the review of income and wealth

Review of Income and Wealth
Series 57, Number 3, September 2011
DOI: 10.1111/j.1475-4991.2010.00435.x

MEASURING AND DECOMPOSING CAPITAL INPUT COST

by Bert M. Balk*

Erasmus University and Statistics Netherlands

The measurement of total factor productivity change (or difference) vis-à-vis labor productivity change crucially depends on the measurement and decomposition of capital input cost. This paper discusses the basics of its measurement and shows that one can dispense with the usual neoclassical assumptions. By virtue of its structural features, the measurement model is applicable to individual establishments and aggregates such as industries, sectors, or economies.

1. Introduction

Measuring the value of capital as stock or service flow and the decomposition in price and quantity components is a thorny issue, with deep historical roots and a large literature, recently summarized by Diewert and Schreyer (2008). A classic paper is Hulten (1990). SNA (2008, chapter 20) provides a non-technical introduction, building on the important OECD (2009) manual.

The focus of this paper is on capital as input of a production process. The background is measurement of total (or multi-) factor productivity change, for which the measurement and decomposition of capital input cost is crucial.

This paper develops the theory with a view to practical implementation. Along the way it is shown that there is no need for the usual neoclassical assumptions.

The contributions of this paper can be summarized in the following points:

1. Though the literature acknowledges the fact that for the treatment of capital input it is necessary to distinguish between time periods as points of time (that is, instances of a real variable) and intervals with a definite length, in the subsequent mathematics this distinction often is not maintained. This paper suggests a notation to deal with this problem.
2. The literature by and large seems to assume that investments that have materialized during a certain time period already have become operational at the beginning of the same period. In this paper it is explicitly assumed that investments become operational at each period's midpoint.
3. The literature *grosso modo* neglects the fact that investments concern not only new assets but also used assets, and that there is a substantial trade in used assets. In this paper it is explicitly assumed that investments can be of various ages.

Note: The views expressed in this paper are those of the author and do not necessarily reflect any policy of Statistics Netherlands. Two referees are thanked for useful comments on a previous version.
*Correspondence to: Bert M. Balk, Rotterdam School of Management, Erasmus University, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands (bbalk@rsm.nl).

Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main St, Malden, MA, 02148, USA.

4. The detailed discussion of implementation issues highlights the role of models, assumptions, and approximations, and is instrumental for the design of various types of sensitivity analysis.

In this paper the *ex post* accounting point of view is used. This is consistent with the statistician's point of view, which is by and large accepted for the components of value added and labor input cost. For revenue, intermediate inputs cost, and labor input cost, one simply uses the observed (money) values, whereby at the lowest level of aggregation prices are computed as ratios of observed values and observed quantities; that is, prices are unit values, *ex post* measured. Of course, a distinctive feature of capital input cost is that this cost as such cannot be observed. Imputations must be made, not only to arrive at (an estimate of) the cost, but also to enable one to split the cost in price and quantity components. Imputations are always more or less arbitrary, and depend on the purpose of the accounting exercise. One has to be clear about this.

The layout of this paper is as follows. In Section 2 the fundamental KLEMS-Y input–output model of a production unit is sketched. This provides the framework for what follows. Section 3 introduces our notation, derives the unit user cost formulas for assets available at the beginning of an accounting period as well as for assets invested during this period, and discusses the decomposition of capital input cost change in price and quantity indices. Section 4 discusses the relation with capital stock measures. Unit user cost depends on prices (or valuations) of the assets, but these prices are not observable. Hence, Section 5 discusses the definition of such prices from expected values of the variables involved. This gives rise to a decomposition of total user cost into four components, namely the cost of waiting, the cost of anticipated time-series depreciation, the cost of unanticipated revaluation, and the cost of tax. An important role in the cost of waiting is played by the interest rate, which is also called the "rate of return." There appear to be several concepts of this rate; they are discussed in Section 6. In Section 7 the issue of aggregation is considered. Section 8 discusses some issues of implementation, and Section 9 concludes.

2. The Basic KLEMS-Y Model

Let us consider a single production unit. This could be an establishment or plant, a firm, an industry, a sector, or even an entire economy.

For the output side as well as for the input side of the unit there is some list of commodities (according to some classification scheme). A commodity is thereby defined as a set of closely related items which, for the purpose of analysis, can be considered as "equivalent," either in the static sense of their quantities being additive or in the dynamic sense of displaying equal relative price or quantity changes. Ideally, then, for any accounting period considered (*ex post*), say a year, each commodity comes with a value (in monetary terms) and a price and/or a quantity. At the output side, the prices must be those received by the unit, whereas at the input side, the prices must be those paid. It is assumed that the unit does not deliver to itself. Put otherwise, all the intra-unit deliveries are netted out.

The inputs are customarily classified according to the KLEMS format. The letter K denotes the class of owned, reproducible capital assets. The commodities here are the asset-types, sub-classified by age category. Cohorts of assets are assumed to be available at the beginning of the accounting period and, in deteriorated form (due to ageing, wear and tear), still available at the end of the period. Investment during the period adds entities to these cohorts, while desinvestment, breakdown, or retirement remove entities. Examples include buildings and other structures, land, machinery, transport and ICT equipment, all sorts of tools. As will be discussed later in detail, accounting rules imply that quantities sought are just the quantities of all these cohorts of assets (together representing the productive capital stock), whereas the relevant prices are their unit user costs (per type–age combination), constructed from imputed interest rates, depreciation profiles, (anticipated) revaluations, and tax rates. The sum of quantities times prices then provides the capital input cost of a production unit.

The letter L denotes the class of labor inputs; that is, all the types of work that are important to distinguish, cross-classified for instance according to educational attainment, gender, and experience (which is usually proxied by age categories). Quantities are measured as hours worked (or paid), and prices are the corresponding wage rates per hour. Where applicable, imputations must be made for the work executed by self-employed persons. The sum of quantities times prices provides the labor input cost (or the labor bill, or labor compensation, as it is sometimes called).

The classes K and L concern so-called primary inputs. The letters E, M, and S denote three, disjunct classes of so-called intermediate inputs. First, E is the class of energy commodities consumed by a production unit: oil, gas, electricity, and water. Second, M is the class of all the (physical) materials consumed in the production process, which could be sub-classified into raw materials, semi-fabricates, and auxiliary products. Third, S is the class of all the business services which are consumed for maintaining the production process. This class includes the services of leased capital assets as well as all the outsourced activities. Though it is not at all a trivial task to define precisely all the intermediate inputs and to classify them, it can safely be assumed that at the end of each accounting period there is a quantity and a price associated with each of those inputs.

Then, for each accounting period, production cost is defined as the sum of primary and intermediate input cost.

At the output side, the letter Y denotes the class of commodities, goods, and/or services, which are produced by the unit. Though in some industries, such as services industries or industries producing mainly unique goods, definitional problems are formidable, it can safely be assumed that for each accounting period there are data on quantities produced. For units operating on the market there are also prices. The sum of quantities times prices then provides the production revenue, and, apart from taxes on production, revenue minus cost yields profit.

The situation as pictured in the preceding paragraphs is typical for a unit operating on a (output) market. Because a non-market unit has no output prices there is no revenue. Though there is cost, like for market units, there is no profit. National accountants usually resolve the problem here by *defining* the revenue of a non-market unit to be equal to its cost, thereby setting profit equal to 0.

Cash flow is defined as revenue minus intermediate inputs cost and labor input cost.[1] Thus, cash flow minus capital input cost equals profit.

Cash flow, when positive, can be seen as the (gross) return to capital input. Balk (2010) called this the K-CF model. Of course, this model only makes sense when the production unit actually owns capital assets. But let us assume that this is indeed the case.

3. Capital Input Cost

The K-CF model provides a good point of departure for a discussion of the measurement of capital input cost.[2] Cash flow, as defined in the foregoing, is the (*ex post* measured) monetary balance of all the flow variables. Capital input cost is different, since capital is a stock variable. Basically, capital input cost is measured as the difference between the book values of the production unit's owned capital stock at beginning and end of the accounting period considered.

Our notation must reflect this. The beginning of period t is denoted by t^-, and its end by t^+. Thus a period is an interval of time $t = [t^-, t^+]$, where $t^- = (t-1)^+$ and $t^+ = (t+1)^-$. Occasionally, the variable t will also be used to denote the midpoint of the period.

