IS THERE ROOM FOR POLARIZATION?

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Polarization is a concept which is defined over the distribution of income. It is clear that it does not fit into the framework of the traditional Bergson-type Social Welfare Function. The aim of this paper is to investigate whether the concept can fit into the framework of the theory of Relative Deprivation. It is suggested that it may be incorporated into this theory as representing the power of groups.

1. Introduction

Polarization is a newly coined concept in the measurement of inequality. It seems to capture the imagination and to have essential properties that may explain social unrest and conflicts (Esteban and Ray, 1994, 1999, 2001, 2007; Wolfson, 1994; Duclos *et al.*, 2004). Alternative formulations of the concept have been suggested by Wang and Tsui (2000) and Zhang and Kanbur (2001). From its birth it was clearly not intended to substitute for inequality measurement but to add another dimension to it (Esteban and Ray, 1999).

Polarization is a measure defined on the distribution of income. In some formulations, it resembles the Gini coefficient, which can be interpreted as a quantitative measure of relative deprivation (Yitzhaki, 1979, 1982; Hey and Lambert, 1980; Chakravarty and Chakraborty, 1984; Duclos, 1998; Chakravarty and Mukherjee, 1999; Ebert and Moyes, 2000; Pedersen, 2004; Bossert and D'Ambrosio, 2006; D'Ambrosio and Frick, 2007; Wodon and Yitzhaki, 2009). Polarization and relative deprivation are therefore related concepts, but it is not clear what is the exact relationship and what are the differences in predictions.[1]

Adding another concept means adding a dimension to a problem. This additional dimension may be associated with existing dimensions, which may mean double counting of the same phenomenon.[2] Double counting diminishes our ability to grasp the additional contribution of the new concept. This is especially relevant to relative deprivation and polarization because both seem to be

[1]It is worth noting that both concepts are not interpreted in a uniform way, so that one has to distinguish the interpretations within and between concepts.

[2]Zhang and Kanbur (2001) find that polarization does not add anything to our understanding of Chinese inequality. However, one has to also add that finding that two concepts yield the same result in one empirical case, does not prove that one should not distinguish between them in general.

borrowed from social psychology and both are based on reference groups. While the declared purpose of Runciman's (1966) concept of relative deprivation (hereafter, RD) is to explain the connection between inequality and grievance, other applications of the theory use it to explain social conflict and struggle (Korpi, 1974; Chandra and Foster, 2005). Since both polarization and relative deprivation can be used to measure the potential for social conflict, and both are quantified by similar measures, the possible relationships between the two concepts are worth investigating.

The aim of this paper is to try to present the similarities and differences between the two concepts. I stress the word try because it is clear to me, and it should be clear to the readers, that it is a complicated task and it is not certain that I have exhausted all possible ways of incorporating the concepts. To keep the comparison as simple as possible, I do not deal with measurement and estimation, but stick to the basic concepts. Also, I am one sided in the sense that I will try to examine whether polarization can fit into RD theory. The reason for this asymmetric view is that RD is based on the individual, and only at a later stage are axioms about the social behavior of the individual imposed. On the other hand, polarization seems to skip the micro-economic base and to start from macro and reference group points of view. Another qualification is that I will rely on my own interpretation of relative deprivation, which is not the same as that which would have been given by Runciman, Chakravarty and Mukherjee, D'Ambrosio, Duclos, Ebert and Moyes, or Hey and Lambert.[3] The major difference between Yitzhaki's approach and the alternative interpretation (Hey and Lambert, 1980, hereafter HY), is that in Yitzhaki's approach there are no externalities in the sense that there is no comparison among individuals that is not channeled through the "market" for redistribution. Therefore, only the social evaluation of the marginal utility is affected so that it can fit into welfare economics, without affecting the main theorems of welfare economics. On the other hand, HY and other axiomatic approaches do assume comparisons between agents, which imply externalities, so that competitive markets cease to be efficient. Hence, there is no contradiction between welfare economics and Yitzhaki's approach.

The structure of the paper is as follows. Section 2 describes the axioms leading to the polarization concept. Section 3 describes the relevant components of RD theory. Section 4 presents the components of the decomposition of the Gini coefficients that are needed in order to deal with the relativity of the concept, while Section 5 relates the components of the decomposition to polarization and RD. Section 6 concludes and offers suggestions for further research.

