# MEASURING PRO-POORNESS: A UNIFYING APPROACH WITH NEW RESULTS

#### BY B. ESSAMA-NSSAH AND PETER J. LAMBERT\*

World Bank Poverty Reduction Group and University of Oregon

Recent economic literature on pro-poor growth measurement is drawn together, using a common analytical framework which lends itself to some significant extensions. First, a new class of propoorness measures is defined, to complement existing classes, with similarities and differences which are fully discussed. Second, all of these measures of pro-poorness can be decomposed across income sources or components of consumption expenditure (depending on the application). This permits the analyst to "unbundle" a pattern of growth, revealing the contributions to overall pro-poorness of constituent parts. Third, all of these pro-poorness measures can be modified to measure pro-poorness at percentiles. An application to consumption expenditures in Indonesia in the 1990s reveals that the poverty reduction achieved remains far below what would have been achieved under distributional neutrality. This can be tracked back to changes in expenditure components.

### 1. INTRODUCTION

In the context of the Millennium Development Goals (MDGs), the international community has declared poverty reduction to be a fundamental objective of development, and therefore poverty reduction has become a metric for assessing the effectiveness of policy. Economic growth that accompanies the process of development is considered a powerful instrument of poverty reduction. Richer countries tend to have lower poverty incidence with respect to both income and non-income dimensions (World Bank, 2001). Yet, countries with the same rates of economic growth do not necessarily have similar achievements in poverty reduction.

The "pro-poorness" of a pattern of income growth measures, in some sense, how "favorably" it impacts upon the poor. Similarly, the "pro-poorness" of an observed pattern of growth of consumption expenditure measures how "tilted" that change has been towards the poor. The interpretation of "favorable" or "tilted" here is essentially a value judgment. Over the past few years, several authors have written academic papers on this issue, including Kakwani and Pernia (2000), Ravallion and Chen (2003), Son (2004), Kraay (2006) and most recently Kakwani and Son (2008b) in this journal. The issue has also been taken up at a

\*Correspondence to: Peter J. Lambert, Department of Economics, University of Oregon, Eugene, OR 97403-1285, USA (plambert@uoregon.edu).

#### © 2009 The Authors

Journal compilation © 2009 International Association for Research in Income and Wealth Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main St, Malden, MA, 02148, USA.

*Note*: The authors are grateful to Jean-Yves Duclos, Bill Griffiths, Stephen Jenkins, Nanak Kakwani, Jim Mirrlees, Martin Ravallion and Panos Tsakloglou for encouragement and useful comments on earlier versions of this paper, and to seminar participants at the Universities of Otago and Tasmania in Australasia and the Universidad Iberoamericana in Mexico City for their interest. The comments of two anonymous referees of this journal were especially valuable in helping us to improve the paper. The views expressed herein are entirely those of the authors or the literature cited, and should not be attributed to the World Bank or to its affiliated organizations.

more populist level in a series of "One Pagers" published by the International Poverty Centre of the United Nations Development Programme: see Zepeda (2004), Kakwani (2004), Ravallion (2004) and Osmani (2005).

The analysis of the impact of income change on income poverty, and of consumption expenditure change on consumption poverty, are two different exercises and they lead to different types of message for the policymaker. Although income and consumption expenditure have each been used as the welfare indicator in poverty analysis, often the choice having been governed by the availability and reliability of survey data,<sup>1</sup> using expenditure is clearly not in principle a substitute for using income. Having said this, and in light of Ravallion's (1996, p. 1331) remark that "In theory, one can define a very broad income concept which provides an exact money metric of almost any concept of 'welfare' one is likely to come up with," in the *theory* part of this paper we will speak of an individual's income *x* as the variable whose distribution is changing, but *x* could perfectly well stand for that person's consumption expenditure, as in one of our two applications to follow.

According to Kakwani and Pernia (2000), economic growth is pro-poor only when the incomes of the poor grow faster than those of the rich. This view also underlies the indicator proposed by Son (2004). The second prevailing interpretation is that growth is pro-poor if it involves poverty reduction for some choice of poverty index. Consistent with this second interpretation, Ravallion and Chen (2003) offer a measure of pro-poor growth based on the Watts (1968) index of poverty. Kraay (2006) has generalized this approach to other poverty measures. Because of its focus on relative gains, the first (Kakwani and Pernia, Son) interpretation is referred to as a relative approach to assessing the pro-poorness of economic growth, while the second (Ravallion and Chen, Kraay) is considered an *absolute* approach because it is based on changes in both the rate of growth and the distribution of gains.<sup>2</sup> Klasen (2008) contains many interesting reflections on the relative/absolute distinction, in particular making an interesting case that a relative approach has "much merit in defining the *state* of pro-poor growth, as it thus gives a sense of how much the opportunities afforded by a given rate of growth disproportionately helped the poor," whilst an absolute approach would be "of particular relevance to policy makers concerned about the pace of income growth among the poor and thus the pace of poverty reduction" (Klasen, 2008, p. 424).

Osmani (2005) argues that pinning the definition of pro-poor growth exclusively on distributional impact adds nothing to the traditional concern with equitable growth that can be traced back at least to Chenery *et al.* (1974). Consensus is now emerging around the absolute approach, due to the fact that pro-poorness is a characteristic of the whole growth process including both the growth rate and the distributional impact. This view underlies Ravallion's (2004) policy recommendation, that to make growth more poverty-reducing involves a combination of

<sup>2</sup>Kakwani and Son (2008b, p. 644) characterize the latter form of growth (that which reduces a poverty index) as "poverty reducing pro-poor growth."

<sup>&</sup>lt;sup>1</sup>Ravallion and Chen (1997) have developed a dataset based upon household survey data from some 67 countries, in more than half of which expenditure was used as the welfare indicator. Interestingly, the authors state that "in developing countries particularly, measurement errors are thought to be greater for income" (p. 359).

policies to induce higher growth rates and to improve the distribution of gains. Furthermore, Osmani insists that poverty-reducing growth should not be declared as inevitably pro-poor (in light of a general dissatisfaction with the scale of poverty reduction achieved by past growth experience in the developing world). He comes out in favor of a recalibrated absolute approach, whereby economic growth is considered pro-poor if it achieves an absolute reduction in poverty greater than would occur in a *benchmark* case. (In a sense, then, this is also a relative approach, which can be traced back to Kakwani and Pernia, 2000.) Such a benchmark could be either a desirable growth pattern or a counterfactual.

Ravallion and Chen (2003) and Kakwani *et al.* (2004) propose definitions and measures of pro-poor growth that address the two issues raised by Osmani. These authors capture pro-poorness using the elasticity of a poverty index with respect to changes in *per capita* income. Ravallion and Chen's "*rate of pro-poor growth*" is based on the Watts index. Kakwani *et al.*'s measure, known as the *poverty equivalent growth rate* (*PEGR*), applies to members of the additive and separable class of poverty indices (including Watts). The respective measures involve correcting the actual growth rate to account for distributional changes induced by the growth process; essentially, they identify growth rates that would induce the same poverty reduction as the observed rate, but under distribution neutrality. Ravallion and Chen (2003) and Kakwani *et al.* (2004) both use distributional neutrality as their benchmark case.<sup>3</sup>

In each of these papers the initial focus is upon the elasticity of a poverty index with respect to mean income, and considerable dexterity in handling these elasticities is displayed, but the notations differ between papers and they are not linked. Our own starting point will be a different and, we think, a more fundamental one. We shall begin with the description of a pattern of income growth, as encapsulated by a function q(x) which measures the elasticity of individual income x with respect to the total (or mean) income. The function q(x) becomes the vehicle by means of which the Ravallion and Chen and Kakwani et al. pro-poorness measures can be introduced, in a common measurement system. But not only that: a new family of measures emerges, which is also consistent with Osmani's (2005) conceptual framework. There are further benefits too, stemming from properties of the elemental function q(x) and applicable to all of the leading measures of pro-poorness. These pro-poorness measures can in fact be decomposed across income sources, permitting the analyst to "unbundle" a pattern of income growth and reveal the contributions to overall pro-poorness of income components; and they can be readily adapted to the measurement of pro-poorness at percentiles in the income distribution. In founding all of our analysis upon the function q(x)

© 2009 The Authors

<sup>&</sup>lt;sup>3</sup>The benchmark could alternatively attribute the same absolute benefits of growth to everybody. See Kakwani and Son (2008b, especially p. 646) for further discussion and comparison with relative approaches. Essama-Nssah (2005) offers a similar framework, but applying a broader social evaluation criterion to the growth rates of all incomes (not just for the poor). His specification of the social weights defining the evaluation criterion respects the Dalton–Pigou principle of transfers, and leads to a pro-poor growth indicator interpreted as the *equally distributed equivalent growth rate*. This is the growth rate that would be socially equivalent to the observed one, for some choice of the degree of aversion to inequality. The idea of a benchmark is embedded in the choice of the degree of aversion to inequality. McKinley (2008) argues that distribution-neutrality should be regarded as an "inclusive" benchmark generally for assessing growth effects without regard to a poverty line.

which describes an underlying growth pattern, we hope to have brought some clarity and unity to the as yet quite disparate pro-poorness literature.

The outline of the paper is as follows. In Section 2, we characterize a pattern of income growth in terms of the way individual incomes change within the growing total, by introducing the function q(x) and considering its main properties. In Section 3, we consider the impact of a growth pattern q(x) on an aggregate poverty index P, using distribution neutrality as benchmark. It is here that the Ravallion and Chen, and Kakwani et al., pro-poorness measures emerge, as well as, quite naturally, a new class of pro-poorness measures with easily understood but distinct properties. In Section 4, two innovations are introduced, for all of the pro-poorness measures considered in the paper. First, they can be decomposed across income components  $x_i$ , because of the simple relationship which exists between an overall growth pattern q(x) and its component growth patterns  $q(x_i)$ . And second, pro-poorness can be measured at percentiles, by simply redoing the analysis of Section 3 at a percentile point on a cumulative poverty curve (rather than for aggregate poverty). An empirical illustration is presented in Section 5, based on data from Indonesia for 1993-2002. In particular, it is found that the overall poverty reduction observed in Indonesia for the period under consideration was not pro-poor because a distributionally neutral growth pattern would have done more. Concluding remarks are made in Section 6.

# 2. FEASIBLE GROWTH PATTERNS

Given our choice of distribution-neutrality as the benchmark growth scenario, we need a calculus within which the effects of distributional shifts induced by the growth process can be quantified and analyzed. We rely on a little-noticed approach developed in Lambert (1984) to define and characterize a "growth pattern" in terms of its feasibility and pro-poorness.

