Review of Income and Wealth
Series 55, Number 2, June 2009

ON THE ESTIMATION OF GROWTH AND INEQUALITY ELASTICITIES OF POVERTY WITH GROUPED DATA

BY FLORENT BRESSON*

Cemafi—Université Nice Sophia Antipolis

After decades of intensive research dedicated to efficient and flexible parametric statistical distributions, the lognormal distribution still enjoys, despite its empirical weaknesses, widespread popularity in the applied literature related to poverty and inequality analysis. In the present study, we emphasize the drawbacks of this choice for the calculation of the elasticities of poverty. For this purpose, we estimate the growth and inequality elasticities of poverty using 1,132 income distributions, and 15 rival assumptions on the shape of the income distributions. Our results confirm that the lognormal distribution is not appropriate in most cases for the analysis of poverty: the magnitude of the elasticities is generally overestimated and the estimation of the relative impact of growth and redistribution on poverty alleviation is biased in favor of the growth objective.

1. INTRODUCTION

When the Millenium Development Goals (MDGs) were defined in the late 1990s, the international community explicitly made poverty alleviation as the prime objective of development policies. However, how best this goal could be achieved is still a matter of discussion. From an analytical point of view, poverty in a given country depends on both the mean income and the degree of inequality within its population.[1] As a result, poverty variations are associated with changes in *per-capita* income and inequality. Such a simple arithmetic statement is at the heart of the consensus that emerged in the 1970s concerning the promotion of growth-with-redistribution policies (Chenery *et al.*, 1974). It also means that the effects of any macro-economic variable (institutions, trade openness, financial development . . .) on poverty are only channeled through growth and redistribution. The knowledge of the linkages between poverty, on one hand, and growth and redistribution, on the other hand, are thus a critical prerequisite to the design of an efficient anti-poverty strategy.

However, on the spirit of the Washington consensus, many studies (Gugerty and Roemer, 1997; Ravallion 2001; Dollar and Kraay 2002) have first attempted

Note: We would like to thank Léandre Bassole, Sylvain Chabé-Ferret, Jean-Louis Combes, Michael Grimm, Patrick Guillaumont, Kenneth Harttgen, Stephan Klasen, Roland Kpodar and the two anonymous referees for their helpful comments.

*Correspondence to: Florent Bresson, CEMAFI, Université Nice Sophia Antipolis, Fac. de Droit, Sc. Pol., Éco. et de Gestion, Av. Doyen Trotabas, 06 050 NICE Cedex 1, France (florent.bresson@unice.fr).

[1] Of course, any poverty measure is also determinated by a poverty line and a set of mathematical constraints that translate ethical preferences of the social evaluator, but these parameters are generally considered as time-invariant.

to show that growth was the main driver of observed poverty alleviation outcomes and, consequently, that policy-makers should principally focus on growth-promoting objectives. This view is perfectly illustrated by the 2005 World Development Report (World Bank, 2005) that extensively relies on the investment–growth–poverty causal relationship. As the practices of the biggest multilateral institutions changed little, this polarization on the growth–poverty relationship led some authors to the conclusion that the MDGs have only resulted in purely rhetorical changes. This feeling was reinforced by some authors' efforts to justify growth-oriented policies in terms of poverty alleviation. However, the efficiency of these policies has been greatly debated. Whereas Dollar and Kraay (2002) argued that the income of the poor grows at the same rate as the mean income, many economists get involved in the estimation of a mean value for the growth elasticity of poverty, in particular for the realization of simulations exercises (Collier and Dollar, 2001). It is worth noting that the estimated values are in a wide range. For instance Besley and Burgess (2003) suggest using a value of –0.7, whereas Bhalla (2004) finds a mean elasticity of –3.4. The difference is economically significant since the achievement of the objective of halving extreme poverty between 1990 and 2015 under the first result implies a rate of growth which is five times larger than the one corresponding to the elasticity calculated by Bhalla (2004).[2]

Because these studies focused on growth, and paid little attention to inequality, recent studies (including Bourguignon, 2003; Heltberg, 2004; Ravallion, 2005) have emphasized the fundamental role of distribution in the determination of poverty variations. Their main message is that growth reduces poverty more if income distributions are less unequal. This effect is of course complemented by the potentially beneficial direct effect of redistribution on poverty. Lopez and Servèn (2006) show that the contribution of inequality reduction to poverty alleviation is higher in richer countries. Consequently, redistribution policies and distributional consequences of growth-promoting policy measures should not be ignored and the diminution of inequalities should be considered as an intermediate objective of poverty alleviation policies in the same way as per-capita income growth.