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[3]Cowell and Ebert (2004) argue that there is a logical agreement between Temkin's "complaints" and deprivation, and they disagree with Chakravarty (1998) who argues for similarity between the two terms. Bossert and D'Ambrosio's (2006) axioms concerning reference groups differ from those of Ebert and Moyes (2000).

2. The Basic Axioms Behind Polarization

In this section I describe the four axioms that lead to the polarization index. The description is based on Duclos, Esteban, and Ray (2004, pp. 1742–3), hereafter DER.[4]

The axioms are:

Axiom 1: If a distribution is composed of a single basic density, then a squeeze of that density cannot increase polarization. A squeeze is a mean-preserving compression whose components are on different sides of the mean.

As far as I can see this axiom fits well into the framework of an inequality measure, like the relative mean deviation.

Axiom 2: If a symmetric distribution is composed of three basic densities with the same root and mutually disjoint supports, then a symmetric squeeze of the side densities cannot reduce polarization.

Unlike Axiom 1, Axiom 2 seems to fit into the definition of equality rather than inequality measurement.

Axiom 3: Consider a symmetric distribution composed of four basic densities with the same root and mutually disjoint supports, ... Slide the two middle densities to the side as shown (keeping all support disjoint). Then polarization must go up.

I must admit that it is hard to follow the above fragmented description of Axiom 3, but it seems that the crucial assumption is the next one.

Let $T(f(x), |x - y|\}$ represent effective antagonism, where x, y are two randomly selected incomes. The Gini-related measure of polarization is defined as:

$$(2.1) \qquad P(F) = \iint T(f(x), |x - y|) f(x) f(y) \, dx dy.$$

Then:

Axiom 4: If $P(F) \geq P(G)$ and $p > 0$, then $P(pF) \geq P(pG)$, where pF and pG represent (identical) population scalings of F and G.

Axiom 4 is the crucial axiom in creating the connection between the Gini coefficient and Gini based polarization.

Those four axioms, and definition (2.1) lead to Theorem 1 (DER, 2004, p. 1744).

Theorem 1: A measure P satisfies Axioms 1–4 if and only if

$$(2.2) \qquad P_\alpha(F) \equiv \iint f(x)^{1+\alpha} f(y) |y - x| \, dy dx,$$

where α_ε [0.25, 1].

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[4]Amiel *et al.* (2007) have examined the acceptance of the axioms by ordinary respondents. They conclude: "The responses suggest that important axioms which serve to differentiate polarisation from inequality—e.g. increased bipolarisation—as well as other distinctive features of polarisation, i.e. the non-monotonous behavior attributed to polarisation, are not widely accepted" (abstract). However, a questionnaire can not give more than a supporting argument, nor does it prove that an argument is not valid.

It is easy to see that if $\alpha = 0$, (2.2) becomes identical to Gini's mean difference. In this sense, polarization is based on an absolute measure of inequality, and unlike typical inequality measures it is sensitive to the fraction of individuals with the same income.

Note that the axioms leading to the index are technical in nature so that it is hard to analyze the behavior of individuals that could lead to such axioms. Having described the measure we move to describe its purpose. As explained by DER: "The idea is simple: polarization is related to the alienation that individuals and groups feel from one another, but such alienation is fuelled by notions of within-group identity . . . We are interested in the correlates of organized, large scale social unrest—strikes, demonstrations, processions, widespread violence, and revolt or rebellion" (p. 1737).

Esteban and Ray (1994) state that "A society that is divided into groups, with substantial intra-group homogeneity and inter-group heterogeneity in, say, incomes, is likely to exhibit the features mentioned above. At the same time, *measured inequality in such a society may be low*" (p. 820; emphasis in original).

An additional characteristic that is important is the following property:

Feature 3: There must be a small number of significantly sized groups. In particular, groups of insignificant size (i.e., isolated individuals) carry little weight. (p. 824).

It is important to note that Feature 3 is related to another dimension of conflicts—power, which may be a function of group size and group identity.