Let an individual's income be x and let  $\mu$  stand for mean income. If f(x) represents the frequency density function for income, then

(2.1) 
$$\mu = \int_0^{m_x} x f(x) dx$$

where  $m_x$  is the maximum income. Denote by q(x) the point elasticity of x with respect to  $\mu$ . The function q(x) is a measure of the instantaneous pattern of change in individual incomes as the total (or mean) grows (we abstract here from population changes). If  $\mu$  grows by a small finite amount, say 1 percent, x grows by (approximately) q(x) percent. Formally:

(2.2) 
$$q(x) = \frac{\mu}{x} \frac{dx}{d\mu} = \frac{d\ln(x)}{d\ln(\mu)}.$$

The function q(x) defines a growth pattern and is the fundamental starting point for our analysis of pro-poorness. There is a strong connection with Ravallion and Chen's (2003) growth incidence curve, call it  $g_{RC}(p)$ , which is defined as the growth rate of income at the *p*-th percentile point of the income distribution:

 $$\cite{C}$$  2009 The Authors Journal compilation  $\cite{C}$  International Association for Research in Income and Wealth 2009

 $g_{RC}(p) = \frac{dx}{x}$  when  $p = \int_0^x f(t) dt$ , i.e.  $g_{RC}(p) = \gamma q(x)$  where  $\gamma = \frac{d\mu}{\mu}$  is the growth rate of mean income. Hence our growth pattern is essentially a normalized growth incidence curve.

The function q(x) must obey a feasibility constraint in order to be considered a legitimate representation of a growth pattern. Suppose the *per capita* income increases by 1 percent, then q(x) is feasible if all of the implied individual income growths add up correctly to this 1 percent. Formally, as shown in Lambert (1984), q(x) must satisfy the following restriction in order to be a potentially observable growth pattern:

(2.3) 
$$\int_0^{m_x} x[q(x)-1]f(x)dx = 0.$$

The demonstration of this feasibility condition, along with the proofs of all subsequent theorems and assertions, can be found in the Appendix.

We spoke of the relative approach to pro-poorness, according to which a growth pattern is pro-poor only when the incomes of the poor grow faster than those of the rich. A sufficient condition for a growth pattern to unambiguously reduce inequality (cause a Lorenz improvement) is that:

$$(2.4) \qquad \exists x_0: q(x) \ge 1 \quad \text{for all} \quad x \le x_0.$$

The counterpart for Ravallion and Chen's growth incidence curve is that if  $g_{RC}(p)$  crosses  $\gamma$ , the growth rate in mean income, once, from above, then inequality is unambiguously reduced.<sup>4</sup>

Under the absolute approach to pro-poorness measurement, calibrated à la Osmani (2005), economic growth is considered pro-poor for a poverty index P if it achieves an absolute reduction in poverty greater than would occur in a benchmark case. The benchmark pattern of growth, call it  $q_0(x)$ , is defined for this paper as the one which is associated with distributional neutrality:

$$(2.5) q_0(x) \equiv 1 \quad \forall x,$$

but some other agreed benchmark pattern could be used, for example to reflect the analyst or decision-maker's inequality aversion.<sup>5</sup> We can consider an observed growth pattern q(x) to be equal to the pattern  $q_0(x)$  associated with distributional neutrality plus an adjustment factor  $[q(x) - q_0(x)]$  which accounts for the extent of change, if any, in inequality. One can compute the growth pattern implied by an observed change in income distribution function as follows. If the distribution function is  $F_1(x)$  before growth takes place, and becomes  $F_2(w)$  afterwards, and if the means are  $\mu_1$  and  $\mu_2$  respectively, then:

<sup>4</sup>This is exactly as Ravallion and Chen (2003) find for China in the period 1990–99 (see their figure 2, where  $\gamma = 0.082$ ).

© 2009 The Authors

<sup>&</sup>lt;sup>5</sup>Our analytical framework could be adapted to allow for a different choice of benchmark. Osmani (2005) points out that the choice of a benchmark will not be critical in determining whether a particular set of policies is more pro-poor than another, as this involves only a comparison of the poverty-reducing effect of the alternative policies relative to the fixed benchmark. People could agree with the conclusions from such a comparison without necessarily agreeing on the choice of the benchmark.

(2.6) 
$$q(x) = \frac{\mu_1}{\mu_2 - \mu_1} \left( \frac{F_2^{-1}[F_1(x)]}{x} - 1 \right)$$

It is readily checked that (2.3) holds for this realization of q(x), and that  $q(x) \equiv 1 \ \forall x$  after a scale change in incomes  $\left(w = \frac{\mu_2}{\mu_1}x\right)$ .

In most of the paper, a scenario of *positive* income growth is assumed ( $\gamma > 0$ ). However, the q(x) concept can equally be applied in situations of *negative* income growth ( $\gamma < 0$ ), to which we shall occasionally refer. In such a case,  $q(x_0) < 0$  if income  $x_0$  increases despite the general decrease in income values that is taking place;  $0 < q(x_0) < 1$  if income  $x_0$  falls, but not by as big a percentage as the mean; and  $q(x_0) > 1$  if income  $x_0$  falls faster than the mean.<sup>6</sup>

# 3. The Pro-Poorness of a Growth Pattern

Assessing the pro-poorness of growth is an exercise in social evaluation that requires a criterion for comparing alternative social states, each characterized by a growth pattern. Pro-poorness thus hinges on the choice of a social evaluation criterion. Here, we focus on a class of additively separable poverty measures. A growth pattern will be declared to be pro-poor for an index P in this class if this growth (here assumed positive) reduces P by more than equiproportionate growth would.<sup>7</sup> In principle, then, a growth pattern may be judged pro-poor for one poverty index but not for another.

The class of poverty indices we shall use for the analysis are those which take the form:

(3.1) 
$$P = \int_0^z \psi(x|z) f(x) dx$$

where z is the poverty line and  $\psi(x|z)$  is a convex and decreasing function which measures individual deprivation and is zero when  $x \ge z$ . This class may be traced back to Atkinson (1987). Its membership includes the Watts (1968) index W, the normalized poverty deficit D and the Foster, Greer and Thorbecke (1984) family of poverty indices  $FGT_{\alpha}$  with  $\alpha \ge 1$ .<sup>8</sup> The headcount index is not in this class.

For the poverty measure defined by (3.1), the growth elasticity of P for the growth pattern q(x) may be written as follows:

(3.2) 
$$\phi_P(q) = \frac{\mu}{P} \frac{dP}{d\mu} = \frac{d\ln(P)}{d\ln(\mu)} = \frac{1}{\gamma} d\ln(P)$$

<sup>6</sup>Ravallion and Chen's growth incidence curve  $g_{RC}(p) = \gamma q(x)$  also needs careful interpretation when  $\gamma < 0$ .

<sup>7</sup>As we shall see, negative growth (recession) will count as pro-poor *à la* Osmani if it raises poverty by *less than* equiproportionate change would. Kakwani and Son (2008b, p. 644) observe that negative growth must *lower* poverty to count as pro-poor in the Ravallion and Chen sense.

<sup>8</sup>The deprivation functions for these measures are  $\psi_w(x|z) = \ell n(z/x)$ ,  $\psi_D(x|z) = 1 - x/z$  and  $\psi_a(x|z) = (1 - x/z)^{\alpha}$  for x < z, and zero otherwise. The case  $\alpha = 2$  in the latter yields the "squared poverty gap."

 $$\cite{C}$$  2009 The Authors Journal compilation  $\cite{C}$  International Association for Research in Income and Wealth 2009

where, as before,  $\gamma = \frac{d\mu}{\mu}$ . The following result expresses this elasticity as a function of the growth pattern.

Theorem 1

For the poverty index *P* defined in (3.1), the growth elasticity for the pattern q(x) is given by  $\phi_P(q) = \frac{1}{P} \int_0^z x \psi'(x|z) q(x) f(x) dx$ .

In particular, the growth elasticity of *P* for distributionally-neutral income growth is  $\phi_P(q_0) = \frac{1}{P} \int_0^z x \psi'(x|z) f(x) dx$ . This result can be found in Kakwani (1993a, p. 125), where the "pure growth effect" on poverty is analyzed for a wide range of poverty measures (and the effects of distributional change are analyzed separately using Lorenz curve methodology).<sup>9</sup>

Although not covered by Theorem 1, the growth elasticity of the headcount ratio  $H = \int_0^z f(t) dt$  can also be expressed in terms of q(x):  $\phi_H(q) = -zq(z)f(z)/H$  (the pure growth version of this,  $\phi_H(q_0) = -zf(z)/H$ , is also to be found in Kakwani, 1993a, p. 123). Only the income density and growth experience *at the poverty line* matter for this (instantaneous, point) elasticity, which has been used by a number of authors to capture the "poverty bias of growth."<sup>10</sup>

If  $q(x) > 0 \forall x < z$  then from Theorem 1,  $\phi_P(q) < 0$ : income growth among the poor reduces poverty. The essence of pro-poorness measurement, though, is to detect whether or not that income growth *favors* the poor—does it reduce poverty more than would be achieved by a reference, benchmark or "neutral" growth pattern? The reduction in poverty associated with a 1 percent increase in mean income according to the growth pattern q(x) is equal to  $-P\phi_P(q)$ . Had this growth been distributionally neutral—our benchmark case—the reduction in poverty would be  $-P\phi_P(q_0)$ . The pro-poorness of q(x) can be captured in the extent to which the former exceeds the latter. There are two obvious ways to do this. One is to measure directly the excess reduction in poverty that q(x) brings, relative to  $q_0(x)$ :

(3.3) 
$$\pi_P(q) = P[\phi_P(q_0) - \phi_P(q)] = \int_0^z \{-x\psi'(x|z)\}[q(x) - 1]f(x)dx$$

(applying Theorem 1), and this is the new measure we wish to introduce. If and only if  $\pi_p(q)$  is positive, is q(x) a pro-poor growth pattern for the poverty index *P*. The other natural way is to measure the excess poverty reduction occasioned by q(x), relative to that of  $q_0(x)$ , in ratio form, as:

<sup>9</sup>See also Kakwani and Son (2008a) on the pro-poorness of government income-generating policies. Klasen and Misselhorn (2006) examine the relationship between income growth and distributional change in the case of the FGT poverty index using a *semi*-elasticity approach, which they prefer to the elasticity approach: see Klasen and Misselhorn (2006) for the case which they make, very cogently, for the semi-elasticity approach.

<sup>10</sup>See Ravallion and Chen (1997), McCulloch and Baulch (2000), Bourguignon (2003), Kalwij and Verschoor (2007) and Besley *et al.* (2006).