All the assets are supposed to be economically born at midpoints of periods, whether this has occurred inside or outside the production unit under consideration. Thus the age of an asset of type i at (the midpoint of) period t is a non-negative integer number $j = 0, \ldots, J_i$. The age of this asset at the beginning of the period is $j - 0.5$, and at the end $j + 0.5$. The economically maximal service life of asset type i is denoted by J_i.[3]

The opening stock of capital assets is the inheritance of past investments and desinvestments; hence, the opening stock consists of cohorts of assets of various types, each cohort comprising a number of assets of the same age. By convention, assets that are discarded (normally retired or prematurely scrapped) or sold during a certain period t are supposed to be discarded or sold at the end of that period; that is, at t^+. Second-hand assets that are acquired during period t from other production units are supposed to be acquired at the beginning of the next period, $(t+1)^-$. However, all other acquisitions of second-hand assets and those of new assets are supposed to happen at the midpoint of the period, and such assets are supposed to be immediately operational.

Hence, all the assets that are part of the opening stock remain active through the entire period $[t^-, t^+]$. The period t investments are supposed to be active through the second half of period t, that is, $[t, t^+]$. Put otherwise, the stock of capital assets at t, the midpoint of the period, is the same as the stock at t^-, the beginning of the period, but 0.5 period older. At the midpoint of the period the investments, of various age, are added to the stock. Notice, however, that the closing stock at t^+,

[1]Cash flow is also called gross or variable profit. The National Accounts term is "gross operating surplus."

[2]This and the next section lean heavily on Balk and van den Bergen (2006).

[3]It is of course a simplification to assume that the economically maximal service life of an asset type is some given constant. Actually a time superscript should be added. See Diewert (2009) for some theoretical considerations.

the end of the period, is not necessarily identical to the opening stock at $(t+1)^-$, because of the convention on sale, acquisition, and discard of assets.

Let K_{ij}^t denote the quantity (number) of asset type i ($i = 1, \ldots, I$) and age j ($j = 1, \ldots, J_i$) at the midpoint of period t. These quantities are non-negative; some of them might be equal to 0. Further, let I_{ij}^t denote the (non-negative) quantity (number) of asset type i ($i = 1, \ldots, I$) and age j ($j = 0, \ldots, J_i$) that is added to the stock at the midpoint of period t. The following relations are useful to keep in mind:

(1) $$K_{i,j-0.5}^{t^-} = K_{ij}^t \quad (j = 1, \ldots, J_i)$$

(2) $$I_{i0}^t = K_{i,0.5}^{t^+}$$

(3) $$I_{ij}^t + K_{ij}^t = K_{i,j+0.5}^{t^+} \quad (j = 1, \ldots, J_i)$$

(4) $$K_{i,(j+1)-0.5}^{(t+1)^-} = K_{i,j+0.5}^{t^+} + B_{i,j+0.5}^{t^+} \quad (j = 0, \ldots, J_i - 1)$$

(5) $$K_{i,(J_i+1)-0.5}^{(t+1)^-} = 0,$$

where $B_{i,j+0.5}^{t^+}$ denotes the balance of sale, acquisition, and discard at t^+. In general, estimates of the K, I, and B variables are generated from detailed investment and desinvestment surveys, combined with the Perpetual Inventory Method. The level of detail can very between units. We are now ready to define the concept of user cost for assets that are owned by the production unit.

The first distinction that must be made is between assets that are part of the opening stock of a period, and investments which are made during this period. Consider an asset of type i that has age j at the midpoint of period t. Its price (or valuation) at the beginning of the period is denoted by $P_{i,j-0.5}^{t^-}$, and its price (or valuation) at the end of the period by $P_{i,j+0.5}^{t^+}$. For the time being, we consider such prices as being given, and postpone their precise definition to a next section. The prices are assumed to be non-negative; some might be equal to 0. In any case, $P_{i,J_i+0.5}^{t^+} = 0$; that is, an asset that has reached its economically maximal age in period t is valued with a zero price at the end of this period.

The (*ex post*) unit user cost over period t of an opening stock asset of type i that has age j at the midpoint of the period is then *defined* as

(6) $$u_{ij}^t \equiv r^t P_{i,j-0.5}^{t^-} + \left(P_{i,j-0.5}^{t^-} - P_{i,j+0.5}^{t^+}\right) + \tau_{ij}^t \quad (j = 1, \ldots, J_i).$$

There are three components here. Let us start with the second, most important, one. This part of expression (6), $P_{i,j-0.5}^{t^-} - P_{i,j+0.5}^{t^+}$, is the value change of the asset between the beginning and end of the accounting period. Among national accountants this value change is called (nominal) time-series depreciation. It combines the effect of the progress of time, from t^- to t^+, with the effect of ageing, from $j - 0.5$ to $j + 0.5$. In general, the difference between the two prices (valuations) comprises the effect of exhaustion, deterioration, and obsolescence.

The third component, τ_{ij}^t, denotes the specific tax(es) that is (are) levied on the use of an asset of type i and age j during period t.

Finally, the first component, $r^t P_{i,j-0.5}^{t^-}$, is the price (or valuation) of this asset at the beginning of the period, when its age is $j - 0.5$, times an interest rate r^t. This component reflects the premium that must be paid by the user of the asset to its owner to prevent it being sold, right at the beginning of the period, and the revenue used for immediate consumption; it is therefore also called the price of "waiting."[4] Another interpretation is to see this component as the actual or imputed interest cost to finance the monetary capital that is tied up in the asset; it is then called "opportunity cost." Anyway, it is a sort of remuneration which, because there might be a risk component involved, is specific for the production unit and the asset, though the last complication is usually not taken into account.[5]

Unit user cost as defined by expression (6) is also called "rental price," because it can be considered as the rental price that the owner of the asset as owner would charge to the owner as user. Put otherwise, unit user cost is like a lease price.[6] Though (6) has been justified from various viewpoints, the expression basically goes back to Walras (1874, p. 269).

One particular justification is worth recalling. Rearranging expression (6) delivers

$$(7) \qquad P_{i,j-0.5}^{t^-} + r^t P_{i,j-0.5}^{t^-} = \left(u_{ij}^t - \tau_{ij}^t\right) + P_{i,j+0.5}^{t^+} \quad (j = 1, \ldots, J_i).$$

Selling the asset at the beginning of the period and generating a return from the proceeds of the sale should cover the rental price minus tax and the money necessary for buying back the asset at the end of the period when it is a full period older. Thus equation (7) could be seen as the outcome of an arbitrage process. There is, however, no behavioral assumption involved, because the rental price is not an independent variable, but defined by the same equation. The rental price is precisely equal to the cost of using this asset during one period, which is the sum of value change, tax, and opportunity cost.[7]

Let us now turn to the unit user cost of an asset of type i and age j that is acquired at the midpoint of period t. To keep things simple, this user cost is, analogous to expression (6), defined as

[4]According to Rymes (1983) this naming goes back to Pigou.

[5]SNA (2008, par. 6.130) implicitly prescribes that for non-market units the interest rate r^t must be set equal to 0.

[6]It should be noted that all the operational costs associated with the use of a particular asset are accounted for as intermediate or labor inputs cost. For investment decisions expected values of such cost components must of course be considered together with expected user cost.

[7]In the case of irreversible investments, such as electricity networks, basically a slight modification of expression (6) must be used, as argued by Diewert *et al.* (2009, chapter 10). The first part remains as it is; in the second part, now called amortization amount, instead of the actual end-of-period price the expected end-of-period price must be used; and the tax component disappears.

$$v_{ij}^{t} \equiv (1/2)\, r^{t} P_{i,j}^{t} + \left(P_{i,j}^{t} - P_{i,j+0.5}^{t^{+}}\right) + (1/2)\,\tau_{ij}^{t} \quad (j = 0, \ldots, J_{i}). \tag{8}$$

The difference from the previous formula is that here the second half of the period instead of the entire period is taken into account.[8]

Total user cost over all asset types and ages, for period t, is then naturally defined by

$$C_{K}^{t} \equiv \sum_{i=1}^{I} \sum_{j=1}^{J_i} u_{ij}^{t} K_{ij}^{t} + \sum_{i=1}^{I} \sum_{j=0}^{J_i} v_{ij}^{t} I_{ij}^{t}. \tag{9}$$

We see that the set of commodities K consists of two subsets, corresponding respectively to the type–age classes of assets that are part of the opening stock and the type–age classes of assets that are acquired later. The dimension of the first set is $\sum_{i=1}^{I} J_i$, and the dimension of the second set is $\sum_{i=1}^{I}(1+J_i)$. The input prices are given by expressions (6) and (8), respectively, while the quantities are given by K_{ij}^{t} and I_{ij}^{t}, respectively.