To summarize this section we conclude that polarization deals with societies that are composed of groups. It may be related to group identity and group power, and is an origin of social tension. It is quantified by a measure of absolute inequality that can be viewed as related to Gini's mean difference.

3. The Theory of Relative Deprivation

There are numerous versions of the theory of relative deprivation and it is not the purpose of this paper to review them. We will follow only one version of the theory, the one presented by Runciman (1966) and its quantification as presented in Yitzhaki (1979, 1982).

Runciman's (1966) theory is based on three dimensions: deprivation, power, and status (prestige). With respect to deprivation, Runciman distinguishes between deprivation of an individual as a member of a group (hereafter, between-group deprivation, in Runciman's term, "relatively deprived because of group's position in the society" (p. 33)), and relatively deprived because of own position in his reference group.

Runciman mentions power in his theory, but he does not analyze its implications. It seems that the main reason for that is that power is used in analyzing social conflicts while Runciman was not interested in that part of the theory. One possible explanation for this approach is that social unrest did not seem a very important factor at that period in Britain.

Yitzhaki (1979) quantified the relative deprivation theory while Yitzhaki (1982) deals with the implications of the existence of reference groups. However, mathematical difficulties prevented Yitzhaki from having a better analysis of the latter. Meanwhile, Frick *et al.* (2006) managed to offer a decomposition of the Gini coefficient according to population subgroups that enables the analysis of the effect of reference groups on deprivation. The rest of this section will set the ground for deprivation theory, with the whole society serving as one reference group, while the effect of different reference groups for different individuals, which is responsible for the relativity of the concept, will be dealt with only in Section 5, following the decomposition of Gini by population subgroups.[5]

Unlike polarization, deprivation is derived from a micro-economic model. Therefore, the description of the considerations leading to the theory is based on three stages: the micro-economic problem; the general equilibrium reached through market activity; and the effect of the resultant income distribution on deprivation.

The Micro-Economic Stage

Consider an individual who maximizes her utility function subject to a given budget constraint. That is:

(3.1)
$$\text{Max } U(x_1, \ldots, x_n)$$
$$\text{s.t. } \Sigma_i p_i x_i = y.$$

As a result of this optimization we can write the indirect utility function as:

(3.2)
$$V(y, p_1, \ldots, p_n).$$

The indirect utility function states that the utility of the individual is a function of her income, and the prices that the individual faces.

The General Equilibrium

We now move into the general equilibrium stage, which describes the role of markets and the resultant national income.[6] At this stage prices and incomes are *simultaneously* determined. Since it is assumed that $p_1, \ldots, p_n$ are equal for all individuals, the only differences among individuals are the utility functions and incomes. This gives the following meaning to prices. Prices enable the society to create units of equal production value. In other words, the n-dimensional commodity-space is converted into units of equal purchasing power value. Each unit of income presents a basket of commodities with equal purchasing power. It is important to note that prices and income distribution are determined simultaneously. This observation is important because the size of the cake and its

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[5]Hopkins (2008) surveys the different approaches toward relativity and the implications of the connection between happiness and inequality. Note, however, that relativity in the Runciman approach arises because deprivation depends on the reference group, and reference groups among individuals may differ. If we were to use the same reference group for the whole population, deprivation ceases to be relative.

[6]Sen (1976) describes the assumptions that lead to national income comparisons.

distribution are simultaneously determined. It implies that the term "real income" is meaningless unless a set of equilibrium prices is attached to it (Yitzhaki, 1982).

Distributional Values

Until now we applied only standard economic theory without referring to deprivation or welfare economics. We now turn to the only assumption needed to apply deprivation theory.

A critical assumption: In evaluating the units of equal purchasing power (i.e. units of income) that he possesses, an individual applies the law of declining marginal utility.

That is, the distributional value of a unit of income y depends on the scarcity of that unit in the population.

Note that we distinguish between production value and distributional value. Production value is the value of the inputs required to produce a dollar value of commodities. The distributional value is the value attached by the individual (or society) to a unit of consumption, the production value of which is one dollar.