© 2009 The Authors

(3.4) 
$$\kappa_{P}(q) = \frac{-P\phi_{P}(q)}{-P\phi_{P}(q_{0})} = \frac{\phi_{P}(q)}{\phi_{P}(q_{0})} = \frac{\int_{0}^{z} x\psi'(x|z)q(x)f(x)dx}{\int_{0}^{z} x\psi'(x|z)f(x)dx}$$

(again applying Theorem 1). If and only if  $\kappa_p(q)$  exceeds unity, is q(x) a pro-poor growth pattern for the poverty index *P*. If  $\pi_p(q) < 0$ , equivalently  $\kappa_p(q) < 1$ , the growth pattern q(x) must be deemed "anti-poor." Notwithstanding that poor incomes may all have increased (which is the case if  $q(x) > 0 \ \forall x < z$ ), poverty does not fall by as much in this case as if the growth process had been distributionally neutral. The characteristics of the growth pattern q(x), and the shape of the deprivation function  $\psi(x|z)$ , together determine pro-poorness. It is clear that if  $q(x) > 1 \ \forall x < z$ , then  $\pi_p(q) > 0$  and  $\kappa_p(q) > 1$  for all poverty indices *P* in the class we are considering. However, it is *not* necessary that  $q(x) > 1 \ \forall x < z$  for a growth pattern to be judged pro-poor *for a specific poverty index*.

We pause to consider the concept of pro-poorness in case there is *negative* income growth. Now, the *increase* in poverty associated with a 1 percent *reduction* in mean income according to the growth pattern q(x) is equal to  $-P\phi_P(q)$ ; and in the benchmark case, the increase in poverty would be  $-P\phi_P(q_0)$ . Clearly, the growth pattern will be pro-poor in such a case *if the benchmark poverty increase* exceeds the actual poverty increase. Hence we should use converse measures  $-\pi_P(q) = P[\phi_P(q) - \phi_P(q_0)]$  and  $\frac{1}{\kappa_P(q)} = \frac{-P\phi_P(q_0)}{-P\phi_P(q)}$  to measure pro-poorness with negative income growth. This observation will become relevant in our main application, to come.

Returning to the case of positive growth, what are the relative attractions of the pro-poorness measures  $\pi_p(q)$  and  $\kappa_p(q)$ , the one measuring the poverty benefit from growth in level terms and the other in ratio form? In fact the second,  $\kappa_p(q)$ , is precisely the measure introduced by Kakwani and Pernia (2000). From (3.4), this measure can be expressed as a weighted average of the individual growth elasticities q(x) along the growth path, up to the poverty line, the weights being determined by the (marginal) deprivation function. Kakwani and Pernia give a more detailed anatomy of pro/anti-poorness than the simple dichotomy we suggested above in terms of whether  $\kappa_p(q) \ge 1$ . As they point out, if  $0 < \kappa_p(q) < 1$  then "growth results in a redistribution against the poor, even though it still reduces poverty incidence. This situation may be generally characterized as trickle-down growth" but they also recognize the other possibility, that  $\kappa_p(q) < 0$ , when growth leads to *increased* poverty. They also suggest, as we did above, that the reciprocal of their index would be a more convenient indicator in times of recession.

The first pro-poorness index,  $\pi_p(q)$ , is new. From (3.3) this measure is a weighted sum of the deviations of a growth pattern from the benchmark values, up to the poverty line, the weights being determined by the (marginal) deprivation function. In terms of Ravallion and Chen's (2003) growth incidence curve, if  $p = \int_0^x f(t)dt$  then  $\pi_p(q) = \frac{1}{\gamma}\int_0^z \{-x\psi'(x|z)(x|z)\}[g_{RC}(p)-\gamma]f(x)dx$ . Pro-poorness obtains for all *P* if income at every percentile up to the poverty line grows faster than mean income. If  $g_{RC}(p)$  crosses  $\gamma$ , the growth rate in mean income, once, from above, then as we have said, inequality is unambiguously reduced, but this does

 $<sup>$\</sup>cite{C}$$  2009 The Authors Journal compilation  $\cite{C}$  International Association for Research in Income and Wealth 2009

not guarantee pro-poorness of the growth pattern: that depends on the weighting scheme, which is determined by the deprivation function inherent in the choice of poverty index.

Pro-poorness measures may be calculated for the Watts, normalized poverty deficit and *FGT* indices using (3.3)–(3.4) and the deprivation functions given in note 8. For the Watts index, for example, the two measures are  $\pi_W(q) = \int_0^z [q(x)-1] f(x) dx$  and  $\kappa_W(q) = \int_0^z q(x) f(x) dx/H$ . Pro-poorness for the headcount ratio is not covered by (3.3)–(3.4), but the relevant measures are  $\pi_H(q) = z[q(z)-1]f(z)$  and  $\kappa_H(q) = q(z)$ .

When a growth pattern is pro-poor according to the Watts index, so that  $\pi_W(q) > 0$ , we must have  $\int_0^z q(x) f(x) dx > H$  or  $\frac{1}{H} \int_0^H g_{RC}(P) dp > \gamma$ . Hence, if the area beneath Ravallion and Chen's growth incidence curve up to the headcount ratio, normalized by the headcount ratio itself, exceeds [is less than] the actual growth rate, then the growth pattern is [is not] pro-poor for the Watts index. Ravallion and Chen (2003) note this result and characterize  $\frac{1}{H} \int_0^H g_{RC}(P) dp$  as "the mean growth rate for the poor" (Ravallion and Chen, 2003, p. 96).

Kakwani *et al.* (2004) develop a measure known as the *poverty equivalent* growth rate (*PEGR*), defined as the uniform growth rate,  $\gamma_e$ , that will induce the same level of poverty reduction as the actual growth, with pattern q(x) and mean income growth rate  $\gamma$ , but under distribution neutrality. Within our framework, the *PEGR* is the solution to the following equation,

(3.5) 
$$\phi_P(q) \cdot \gamma = \phi_P(q_0) \cdot \gamma_e$$

from which it is immediate that the *PEGR* can be written as:

(3.6) 
$$\gamma_e = \kappa_P(q) \cdot \gamma.$$

In this form,  $\gamma_e$  expresses the *PEGR* as the product of a distribution–correction factor, which is none other than the pro-poorness measure  $\kappa_P(q)$ , and the actual growth rate  $\gamma$ . The correction factor adjusts the actual growth rate up or down according to whether the distributional changes induced by the growth process favor the poor or not.<sup>11</sup> In Kakwani and Son (2003), a "monotonicity axiom" is developed, whereby the proportional reduction in poverty should be a monotonically increasing function of the pro-poor growth measure. The *PEGR* evidently has this property, taking into account both the magnitude of growth and how the benefits of growth are distributed to the poor and the non-poor. As (3.6) shows, to achieve rapid poverty reduction, it is the *PEGR*  $\gamma_e$ —rather than the actual growth rate  $\gamma$ —which should be maximized.

The *PEGR* can also be expressed in terms of Ravallion and Chen's (2003) growth incidence curve, as  $\gamma_e = \frac{\int_0^z x \psi'(x|z) g_{RC}(p) f(x) dx}{\int_0^z x \psi'(x|z) f(x) dx}$ , where, as before,

<sup>11</sup>There is an analogy here with the measurement of the distributional effect of an income tax à la Kakwani (1977), whereby the inequality reduction caused by the tax is a function of its level and its progressivity. Here, poverty reduction is a function of the growth level  $\gamma$  and the distributional effect of growth  $\kappa_p(q)$ . We thank Nanak Kakwani for this insight.

© 2009 The Authors

 $p = \int_0^x f(t) dt$  is the percentile point at which income *x* occurs. That is, the *PEGR* is a weighted average of the growth rates of incomes,  $g_{RC}(p)$ , along the growth path up to the poverty line—just as Ravallion and Chen's mean growth rate for the poor is, but in this case the weights are specified in terms of marginal deprivations.

The respective pro-poorness indices  $\kappa_P(q)$  and  $\pi_P(q)$  use different calibrations, but both are in line with Osmani's proposal to compare the actual growth experience with what would occur in the benchmark case.<sup>12</sup> For comparisons of alternative growth patterns with respect to a fixed income distribution f(x),  $\pi_P(q)$  and  $\kappa_{P}(q)$  move together. But for comparisons between regimes in which the income distributions differ-for example, in international comparisons-we can expect that in general the additive and ratio measures will lead to different conclusions. We illustrate this with a somewhat back-of-the-envelope comparison of growth patterns, based loosely on Denmark and Portugal, using lognormal approximations and simulation. The lognormal distribution  $LN(\theta, \sigma)$  is such that  $x \in LN(\theta, \sigma)$  $\sigma$ ) if and only if  $\ln(x) = \theta + n\sigma$  where  $n \in N(0, 1)$  is a standard normal variate. The mean is  $\mu = \exp\{\theta + \frac{1}{2}\sigma^2\}$  and the Gini coefficient in percentage terms (0 < G < 100) is  $G = 100 \left[ 2\Phi \left( \frac{\sigma}{\sqrt{2}} \right) - 1 \right]$  where  $\Phi$  is the distribution function for N(0, 1). The values  $\theta_{Den} = 10$  and  $\sigma_{Den} = 0.5$ ,  $\theta_{Por} = 8$  and  $\sigma_{Por} = 0.7$  are reasonably close to those respectively observed for the Danish and Portuguese distributions of household disposable income per capita in the year 2000.13 Suppose the pattern of income growth in each is such that mean income rises by 4.5 percent and the Gini coefficient falls by half a percentage point. This is achieved by setting  $\sigma$  to yield the desired change in the Gini, and making a compensating change in  $\theta$ .<sup>14</sup> By drawing 5,000 random values  $n \in N(0, 1)$  and generating the relevant income distributions, we computed the relevant q(x) functions (see Figure 1).

Plainly the growth is pro-poor whatever (reasonable) poverty lines we might set in the two distributions. We chose poverty lines equal to one half of median income in each country's base distribution,<sup>15</sup> and calculated the pro-poorness measures  $\pi_P(q)$  and  $\kappa_P(q)$  for the headcount ratio, Watts index and normalized poverty deficit (see Table 1).

Each of the additive measures in Table 1 judges the Portuguese income growth pattern as more pro-poor than the Danish growth pattern, and each ratio measure does the opposite.

It might aid the reader's understanding of our results for the Indonesian growth experience which are to come, if we expand upon some of these findings. For Denmark and the Watts index, the poverty elasticities are in fact

<sup>15</sup>Our choice of poverty lines for these simulated data leads to headcount ratios of 8.72 and 16.44 percent respectively for Denmark and Portugal. Figure 1 shows that 70 and 76 percent of the population for Denmark and Portugal respectively have q(x) greater or equal to 1.

<sup>&</sup>lt;sup>12</sup>The measures  $\pi_p(q)$  do not satisfy Kakwani and Son's monotonicity axiom; neither does Ravallion and Chen's mean growth rate for the poor, nor the headcount-based "poverty bias of growth" measure mentioned earlier.

<sup>&</sup>lt;sup>13</sup>We thank Panos Tsakloglou for supplying the accurate estimates in which these ball-park figures are based. See also Lambert (2009). The ball-park values of the mean and Gini coefficient are €16131 and 37.9 for Denmark, and €4805 and 27.6 for Portugal.