The next task is to split, for any two periods $t = 0, 1$, the cost ratio C_K^1 / C_K^0 into a price index and a quantity index, or the cost difference $C_K^1 - C_K^0$ into a price indicator and a quantity indicator. There is some choice here and guidance can be obtained from Balk (2008). For example, the Laspeyres quantity index reads

$$Q_{K}^{L}(1,0) \equiv \frac{\sum_{i=1}^{I} \sum_{j=1}^{J_i} u_{ij}^{0} K_{ij}^{1} + \sum_{i=1}^{I} \sum_{j=0}^{J_i} v_{ij}^{0} I_{ij}^{1}}{\sum_{i=1}^{I} \sum_{j=1}^{J_i} u_{ij}^{0} K_{ij}^{0} + \sum_{i=1}^{I} \sum_{j=0}^{J_i} v_{ij}^{0} I_{ij}^{0}}. \tag{10}$$

When all or a number of the maximal life times J_i are time-dependent, a detour might be necessary. First, one computes a price index $P_K(1, 0)$ on the common vintages, and next an implicit quantity index as $\left(C_K^1 / C_K^0\right) / P_K(1,0)$.

If all the variables occurring in expression (9) were observable, then our story would almost end here. However, this is not the case. Though the quantity variables are in principle observable, the price variables are not. To start with, the expressions (6) and (8) contain prices (valuations) for all asset types and ages, but, except for new assets and where markets for second-hand assets exist, such prices are not observable. Thus, we need models.

But first we turn to the relation between measures of capital input cost and capital stock.

[8]The factor $(1/2)r^t$ is meant as an approximation to $(1 + r_t)^{1/2} - 1$, and the factor $(1/2)\,\tau_{ij}^{t}$ as an approximation to $\left(\left(1 + \tau_{ij}^{t} / P_{i,j-0.5}^{t^{-}}\right)^{1/2} - 1\right) P_{i,j}^{t}$.

4. The Relation with Capital Stock Measures

The set of quantities $\{K_{ij}^t,\ I_{ij}^t;\ i = 1, \ldots, I;\ j = 0, \ldots, J_i\}$ represents the so-called productive capital stock of the production unit. This is an enumeration of the assets that make production during period t possible. The total value of these assets at the midpoint of period t can be calculated as

$$\text{(11)} \qquad NCS^t = \sum_{i=1}^{I}\sum_{j=1}^{J_i} P_{ij}^t K_{ij}^t + \sum_{i=1}^{I}\sum_{j=0}^{J_i} P_{ij}^t I_{ij}^t.$$

This value is called the net (or wealth) capital stock. Notice that, though the quantities in expression (11) are the same as those occurring in (9), the prices are different. For any two periods $t = 0, 1$ the ratio NCS^1/NCS^0 can also be split into a price index and a quantity index. For example, the Laspeyres quantity index reads

$$\text{(12)} \qquad Q_{NCS}^L(1,0) \equiv \frac{\sum_{i=1}^{I}\sum_{j=1}^{J_i} P_{ij}^0 K_{ij}^1 + \sum_{i=1}^{I}\sum_{j=0}^{J_i} P_{ij}^0 I_{ij}^1}{\sum_{i=1}^{I}\sum_{j=1}^{J_i} P_{ij}^0 K_{ij}^0 + \sum_{i=1}^{I}\sum_{j=0}^{J_i} P_{ij}^0 I_{ij}^0}.$$

Both quantity indices, $Q_{NCS}^L(1,0)$ and $Q_K^L(1,0)$, measure the volume change of the productive capital stock. The prices, used to weight the quantities, are different, however. The quantity index $Q_{NCS}^L(1,0)$ is called a volume index of the capital *stock*, whereas $Q_K^L(1,0)$ is called a volume index of capital *services*. Their numerical difference can be appreciable (see, for example, Coremberg, 2008).

If there are no transactions in second-hand assets, then the number of assets K_{ij}^t is equal to the number of new investments of j periods earlier, I_{i0}^{t-j}, adjusted for the probability of survival. Then expression (9) reduces to

$$\text{(13)} \qquad C_K^t = \sum_{i=1}^{I}\left(\sum_{j=1}^{J_i} u_{ij}^t I_{i0}^{t-j} + v_{i0}^t I_{i0}^t\right).$$

If, in addition, new investments are supposed to start their economic life at the beginning of the first-next period, then the last expression reduces further to

$$\text{(14)} \qquad C_K^t = \sum_{i=1}^{I}\left(\sum_{j=1}^{J_i} u_{ij}^t I_{i0}^{t-j}\right),$$

which is the classical formula for the value of capital services. This formula can be rewritten as

$$\text{(15)} \qquad C_K^t = \sum_{i=1}^{I} u_{i1}^t\left(\sum_{j=1}^{J_i} \left(u_{ij}^t / u_{i1}^t\right) I_{i0}^{t-j}\right),$$

provided that $u_{i1}^t \neq 0$. If it could be assumed that, for each asset type, the unit user cost ratios are independent of time,[9] that is,

$$u_{ij}^t / u_{i1}^t = \phi_{ij} \quad (i = 1, \ldots, I;\ j = 1, \ldots, J_i), \tag{16}$$

then expression (15) reduces to

$$C_K^t = \sum_{i=1}^{I} u_{i1}^t \left(\sum_{j=1}^{J_i} \phi_{ij} I_{i0}^{t-j} \right). \tag{17}$$

Notice that $\phi_{i1} = 1$ $(i = 1, \ldots, I)$. Moreover, common sense suggests that $0 < \phi_{ij} \leq 1$ $(i = 1, \ldots, I; j = 1, \ldots, J_i)$. For each asset $i = 1, \ldots, I$, the coefficients ϕ_{ij} serve to transform (linearly) assets of age 2 to J_i into assets of age 1. The set $\{\phi_{i1}, \ldots, \phi_{iJ_i}\}$ is called the age-efficiency profile of asset type i. The part between brackets in expression (17) is called the productive capital stock of asset type i, measured in efficiency units. Put otherwise, given the age–efficiency profile, the units of measurement become units of age 1, and the only unit user cost that is needed for the computation of (17) is the user cost of a one year old asset.

Under our two assumptions—no transactions in second-hand assets and new investments start their economic life at the beginning of the first-next period—expression (11) reduces to

$$NCS^t = \sum_{i=1}^{I} \left(\sum_{j=1}^{J_i} P_{ij}^t I_{i0}^{t-j} \right) \tag{18}$$

which is the classical formula for the net capital stock value. This formula can be rewritten as

$$NCS^t = \sum_{i=1}^{I} P_{i0}^t \left(\sum_{j=1}^{J_i} (P_{ij}^t / P_{i0}^t) I_{i0}^{t-j} \right), \tag{19}$$

provided that $P_{i0}^t \neq 0$. If it could be assumed that, for each asset type, the price (valuation) ratios are independent of time, that is,

$$P_{ij}^t / P_{i0}^t = \varphi_{ij} (i = 1, \ldots, I; k = 1, \ldots, J_i), \tag{20}$$

then expression (19) reduces to

$$NCS^t = \sum_{i=1}^{I} P_{i0}^t \left(\sum_{j=1}^{J_i} \varphi_{ij} I_{i0}^{t-j} \right). \tag{21}$$

[9]Authors such as Hulten (1990) suggest that "in equilibrium" this will be the case. Hulten (2009) calls it a "strong assumption."

Since ageing in general reduces the value of an asset, it is to be expected that the coefficients φ_{ij} are smaller than 1 and their successive magnitudes declining. The set $\{\varphi_{i1}, \ldots, \varphi_{iJ_i}\}$ is called the age–price profile of asset type i.

According to the literature, the productive capital stock at current prices is defined as

$$PCS^t = \sum_{i=1}^{I} P_{i0}^t \left(\sum_{j=1}^{J_i} \phi_{ij} I_{i0}^{t-j} \right), \tag{22}$$

where P_{i0}^t is the price (valuation) at t of an asset of type i and age 0; that is, a new asset. It is interesting to compare expressions (21) and (22). While the value of the productive capital stock is obtained by aggregating the various vintages by the age–efficiency profile, the net value of the capital stock is obtained by aggregating the vintages by the age–price profile. These two profiles are in general different.[10] As is clear from expression (17), it is the productive capital stock that plays a role in the determination of the value of capital services. However, the more primitive expression for the value of capital services is expression (14), where the vintages are aggregated by their unit user costs.[11]

We now return to where we left off at the end of the previous section.