Let y, $F(y)$ represent income and the cumulative distribution of income respectively. Each dollar of income represents a different bundle of commodities and its distributional value is determined by its scarcity. The abundance of the unit y in the population is $(1 - F(y))$, because $1 - F(y)$ is the share of individuals who do have the y unit of income (which represents a bundle of commodities consumed by someone with y units of income). The cumulative distribution $F(y)$ represents the scarcity of the y unit in the population while $1 - F(y)$ represents its abundance. I will refer to this value as the distributional value of the y unit. The individual can be deprived of the y unit (if he does not have it), or satisfied with having the y unit, if he has it. Note that the distributional value is determined independently of whether the individual has or has not that unit.[7]

The total value of the units an individual with income y is deprived of is:

$$(3.3) \qquad d(y) = \int_y^\infty (1 - F(t))\, dt,$$

while the total value of the units of y the individual has is:

$$(3.4) \qquad s(y) = \int_0^y [1 - F(t)]\, dt,$$

and the sum of (3.3) and (3.4) is equal, by definition, to μ, the mean income in the society.

As shown in Yitzhaki (1979), total deprivation in the society, which is the average of (3.3) over all individuals, is

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[7]See Bossert *et al.* (2007) for a different view.

(3.5) $$D = \mu G,$$

where G is the Gini coefficient. Total satisfaction in the society is:

(3.6) $$S = \mu(1 - G).$$

To give a concrete example, imagine the market for stamps. Consider a group of collectors, each one of them being interested in maximizing the value of his collection. The prices of each type of stamps are determined in a market that takes into account tastes and scarcity of stamps. The result of the market activity is a set of prices and distribution of the values of stamp collections. Real income can be defined in terms of a specific stamp, with each unit of income representing a bundle of stamps of equal exchange value. The distributional value attached to each unit of real income depends on the scarcity of that unit. According to deprivation theory each collector feels deprived of units that he does not have and satisfied of each dollar he possesses. The units of income he is deprived of, are those units that would have enabled him to have a larger stamp collection.

Relative Deprivation

Until now society was viewed as one reference group. As we have demonstrated, the deprivation/satisfaction theory differs from the Bergson type social welfare function (SWF) only in one point: the way the social evaluation of the marginal utility of income is determined. While in the Bergson type SWF the social evaluation of the marginal utility of income is determined without reference to incomes of other persons, in the deprivation/satisfaction theory the social evaluation of the marginal utility of income is a function of the income distribution. Specifically, it is determined by the law of diminishing marginal utility applied to each unit of income. Note that deprivation theory does not have an element of envy or altruism. The way an individual determines the distributional value of the dollars he has is identical to the way a stamp collector, and the market, determines its value—as an inverse function of its scarcity.

We turn now to the relativity dimension of the concept. The individual determines the distributional value of a dollar with reference to a reference group. The reference group may be composed of the whole society or any subgroup of members of the society.

In a general framework, reference groups should be determined endogenously by the individual. In a dynamic model, especially when the society is changing and the individual changes his position, we should expect movement from one reference group to another. Even if we ignore the time dimension, we still face complications that arise from the individual having several reference groups, and the reference groups being open rather than closed. By closed groups I mean that if individual A belongs to the reference group of B, then B also belongs to the reference group of A. That the individual is influenced by subpopulations is well recognized by Runciman. This is the basis of the relativity of the concept of deprivation. Ben-Porath (1980) has coined the F-connection as the natural base of reference groups: families, friends, and firms. Ethnic origin, common language,

common religion, neighborhoods, and nationalities are also the base of group identity. However, technical difficulties prevent the analysis of a complicated division of the society into reference groups.[8]

An additional point worth mentioning is Runciman's attitude toward the role of reference groups. Pedersen (2004, p. 39) uses the following quote to describe Runciman's theory:

> Most people's lives are governed more by the resentment of narrow inequalities, the cultivation of modest ambitions and the preservation of small differentials than by attitudes to public policy or the social structure as such. (Runciman, 1966, p. 285)

In this respect, the reference group in Runciman's approach seems to be the group that causes the feelings of deprivation rather than the group with which an individual identifies himself. This is different from the polarization approach. It hints that the reference group for comparisons and the group with which one identifies himself may be different. In this sense it seems that Zhang and Kanbur's (2001) reasoning for the alternative polarization index agrees with Runciman. Note the following: "The three polarization measures discussed so aim to capture the 'clustering' along the income dimension into high and low income groups. However, debates on polarizations are often conducted in the framework of recognized and accepted non-income grouping. In the U.S., for example, clustering of black and white income levels is as much concern as 'the disappearing middle class'. In China, as discussed in the introduction, geographical clustering of income is a major policy concern. (Zhang and Kanbur, 2001, p. 93)