<sup>&</sup>lt;sup>14</sup>The relevant  $\sigma$ 's are 0.49 for Denmark and 0.69 for Portugal. The Gini coefficient falls from 27.6 to 27.1 for Denmark, and from 37.9 to 37.4 in the case of Portugal.

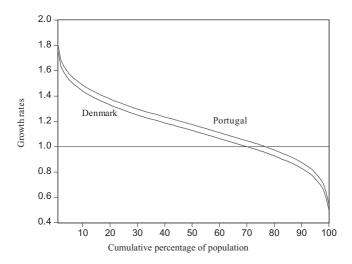


Figure 1. Simulated Growth Patterns for Denmark and Portugal

TABLE 1
SIMULATED MEASURES OF PRO-POORNESS FOR DENMARK AND
Portugal

	$\pi_P(q)$	$\kappa_P(q)$
Denmark		
Headcount ratio	0.135	1.441
Watts	0.051	1.547
Poverty gap	0.040	1.535
Portugal		
Headcount ratio	0.138	1.395
Watts	0.092	1.521
Poverty gap	0.062	1.495

Source: Authors' calculations.

 $\phi_W(q) = -6.719$  and  $\phi_W(q_0) = -4.342$ . This means that the Watts index *W* is reduced by an absolute amount of  $0.00051 = \frac{1}{100} W[\phi_W(q_0) - \phi_W(q)]$  more by the growth pattern q(x) than it would be by distributionally-neutral growth, for each 1 percent growth in mean income (here, W = 0.0215). The measure  $\pi_P(q)$  records this premium. The ratio measure  $\kappa_P(q)$  records that the Watts index falls 1.547 times faster under q(x) than under equiproportionate growth:  $\kappa_P(q) = \frac{6.719}{4.342} \approx 1.547$ . For Portugal,  $\phi_W(q) = -4.030$  and  $\phi_W(q_0) = -2.649$ . Poverty is reduced by an absolute amount of  $0.00092 = \frac{1}{100} W[\phi_W(q_0) - \phi_W(q)]$  more by q(x) than by distributionally-neutral growth, per 1 percent growth in the mean. The measure  $\pi_P(q)$  records this as superior, whilst the ratio measure  $\kappa_P(q)$  records it as inferior, since poverty falls only 1.521 times faster than under equiproportionate growth in this scenario:  $\kappa_P(q) = \frac{4.029}{2.649} \approx 1.521$  (poverty having started at a high level in Portugal: W = 0.0666).

© 2009 The Authors

A referee suggested at this point that the reason the additive and ratio comparisons differ in this example is that the additive one depends on the poverty *level*, and that if this measure were normalized, to become

(3.7) 
$$\widetilde{\pi}_{P}(q) = \frac{1}{P} \pi_{P}(q) = \phi_{P}(q_{0}) - \phi_{P}(q)$$

(thus measuring the poverty improvement, or reduction in *per capita* deprivation, in *percentage* rather than level terms), then the ratio and (modified) additive indices would both rate Danish income growth as more pro-poor than Portuguese income growth. This is indeed the case for the data at hand,<sup>16</sup> but it is not generally true that the use of  $\tilde{\pi}_p(q) = \phi_p(q_0) - \phi_p(q)$  and  $\kappa_p(q) = \frac{\phi_p(q)}{\phi_p(q_0)}$  will always result in the same conclusions about relative pro-poorness between regimes. Each of  $\tilde{\pi}_p(q)$ ,  $\pi_p(q)$  and  $\kappa_p(q)$  measures pro-poorness à *la* Osmani, but all three in fact rank growth patterns across regimes differently, as simple numerical examples show.<sup>17</sup>

The additive measures  $\tilde{\pi}_p(q)$  and  $\pi_p(q)$  can of course only provide different rankings when used to make pro-poorness comparisons across regimes which have different poverty levels. The analyst's choice, between using  $\tilde{\pi}_p(q)$  and  $\pi_p(q)$  for such comparisons, provides yet another context in which "absolute" and "relative" considerations come into play in measuring pro-poorness. For  $\pi_p(q)$  measures the effect of differential growth on the poverty *level* (an absolute effect, equal to the absolute reduction in *per capita* deprivation), whilst  $\tilde{\pi}_p(q)$  measures the effect on poverty in percentage terms (a relative effect). Note, though, that neither measure accords with the ratio measure  $\kappa_p(q)$  in general. More research, in a more abstract setting (such as that of Duclos, 2009), will surely be needed to determine the class of pro-poorness rankings which will be most suitable for international and intertemporal pro-poorness comparisons.

### 4. Some Further New Developments

Our anatomy of pro-poorness has thus far expressed a number of existing measures in terms of the elemental growth pattern function q(x), and generated a new family of measures— $\pi_p(q)$  in level terms and  $\tilde{\pi}_p(q)$  when normalized—one for each member P of the additively separable class of poverty indices. Here we push forward with two further issues, not currently addressed in any literature we know of: (a) decomposing pro-poorness across income components, and (b) measuring pro-poorness at percentiles points in the income distribution. In what follows, results for the additive measure are derived for  $\pi_p(q)$  only, for

© 2009 The Authors Journal compilation © International Association for Research in Income and Wealth 2009

<sup>&</sup>lt;sup>16</sup>As the reader may verify using statistics already quoted,  $\tilde{\pi}_w(q) = 2.337$  for Denmark, 1.381 for Portugal; and  $\tilde{\pi}_H(q) = 0.015$  for Denmark, 0.008 for Portugal.

<sup>&</sup>lt;sup>17</sup>Consider regimes A, B and C, in which the poverty levels and elasticities are A: {P = 0.8,  $\phi_P(q) = -2.5$ ,  $\phi_P(q_0) = -0.625$ }, B: {P = 0.4,  $\phi_P(q) = -3.75$ ,  $\phi_P(q_0) = -1.25$ }, C: {P = 0.2,  $\phi_P(q) = -2.5$ ,  $\phi_P(q_0) = -0.5$ }. The pro-poorness values are A: { $\tilde{\pi}_p(q) = 1.875$ ,  $\pi_P(q) = 1.5$ ,  $\kappa_P(q) = 4$ }, B: { $\tilde{\pi}_p(q) = 2.5$ ,  $\pi_P(q) = 1.0$ ,  $\kappa_P(q) = 3$ }, C: { $\tilde{\pi}_p(q) = 2.0$ ,  $\pi_P(q) = 0.4$ ,  $\kappa_P(q) = 5$ }. Hence the rankings are B > C > A for  $\tilde{\pi}_P(q)$ , A > B > C for  $\pi_p(q)$  and C > A > B for  $\kappa_P(q)$ . Notice that  $\tilde{\pi}_P(q)$  and  $\kappa_P(q)$  disagree on A and B.

compactness, but they can all be replicated for  $\tilde{\pi}_{P}(q)$  simply by rescaling all terms by the poverty level *P*.

# 4.1. Decomposability of Pro-Poorness Measures Across Income Components

Suppose that each person's income x has two components,  $x = x_1 + x_2$ . The overall growth pattern is then an income-share-weighted average of the component growth patterns:  $q(x) = \frac{\mu}{x} \cdot \frac{dx}{d\mu} = \frac{\mu}{x_1 + x_2} \cdot \frac{dx_1 + dx_2}{d\mu} = \alpha(x) \cdot \frac{\mu}{x_1} \cdot \frac{dx_1}{d\mu} + (1 - \alpha(x)) \cdot \frac{\mu}{x_2} \cdot \frac{dx_2}{d\mu}$  where  $\alpha(x) = \frac{x_1}{x}$  is the share of total income coming from the first source. That is, we can write:

(4.1) 
$$q(x) = \alpha(x) \cdot q(x_1) + (1 - \alpha(x)) \cdot q(x_2)$$

where  $q(x_1)$  and  $q(x_2)$  are the growth patterns of the income components. Multiplying through by *x*, we get

(4.2) 
$$xq(x) = x_1 \cdot q(x_1) + x_2 \cdot q(x_2)$$

and, just as easily,

(4.3) 
$$x[q(x)-1] = x_1 \cdot [q(x_1)-1] + x_2 \cdot [q(x_2)-1].$$

By substituting from these expressions into the integrals defining  $\phi_P(q)$  and the indices  $\pi_P(q)$  and  $\kappa_P(q)$  defined in (3.3)–(3.4), we arrive at the following result.

# Theorem 2

If income *x* has two components,  $x = x_1 + x_2$ , the growth elasticity  $\phi_P(q)$  and the pro-poorness measures  $\pi_P(q)$  and  $\kappa_P(q)$  for the poverty index *P* defined in (3.1) can be decomposed, as

$$\phi_{P}(q) = \phi_{1P}(q) + \phi_{2P}(q),$$
  
$$\pi_{P}(q) = \pi_{1P}(q) + \pi_{2P}(q) \text{ and } \kappa_{P}(q) = \beta_{1}\kappa_{1P}(q) + \beta_{2}\kappa_{2P}(q)$$

respectively, where  $\beta_i = \frac{\int_0^z x_i \psi'(x|z) f(x) dx}{\int_0^z x \psi'(x|z) f(x) dx}$ , i = 1,2 (and the component measures are the natural analogues of the overall ones).<sup>18</sup> Similarly, the *PEGR* decomposes, as  $\gamma_e = \beta_1 \gamma_{1e} + \beta_2 \gamma_{2e}$ .

Note that the weights determining the composition of overall pro-poorness in terms of components for the multiplicative measure  $\kappa_P(q)$ , and for the *PEGR*, are given by the shares of the components in overall pro-poorness in the benchmark

<sup>18</sup>That is,  $\phi_{ip}(q) = \int_0^z \frac{x_i \psi'(x|z) q(x_i) f(x) dx}{P}$ ,  $\pi_{ip}(q) = \int_0^z \{-x_i \psi'(x|z)\} [q(x_i) - 1] f(x) dx$  and  $\kappa_{ip}(q) = \int_0^z x_i \psi'(x|z) q(x_i) f(x) dx / \int_0^z x_i \psi'(x|z) f(x) dx$ , i = 1, 2.

© 2009 The Authors

case, which provides the standard of comparison. If there are n > 2 income sources,  $x = \sum_{i=1}^{n} x_i$ , then because  $xq(x) = \sum_{i=1}^{n} x_i q(x_i)$  and  $x[q(x)-1] = \sum_{i=1}^{n} x_i[q(x_i)-1]$  as in (4.2)-(4.3), essentially the same decompositions apply:  $\phi_p(q) = \sum_{i=1}^{n} \phi_{ip}(q)$ ,  $\phi_p(q) = \sum_{i=1}^{n} \pi_{ip}(q)$ ,  $\kappa_p(q) = \sum_{i=1}^{n} \beta_i \kappa_p(q)$  and  $\gamma_e = \sum_{i=1}^{n} \beta_i \gamma_{ie}$ . Using these decompositions, we may "unbundle" a pattern of income growth and thereby identify the contributions of income components to overall pro-poorness.<sup>19</sup>

#### 4.2. Pro-Poorness at Percentiles

The focus in existing literature has been entirely on the formulation of an *overall* judgment about the pro-poorness of a growth pattern. We now consider a way of assessing pro-poorness at certain segments in the distribution of income among the poor. We base this assessment on a poverty curve akin to the so-called *three I's of poverty (TIP) curve* of Jenkins and Lambert (1997) or the *absolute poverty gap profile (APGP) curve* of Shorrocks (1998). The curve is obtained by partially cumulating individual contributions to overall poverty from the biggest one downwards (i.e. from the poorest individual to the richest). Individual contributions are computed on the basis of the deprivation function  $\psi(x|z)$  which defines the poverty index *P*.

