic function. To do this we consider how the index for soft drinks is constructed and look at the bias that results under two hypothetical scenarios of quantity–quality change. Indexes for soft drinks are calculated as matched samples. Here a set of items is sampled in some base period and these items are followed over time until they disappear (Greenlees and McClelland, 2011). When they disappear they are replaced with a similar item and some quality adjustment is often done to ensure the old and replacement items are directly comparable. In order to focus wholly on the impact of quantity–quality adjustment we will consider the hypothetical situation where some share of price quotes disappears from the index, the replacement quotes all require the same size adjustment, and there are no other quality differences between the items. We compare the index when the items are quality-adjusted using the linear technique, favored by statistical agencies, and when our more sophisticated method is used.

We suppose that the soft drinks index is constructed using a simple geometric mean (Jevons) matched model approach. The geometric mean approach is widely used by many leading statistical agencies at the elementary level of aggregation, including the U.S. Bureau of Labor Statistics; see ILO (2004, chapter 20). Suppose that in period $t-1$ there are $i \in I_{t-1}$ prices sampled, where I_{t-1} is an index set. In period t some of these products disappear and new ones are added so we have the sample $i \in I_t$. The matched items are in the set $I_{t,t-1} = I_{t-1} \cap I_t$. Given this we may then write the Jevons index, $P^J_{t,t-1}$, in two parts as,

$$P^J_{t,t-1} = \prod_{i \in I_{t,t-1}} \left(\frac{p_{imt}}{p_{imt-1}} \right)^{w_{im}} \prod_{i \in I_t \setminus I_{t-1}} \left(\frac{p^*_{imt}}{p_{imt-1}} \right)^{w_{im}}. \tag{3}$$

Here p^*_{imt} is the price in period t, which has been used to replace item i, and has been quality adjusted. This could be adjusted linearly or using our method. We compare the non-linear and linear adjustment methods across all cities in January 2005 for two changes. First, all the items that disappear are 2-liter bottles and they are replaced by 1-liter bottles. Second, the items that disappear are all 2-pack 12-oz

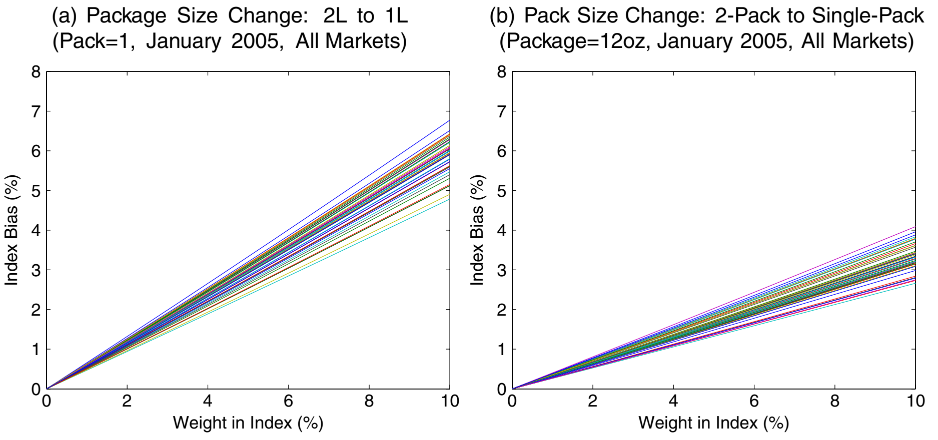

Figure 6. Index Bias Estimates

cans and they are replaced by single 12-oz cans. The bias introduced into the index in such cases, where the missing items have various weights in the index, is illustrated in Figure 6.

The bias is upward because the new prices have been multiplied by two under the linear adjustment case to make them comparable to the old prices. However, according to our results such a large upward adjustment is not warranted. What is clear is that even when the weight on such changes is relatively low—just a few observations are affected—the index can still exhibit significant bias. For example, in the case of a change from a 2-liter to a 1-liter bottle where just 1 percent of the index is affected, the bias is nevertheless equal to an average of 0.58 percent across all the markets we examine. If the weight rises to 5 percent then bias increases to 2.92 percent. The large bias is the result of the fact that the non-linear pricing effects are large. In cases where there is significant market-wide changes in pack or package size—as can happen, say, if a brand decides to stop producing a particular size—then the weight of these quality adjustments in the index can be much larger and they can have very significant effects on overall measured price change.

5. Conclusion

Our primary objective has been to investigate empirically the relationship between package size, pack size, and price with reference to the practices of statistical agencies. Using a flexible hedonic regression framework our empirical results indicate that prices, package, and pack size do not exhibit a simple one-for-one relationship. That is, the price of a product does not double if the package size doubles or if the number of units in a pack doubles. In fact we found that the relationship between price and package size was significantly flatter than this, especially for package size.