5. The Relation Between Asset Price and Unit User Cost

Consider expression (6) and rewrite it in the form

$$u_{ij}^t - \tau_{ij}^t = (1+r^t) P_{i,j-0.5}^{t^-} - P_{i,j+0.5}^{t^+} (j = 1, \ldots, J_i). \tag{23}$$

For any asset that is not prematurely discarded it will be the case that its value at the end of period t is equal to its value at the beginning of period $t + 1$; formally, $P_{i+j+0.5}^{t^+} = P_{i,(j+1)-0.5}^{(t+1)^-}$. Substituting this into expression (23), and rewriting again, one obtains

$$P_{i,j-0.5}^{t^-} = \frac{1}{1+r^t} \left(P_{i,(j+1)-0.5}^{(t+1)^-} + u_{ij}^t - \tau_{ij}^t \right) \quad (j = 1, \ldots, J_i). \tag{24}$$

This expression links the price of an asset at the beginning of period t with its price at the beginning of period $t + 1$, being then 1 period older. But a similar relation links its price at the beginning of period $t + 1$ with its price at the beginning of period $t + 2$, being then again 1 period older,

$$P_{i,(j+1)-0.5}^{(t+1)^-} = \frac{1}{1+r^{t+1}} \left(P_{i,(j+2)-0.5}^{(t+2)^-} + u_{i,j+1}^{t+1} - \tau_{i,j+1}^{t+1} \right) \quad (j = 1, \ldots, J_i). \tag{25}$$

[10]Under mild regularity conditions it can be shown that a geometric age–efficiency profile, $\phi_{ij} = (1 - \delta_i)^{j-1}$, implies a geometric age–price profile, $\varphi_{ij} = (1 - \delta_i)^j$, and vice versa. See SNA (2008, par. 20.22–24) for a simple proof.

[11]Using expressions (14) and (18) as points of departure for the construction of price and quantity indices was first suggested by Diewert and Lawrence (2000).

This can be continued until

$$(26)\qquad P_{i,J_i-0.5}^{(t+J_i-j)^-}=\frac{1}{1+r^{t+J_i-j}}\left(P_{i,J_i+0.5}^{(t+J_i-j+1)^-}+u_{i,J_i}^{t+J_i-j}-\tau_{i,J_i}^{t+J_i-j}\right)\quad (j=1,\ldots,J_i),$$

since we know that $P_{i,J_i+0.5}^{(t+J_i-j+1)^-}=P_{i,J_i+0.5}^{(t+J_i-j)^+}=0$. Substituting expression (25) into (24), etc., one finally obtains

$$(27)\qquad P_{i,j-0.5}^{t}=\frac{u_{ij}^t-\tau_{ij}^t}{1+r^t}+\frac{u_{i,j+1}^{t+1}-\tau_{i,j+1}^{t+1}}{(1+r^t)(1+r^{t+1})}+\ldots+\frac{u_{i,J_i}^{t+J_i-j}-\tau_{i,J_i}^{t+J_i-j}}{(1+r^t)\ldots(1+r^{t+J_i-j})}.$$

Here materializes what is known as the fundamental asset price equilibrium equation. Notice, however, that there was no equilibrium—whatever that may mean—assumed here, and that there are no other economic behavioral assumptions involved; it is just a mathematical result. Expressions (23) and (27) are dual. The first derives the (ex tax) unit user cost from discounted asset prices, while the second derives the asset price as the sum of discounted future (ex tax) unit user costs; the discounting is executed by means of future interest rates.

A mathematical truth like expression (27), however, is not immediately helpful in the real world. At the beginning, or even at the end of period t, most if not all of the data that are needed for the computation of the asset prices $P_{i,j-0.5}^{t^-}$ and $P_{i,j+0.5}^{t^+}$ are *not* available. Thus, in practice, expression (27) must be filled in with expectations, and these depend on the point of time from which one looks at the future. A rather natural vantage point is the beginning of period t; thus, the operator $\mathcal{E}^{t^-}$ placed before a variable means that the expected value of the variable at t^- is taken. Modifying expression (27), the price at the beginning of period t of an asset of type i and age $j-0.5$ is given by

$$(28)\qquad P_{i,j-0.5}^{t^-}\equiv\frac{\mathcal{E}^{t^-}(u_{ij}^t-\tau_{ij}^t)}{1+\mathcal{E}^{t^-}r^t}+\frac{\mathcal{E}^{t^-}(u_{i,j+1}^{t+1}-\tau_{i,j+1}^{t+1})}{(1+\mathcal{E}^{t^-}r^t)(1+\mathcal{E}^{t^-}r^{t+1})}+\ldots+\frac{\mathcal{E}^{t^-}\left(u_{i,\mathcal{E}^{t^-}J_i}^{t+\mathcal{E}^{t^-}J_i-j}-\tau_{i,\mathcal{E}^{t^-}J_i}^{t+\mathcal{E}^{t^-}J_i-j}\right)}{(1+\mathcal{E}^{t^-}r^t)\ldots\left(1+\mathcal{E}^{t^-}r^{t+\mathcal{E}^{t^-}J_i-j}\right)}.$$

Notice in particular that in this expression the economically maximal age, as expected at the beginning of period t, $\mathcal{E}^{t^-}J_i$, occurs. Put otherwise, at the beginning of period t the remaining economic lifetime of the asset is expected to be $\mathcal{E}^{t^-}J_i-j-0.5$ periods.[12] For each of the coming periods there is an expected (ex tax) rental, and the (with expected interest rates) discounted rentals are summed. This sum constitutes the price (value) of the asset.

[12]See Erumban (2008) on the estimation of expected lifetimes for three types of assets in a number of industries.

Similarly, the price at the end of period t of an asset of type i and age $j + 0.5$ is given by

$$(29)\quad P^{t^+}_{i,j+0.5} = P^{(t+1)}_{i,(j+1)-0.5} \equiv \frac{\mathcal{E}^{(t+1)^-}\left(u^{t+1}_{i,j+1} - \tau^{t+1}_{i,j+1}\right)}{1+\mathcal{E}^{(t+1)^-} r^{t+1}} + \frac{\mathcal{E}^{(t+1)^-}\left(u^{t+2}_{i,j+2} - \tau^{t+2}_{i,j+2}\right)}{\left(1+\mathcal{E}^{(t+1)^-} r^{t+1}\right)\left(1+\mathcal{E}^{(t+1)^-} r^{t+2}\right)} + \ldots + \frac{\mathcal{E}^{(t+1)^-}\left(u^{t+\mathcal{E}^{(t+1)^-} J_i - j}_{i,\mathcal{E}^{(t+1)} J_i} - \tau^{t+\mathcal{E}^{(t+1)^-} J_i - j}_{i,\mathcal{E}^{(t+1)} J_i}\right)}{\left(1+\mathcal{E}^{(t+1)^-} r^{t+1}\right)\ldots\left(1+\mathcal{E}^{(t+1)^-} r^{t+\mathcal{E}^{(t+1)^-} J_i - j}\right)}.$$

Notice that this price depends on the economically maximal age, as expected at the beginning of period $t + 1$ (which is the end of period t), $\mathcal{E}^{(t+1)^-} J_i$, which may or may not differ from the economically maximal age, as expected one period earlier, $\mathcal{E}^{t^-} J_i$. The last mentioned expected age plays a role in the price at the end of period t of an asset of type i and age $j + 0.5$, as expected at the beginning of this period,

$$(30)\quad \mathcal{E}^{t^-} P^{t^+}_{i,j+0.5} \equiv \frac{\mathcal{E}^{t^-}\left(u^{t+1}_{i,j+1} - \tau^{t+1}_{i,j+1}\right)}{1+\mathcal{E}^{t^-} r^{t+1}} + \frac{\mathcal{E}^{t^-}\left(u^{t+2}_{i,j+2} - \tau^{t+2}_{i,j+2}\right)}{\left(1+\mathcal{E}^{t^-} r^{t+1}\right)\left(1+\mathcal{E}^{t^-} r^{t+2}\right)} + \ldots + \frac{\mathcal{E}^{t^-}\left(u^{t+\mathcal{E}^{t^-} J_i - j}_{i,\mathcal{E}^{t^-} J_i} - \tau^{t+\mathcal{E}^{t^-} J_i - j}_{i,\mathcal{E}^{t^-} J_i}\right)}{\left(1+\mathcal{E}^{t^-} r^{t+1}\right)\ldots\left(1+\mathcal{E}^{t^-} r^{t+\mathcal{E}^{t^-} J_i - j}\right)}.$$

Expression (30) was obtained from expression (28) by deleting its first term as well as the first period discount factor $1+\mathcal{E}^{t^-} r^t$. This reflects the fact that at the end of period t the asset's remaining lifetime has become shorter by one period. Generally one may expect that $\mathcal{E}^{t^-} P^{t^+}_{i,j+0.5} \le P^{t^-}_{i,j-0.5}$.