Formally, let F(x) be the distribution function for income and let  $p = F(t) \in [0, 1]$ . We may define a generalized *TIP* curve  $J_P(.)$  for the poverty index *P* by

(4.4) 
$$J_p(p) = \int_0^t \psi(x|z) f(x) dx, \quad 0 \le p \le 1.$$

Clearly, this poverty curve is upward sloping and concave, and  $J_P(p) = P$  $\forall p \ge H = F(z)$ . For the normalized poverty gap *D*, the curve  $J_D(p)$  is precisely the *normalized TIP* or *APGP curve* of Jenkins and Lambert (1997) and Shorrocks (1998). The reader may use (4.4), along with the specifications of the deprivation functions for the Watts, normalized poverty gap and squared poverty gap indices given in note 8, to obtain analytical expressions for the curves  $J_W(p)$ ,  $J_D(p)$  and  $J_{FGT_5}(p)$ .

The following theorem identifies the form of a measure of *pro-poorness at percentile* p, when this is expressed as the effect of a growth pattern q(x) on  $J_P(p)$  over and above the effect that would obtain under distribution neutrality. This measure also decomposes across components.

### Theorem 3

If  $p = F(t) \in [0, 1]$ , then the pro-poorness measures defined in (3.3)–(3.4), evaluated for  $J_P(p)$  in (4.4) rather than for P, take the forms

<sup>19</sup>Examples of (gross) income components include employment and self-employment income, investment income, pensions, family support and unemployment benefits. Income taxes would be a negative component. In Kakwani *et al.* (2006) the *PEGRs*  $\gamma_e$  and actual growth rates  $\gamma$  for labor and non-labor income components between 1995 and 2004 in Brazil are computed and compared, and a decomposition methodology is laid out for examining labor income pro-poorness in terms of labor market characteristics, but pro-poorness is not itself decomposed across income sources. The decompositions can also be used for expenditure-based poverty analysis. In Son (2006), pro-poorness of income and expenditure components in Thailand are investigated, albeit without allowing for growth patterns which are non-equiproportionate.

 $\pi_{J}(q|p) = \int_{0}^{t} \{-x\psi'(x|z)\}[q(x)-1]f(x)dx \text{ and } \kappa_{J}(q|p) = \frac{\int_{0}^{t} x\psi'(x|z)q(x)f(x)dx}{\int_{0}^{t} x\psi'(x|z)f(x)dx}.$  In the case of  $n \ge 2$  income components,  $x = \sum_{i=1}^{n} x_{i}$ , we have  $\pi_{J}(q|p) = \sum_{i=1}^{n} \pi_{iJ}(q|p)$  and  $\kappa_{J}(q|p) = \sum_{i=1}^{n} \beta_{i}\kappa_{iJ}(q|p)$ , and similarly for the *PEGR*.<sup>20</sup>

When  $p \ge H$ , equivalently  $t \ge z$ , Theorem 3 leads to formulae for the percentile indicators of pro-poorness that are identical to the aggregate ones in (3.3)–(3.4). The condition for a growth pattern q(x) to be pro-poor at every percentile  $p \le H$  for the poverty index P is the following.

(4.5) 
$$\int_0^t \{-x\psi'(x|z)\}[q(x)-1]f(x)dx > 0 \ \forall t \le z.$$

Clearly, this condition is stronger than merely requiring that q(x) be pro-poor for the overall poverty measure P (for which (4.5) need hold at t = z only).

The condition  $q(x) > 1 \quad \forall x < z$  is of course sufficient for growth to be judged pro-poor at every percentile *p* for all the poverty indices *P* in the class we are considering. Suppose that growth is judged pro-poor for a certain index *P* at the lowest percentiles. The following theorem shows that this growth must be experienced more than proportionately by the poorest individuals.

### Theorem 4

If there is a cut-off percentile  $p_0 > 0$  such that growth is pro-poor  $(\pi_j(q|p) > 0)$  for all percentiles below and up to  $p_0$ , then there must be an income level v, below the poverty line, such that the elasticity of all individual incomes x < v with respect to the overall mean is greater than one. Formally,

$$\exists p_0 > 0: \pi_I(q|p) > 0 \ \forall p \le p_0 \Leftrightarrow \exists v < z: q(x) > 1 \ \forall x \in [0, v].$$

Notice that this result goes both ways. Pro-poorness for the very poorest *for one index P* is only securable if income growth at the very bottom is more than proportionate to overall growth—and then pro-poorness at the very bottom follows *for every index P* in our class. This Rawlsian-type condition is intuitively reasonable.

What does the local pro-poorness measure tell us, for p < H, that the global one (obtained when p = H) cannot? We examine this question for a particular poverty index, the normalized poverty deficit D which generates the regular TIP curve. Let us denote the percentile pro-poorness measures by  $\pi_{TIP}(q|p)$  and  $\kappa_{TIP}(q|p)$  in this case.

From note 8 and Theorem 3,  $\kappa_{TIP}(q|p) = \int_0^t xq(x)f(x)dx / \int_0^t xf(x)dx$  (where  $p = F(t) \in [0, 1]$ ). For  $p \le H$  this construct is a scaled version of Son's (2004) *poverty growth curve*, call it  $g_s(p)$ , which plots the growth rate of mean income among the 100*p* percent poorest against position *p* in the income distribution. In fact,  $g_s(p) = \gamma \cdot \kappa_{TIP}(q|p)$  for  $p \le H$ . As Son remarks, growth is unambiguously

© 2009 The Authors

<sup>&</sup>lt;sup>20</sup>If the underlying growth elasticity of  $J_P(p)$  for the growth pattern q(x) is  $\zeta_P(q|p)$ , then  $\pi_J(q|p)$  and  $\kappa_J(q|p)$  are defined by  $\pi_J(q|p) = J_P(p) \cdot [\zeta_P(q_o|p) - \zeta_P(q|p)]$  and  $\kappa_J(q|p) = \zeta_P(q|p) / \zeta_P(q_o|p)$  in the same way that (3.3) and (3.4) define the overall pro-poorness measures  $\pi_P(q)$  and  $\kappa_P(q)$ . The normalized additive measure  $\tilde{\pi}_J(q|p)$  would be defined as  $\zeta_P(q_o|p) - \zeta_P(q|p)$  (see 3.7).

pro-poor if  $g_s(p) > \gamma \forall p$ ; unambiguously anti-poor if  $g_s(p) < \gamma \forall p$ ; and will display negative values in times of recession ( $\gamma < 0$ ).

The additive measure of aggregate pro-poorness for the normalized poverty deficit is  $\pi_D(q) = \pi_{TIP}(q|p = H)$  and the corresponding "local" measure is  $\pi_{TIP}(q|p) = \frac{1}{z} \int_0^t x[q(x)-1]f(x)dx$ . Although  $\pi_D(q)$  depends only on the growth pattern q(x) for x < z, and compresses all of that information into a single number, in fact,  $\pi_D(q)$  reveals quite a lot about income growth *among the non-poor*, as Theorem 5 shows.

#### Theorem 5

Suppose that  $\pi_D(q)$  is explained by a growth pattern q(x). Then  $\pi_D(q)$  is also explained by a modified growth pattern  $q^*(x)$  such that for x < z,  $q^*(x) = q(x)$  and for x > z,  $q^*(x)$  is constant, with value  $q^* = 1 - \frac{z}{\mu} \cdot \frac{\pi_D(q)}{\theta}$ , where  $\mu$  is mean income and  $\theta$  is the fraction of all income held by the non-poor.

Thus  $q^*$  enjoys a similar relationship with  $\pi_D(q)$  as Ravallion and Chen's (2003) mean growth rate for the poor does with  $\pi_W(q)$ , the pro-poorness measure for the Watts index. The number  $q^*$  could be defined as the "pro-poorness equivalent uniform growth rate among the non-poor." Clearly,  $q^* \ge 1 \Leftrightarrow \pi_D(q) \le 0$ . If growth is overall pro-poor/anti-poor for *D*, then the equivalent smooth growth for the non-poor must be less than proportionate/more than proportionate—and vice versa. Corollaries are that (a) pro-poorness for the normalized poverty deficit *D* rules out growth patterns with  $q(x) > 1 \forall x > z$ ; and (b) growth patterns for which  $q(x) < 1 \forall x > z$  are necessarily pro-poor for *D*.

For a growth pattern q(x) which is continuous in x, constant for x > z and such that [q(x) - 1] changes sign *once* or *twice* on [0, z], being first positive (so that that the growth pattern is pro-poor at the poorest percentiles whatever the poverty measure used in the evaluation, by Theorem 4), it is easy to see that overall pro-poorness for the normalized poverty deficit occurs if there is a single sign change and overall anti-poorness if there are two sign changes.<sup>21</sup> In general, we can expect to see *overall pro-poorness* associated with *anti-poorness at some percentiles* among the poor (and conversely). The percentile pro-poorness measures  $\pi_{TIP}(q|p)$ can reveal facets of a growth pattern that the overall measure  $\pi_D(q)$  cannot.

#### 5. Empirical Considerations

The theoretical framework we have developed will help the analyst to assess the pro-poorness of economic growth, both in a particular country across time, and (as already shown) between countries. Here, we use consumption expenditure

<sup>21</sup>This is because the constant value of q(x) for x > z is  $q^* = 1 - \frac{z}{\mu} \cdot \frac{\pi_D(q)}{\theta}$  from Theorem 6, and  $q(z) = q^*$  can be inferred from continuity. Hence, for such a growth pattern, pro-poorness can be read simply from the value q(z):  $\pi_D(q) = [1 - q(z)] \cdot \frac{\mu \theta}{z}$ . Shortly, we shall encounter a growth pattern for Indonesia for which [q(x) - 1] changes sign twice on [0, z], being first negative, and with three sign changes overall. Note that in all scenarios, [q(x) - 1] must change sign at least once, because of (2.3).