Zhang and Kanbur (2001) call for a decomposition of the concept into groups that are defined over additional dimensions. This task is handled in Section 5. Before doing that it is worth mentioning a technical restriction: in this paper, I restrict myself to the analysis of the impact of having closed reference groups, with each individual belonging to one and only one reference group, on deprivation. I emphasize the difference in results between societies organized with reference groups that are based on income, and societies with reference groups organized on the basis of another variable. The restriction on the individual being associated with only one reference group and the restriction on closed reference groups are intended to make the analysis tractable. This enables us to also deal with group deprivation, and to compare the results with polarization theory. To do that, we have to survey the results on the decomposition of the Gini according to subgroups. Readers who are familiar with this decomposition can skip the following section.

4. ANOGI: The Decomposition of the Gini Coefficient

This section replicates the basic components in the decomposition of the Gini coefficient as presented by Frick *et al.* (2006).[9] Let y_i, $F_i(y)$, $f_i(y)$, μ_i, and p_i represent

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[8]The classification of a society into classes is the cornerstone of Marxist theory, but is rarely done in the measurement of inequality. For a recent example, see Wolff and Zacharias (2007).

[9]The decomposition is applied to the Gini coefficient only. Although the extended Gini can also represent deprivation and other theories (Moyes, 2007), it is not decomposable in the same way as the Gini.

the income, the cumulative distribution, the density function, the expected value, and the share of subpopulation i in the overall population, respectively. Let $s_i = p_i \mu_i / \mu_u$ denote the share of group i in the overall income. The overall population is composed of the union of the subpopulations. That is: $Y_u = Y_1 \cup Y_2 \cup \ldots \cup Y_n$, where Y_u is the union of subpopulations Y_i, $i = 1, \ldots, n$.

Note that:

$$(4.1) \qquad F_u(y) = \sum_i p_i F_i(y).$$

That is, the cumulative distribution of the overall population is the weighted average of the cumulative distributions of the subpopulations, weighted by the relative sizes of the populations. The formula of the Gini used in this paper is (Lerman and Yitzhaki, 1984):

$$(4.2) \qquad G = \frac{2 \operatorname{cov}(y, F(y))}{\mu},$$

which is twice the covariance between the income y and the cumulative distribution $F(y)$, standardized by mean income μ. The Gini of the entire population, G_u, can be decomposed as:

$$(4.3) \qquad G_u = \sum_{i=1}^{n} s_i G_i O_i + G_b,$$

where O_i is the overlapping index of subpopulation i with the entire population (explained below), and G_b measures the between-group inequality. Equation (4.3) decomposes the Gini of the union into two related components: intra- and inter-group components, connected in a way, which is relatively complicated.[10] We will return to the implications of the overlapping index following the explanation of the individual components.

4.1. *The Overlapping Index and its Properties*[11]

The index of overlapping is the one that distinguishes the decomposition of the Gini (ANOGI) from the decomposition of the variance (ANOVA).

Overlapping should be interpreted as the inverse of stratification. Stratification is a concept used by sociologists. We follow Lasswell's (1965, p. 10) definition as: "In its general meaning, a stratum is a horizontal layer, usually thought of as between, above or below other such layers or strata. Stratification is the process of forming observable layers, or the state of being comprised of layers. Social stratification suggests a model in which the mass of society is constructed of layer upon layer of congealed population qualities."

According to Lasswell, perfect stratification occurs when the observations of each subpopulation are confined to a specific range of income, and the ranges of

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[10]The Gini coefficient is not additively decomposable, a property that is criticized by Shorrocks (1984).

[11]The proofs of all statements in this section are given in Yitzhaki (1994).

incomes do not overlap. An example of a perfect stratification is the division of the society into deciles. An additional example of perfect stratification is the division of sports teams into leagues that are defined by ability. Stratification plays an important role in the theory of relative deprivation (Runciman, 1966), which argues that stratified societies can tolerate greater inequalities than non-stratified ones (Yitzhaki, 1982).