This has important implications for the methods which are used to construct official price indexes. While the sign of the error introduced into the index will depend on whether package and pack sizes are increasing or decreasing, it is clear that the difference can potentially be large enough to contaminate index comparisons. For this reason the recommendations of this paper are that statistical agencies discontinue the procedure of scaling prices by changes in the package size, or the number of packs, on a one-for-one basis. There are two main alternatives. The first approach is to exclude the prices from the index altogether. This will reduce the bias that results from quantity–quality adjustment but, as Silver and Heravi (2005) have argued, dropping items in this way may introduce another bias due to selection. It will certainly be the case that removing items from the index will increase the index's variance as less observations are used in constructing it. In some cases, where there are widespread package size changes introduced by a major manufacturer, exclusion may not be possible as it would leave the agency with too few observations. These various factors make exclusion somewhat undesirable. A second approach is to use hedonic regression methods to inform the package and pack size adjustments. This article has outlined a methodology for pursuing this second option and demonstrated that it represents a viable alternative which could potentially be applied in real time. However, our results also provide some cautionary notes. It is clear that the hedonic size function is complex and varies both over markets and to a lesser extent across time. This means that some care must be taken in estimating and implementing these quantity–quality adjustments in practice.

References

Armknecht, P. A. and F. Maitland-Smith, "Price Imputation and Other Techniques for Dealing with Missing Observations, Seasonality and Quality Change in Price Indices," International Monetary Fund, Statistics Department, WP/99/78, 1999.

Bils, M., "Do Higher Prices for New Goods Reflect Quality Growth or Inflation?" *Quarterly Journal of Economics*, 124(2), 637–75, 2009.

Bronnenberg, B. J., M. W. Kruger, and C. F. Mela, "Database Paper: The IRI Marketing Data Set," *Marketing Science*, 27(4), 745–48, 2008.

Clements, K. W., "Pricing and Packaging: The Case of Marijuana," *Journal of Business*, 79, 2019–44, 2006.

Cleveland, W. S., "Robust Locally Weighted Regression and Smoothing Scatterplots," *Journal of the American Statistical Association*, 74(368), 829–36, 1979.

Cohen, A., "Package Size and Price Discrimination in the Paper Towel Market," *International Journal of Industrial Organization*, 26(2), 502–16, 2008.

Diewert, W. E., "Index Number Issues in the Consumer Price Index," *Journal of Economic Perspectives*, 12(1), 47–58, 1998.

———, "Hedonic Regressions: A Consumer Theory Approach," in R. C. Feenstra and M. D. Shapiro (eds), *Scanner Data and Price Indexes*, *Review of Income and Wealth Conference*, Vol. 64, National Bureau of Economic Research, University of Chicago Press, 317–48, 2003a.

———, "Hedonic Regressions: A Review of Some Unresolved Issues," Paper Presented at the Seventh Meeting of the Ottawa Group, Paris, France, 2003b.

Gertsner, E. and J. D. Hess, "The Effect of Consumer Heterogeneity on Package Size and Unit Price Variations," Working Paper No. 66, Department of Economics and Business, North Carolina State University, 1986.

Greenlees, J. S. and R. McClelland, "Does Quality Adjustment Matter for Technologically Stable Products? An Application to the CPI for Food," *American Economic Review*, 101(3), 200–05, 2011.

Hausman, J., "Sources of Bias and Solutions to Bias in the Consumer Price Index," *Journal of Economic Perspectives*, 17(1), 23–44, 2003.

Hong, P., R. P. McAfee, and A. Nayyar, "Equilibrium Price Dispersion with Consumer Inventories," *Journal of Economic Theory*, 105(2), 503–17, 2002.

Huang, R. and R. A. Lopez, "Are You Are What You Eat? Overweight Status and Soft Drink Choices," Selected Paper prepared for presentation at the Agricultural and Applied Economics Associations 2009 AAEA and ACCI Joint Annual Meeting, Milwaukee, 2009.

Hwang, M., B. J. Bronnenberg, and R. Thomadsen, "An Empirical Analysis of Assortment Similarities Across U.S. Supermarkets," *Marketing Science*, 29(5), 858–79, 2010.
ILO, *Consumer Price Index Manual: Theory and Practice*, compiled by ILO in conjunction with the Technical Expert Group on Consumer Price Indices, 2004.
Lal, R. and C. Matutes, "Retail Pricing and Advertising Strategies," *Journal of Business*, 67(3), 345–70, 1994.
Maskin, E. and J. Riley, "Monopoly with Incomplete Information," *Rand Journal of Economics*, 15(2), 171–96, 1984.
McManus, B., "Nonlinear Pricing in an Oligopoly Market: The Case of Speciality Coffee," *RAND Journal of Economics*, 38(2), 512–32, 2007.
Pakes, A., "A Reconsideration of Hedonic Price Indices with an Application to PC's," *American Economic Review*, 93(5), 1578–96, 2003.
Rosen, S., "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition," *Journal of Political Economy*, 82(1), 34–55, 1974.
Silver, M. and S. Heravi, "A Failure in the Measurement of Inflation: Results from a Hedonic and Matched Experiment Using Scanner Data," *Journal of Business and Economic Statistics*, 23(3), 269–81, 2005.
Spence, M., "Nonlinear Prices and Welfare," *Journal of Public Economics*, 8(1), 1–18, 1976.
Triplett, J. E., *Handbook on Hedonic Indexes and Quality Adjustments in Price Indexes: Special Application to Information Technology Products*, prepared for Directorate for Science, Technology and Industry, OECD, 2004.