	1993	1996	1999	2002
Average annual growth rate (real per capita expenditure)		8.12	-8.59	7.34
Headcount	61.53	50.49	55.29	52.41
Poverty gap	21.03	15.33	16.56	15.68
Squared poverty gap	9.16	6.02	6.49	6.09
Watts	28.12	19.73	21.33	20.12
Gini	32	36	31	34

 TABLE 2

 A Profile of Growth, Poverty and Inequality in the 1990s

Source: Authors' calculations.

data for Indonesia to illustrate the insights our constructs bring in regard to *expenditure pro-poorness* in that country in the period 1993–2002. We choose 1993 as our base year as it is the year when Indonesia became a middle-income country according to the World Bank's classification.

Indonesia has a long reputation of high achievements in growth and poverty reduction, even in the face of adverse conditions. In the 1980s, for instance, the country suffered a serious deterioration in its terms of trade with the rest of the world. The government responded with an aggressive adjustment program. The rate of economic growth slowed down during the adjustment period, but remained positive and above the rate of growth of the population. Poverty continued to decline over the period.

It is commonly believed that the poverty reduction experienced by the country between 1993 and 1996 (see Table 2) is part of the trend that started in the mid-1980s. Based on an absolute poverty line of about 2 dollars a day, poverty incidence fell from 61.5 percent in 1993 to 50.5 percent in 1996. Over that period of time, the country also successfully adjusted to the oil price shock, the overheating of the economy early in the 1990s, and the increased domestic interest rates to protect the rupiah against the contagion effect of the 1995 Mexican crisis (see World Bank, 1995, 1996).

This track record was briefly reversed by the 1997 Asian financial crisis that took the form of rapid currency depreciation. As a result, real GDP declined by 13 percent in 1998, and real *per capita* expenditure declined almost 9 percent per year between 1996 and 1999, while the incidence of poverty increased from 50.49 percent in 1996 to about 55 percent in 1999. Since 1999, the country has made some progress in restoring macroeconomic stability and in reducing the economy's vulnerability to external shocks. Real *per capita* expenditure grew on average 7 percent per year between 1999 and 2002, and poverty incidence started heading back to the pre-crisis level, reaching about 52 percent in 2002.

Focusing on the 1993–2002 period, we address the following question: "Is the observed poverty reduction more or less than what the country would have experienced, had economic growth been distributionally neutral?" To answer this question, we consider the following sub-periods: 1993–96, 1996–99, and 1999–2002. We use aggregate data, in the form of the distributions of *per capita* expenditure by decile, to recover the underlying size distributions of expenditure from the means and parameterized Lorenz curves. Our parameterization is based on the General Quadratic model described in Datt (1998). For all sub-periods, except 1999–2002,

<sup>© 2009</sup> The Authors

Journal compilation © International Association for Research in Income and Wealth 2009

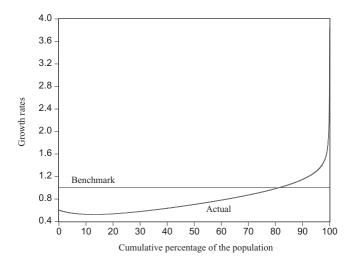


Figure 2. Growth Patterns (normalized growth incidence curves), 1996-99

the poverty elasticity underlying the reported measures of pro-poorness is computed as the ratio of the annualized percentage change in poverty to the average percentage change in real *per capita* expenditure. The measures reported for 1999–2002 are computed on the basis of more detailed information from two household surveys, known as SUSENAS (1999, 2002) produced by Statistics Indonesia (BPS).<sup>22</sup> This information helps us demonstrate the decomposability of our measure of propoorness. To ensure consistency with available aggregate data, we convert the household survey data from rupiahs into 1993 PPP dollars.

Figure 2 shows the observed and benchmark growth patterns for Indonesia for the 1996–99 period. The curve representing the actual growth pattern lies entirely below the benchmark for all percentiles way past the headcount ratio (50.49 percent). In fact, the growth patterns for all other sub-periods (not shown here) also lie below the benchmark, meaning that growth was unambiguously anti-poor in those other cases, but the 1996–99 period requires a different interpretation, since aggregate expenditure *fell* over that period. Namely, the expenditures of the poor fell by less than the average during the recession, which counts as a pro-poor growth experience. In all other sub-periods, economic growth in Indonesia in the 1990s was not pro-poor in the sense defined in this paper: positive economic growth did not deliver a significant reduction in poverty, where the level of significance is set at the amount of poverty reduction that would have been achieved under distributional neutrality.

To further illustrate this point empirically, we computed our additive measure of pro-poorness  $\pi_P(q)$  for the headcount index, Watts index, normalized poverty deficit and squared poverty gap. The estimates, presented in Table 3, are calculated using the growth patterns already described. It is clear from these results that in the 1990s economic growth has not been significantly pro-poor in Indonesia. This

<sup>22</sup>SUSENAS stands for Survei Sosial Ekonomi Nasional (National Socio-Economic Household Survey), and BPS stands for Badan Pusat Statistik (Central Bureau of Statistics).

	Headcount	Poverty Gap	Squared Poverty Gap	Watts
1993–96	-0.28	-0.16	-0.09	-0.23
1996-99	0.41	0.27	0.16	0.44
1999-2002	-0.12	-0.06	-0.04	-0.02
1993-2002	-0.23	-0.16	-0.09	-0.22

 TABLE 3

 Additive Measures of Pro-Poorness for the 1990s

Source: Authors' calculations.

 TABLE 4

 Ratio Measures of Pro-Poorness (correction factors for the *PEGR*) for the 1990s

	Headcount	Poverty Gap	Squared Poverty Gap	Watts
1993–96	0.64	0.62	0.62	0.63
1996–99	3.26	6.89	9.80	8.36
1999-2002	0.28	0.14	0.12	0.10
1993-2002	0.71	0.68	0.69	0.70

Source: Authors' calculations.

conclusion holds for all sub-periods with positive growth and all poverty measures. This could be expected from Theorem 4, since the observed growth pattern curves for these periods lie below the benchmark.

Another empirical illustration of our theoretical results involves the computation of the ratio measures  $\kappa_P(q)$  which are the correction factors associated with the *PEGR* (see equation (3.6)). The results are presented in Table 4. The adjustment factor is less than one for all periods associated with positive growth, confirming that growth was generally not pro-poor in Indonesia in the 1990s.

The interpretation of the results presented in Tables 3 and 4 follows the same logic as for the numerical example involving Denmark and Portugal discussed earlier. The period 1996–99 deserves special attention because this is a period of negative growth. Per capita expenditure fell by about 8.6 percent on average in real terms. All additive measures of pro-poorness presented in Table 3 are positive while all the ratio measures in Table 4 are greater than one. According to our analytics, this means that the negative growth experienced in 1996–99 was actually *pro*-poor—in the sense that the expenditure reductions experienced by the poor were less than the mean reduction, and accordingly the benchmark increase in poverty was greater than the actual increase in poverty. For instance, the additive pro-poorness value associated with the Watts index is equal to 0.44. The underlying elasticities are  $\phi_w(q_0) = -2.5587$  and  $\phi_w(q) = -0.3060$ . In other words, for every 1 percent decrease in per capita expenditure, a distributionally-neutral process would have increased poverty by 0.44 percent more than the observed pattern. In terms of the ratio pro-poorness measure (Kakwani *et al.*'s adjustment factor), we may say that distributionally neutral growth would have increased poverty 8.36 times faster than the observed outcome.<sup>23</sup>

<sup>23</sup>These conclusions can be confirmed by considering the results of a Shapley decomposition of poverty outcomes into their growth and distributional components over the period. See Essama-Nssah and Lambert (2006) for further details, and Kakwani (2000) for an axiomatic treatment establishing the sufficiency of pure growth and inequality effects to determine the overall effect on poverty as a sum.

© 2009 The Authors

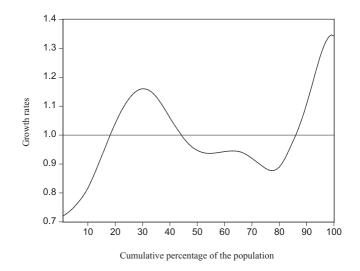


Figure 3a. Aggregate Pattern of Growth (1999–2002)

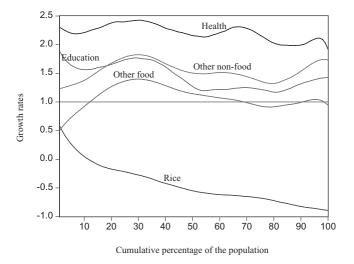


Figure 3b. Incidence of Growth on Expenditure Components (1999-2002)

The available household survey data for 1999 and 2002 allow us to look beyond aggregate results for this period and consider the contribution of expenditure components to the observed outcome. We also use the same data to look at pro-poorness at percentiles. Figure 3a shows the aggregate pattern of growth for the period under consideration while Figure 3b represents a disaggregation of this overall pattern and is interpreted as the incidence of economic growth on five

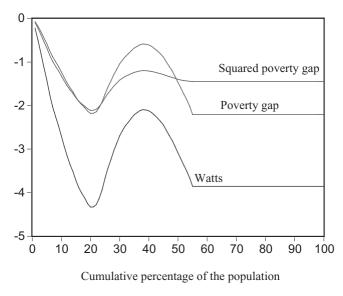


Figure 4. Pro-Poorness at Percentiles (1999–2002)

expenditure components: (1) rice, (2) other food, (3) education, (4) health, and (5) other non-food.<sup>24</sup>

Finally, we point to an interesting feature, apparent in the aggregate growth curve (Figure 3a), which is further illuminated by resorting to our percentile measures. This growth curve crosses the benchmark case twice before the head count ratio (55 percent). This means that we cannot use Theorem 4 to infer overall pro-poorness. However, the curve lies completely below the benchmark up to the 20th percentile and between the 43rd and 55th percentiles. This represents about 60 percent of the poor for whom expenditure per capita grew less than average. The underlying data show that for this segment of the poor population, on average, *per* capita expenditure grew 14 percent less than it would have, had growth been distributionally neutral. The other 40 percent of the poor enjoyed an average increase in expenditure 17 percent above the hypothetical case. Figure 4 shows plots of pro-poorness at percentiles  $[\pi_i(q|p)]$  computed according to Theorem 3 for three poverty measures. The fact that all these curves lie below zero indicates that economic growth has not been pro-poor at any percentile up to the headcount. Thus, according to our metric, the benefits enjoyed by the poor located between the 20th and the 43rd percentiles are not high enough to compensate for the loss experienced by those who came before.

Information on the contributions of expenditure components to pro-poorness is contained in Table 5. This tells us that the outcome in Figure 4 is driven mainly by what happened to expenditure on rice, with some help from expenditure on

© 2009 The Authors Journal compilation © International Association for Research in Income and Wealth 2009

<sup>&</sup>lt;sup>24</sup>The growth pattern curves presented in Figure 3a and 3b are more refined than those presented in Figure 2: they are computed directly from household survey data (and smoothed using the Epanechnikov kernel function) whilst the curves in Figure 2 come from a parameterization of the Lorenz curve based on aggregate data.