Expression (29) differs from expression (30) in that expectations are at $(t + 1)^-$ instead of t^-. Since one may expect that, due to technological progress, the remaining economic lifetime of any asset shortens, that is, $\mathcal{E}^{(t+1)^-} J_i < \mathcal{E}^{t^-} J_i$, expression (29) contains fewer terms than expression (30). Generally one may expect that $P^{t^+}_{i,j+0.5} < \mathcal{E}^{t^-} P^{t^+}_{i,j+0.5}$; that is, the actual price of an asset at the end of a period is less than or equal to the price as expected at the beginning.

Armed with these insights we return to the unit user cost expressions (6) and (8). Natural decompositions of these two expressions are

$$(31)\quad u^t_{ij} = r^t P^{t^-}_{i,j-0.5} + \left(P^{t^-}_{i,j-0.5} - \mathcal{E}^{t^-} P^{t^+}_{i,j+0.5}\right) + \left(\mathcal{E}^{t^-} P^{t^+}_{i,j+0.5} - P^{t^+}_{i,j+0.5}\right) + \tau^t_{ij} \quad (j = 1, \ldots, J_i),$$

and

$$(32)\quad v^t_{ij} = (1/2) r^t P^t_{i,j} + \left(P^t_{i,j} - \mathcal{E}^t P^{t^+}_{i,j+0.5}\right) + \left(\mathcal{E}^t P^{t^+}_{i,j+0.5} - P^{t^+}_{i,j+0.5}\right) + (1/2)\,\tau^t_{ij} \quad (j = 0, \ldots, J_i).$$

As before, the first term at either right-hand side represents the price of waiting. The second term, between brackets, is called anticipated time-series depreciation, and could be decomposed further into the anticipated effect of time (or, anticipated revaluation) and the anticipated effect of ageing (or, anticipated cross-sectional depreciation). The third term, also between brackets, is called unanticipated revaluation. We will come back to these terms later.

The underlying idea is that, at the beginning of each period or, in the case of investment, at the midpoint, economic decisions are based on anticipated rather than realized prices. The fourth term in the two decompositions is again the tax term. It is here assumed that with respect to waiting and tax, anticipated and realized prices coincide.

Substituting expressions (31) and (32) into expression (9), one obtains the following aggregate decomposition:

$$\begin{aligned}
(33)\qquad C_K^t = {} & \sum_{i=1}^{I}\sum_{j=1}^{J_i} r^t P_{i,j-0.5}^{t^-} K_{ij}^t + \sum_{i=1}^{I}\sum_{j=0}^{J_i} (1/2) r^t P_{i,j}^t I_{ij}^t + \\
& \sum_{i=1}^{I}\sum_{j=1}^{J_i}\left(P_{i,j-0.5}^{t^-} - \mathcal{E}^{t^-} P_{i,j+0.5}^{t^+}\right)K_{ij}^t + \sum_{i=1}^{I}\sum_{j=0}^{J_i}\left(P_{i,j}^t - \mathcal{E}^t P_{i,j+0.5}^{t^+}\right)I_{ij}^t + \\
& \sum_{i=1}^{I}\sum_{j=1}^{J_i}\left(\mathcal{E}^{t^-} P_{i,j+0.5}^{t^+} - P_{i,j+0.5}^{t^+}\right)K_{ij}^t + \sum_{i=1}^{I}\sum_{j=0}^{J_i}\left(\mathcal{E}^t P_{i,j+0.5}^{t^+} - P_{i,j+0.5}^{t^+}\right)I_{ij}^t + \\
& \sum_{i=1}^{I}\sum_{j=1}^{J_i} \tau_{ij}^t K_{ij}^t + \sum_{i=1}^{I}\sum_{j=0}^{J_i}(1/2)\,\tau_{ij}^t I_{ij}^t.
\end{aligned}$$

On the first line after the equality sign we have the aggregate cost of waiting,

$$(34)\qquad C_{K,w}^t \equiv r^t\left(\sum_{i=1}^{I}\sum_{j=1}^{J_i} P_{i,j-0.5}^t K_{ij}^t + \sum_{i=1}^{I}\sum_{j=0}^{J_i}(1/2)\,P_{i,j}^t I_{ij}^t\right).$$

Notice that the part between brackets differs slightly from the net capital stock as defined by expression (11). It can be interpreted as the value of the production unit's productive capital stock *as used* during period t.

On the second line after the equality sign in expression (33) we have the aggregate cost of anticipated time-series depreciation,

$$(35)\qquad C_{K,e}^t \equiv \sum_{i=1}^{I}\sum_{j=1}^{J_i}\left(P_{i,j-0.5}^{t^-} - \mathcal{E}^{t^-} P_{i,j+0.5}^{t^+}\right)K_{ij}^t + \sum_{i=1}^{I}\sum_{j=0}^{J_i}\left(P_{i,j}^t - \mathcal{E}^{t^-} P_{i,j+0.5}^{t^+}\right)I_{ij}^t.$$

On the third line we have the aggregate cost of unanticipated revaluation,

$$(36)\qquad C_{K,u}^t \equiv \sum_{i=1}^{I}\sum_{j=1}^{J_i}\left(\mathcal{E}^{t^-} P_{i,j+0.5}^{t^+} - P_{i,j+0.5}^{t^+}\right)K_{ij}^t + \sum_{i=1}^{I}\sum_{j=0}^{J_i}\left(\mathcal{E}^t P_{i,j+0.5}^{t^+} - P_{i,j+0.5}^{t^+}\right)I_{ij}^t.$$

Finally, on the fourth line we have the aggregate cost of tax,

(37) $$C^t_{K,tax} \equiv \sum_{i=1}^{I}\sum_{j=1}^{J_i} \tau^t_{ij} K^t_{ij} + \sum_{i=1}^{I}\sum_{j=0}^{J_i} (1/2)\, \tau^t_{ij} I^t_{ij}.$$

Using all these definitions, expression (33) reduces to

(38) $$C^t_K = C^t_{K,w} + C^t_{K,e} + C^t_{K,u} + C^t_{K,tax}.$$

Thus, capital input cost can rather naturally be split into four meaningful components.

6. Rates of Return

The K-CF model is governed by the following accounting identity, where input categories are placed left and output categories are placed right of the equality sign:

(39) $$C^t_{K,w} + C^t_{K,e} + C^t_{K,u} + C^t_{K,tax} + \Pi^t = CF^t,$$

where CF^t denotes *ex post* cash flow generated by the operations of the production unit during period t, and profit Π^t is defined by this identity.

The next model is based on the idea that the (*ex post*) cost of time-series depreciation plus tax should be treated in the same way as the cost of intermediate inputs, and thus subtracted from cash flow. Hence, the output concept is called net cash flow, and the remaining input cost is the waiting cost of capital. This is called the K-NCF model, which is governed by

(40) $$C^t_{K,w} + \Pi^t = CF^t - \left(C^t_{K,e} + C^t_{K,u} + C^t_{K,tax}\right) \equiv NCF^t.$$

This accounting relation provides an excellent point of departure for a discussion of the interest rate r^t, which determines the aggregate cost of waiting or opportunity cost $C^t_{K,w}$ according to expression (34). Using expression (34), equation (40) can be rewritten as

(41) $$r^t UCS^t + \Pi^t = NCF^t,$$

where

(42) $$UCS^t \equiv \sum_{i=1}^{I}\sum_{j=1}^{J_i} P^{t^-}_{i,j-0.5} K^t_{ij} + \sum_{i=1}^{I}\sum_{j=1}^{J_i} (1/2)\, P^t_{i,j} I^t_{ij},$$

which can be interpreted as the (value of the) production unit's capital stock as used during period t. Provided that $NCF^t \geq \Pi^t \geq 0$, equation (41) then says that, apart from profit, net cash flow provides the return to the (owner of the) capital stock. This is the reason why r^t is also called the "rate of return."