Nonetheless, even if there is general agreement on the necessity of taking into account the distribution issue in poverty analysis, no consensus has emerged about the relative contributions of growth and inequality reduction to poverty alleviation. Considering factors that both stimulate growth and narrow inequality, it may be of little interest to look for these relative contributions, as their impact on poverty is most likely to be positive. However, those factors for which the growth and inequality effects work in opposite directions—trade openness and financial development are frequently accused of contributing to development at the expense

[2]According to Besley and Burgess (2003), halving extreme poverty requires income *per capita* to grow at an annual rate of 3.8 percent between 1990 and 2015 in the developing world. With the elasticities suggested by Collier and Dollar (2001) and Bhalla (2004), the needed growth rates are only 1.4 and 0.8 percent, respectively.

of widening inequalities—it is crucial to know more about the trade-off faced by policy-makers (McKay, 1997).[3]

The calculation of growth and inequality elasticities of poverty is an elegant way to present the relationships between growth, inequality and poverty, and to assess the terms of this growth–inequality trade-off. From a practical point of view, a direct estimation of these elasticities for different values of mean income and different degrees of inequality can easily be achieved under the assumption that the observed distributions can be described by a known statistical distribution. In most studies (Bourguignon, 2003; Epaulard, 2003; Kalwij and Verschoor, 2005; Lopez and Servèn, 2006) the lognormal distribution is used. This can be seen as a peculiar choice since these authors choose to set aside all the 20th century debates on the statistical distributions of income.[4] Since the late 19th century and the pioneering works of Pareto, research has been extremely active to find the functional form that best fits the observed distributions. Practical considerations and the considerable influence of the study of Aitchison and Brown (1957) may still explain the current popularity of the lognormal distribution, but cannot justify its systematic use in empirical studies. Many authors have pointed out its empirical weaknesses and have suggested alternative functional forms (Maddala and Singh, 1976; Dagum, 1977) or the generalized beta 2 distribution (McDonald, 1984; Jenkins, 2009).[5] For instance, Bandourian *et al.* (2002) showed that the lognormal distribution was outperformed by many alternative functional forms, even within the set of two-parameter distributions, for a relatively large sample of developed countries. If the lognormal distribution is such a poor approximation of observed income distributions, elasticities obtained through the lognormality hypothesis are then questionable. In particular it is necessary to ask whether relying on the normality assumption is not a source of bias when we try to appreciate the growth–inequality trade-off in the context of poverty alleviation.

In the present paper, we intend to shed light on the consequences of the use of a potentially inadequate distributional hypothesis for the estimation of growth and inequality elasticities of poverty. For this purpose we estimate these elasticities for a sample of 1,132 income distributions in 120 countries between 1960 and 2005 using 15 alternative statistical distribution assumptions. Our results confirm the intuition that moving to more flexible functional forms dramatically improves the quality of the fit, and show that the corresponding elasticities differ from the one obtained under the lognormal hypothesis. Moreover, we find that the estimated elasticities under the lognormal assumption tend to overestimate the "real" value of these elasticities, and may cause a bias

[3]Dollar and Kraay's (2002) results suggest that the growth process is distribution-neutral on average. This feeling is reinforced by the relative stability of inequality measure over time (Li *et al.*, 1998). However, as emphasized by Kanbur and Lustig (2000), if the combination of different policies is generally distribution-neutral, it may not be the case for each factor considered independently.

[4]In the present paper, the expression "statistical distributions" corresponds to parametric statistical distributions. It should not be confused with observed distributions.

[5]Among the most widely known attempts to propose an alternative functional form, we can cite Stacy (1962), Metcalf (1969), Thurow (1970) and Mount and Salem (1974). Kleiber and Kotz (2003) also provide a comprehensive survey.

against poverty-reduction strategies and in favor of growth intermediate objectives.

The paper is organized as follow. The next section introduces the methodology used for the estimation of the desired growth and inequality elasticities of poverty. The data and the raw results are presented in Section 3. Section 4 is concerned with the criterion used for the choice of an adequate functional form for income distributions and Section 5 deals with the drawbacks of the lognormal hypothesis. Section 6 concludes.