	Headcount	Watts	Poverty Gap	Squared Poverty Gap
Rice	-0.185	-0.172	-0.124	-0.061
Other food	-0.020	0.033	0.029	0.011
Education	-0.006	0.004	0.003	0.002
Health	0.017	0.008	0.006	0.003
Other non-food	0.073	0.070	0.051	0.025
Aggregate	-0.122	-0.056	-0.035	-0.021

TABLE 5 A Decomposition of Aggregate Measure  $\pi_P(q)$  (1999–2002)

Source: Authors' calculations.

other food items. The underlying data reveal that rice represents 26 percent of total expenditure for the poor (total food expenditure including rice is about 73 percent of household expenditures for the poor). The growth pattern for rice in fact lies entirely below the benchmark, while that for the other food items lies below only for the 20 percent poorest.

This important finding can be understood on the basis of the following considerations. The 1997 currency crisis combined with rice crop reduction induced by the drought and the Sumatra fires that occurred the same year led to skyrocketing prices for rice and other food. Between February 1996 and February 1999 the price of rice increased by 184 percent. During the same period, food inflation was estimated at 160 percent compared with 81 percent for non-food items (Suryahadi *et al.*, 2003). Indonesia is the world's largest net rice importer (18 percent of the world's total imports between 1998 and 2001). Since the financial crisis, the government has increasingly sought to limit rice imports using both tariff and non-tariff barriers (such as licensing and temporary bans). From about 2000 until late 2004, it is estimated that the domestic price of rice has settled around 40–50 percent above the import price. This level is above any reached in the previous three decades (Warr, 2005).

# 6. CONCLUDING REMARKS

Poverty reduction is considered a fundamental objective of development, and has become a metric for assessing the effectiveness of development interventions. Measuring the pro-poorness of a growth process is an exercise in social evaluation, the outcome of which hinges on the underlying value judgments. In this paper we have examined the elasticity-based measurement of pro-poorness, using an analytical framework with three key elements: (1) the definition of a growth pattern in terms of the point elasticity of individual incomes or consumption expenditures with respect to the mean; (2) the use of individual poverty contributions and members of the class of additively separable poverty measures in the construction of a social evaluation criterion; and (3) the selection of the poverty reduction obtainable under distributional neutrality as the threshold that must be crossed in order to declare a growth pattern pro-poor.

What emerges from the analysis in these terms is a full taxonomy of existing elasticity-based pro-poorness measures, and a new measure, expressible in both level and percentage terms, along with a method to decompose overall pro-

poorness across income sources, or components of consumption expenditure, and an adaptation of the entire methodology to permit the "local" measurement of pro-poorness at percentile points in the distribution among the poor.

An application of this methodology to expenditure data for Indonesia for the period 1993–2002 shows that the reduction in expenditure poverty achieved over that period remains generally far below what distributionally-neutral growth would have produced. However the 1997 economic crisis must be considered pro-poor, since the losses for the non-poor outweigh those sustained by the poor. These conclusions are robust to the choice of both a poverty measure among members of the additively separable class, and a poverty line up to about 2 dollars a day. The behavior of five categories of expenditure over the 1999–2002 period suggests that the weak performance is due mainly to changes in food expenditure.

Pro-poorness decompositions across income components can identify income sources (for example, labor income) whose growth pattern may be anti-poor or weakly pro-poor, enabling government to put forward appropriate incomeenhancing policies. The ultimate objective of pro-poor income-generating policies may well be to increase people's expenditures, and pro-poorness decompositions across expenditure components clearly have a complementary role in aiding propoor policy design. For example, Besley and Kanbur (1988) discuss the targeting of food subsidies to alleviate poverty. Poverty alleviating policy and pro-poor policy are not quite the same thing. It is worth reminding ourselves at this point that pro-poorness has a "relative" dimension: a pro-poor policy need not favor the poor "more than" the rich, but it does favor them relative to a benchmark situation in which the gains go in equal proportions to both the poor and the rich.

Klasen (2008) shows how to extend some of the tools of pro-poor growth measurement to non-income dimensions of poverty, and suggests that an absolute approach may be particularly suitable for this. He sees the lack of consideration by economists of non-income pro-poorness as "highly lamentable . . . and . . . quite contrary to the spirit of the MDGs which consider non-income dimensions of well-being (particularly education, health, and gender equity) as being of equal importance to income poverty" (p. 424). Klasen shows how Ravallion and Chen's growth incidence curve can be adapted to non-income dimensions, and he also explains the nature of the insights that can then be derived for measuring and monitoring, as well for targeting and other policy priorities (see also Grosse *et al.*, 2008).

We hope that our approach may open up other new lines of investigation, in which the analysis will be conducted in terms of the elemental function q(x) which specifies the growth pattern. Standard errors for the poverty measures used in this paper are easy to calculate in simple random samples (Kakwani, 1993b); similar work could hopefully also establish standard errors for q(x) and the pro-poorness measures  $\pi_P(q)$  and  $\kappa_P(q)$ . Foster and Szekely's (2000) conception of pro-poorness uses a growth elasticity for the Atkinson (1970) inequality index, and this could be cast in terms of the underlying q(x). Bourguignon (2003) suggests in respect of pro-poorness that "instead of considering poverty measures, it would be interesting to consider aggregate measures of social welfare" (p. 25); that could also be done. Bourguignon gives encouragement for using the micro-level ingredient q(x) in the determination of pro-poorness: "the sooner poverty specialists will get used

<sup>© 2009</sup> The Authors

Journal compilation © International Association for Research in Income and Wealth 2009

to dealing systematically with distribution data, rather than with inequality or poverty summary measures, at the national level, the better it will be" (p. 19).

We conclude with a remark that is prompted by a recent and very elegant analysis of pro-poorness by Grimm (2007). All of the measures discussed in this paper have in common that they are based on the anonymity axiom, because the elemental function q(x) (equivalently, the growth incidence curve of Ravallion and Chen, 2003) describing the pattern of income growth takes no account of who is at what income level x before and after the growth experience (recall our equation (2.6)). As Grimm points out, the same growth pattern could reflect ongoing (chronic) poverty or transient poverty, depending on mobility among the poor. He advocates "looking at . . . group-specific trajectories" (Grimm, 2007, p. 180), and defines a modified growth incidence curve, à la Ravallion and Chen but defined for the individuals who started at a specific percentile. He shows how, by using this curve, one can decompose changes in the Watts index into components accounted for by income changes among the initially poor who crossed the poverty line, the initially poor who did not cross the poverty line, and the initially non-poor who crossed the poverty line (see also Jenkins and Van Kerm, 2006). Here is a domain in which the careful refinement of the q(x) modeling of this paper could have a significant pay-off.

### APPENDIX: MATHEMATICAL RESULTS

Let  $U = \int_0^{m_x} u(x) f(x) dx$  be the average of an attribute u(x) across the population. Now let mean income  $\mu$  grow by a small amount  $\Delta \mu$ , so that individual income x grows to  $x \left[ 1 + q(x) \cdot \frac{\Delta \mu}{\mu} \right] = x + \Delta x$ , say. The income value  $x + \Delta x$  now occurs with frequency density f(x), and U changes to  $U + \Delta U = \int_0^{m_x} u(x + \Delta x) f(x) dx$ . Writing  $u(x + \Delta x) = u(x) + u'(x) \Delta x = u(x) + xu'(x)q(x) \cdot \frac{\Delta \mu}{\mu}$ , we have  $\Delta U = \frac{\Delta \mu}{\mu} \cdot \int_0^{m_x} xq(x)u'(x)f(x) dx$ , or:

(A.1) 
$$\frac{\mu}{U} \cdot \frac{\Delta U}{\Delta \mu} = \frac{\int_0^{m_x} xq(x)u'(x)f(x)dx}{\int_0^{m_x} u(x)f(x)dx}.$$

Putting u(x) = x and  $U = \mu$  in (A.1),  $1 = \frac{\int_0^{m_x} xq(x)f(x)dx}{\int_0^{m_x} xf(x)dx}$  which reduces to equation (2.3). For Theorem 1, just put  $u(x) = \psi(x|z)$  in (A.1). Equations (3.3) and (3.4) follow directly.

For the effect on the headcount *H* of a growth pattern q(x), let  $H \to H^*$  when all incomes grow according to the growth pattern q(x), so that H = F(z). Let  $x + xq(x)\frac{\Delta y}{y} = v(x)$ , assumed increasing in *x*, so that growth does not cause reranking of income units, and let *w* be the inverse function of *v*, also increasing.

 $$\ensuremath{\mathbb{C}}\xspace$  2009 The Authors Journal compilation  $\ensuremath{\mathbb{C}}\xspace$  International Association for Research in Income and Wealth 2009

Then 
$$H^* = F(w(z))$$
 and  $\phi_H(q) = \lim_{\Delta\mu\to 0} \frac{(H^* - H)/H}{\Delta\mu/\mu} = \lim_{\Delta\mu\to 0} \frac{\frac{\mu}{\Delta\mu} [F(w(z)) - F(z)]}{F(z)}$ . Set  
 $w(z) = w_0$ , so that  $z = w_0 + w_0 q(w_0) \frac{\Delta\mu}{\mu}$ . Then  $\phi_H(q) = \lim_{\Delta\mu\to 0} \frac{-\frac{\mu}{\Delta\mu} \left[ w_0 q(w_0) \frac{\Delta\mu}{\mu} f(w_0) \right]}{F(z)}$   
 $= -\frac{zq(z)f(z)}{H}$  since  $w_0 \to z$  as  $\Delta\mu \to 0$ .

H For Theorem 3, let  $p = F(t) \in [0, 1]$  and define  $U(t) = \int_0^t u(x)f(x)dx$ . Arguing as before, the change in U(t) is  $\Delta U(t) = \frac{\Delta \mu}{\mu} \cdot \int_0^t xq(x)u'(x)f(x)dx$ , so that:

(A.2) 
$$\frac{\mu}{U(t)} \cdot \frac{\Delta U(t)}{\Delta \mu} = \frac{\int_0^t xq(x)u'(x)f(x)dx}{\int_0^t u(x)f(x)dx}.$$

Putting u(x) = x in (A.2), so that  $U(t) = \mu L(p)$  where L(p) is the Lorenz curve, we have

(A.3) 
$$\frac{\mu}{L(p)} \cdot \frac{dL(p)}{d\mu} = \int_0^t x[q(x)-1]f(x)dx / \mu.$$

By the same token, with  $u(x) = \frac{x}{p}$ , so that  $U(t) = \mu \frac{L(p)}{p} = \mu(p)$ , (A.2) yields

(A.4) 
$$\frac{\mu}{\mu(p)} \cdot \frac{d\mu(p)}{d\mu} = \frac{\int_0^t xq(x)f(x)dx}{\int_0^t xf(x)dx}.$$

Property (2.4) tells us that the function of p (or x) defined in (A.3) has an inverted U-shape, and is zero at each end of its range, implying inequality reduction, as claimed of property (2.4).