One can rarely find a perfect stratification, and an index describing the degree of stratification is called for. The index of overlapping is actually an index describing the extent to which the overall population is stratified.

Formally, overlapping of the overall population by subpopulation i is defined as:

$$(4.4) \qquad O_i = O_{ui} = \frac{\text{cov}_i(y, F_u(y))}{\text{cov}_i(y, F_i(y))},$$

where, for convenience, the index u is omitted and cov_i means that the covariance is according to distribution i, i.e.

$$(4.5) \qquad \text{cov}_i(y, F_u(y)) = \int (y - \mu_i)(F_u(y) - \bar{F}_{ui}) f_i(y)\, dy,$$

where $\bar{F}_{ui}$ is the expected rank of population i in the union (all observations of population i are assigned their union's ranks $F_u(y)$, and $\bar{F}_{ui}$ represents their expected value).[12] The overlapping (4.4) can be further decomposed to identify the overlapping of subpopulation i with all subpopulations that comprise the union. In other words, total overlapping of subpopulation i, O_i, is composed of the overlapping of i with all subpopulations, including group i itself. This further decomposition of O_i is:

$$(4.6) \qquad O_i = \sum_j p_j O_{ji} = p_i O_{ii} + \sum_{j \neq i} p_j O_{ji} = p_i + \sum_{j \neq i} p_j O_{ji}$$

where $O_{ji} = \dfrac{\text{cov}_i(y, F_j(y))}{\text{cov}_i(y, F_i(y))}$ is the overlapping of group j by group i.

The properties of the overlapping index O_{ji} are as follows:
(a) $O_{ji} \geq 0$. The index is equal to zero if no member of the j distribution lies in the range of distribution i (i.e. group i is a perfect stratum).
(b) O_{ji} is an increasing function of the fraction of population j that is located in the range of population i.
(c) For a given fraction of distribution j that is in the range of distribution i, the closer the observations belonging to j are to the expected value of distribution i, the higher is O_{ji}.
(d) If the distribution of group j is identical to the distribution of group i, then $O_{ji} = 1$. Note that by definition $O_{ii} = 1$. This result explains the second

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[12]It is worth noting that the O_i is a kind of a Gini correlation. See Schechtman and Yitzhaki (1987, 1999) for the properties of Gini correlations.

equality in (4.6). Using (4.6), it is easy to see that $O_i \geq p_i$ is a result to be borne in mind when comparing different overlapping indices of groups with different sizes.

(e) $O_{ji} \leq 2$. That is, O_{ji} is bounded from above by 2. This maximum value will be reached if all observations belonging to distribution j that are located in the range of i, are concentrated at the mean of distribution i. Note, however, that if distribution i is given then it may be that the upper limit is lower than 2 (see Schechtman, 2005). That is, if we confine distribution i to be of a specific type, such as normal, then it may be that the upper bound will be lower than 2, depending on the assumption of the distribution.

(f) In general, the higher the overlapping index O_{ji}, the lower will O_{ij} be, that is, the more group j is included in the range of distribution i, the less distribution i is expected to be included in the range of j.

Properties (a) to (f) show that O_{ji} is an index that measures the extent to which population j is included in the range of population i. Note that the indices O_{ji} and O_{ij} are not inter-related by a simple relationship. However, it is clear that the indices of overlapping are not independent.

4.2. *Between Group Component G_b and its Properties*

As will be seen later, we are interested in two complementary forms for representing between-group Gini. We start with the one appearing in equation (4.3). The between groups inequality G_b is defined in Yitzhaki and Lerman (1991) as:

$$(4.7) \qquad G_b = \frac{2 \operatorname{cov}\left(\bar{Y}, \bar{F}_u\right)}{\mu_u}.$$

G_b is twice the covariance between the mean incomes of subpopulations and the subpopulations' mean ranks in the overall population, divided by overall expected income. That is, each subpopulation is represented by its mean income, and the mean rank of its members in the overall distribution. The term G_b equals zero if either the mean incomes or the mean ranks are equal for all subpopulations. In extreme cases, G_b can be negative, which occurs when the mean income is negatively correlated with mean rank.[13]