In principle, the value of the capital stock as well as the net cash flow are empirically determined. That leaves an equation with two unknowns, namely the rate of return r^t and profit Π^t.

Setting $\Pi^t = 0$ and solving equation (41) for r^t delivers the so-called "endogenous," or "internal," or "balancing," rate of return. This solution is, of course, specific for the production unit. Net cash flow is calculated *ex post*, since it contains total time-series depreciation. Thus, the endogenous rate of return as calculated from expression (41) is also an *ex post* concept. The alternative is to specify some reasonable, *exogenous* value for the rate of return, say the annual percentage of headline CPI change plus something. Then, of course, profit follows from equation (41) and will in general be unequal to 0.

The endogenous rate of return is defined by the equation

$$r_{endo}^t UCS^t = NCF^t. \tag{43}$$

Combining this with expression (41) delivers the following relation between the endogenous and some exogenous rate of return:

$$r_{endo}^t = r^t + \Pi^t / UCS^t. \tag{44}$$

Hence, profit Π^t is positive if and only if $r_{endo}^t > r^t$. Then relation (44) can be interpreted as saying that r_{endo}^t absorbs profit.

A variant of the K-NCF model is obtained by considering unanticipated revaluation, which is the unanticipated part of time-series depreciation, as a component that must be added to profit:[13]

$$r_{K,w}^t + \Pi^{*t} = CF^t - \left(C_{K,e}^t + C_{K,tax}^t\right) \equiv NNCF^t. \tag{45}$$

This identity at the same time serves as the definition of profit from normal operations Π^{*t}. The relation between the two profit concepts is

$$\Pi^{*t} = \Pi^t + C_{K,u}^t. \tag{46}$$

The accounting identity of the K-NNCF model, given by expression (45), can be rewritten as

$$r^t UCS^t + \Pi^{*t} = NNCF^t. \tag{47}$$

Now, provided that $NNCF^t \geq \Pi^{*t} \geq 0$, normal net cash flow is seen as the return to the (owner of the) capital stock. Setting $\Pi^{*t} = 0$ and solving equation (47) for r^t delivers what can be called the "normal endogenous" rate of return, defined by

$$r_{endo}^{*t} UCS^t = NNCF^t. \tag{48}$$

[13]In the model of Hulten and Schreyer (2006), total (= unanticipated plus anticipated) revaluation is added to profit. This is consistent with SNA (2008)'s prescription for non-market units.

Using expressions (47), (46), and (44) it appears that

$$r^{*t}_{endo} = r^{t}_{endo} + C^{t}_{K,u} \big/ UCS^{t}. \tag{49}$$

Thus, the normal endogenous rate of return absorbs not only profit, but also the monetary value of all unanticipated asset revaluations. Alternatively, as in the previous model, one can specify some reasonable, exogenous value for the rate of return. Then, of course, Π^{*t} follows from equation (47), and by subtracting the value of all unanticipated asset revaluations, $C^{t}_{K,u}$, one obtains profit.

The two expressions (41) and (47) and their underlying models are to be considered as polar cases. In the first all the unanticipated revaluations (that is, the whole of $C^{t}_{K,u}$) are considered as intermediate cost, whereas in the second they are considered as belonging to profit. Clearly, positions in between these two extremes are thinkable. For some asset types unanticipated revaluations might be considered as intermediate cost and for other types these revaluations might be considered as belonging to profit.

A number of conclusions can be drawn. First, there is no single concept of the endogenous rate of return. There is rather a continuum of possibilities, depending on the way one wants to deal with unanticipated revaluations.

Second, an endogenous rate of return, of whatever variety, can only be calculated *ex post*. Net cash flow as well as normal net cash flow require for their computation that the accounting period has expired.

Third, we are assuming that all the inputs and outputs are correctly observed. Unobserved inputs and outputs and measurement errors lead to a distorted profit figure. Since an endogenous rate of return can be said to absorb profit—see expressions (44) and (49)—the extent of undercoverage also has implications for the interpretation of the rate of return (see also Schreyer, 2010). Put otherwise, since an endogenous rate of return closes the gap between the input and the output side of the production unit, it is influenced by all sorts of measurement errors.

Finally, the concept of an endogenous rate of return does not make sense for non-market units, since there is no accounting identity based on independent measures at the input and the output side. Yet, non-market units use capital like market units.

The accounting point of view as used in this paper implies an *ex post* user cost concept. However, economic decision processes are usually based on anticipated magnitudes of certain variables. In particular, investment and other production decisions are based on *ex ante* user costs. How do the two concepts relate? This is one of the questions considered in an intriguing article by Oulton (2007). He proposed a "hybrid approach," which in our setup can be summarized as follows.

Ex ante capital input cost is calculated as

$$\hat{C}^{t}_{K} \equiv C^{t}_{K,w} + C^{t}_{K,e} + C^{t}_{K,tax}, \tag{50}$$

where $C^{t}_{K,w}$ is based on some required rate of return r^{t} (perhaps derived from past endogenous rates of return) and $C^{t}_{K,e}$ is based on expected end-of-period prices.

The *ex ante* capital input cost ratio for period 1 relative to period 0, $\hat{C}_K^1/\hat{C}_K^0$, can be decomposed into a price index $\hat{P}_K(1, 0)$ and a quantity index $\hat{Q}_K(1, 0)$, and it is this quantity index that is supposed to act as "the" capital input quantity index.

Ex post capital input cost plus profit appears to be

$$C_{K,w}^t + C_{K,e}^t + C_{K,u}^t + C_{K,tax}^t + \Pi^t, \tag{51}$$

and this is by definition equal to CF^t. The cash-flow based total factor productivity (TFP) index for period 1 relative to period 0 is generically defined as $Q_{CF}(1, 0)/Q_K(1, 0)$, where $Q_{CF}(1, 0)$ is a cash-flow based output quantity index and $Q_K(1, 0)$ is the quantity index component of the *ex post* capital input cost ratio (see Balk, 2010, p. S243).

If I interpret Oulton (2007) correctly, his suggestion is to calculate the TFP index instead by the following formula:

$$\frac{Q_{CF}(1, 0)}{\left(\hat{C}_K^0/CF^0\right)\hat{Q}_K(1, 0)}. \tag{52}$$

We see here that in the denominator the *ex ante* capital input quantity index is multiplied by the share of *ex ante* capital input cost in *ex post* cash flow (which is equal to *ex post* capital input cost under an endogenous rate of return). Using the familiar product relations linking price index, quantity index, and value ratio, Oulton's TFP index can be written as

$$\frac{Q_{CF}(1, 0)}{\left(\hat{C}_K^0/CF^0\right)\hat{Q}_K(1, 0)} = \frac{CF^1/P_{CF}(1, 0)}{\hat{C}_K^0/\hat{P}_K(1, 0)}. \tag{53}$$

This is deflated *ex post* cash flow divided by deflated *ex ante* capital input cost. It is not completely clear what this ratio is supposed to measure.

7. Aggregation

Let us now consider an ensemble of production units $\mathcal{K}$. Let there be a common classification of asset types and ages. All the price, quantity, and value variables discussed in the foregoing should be adorned with the superscript k. Then, for each production unit the K-NCF accounting relation reads

$$r^{kt}UCS^{kt} + \Pi^{kt} = NCF^{kt}\,(k \in \mathcal{K}). \tag{54}$$

If there are no tax wedges, so that the values of outgoing and incoming trade flows between the production units cancel, then aggregation reduces to simple addition, and the K-NCF accounting relation for the aggregate, considered as a big production unit, reads

$$r^{\mathcal{K}t}\sum_{k \in \mathcal{K}} UCS^{kt} + \Pi^{\mathcal{K}t} = \sum_{k \in \mathcal{K}} NCF^{kt}. \tag{55}$$

After having empirically filled in the capital stock and cash flow variables we are left with a large number of interrelated unknowns. For reaching consistency, there are two approaches, a bottom-up approach and a top-down approach, respectively.