2. Methodology

2.1. *Calculation of the Elasticities of Poverty*

In the present paper, we focus on the absolute approach of poverty measurement. A common practice is to express poverty measures as nonlinear functions of the poverty line z, the mean income μ, a set of inequality parameters which fully describes the Lorenz curve L and some parameters which reflect ethical preferences.[6] For instance, the widely used (Foster *et al.*, 1984) class of poverty measures P_α is defined by:

$$P_\alpha = \int_0^z \left(\frac{z-y}{z}\right)^\alpha f(y)\,dy, \quad \alpha \geqslant 0, \tag{1}$$

where y corresponds to income, $f(.)$ is the income density function and α is a parameter of inequality aversion. For $\alpha = \{0,1,2\}$, P_α is respectively the headcount, the poverty gap and the squared poverty gap index. If we assume that incomes are distributed according to a known statistical distribution, a functional form can be attributed to $f(.)$ and the corresponding Lorenz curve can be fully described using some reduced set of inequality parameters. With the help of derivative tools, such an assumption allows a direct estimation of the required elasticities (Bourguignon, 2003). The approach is appealing since the elasticities can be estimated for each observed income distribution with few information requirements. Moreover, growth and redistribution effects are orthogonal, and, as shown further below, inequality elasticities of poverty can be directly compared in cross-section analysis.

In the present study, we choose to consider the following traditional statistical distributions: Pareto, lognormal, gamma, Weibull, Fisk, Singh–Maddala, Dagum and beta of the second kind.[7] Each one of these functional forms has been proposed for the estimation of income distributions and successfully tested using observed income distributions (see Kleiber and Kotz, 2003, for a quite

[6]See, for instance, the class of decomposable poverty measures defined by Kakwani (1980). For a comprehensive survey of absolute poverty measures, see Zheng (1997).

[7]We also performed estimations with the generalized gamma, the beta of the first kind and the generalized beta of the second kind. In each case, the estimators of the non-linear least-squares were not convergent because of the existence of multiple local optima. Hence, we dropped these distributions from the set of tested functional forms. For a closer look at the linkages between all these distributions, see McDonald (1984).

TABLE 1
"Classical" Distribution Functions

Name	Cumulative Distribution Function	Lorenz Curve	Scale Parameter
Two-parameter distributions			
Pareto	$F(y)=1-\left(\frac{y}{y_0}\right)^{-\gamma}$	$L(p)=-(1-p)^{1-\frac{1}{\alpha}}$	$y_0=\frac{\mu(\alpha-1)}{\alpha}$
Lognormal	$F(y)=\Phi\left(\frac{\log y-\bar{y}}{\sigma}\right)$	$L(p)=\Phi(\Phi^{-1}(p)-\sigma)$	$\bar{y}=\log\mu-\frac{\sigma^2}{2}$
Gamma	$F(y)=G(y,\pi,\theta)$	$L(p)=G(G^{-1}(p,c,\gamma),c,\gamma+1)$	$\rho=\frac{\mu}{\gamma}$
Weibull	$F(y)=1-e^{-\left(\frac{y}{\pi}\right)^{\beta}}$	$L(p)=G_G\left(W^{-1}(p,c,\beta),c,\beta,1+\frac{1}{\beta}\right)$	$\rho=\frac{\mu}{\Gamma\left(1+\frac{1}{\beta}\right)}$
Fisk	$F(y)=\left(1+\left(\frac{y}{\kappa}\right)^{-\tau}\right)^{-1}$	$L(p)=B_1\left(p,1+\frac{1}{\tau},\frac{\tau-1}{\tau}\right)$	$\kappa=\frac{\mu}{\Gamma\left(1+\frac{1}{\tau}\right)\Gamma\left(1-\frac{1}{\tau}\right)}$
Three-parameter distributions			
Singh–Maddala	$F(y)=1-\left(1+\left(\frac{y}{\kappa}\right)^{\tau}\right)^{-\lambda}$	$L(p)=B_1\left(1-(1-p)^{\frac{1}{\lambda}},1+\frac{1}{\tau},\lambda-\frac{1}{\tau}\right)$	$\kappa=\frac{\mu\Gamma(\lambda)}{\Gamma\left(1+\frac{1}{\tau}\right)\Gamma\left(\lambda-\frac{1}{\tau}\right)}$
Dagum	$F(y)=\left(1+\left(\frac{y}{\kappa}\right)^{-\tau}\right)^{-\theta}$	$L(p)=B_1\left(p^{\frac{1}{\theta}},\theta+\frac{1}{\tau},1-\frac{1}{\tau}\right)$	$\kappa=\frac{\mu\Gamma(\theta)}{\Gamma\left(\theta+\frac{1}{\tau}\right)\Gamma\left(1-\frac{1}{\tau}\right)}$
Beta 2	$F(y)=B_2(y,\kappa,\lambda,\theta)$	$L(p)=B_{G2}(B_2^{-1}(p,c,\lambda,\theta),c,1,\lambda+1,\theta-1)$	$\kappa=\frac{\mu\Gamma(\theta)\Gamma(\lambda)}{\Gamma(\theta+1)\Gamma(\lambda-1)}$