An alternative between-group Gini (G_{bp}) was defined by Pyatt (1976); Mookherjee and Shorrocks (1982), Shorrocks (1984), and Silber (1989) also follow Pyatt. In this definition, the between-group Gini is based on the covariance between mean income in each subpopulation and its rank among the mean incomes of subpopulations. The difference between the two definitions is in the rank that is used to represent the group: under Pyatt's approach it is the rank of the

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[13]To see that, imagine one group of poor people that also includes a very rich person. Therefore, the mean rank in the overall population may be low while the mean income can be high.

mean income of the subpopulation, while under Yitzhaki–Lerman it is the mean rank of all members. Generally, it can be shown that:[14]

$$(4.8) \qquad\qquad G_b \le G_{bp}.$$

The upper limit is reached and (4.8) holds as an equality, if the ranges of incomes that groups occupy do not overlap (i.e. perfect stratification).

Having explained the different components we now present a variation of decomposition (4.3) that will be used in this paper as:

$$(4.9) \qquad\qquad G_u = \sum_{i=1}^{n} s_i G_i + \sum_{i=1}^{n} s_i G_i (O_i - 1) + G_{bp} + (G_b - G_{bp}).$$

Section 5 interprets the various components of (4.9).

5. Stratification and Deprivations of Groups and Individuals

Equation (4.9) will be the only equation that will be used to incorporate polarization into different versions of relative deprivation theory.

Relative deprivation is modeled in equation (3.5)

$$(5.1) \qquad\qquad D = \mu G,$$

where μ is mean income while G is the Gini coefficient. Assuming that the society is composed of subgroups, then by using (4.9) we get:

$$(5.2) \qquad D_u = \mu_u G_u = \sum_{i=1}^{n} p_i \mu_i G_i + \sum_{i=1}^{n} p_i \mu_i G_i (O_i - 1) + \mu_u G_{bp} + \mu_u (G_b - G_{bp}).$$

To be able to illustrate the implications of (5.2), let us distinguish between intra- and inter-group deprivations, and the role played by stratification:

- *Intra-group deprivation*: hereafter IG. The first component to the right, i.e. $\sum_{i=1}^{n} p_i \mu_i G_i$, is the intra-group component of deprivation. Note that the contribution of each group to overall deprivation is a function of its size, average income, and inequality among its members. IG is a weighted average of the intra-group Ginis.
- *Between-group Gini*: hereafter BG. This term is the between-group Gini as defined by Pyatt (1976), i.e. $\mu_u G_{bp}$; it is calculated as if all members of the group received the same income which is equal to the mean income of the group. This term is the "closest" to the polarization index; the main difference is that while BG is homogeneous of degree zero in the share of the population in each group, the polarization index is not.

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[14]The term G_{bp} is always non-negative, while G_b can be negative in extreme cases. For example, one group may be composed of many poor individuals and a few rich ones, so that the average rank is low while average income is high. In this sense, one can argue that G_b is not a Gini.

- *Stratification*: Stratification, which is the inverse of overlapping, has two effects: one is the effect on IG and the other is the effect on the BG component. Assuming that the society is stratified, both terms of stratification vanish and the Gini decomposes neatly into intra- and inter-group components. Existence of non-stratification, i.e. overlapping between distributions, reduces the between-group component because $\mu_u(G_b - G_{bp})$ is always non-positive. The other effect is on intra-group deprivation which is equal to $\sum_{i=1}^{n} p_i \mu_i G_i (O_i - 1)$; its components can be positive, zero, or negative depending on the distributions. However, since large values of overlapping tend to be associated with large values of Gini the overall term is positive. An alternative way to see it, is that since the overall and Pyatt's BG are given, and $\mu_u(G_b - G_{bp})$ is negative, by definition the overall overlapping term is positive. Hence, overlapping increases the intra-group deprivation component and decreases the inter-group component.

We are now ready to explore different structures of reference groups on relative deprivation.

Consider first the case where individuals' deprivations arise only from the intra-group component. We can analyze the implications of different scenarios depending on stratification.