The bottom-up approach starts, not unexpectedly, with relations (54). Aggregate profit is then set equal to the sum of individual profits,

$$\Pi^{\mathcal{K}t} = \sum_{k \in \mathcal{K}} \Pi^{kt}, \tag{56}$$

and the aggregate rate of return is set such that equation (55) holds. But this means that

$$r^{\mathcal{K}t} \sum_{k \in \mathcal{K}} UCS^{kt} = \sum_{k \in \mathcal{K}} r^{kt} UCS^{kt}, \tag{57}$$

or

$$r^{\mathcal{K}t} = \frac{\sum_{k \in \mathcal{K}} r^{kt} UCS^{kt}}{\sum_{k \in \mathcal{K}} UCS^{kt}}. \tag{58}$$

Thus the rate of return for the aggregate is a weighted mean of the rates of return for the individual units, the weights being shares of the value of the productive capital stock as used during period t.

The top-down approach starts with relation (55). For each individual production unit we then set

$$r^{kt} = r^{\mathcal{K}t} (k \in \mathcal{K}). \tag{59}$$

One checks immediately that this implies that equation (56) holds; that is, aggregate profit is the sum of all the individual profits. When $r^{\mathcal{K}t}$ is the endogenous rate of return, then $\Sigma_{k \in \mathcal{K}} \Pi^{kt} = 0$, but notice that the individual Π^{kt} need not be equal to zero; put otherwise, the endogenous rate of return for the aggregate does not necessarily coincide with the endogenous rates of return for the individual production units.

8. Some Implementation Issues

There remain a number of implementation issues to discuss. For this, the reader is invited to return to expression (33). To ease the presentation, a period is now set equal to a year.

The quantities $\{K_{ij}^t;\ i = 1, \ldots, I;\ j = 1, \ldots, J_i\}$ and $\{I_{ij}^t;\ i = 1, \ldots, I;\ j = 0, \ldots, J_i\}$ are usually not available. Instead, as is usually the case, the Perpetual Inventory Method generates estimates of the opening stock of assets at period $t - 1$

prices $\{P^{t}_{i,j-0.5}K^{t}_{ij} = P^{t-1}_{i,j-0.5}K^{t^-}_{i,j-0.5};\ i = 1, \ldots, I;\ j = 1, \ldots, J_i\}$, and the Investment Survey generates estimates of mid-period values $\{P^{t}_{ij}I^{t}_{ij};\ i = 1, \ldots, I; j = 0, \ldots, J_i\}$.

Models for time-series depreciation are briefly discussed in the Appendix. The time-series depreciation of an asset of type i and age j that is available at the beginning of period t is in practice frequently modeled as

$$\text{(60)} \qquad \frac{P^{t^+}_{i,j+0.5}}{P^{t^-}_{i,j-0.5}} = \frac{PPI^{t^+}_i}{PPI^{t^-}_i}(1-\delta_{ij}),$$

where PPI^{t}_i denotes the Producer Price Index (or a kindred price index) that is applicable to new assets of type i, and δ_{ij} is the annual cross-sectional depreciation rate that is applicable to an asset of type i and age j. This depreciation rate ideally comes from an empirically estimated age–price profile.

Thus, time-series depreciation is modeled as a simple, multiplicative function of two, independent factors. The first, $PPI^{t^+}_i / PPI^{t^-}_i$, which is 1 plus the annual rate of price change of new assets of type i, concerns the effect of the progress of time on the value of an asset of type i and age j. The second, $1 - \delta_{ij} > 0$, concerns the effect of ageing by one year on the value of an asset of type i and age j. Ageing by one year causes the value to decline by $\delta_{ij} \times 100$ percent.

Similarly, anticipated time-series depreciation is modeled as

$$\text{(61)} \qquad \frac{\mathcal{E}^{t^-} P^{t^+}_{i,j+0.5}}{P^{t^-}_{i,j-0.5}} = \mathcal{E}^{t^-}\left(\frac{PPI^{t^+}_i}{PPI^{t^-}_i}\right)(1-\delta_{ij}).$$

In this expression, instead of the annual rate of price change of new assets, as observed *ex post*, the annual rate as expected at the beginning of period t is taken.

But what to expect? There are, of course, several options here. The first that comes to mind is to use some past, observed rate of change of PPI_i or a more general PPI. Second, one could assume that expectedly the rate of price change of new assets is equal to the rate of change of a (headline) CPI, and use the "realized expectation":[14]

$$\text{(62)} \qquad \mathcal{E}^{t^-}\left(\frac{PPI^{t^+}_i}{PPI^{t^-}_i}\right) = \frac{CPI^{t^+}}{CPI^{t^-}}.$$

Under the last assumption anticipated time-series depreciation is measured as

$$\text{(63)} \qquad \frac{\mathcal{E}^{t^-} P^{t^+}_{i,j+0.5}}{P^{t^-}_{i,j-0.5}} = \frac{CPI^{t^+}}{CPI^{t^-}}(1-\delta_{ij}),$$

[14]This corresponds to the SNA (2008, par. 12.87) advice on calculating "neutral holding gains."

and, combining expressions (60) and (63), unanticipated revaluation is measured by

$$\text{(64)} \qquad \frac{\mathcal{E}^{t^-} P_{i,j+0.5}^{t^+}}{P_{i,j-0.5}^{t^-}} - \frac{P_{i,j+0.5}^{t^+}}{P_{i,j-0.5}^{t^-}} = \left(\frac{CPI^{t^+}}{CPI^{t^-}} + \frac{PPI_i^{t^+}}{PPI_i^{t^-}} \right) (1-\delta_{ij}).$$

Similar expressions hold for assets that are acquired at the midpoint of period t, except that we must make a distinction between new and used assets. The time-series depreciation for an asset of type i and age j is modeled as

$$\text{(65)} \qquad \begin{aligned} \frac{P_{i,0.5}^{t^+}}{P_{i,0}^{t}} &= \frac{PPI_i^{t^+}}{PPI_i^{t}}(1-\delta_{i0}) \\ \frac{P_{i,j+0.5}^{t^+}}{P_{i,j}^{t}} &= \frac{PPI_i^{t^+}}{PPI_i^{t}}(1-\delta_{ij}/2) \quad (j=1,\ldots,J_i). \end{aligned}$$

The anticipated time-series depreciation is measured by

$$\text{(66)} \qquad \begin{aligned} \frac{\mathcal{E}^{t} P_{i,0.5}^{t^+}}{P_{i,0}^{t}} &= \frac{CPI^{t^+}}{CPI^{t}}(1-\delta_{i0}) \\ \frac{\mathcal{E}^{t} P_{i,j+0.5}^{t^+}}{P_{i,j}^{t}} &= \frac{CPI^{t^+}}{CPI^{t}}(1-\delta_{ij}/2) \quad (j=1,\ldots,J_i), \end{aligned}$$

and unanticipated revaluation is measured by

$$\text{(67)} \qquad \begin{aligned} \frac{\mathcal{E}^{t} P_{i,0.5}^{t^+}}{P_{i,0}^{t}} - \frac{P_{i,0.5}^{t^+}}{P_{i,0}^{t}} &= \left(\frac{CPI^{t^+}}{CPI^{t}} - \frac{PPI_i^{t^+}}{PPI_i^{t}} \right)(1-\delta_{i0}) \\ \frac{\mathcal{E}^{t} P_{i,j+0.5}^{t^+}}{P_{i,j}^{t}} - \frac{P_{i,j+0.5}^{t^+}}{P_{i,j}^{t}} &= \left(\frac{CPI^{t^+}}{CPI^{t}} - \frac{PPI_i^{t^+}}{PPI_i^{t}} \right)(1-\delta_{ij}/2) \quad (j=1,\ldots,J_i). \end{aligned}$$

An important question is in what circumstances do the unit user costs u_{ij}^t and v_{ij}^t become non-positive? Consider, for instance, expression (31), and substitute expressions (63) and (64). This yields

$$\text{(68)} \qquad \begin{aligned} \frac{u_{ij}^t}{P_{i,j-0.5}^{t^-}} &= r^t + 1 - \frac{CPI^{t^+}}{CPI^{t^-}}(1-\delta_{ij}) + \left(\frac{CPI^{t^+}}{CPI^{t^-}} - \frac{PPI_i^{t^+}}{PPI_i^{t^-}} \right)(1-\delta_{ij}) + \frac{\tau_{ij}^t}{P_{i,j-0.5}^{t^-}} \\ &= r^t + 1 - \frac{PPI_i^{t^+}}{PPI_i^{t^-}}(1-\delta_{ij}) + \frac{\tau_{ij}^t}{P_{i,j-0.5}^{t^-}}. \end{aligned}$$

Hence, $u_{ij}^t \le 0$ if and only if

$$\text{(69)} \qquad \frac{PPI_i^{t^+}}{PPI_i^{t^-}} \geq \frac{1+r^t+\tau_{ij}^t \big/ P_{i,j-0.5}^{t^-}}{1-\delta_{ij}}$$

In certain, extreme cases this could indeed happen. Consider assets with a very low cross-sectional depreciation rate (such as certain buildings or land) and a very high revaluation rate (or rate of price increase). A low interest-plus-tax rate then could lead to negative unit user costs. Put otherwise, when the *ex post* revaluation (as measured by a *PPI*) more than offsets interest plus tax plus depreciation then the unit user cost of such an asset becomes negative.