Now put  $u(x) = \psi(x|z)$  and  $U(t) = J_P(p)$  in (A.2). The growth elasticity of  $J_P(p)$ is  $\zeta_P(q|p) = \frac{\int_0^t xq(x)\psi'(x|z)f(x)dx}{J_P(p)}$ , from which Theorem 3 follows using note 18. Son's  $g_S(p)$  is defined as  $\frac{d\mu(p)}{\mu(p)}$  which is just  $\gamma = \frac{d\mu}{\mu}$  times  $\kappa_{TIP}(q|p)$  from (A.4). For Theorem 4, if  $\pi_I(q|p) > 0 \quad \forall p \le p_0$  then, as a function of  $t \in [0, 1]$ ,  $\int_0^t \{-x\psi'(x|z)\}[q(x)-1]f(x)dx$  is initially upward-sloping. Taking the derivative,  $\exists v < z : q(x) > 1 \quad \forall x \in [0, v]$ . Conversely, if  $\exists v < z : q(x) > 1 \quad \forall x \in [0, v]$ , then  $\int_0^t \{-x\psi'(x|z)\}[q(x)-1]f(x)dx$  is initially upward-sloping, whence  $\exists p_0 < 1 : \pi_J(q|p) > 0 \quad \forall p \le p_0$ . The result follows.

Finally, using the deprivation function for *D* given in note 8 and the feasibility property (2.3), (3.3)–(3.4) imply that  $\pi_D(q) = \frac{-1}{z} \int_z^{m_x} x[q(x)-1]f(x)dx$ . Setting

© 2009 The Authors

 $q(x) = q^* \quad \forall x > z$  in this, and noting that  $\int_z^{m_x} xf(x) dx = \mu \theta$ , we have  $\frac{1-q^*}{z} \cdot \mu \theta = \pi_D(q)$ , from which Theorem 5 follows.

#### References

Atkinson, Anthony B., "On the Measurement of Inequality," *Journal of Economic Theory*, 2, 244–63, 1970.

, "On the Measurement of Poverty," Econometrica, 55, 749-64, 1987.

- Besley, Timothy and Ravi Kanbur, "Food Subsidies and Poverty Alleviation," *The Economic Journal*, 98, 701–19, 1988.
- Besley, Timothy, Robin Burgess, and Berta Esteve-Volart, "The Policy Origins of Poverty and Growth in India," in Timothy Besley and Louise Cord (eds), *Delivering on the Promise of Pro-Poor Growth*, Palgrave, New York, 2006.
- Bourguignon, Francois, "The Growth Elasticity of Poverty Reduction: Explaining Heterogeneity Across Countries and Time Periods," in Theo S. Eicher and Stephen J. Turnovsky (eds), *Inequality* and Growth: Theory and Policy Implications, MIT Press, Cambridge, MA, 3–26, 2003.
- Chenery, Hollis, Móntele S. Ahluwalia, Clive L. G. Bell, John H. Duloy, and Richard Jolly, *Redistribution with Growth: Policies to Improve Income Distribution in Developing Countries in the Context of Economic Growth*, Oxford University Press, New York and Oxford, 1974.
- Datt, Gaurav, "Computational Tools for Poverty Measurement and Analysis," International Food Policy Research Institute (IFPRI) Discussion Paper No. 50 (Food Consumption and Nutrition Division), Washington, DC, 1998.

Duclos, Jean-Yves, "What is Pro-Poor?" Social Choice and Welfare, 32, 37-58, 2009.

- Essama-Nssah, Boniface, "A Unified Framework for Pro-Poor Growth Analysis," *Economics Letters*, 89, 216–21, 2005.
- Essama-Nssah, Boniface and Peter J. Lambert, "Measuring the Pro-Poorness of Income Growth Within an Elasticity Framework," Economics Discussion Paper No. 2006-12, University of Oregon, and Policy Research Working Paper No. WPS 4035, World Bank, 2006.
- Foster, James E. and Miguel Szekely, "How Good is Growth?" *Asian Development Review*, 18, 59–73, 2000.
- Foster, James, Joel Greer, and Erik Thorbecke, "A Class of Decomposable Poverty Measures," *Econometrica*, 52, 761–6, 1984.
- Grimm, Michael, "Removing the Anonymity Axiom in Assessing Pro-Poor Growth," Journal of Economic Inequality, 5, 179–97, 2007.
- Grosse, Melanie, Kenneth Harttgen, and Stephan Klasen, "Measuring Pro-Poor Growth in Non-Income Dimensions," *World Development*, 36, 1027–47, 2008.
- Jenkins, Stephen P. and Peter J. Lambert, "Three 'I's of Poverty Curves, with an Analysis of U.K. Poverty Trends," *Oxford Economic Papers*, 49, 317–27, 1997.

Jenkins, Stephen P. and Philippe Van Kerm, "Trends in Income Inequality, Pro-Poor Income Growth, and Income Mobility," *Oxford Economic Papers*, 58, 531–48, 2006.

- Kakwani, Nanak, "Poverty and Economic Growth with Application to Côte d'Ivoire," The Review of Income and Wealth, 39, 121–38, 1993a.
  - —, "Measuring Poverty: Definitions and Significance Tests with Application to Côte d'Ivoire," in M. Lipton and J. Van Der Gaag (eds), *Including the Poor*, The World Bank, Washington, DC, 1993b.
  - —, "Measurement of Tax Progressivity: An International Comparison," *The Economic Journal*, 87, 71–80, 1977.
- ——, "On Measuring Growth and Inequality Components of Poverty with Application to Thailand," *Journal of Quantitative Economics*, 16, 67–80, 2000.
- ——, Methods in Measuring Poverty Matter: An Indian Story, One Pager Number 2, International Poverty Center, Brazil, 2004.
- Kakwani, Nanak and Ernesto M. Pernia, "What is Pro-Poor Growth?" Asian Development Review, 18, 1–16, 2000.

Kakwani, Nanak and Hyun H. Son, "Pro-Poor Growth: Concepts and Measurement with Country Case Studies," *Pakistan Development Review*, 42, 417–44, 2003.

—, "On Assessing Pro-Poorness of Government Programmes: International Comparisons," in Nanak Kakwani and Jacques Silber (eds), *Many Dimensions of Poverty*, Palgrave Macmillan, Basingstoke and New York, 251–74, 2008a.

© 2009 The Authors

<sup>-, &</sup>quot;Poverty Equivalent Growth Rate," The Review of Income and Wealth, 54, 643-55, 2008b.

- Kakwani, Nanak, Shahid Khandker, and Hyun H. Son, "Pro-Poor Growth: Concepts and Measurement with Country Case Studies," Working Paper Number 2004-1, International Poverty Center, Brazil, 2004.
- Kakwani, Nanak, Marcelo Neri, and Hyun H. Son, "Linkages Between Pro-Poor Growth, Social Programmes and the Labour Market: The Recent Brazilian Experience," Working Paper No. 26, International Poverty Centre, Brazil, 2006.
- Kalwij, Adriaan and Arjan Verschoor, "Not by Growth Alone: The Role of the Distribution of Income in Regional Diversity in Poverty Reduction," European Economic Review, 51, 805-29, 2007.
- Klasen, Stephan, "Economic Growth and Poverty Reduction: Measurement Issues Using Income and Non-Income Indicators," World Development, 36, 420-45, 2008.
- Klasen, Stephan and Mark Misselhorn, "Determinants of the Growth Semi-Elasticity of Poverty Reduction," No. 15 Proceedings of the 2006 German Development Economics Conference, Berlin, 2006.
- Kraay, Aart, "When is Growth Pro-Poor? Evidence From a Panel of Countries," Journal of Development Economics, 80, 198-227, 2006.
- Lambert, Peter J., "Non-Equiproportionate Income Growth, Inequality and the Income Tax," Public Finance/Finances Publiques, 39, 104–18, 1984.

, "Positional Equity and Equal Sacrifice: Design Principles for an EU-Wide Income Tax?" in Marc Fleurbaey, Maurice Salles, and John A. Weymark (eds), Social Ethics and Normative Economics, Studies in Choice and Welfare series, Springer, forthcoming, 2009.

McCulloch, Neil and Bob Baulch, Tracking Pro-Poor Growth: New Ways to Spot the Biases and Benefits, ID21 Insights No. 31, Institute of Development Studies, Sussex, 2000.

- McKinley, Terry, How Inclusive is "Inclusive Growth"? Development Viewpoint No. 6, School of Oriental and African Studies, London, 2008.
- Osmani, Siddiq, Defining Pro-Poor Growth, One Pager Number 9, International Poverty Center, Brazil, 2005.
- Ravallion, Martin, "Issues in Measuring and Modelling Poverty," The Economic Journal, 106, 1328-43, 1996.
- , Defining Pro-Poor Growth: A Response to Kakwani, One Pager Number 4, International Poverty Center, Brazil, 2004.
- Ravallion, Martin and Shaohua Chen, "What Can New Survey Data Tell Us About Recent Changes in Distribution and Poverty?" World Bank Economic Review, 11, 357-82, 1997.

, "Measuring Pro-Poor Growth," Economics Letters, 78, 93-9, 2003.

Shorrocks, Anthony F., "Deprivation Profiles and Deprivation Indices," in Stephen Jenkins, Arie Kapteyn, and Bernard Van Praag (eds), The Distribution of Welfare and Household Production: International Perspectives, Cambridge University Press, Cambridge, 250-67, 1998.

- "Assessing the Pro-Poorness of Government Fiscal Policy in Thailand," Public Finance Review, 34, 427-49, 2006.
- Suryahadi, Asep, Sudarno Sumarto, and Lant Pritchett, "Evolution of Poverty During the Crisis in Indonesia," *Asian Economic Journal*, 17, 221–41, 2003. Warr, Peter G., "Food Policy and Poverty in Indonesia: A General Equilibrium Analysis," *Australian*
- Journal of Agricultural and Resource Economics, 49, 429-51, 2005.
- Watts, Harold, "An Economic Definition of Poverty," in Daniel P. Moynihan (ed.), On Understanding Poverty: Perspectives from the Social Sciences, Basic Books, New York, 316-29, 1968.
- World Bank, Indonesia: Improving Efficiency and Equity-Changes in the Public Sector's Role, Report No. 14006-IND, Country Department III, East Asia and Pacific Region, The World Bank, Washington, DC, 1995.

, Indonesia: Dimensions of Growth, Report No. 15383-IND, Country Department III, East Asia and Pacific Region, The World Bank, Washington, DC, 1996.

-, World Development Report 2000/2001: Attacking Poverty, The World Bank and Oxford University Press, Washington, DC and New York, 2001.

Zepeda, Eduardo, Pro-Poor Growth: What is it? One Pager Number 1, International Poverty Center, Brazil, 2004.

Son, Hyun H. "A Note on Pro-Poor Growth," Economics Letters, 82, 307-14, 2004.