The first case to be analyzed is Runciman's (1966) approach, which, although it mentions between-group deprivation, seems to stress the role of intra-group deprivation. That is, it is assumed that deprivation arises from comparisons in the reference group of the individual. This is the easiest to handle with clear cut results. We divide it to two cases: with perfect stratification and an imperfect one.

Consider perfect stratification: the society is divided into "leagues," with the aspirations of members of each "league" limited to that league. In this case, it is easy to see that deprivation is low even if inequality is high. Also, the larger the number of groups, the larger the BG term, and the lower the relative deprivation, even if inequality and mean income are given. In the extreme case where the number of groups approaches the number of members in the society, high inequality can prevail with zero deprivation. This case is analyzed in detail in Yitzhaki (1982). As far as I can see, this classification, if accepted by the members of the society, can allow inequalities without deprivation. Therefore, we should expect the upper class in the society to convince others to restrict their aspirations to their reference groups. Examples of such behavior include: the separation in the army into soldiers and officers, with officers eating in a separate dining room, the tendency to have ranks at work, and the existence of different ranks in universities. In an extreme and disgusting case, this theory can supply the rationale for an Apartheid policy. If a policy designer can convince each group to stick to their own folks, so no cross-group comparison is done, society can tolerate large inequalities with low level of deprivation.

The second case considered here is when individuals' deprivations arise only from the intra-group components, but stratification is not perfect. The overlapping component means that although there are no between-group deprivations, members of each group can see members of other groups and mingle with them. As

a result it is possible that deprivation in one group is greater than deprivation in the whole society, and in extreme cases the sum of deprivation in all subgroups may be greater than the deprivation in a society that is not divided into reference groups.[15] In this case the "revolution" may start among the richest class. In some sense, deprivation arises not from an increase in inequality but from the collapse of the reference group. For example, in the past women used to compare themselves to other women only. As a result of a greater participation in the labor force, however, the reference group of women was extended to include men. Deprivation then increased without a change in gender inequality and moreover, it may increase even if inequality between men and women declines (Gurin, 1985). Clearly, mass media, television, and globalization tend to widen the reference groups of individuals and therefore can increase deprivation even if inequality does not increase. Our conclusion is that if we accept Runciman's view that group identity is less important than the feelings of the individual with respect to its own reference group, then polarization does not play a role.

We now move to the alternative view that seems to be stressed in polarization: namely, that group identity is the main determinant of deprivation so that the appropriate element to concentrate on is between-group inequality. However, it is easy to see that this term is always lower than the deprivation in an undivided society. Therefore, we argue that as far as relative deprivation theory is concerned, polarization has no effect.

6. Conclusions and Suggestions for Further Research

According to Runciman, the list of factors that lead to social deprivation includes status, power, and income. The factors that lead to social clashes and unrest are deprivation and power. We have analyzed the implications of between-group inequality, a concept that resembles polarization in a relative deprivation context, and found that it does not lead to an increase in deprivation. However, we did not analyze differences in power, and it may well be that introducing power as a non-linear increasing function of the group size will be the appropriate way to explain social unrest. If this is the case, then polarization should be associated with power rather than deprivation or inequality in economic well-being. Esteban and Ray (1999) associate polarization with ethnic conflict. Montalvo and Reynal-Querol (2003, 2005) associate polarization with ethnic and religious conflicts, which is in line with the suggestion advocated in this paper. Another topic that is missing from this paper is the endogenous formation of reference groups. In this paper we have assumed that reference groups are given. Future research is needed to make the formation of reference groups an endogenous decision made by the individuals. Alesina and La Ferrara (2005) survey some of the approaches, Shayo (2004) presents an additional quantitative aspect, while Benabou (2000) seems to suggest the possibility of multiple equilibria. An additional area that calls for additional research is the effect of economic growth on polarization and

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[15]To see that, note that G_b can be negative while G_{bp} terms cancel each other out. G_b can be negative if one group includes poor people and a small number of extremely rich people so that the covariance between average rank and average income among groups is negative.

deprivation. A first step in this direction is offered in Wodon and Yitzhaki (2009), who argue that economic growth may lead to higher well-being but also to higher deprivation because it increases the spectrum of commodities that the individual feels she is deprived of.

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