If the unanticipated revaluation is excluded from the user cost, that is, unit user cost is measured by

$$\text{(70)} \qquad \frac{u_{ij}^t}{P_{i,j-0.5}^{t^-}} = r^t + 1 - \frac{CPI^{t^+}}{CPI^{t^-}}\left(1-\delta_{ij}\right) + \frac{\tau_{ij}^t}{P_{i,j-0.5}^{t^-}},$$

then $u_{ij}^t \leq 0$ if and only if

$$\text{(71)} \qquad \frac{CPI^{t^+}}{CPI^{t^-}} \geq \frac{1+r^t+\tau_{ij}^t \big/ P_{i,j-0.5}^{t^-}}{1-\delta_{ij}}.$$

The likelihood that such a situation will occur is small. For this to happen, expected revaluation (as measured by a *CPI*) must more than offset interest plus tax plus depreciation.

9. Conclusion

The approach followed in this paper is that capital input cost is the cost of using capital assets, much like labor input cost is the cost of employing workers. However, unlike the building blocks for labor input cost, quantities of assets and unit user costs cannot simply be observed. The usual, neoclassically inspired approach to measuring and decomposing capital input cost seems to rest on a number of daring, behavioral assumptions. We are following here the accounting approach which on the one hand avoids such assumptions and on the other hand shows the degrees of freedom one has in implementing measurement. Some would say that freedom means arbitrariness, or even anarchy. I hope to have made clear that this is not the case, however. Surely, there is a lot of freedom, but that forces us to think about the choices that must be made. If choices are made to accommodate neoclassical behavioral assumptions, fine. But within the framework explicated in this paper alternative choices can be argued for. Put otherwise, the accounting framework is so general that the consequences of variant assumptions can be explored and the assumptions themselves be tested at their relevance.

To wrap up it is useful to revisit our main steps. As said, the point of departure is that capital input cost is the cost of using (owned) assets. The quantities of those assets, according to type–age classes at some level of detail, at the beginning and end of each accounting period are estimated from investment surveys and other data sources, combined with some variant of the Perpetual Inventory Method. These quantities constitute the productive capital stock of the production unit under consideration. There appears to be no need for the concept of capital service, or the assumption that the service rendered by a certain asset is proportional to its quantity. This concept is as it were replaced by the user cost concept.

The unit user cost of an asset of certain type and age is basically determined by the difference between its beginning- and end-of-period price, plus an opportunity cost determined by a certain interest rate, and an amount of tax. There is no behavioral assumption underlying this specification, but there are a number of as yet undetermined variables. Prices and unit user costs appear to be mutually determined by the fundamental asset price equation, which here materializes as a purely mathematical result. Prices appear to depend on future unit user costs, but the future is unknown; thus, this is where expectations enter the picture. It appears useful to distinguish between actual and expected prices, and to decompose the difference between beginning- and end-of-period actual price into an expected and an unexpected part, where it is assumed that expectations are formed at the beginning of the period. The difference between beginning- and (expected) end-of-period prices is then conventionally modelled by a simple multiplicative relation, in which Producer and Consumer Price Indices figure, as well as empirically determined asset-specific depreciation rates.

The important remainder is the determination of the opportunity cost component, which turns out to be equal to the value of the productive capital stock times an interest rate. The key assumption of the neoclassical setup is to determine this interest rate, called rate of return, endogenously by setting capital input cost equal to cash flow (or gross operating surplus); put otherwise, by setting profit, as the difference between cash flow and capital input cost, equal to zero. This of course accommodates the assumption of competitive profit maximization under a constant-returns-to-scale technology, but there is nothing in the accounting system that necessitates such an assumption. And the assumption itself is at variance with much empirical evidence. Moreover, as demonstrated in Section 6, there is not a unique endogenous rate of return, but a continuum of possibilities, depending on the way one wants to deal with unexpected revaluations.

The bottom line could be the advice to statistical agencies to use the degrees of freedom available in the measurement system to accommodate different sorts of users.

References

Balk, B. M., *Price and Quantity Index Numbers: Models for Measuring Aggregate Change and Difference*, Cambridge University Press, New York, 2008.

———, "An Assumption-Free Framework for Measuring Productivity Change," *Review of Income and Wealth*, 56, Special Issue 1, S224–56, 2010.

Balk, B. M. and D. A. van den Bergen, "The Cost of Capital Input: Calculation Methods," Revised version of a paper presented at the Capital Measurement Workshop, Ottawa, May 22, 2006.

Coremberg, A., "The Measurement of TFP in Argentina, 1990–2004: A Case of the Tyranny of Numbers, Economic Cycles and Methodology," *International Productivity Monitor*, 17, 52–74, 2008.

Diewert, W. E., "The Aggregation of Capital over Vintages in a Model of Embodied Technical Progress," *Journal of Productivity Analysis*, 32, 1–19, 2009.
Diewert, W. E. and D. Lawrence, "Progress in Measuring the Price and Quantity of Capital," in L. Lau (ed.), *Econometrics and the Cost of Capital: Essays in Honor of Dale W. Jorgenson*, MIT Press, Cambridge, MA, 2000.
Diewert, W. E. and P. Schreyer, "The Measurement of Capital," in S. N. Durlauf and L. E. Blume (eds), *The New Palgrave Dictionary of Economics*, 2nd edition, Palgrave Macmillan, 2008.
Diewert, W. E., D. Lawrence, and J. Fallon, *The Theory of Network Regulation in the Presence of Sunk Costs*, Economic Insights, Hawker ACT, Australia, 2009.
Erumban, A. A., "Lifetimes of Machinery and Equipment: Evidence from Dutch Manufacturing," *Review of Income and Wealth*, 54, 237–68, 2008.
Hulten, C. R., "The Measurement of Capital," in E. R. Berndt and J. E. Triplett (eds), *Fifty Years of Economic Measurement*, Studies in Income and Wealth, Volume 54, University of Chicago Press, Chicago and London, 1990.
———, "Growth Accounting," Working Paper 15341, National Bureau of Economic Research, Cambridge, MA, 2009.
Hulten, C. R. and P. Schreyer, "Income, Depreciation and Capital Gains in an Intertemporal Economic Model," Fifth Ottawa Productivity Workshop, May 23–24, 2006.
OECD, *Measuring Capital; OECD Manual*, 2nd edition, Organisation for Economic Co-operation and Development, Paris, 2009.
Oulton, N., "Ex Post versus Ex Ante Measures of the User Cost of Capital," *Review of Income and Wealth*, 53, 295–317, 2007.
Rymes, T. K., "More on the Measurement of Total Factor Productivity," *Review of Income and Wealth*, 29, 297–316, 1983.
Schreyer, P., "Measuring Multi-Factor Productivity when Rates of Return are Exogenous," in W. E. Diewert, B. M. Balk, D. Fixler, K. J. Fox, and A. O. Nakamura (eds), *Price and Productivity Measurement*, Volume 6, Trafford Press, 2010 (available at www.vancouvervolumes.com).
SNA, *System of National Accounts*, European Commission, International Monetary Fund, Organisation for Economic Co-operation and Development, United Nations, World Bank, 2008.
Walras, L., *Elements of Pure Economics* (translated by W. Jaffé, Richard D. Irwin, Homewood, IL, 1954), 1874.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix: Decompositions of Time-Series Depreciation.

Please note: Wiley-Blackwell are not responsible for the content or functionality of any supporting materials supplied by the authors. Any queries (other than missing material) should be directed to the corresponding author for the article.