Notes: $\Phi(.)$ stands for the c.d.f. of the standard normal distribution, c for any constant term, $G(.)$ for the c.d.f. of the gamma distribution, $G_G(.)$ for the c.d.f. of the generalized gamma distribution, $W(.)$ for the c.d.f. of the Weibull distribution, $B_1(.)$ for the c.d.f. of the beta distribution of the first kind, $B_2(.)$ for the c.d.f. of the beta distribution of the second kind, $B_{G2}(.)$ for the c.d.f. of the generalized beta distribution of the second kind. More details on the last distributions can be found in Kleiber and Kotz (2003).

comprehensive survey). The corresponding cumulative distribution functions (c.d.f.) $F(.)$ are presented in Table 1.

The estimation of the parameters of these different functional forms depends to a great extent on data availability. In the context of cross-section or panel studies, it is currently very difficult to get a reasonable number of micro-data income series. Hence, the most common practice is to use the mean income and a set of inequality measures so as to define a system of m linearly independent equations for m parameters to be estimated (Bourguignon, 2003). For example, with two-parameter distributions like the Pareto, lognormal, gamma, Weibull and

Fisk distributions, the desired parameters can be easily found using the mean income and the Gini coefficient.[8]

However, the information requirements increase as the functional form becomes more flexible. Moreover, with three- (or more) parameter distributions, the system of equations is often intractable, yet difficult to solve.[9] For these reasons, it is generally more convenient to opt for an approach based on the estimation of the Lorenz curve corresponding to each distribution. Data availability is not a matter of concern since one can easily get some points of the Lorenz curve for a large number of countries. Moreover, as the number of points is usually greater than the number of parameters of the chosen functional form, standard errors can be estimated so as to compute confidence interval for the desired elasticities of poverty. Since the Lorenz curve does not depend on the mean income value, another useful feature is that the estimation of the inequality parameters can be separated from the estimation of the scale parameter. Finally, as criteria based on prediction errors are necessary to assess the performance of each functional form (see Section 4), there is a need for degrees of freedom.

The equations of the Lorenz curve corresponding to the aforementioned "classical" distributions are presented in Table 1. It can be seen that the estimation

[8]In the case of the lognormal distribution, the system to be solved is:

$$\begin{cases} \sigma = \sqrt{2}\Phi^{-1}\left(\dfrac{I+1}{2}\right), \\ \bar{y} = \log \mu - \dfrac{\sigma^2}{2}, \end{cases}$$

where I is the Gini coefficient and μ is mean income. It is worth stressing that simplicity is not the privilege of the lognormal distribution. For instance, using the Fisk distribution yields the following system:

$$\begin{cases} \tau = I^{-1}, \\ \kappa = \dfrac{\mu}{\Gamma\left(1+\dfrac{1}{\tau}\right)\Gamma\left(1-\dfrac{1}{\tau}\right)}. \end{cases}$$

[9]For instance, the parameters of the Singh–Maddala distribution can be estimated using the mean income μ, the Gini coefficient I and the Theil index T:

$$\begin{cases} I = 1 - \dfrac{\Gamma(\lambda)\Gamma\left(2\lambda-\dfrac{1}{\tau}\right)}{\Gamma\left(\lambda+\dfrac{1}{\tau}\right)\Gamma\left(1-\dfrac{1}{\tau}\right)} \\ T = \dfrac{1}{\tau}\left(\psi\left(1+\dfrac{1}{\tau}\right)-\psi\left(\lambda-\dfrac{1}{\tau}\right)\right)-\log\left(\lambda B\left(1+\dfrac{1}{\tau},\lambda-\dfrac{1}{\tau}\right)\right), \\ \kappa = \dfrac{\mu\Gamma(\lambda)}{\Gamma\left(1+\dfrac{1}{\tau}\right)\Gamma\left(\lambda-\dfrac{1}{\tau}\right)} \end{cases}$$

where $\psi(.)$ and $B(.)$ are the digamma and beta functions.

of the parameters of these different functional forms entails the use of non-linear estimators that may not be convergent contrary to traditional linear estimators.[10] Indeed, their use requires choosing an initial value for each parameter to be estimated. In the presence of multiple optima, these estimators converge to different values depending on the chosen initial values. Thus, it may be difficult to get the true value of the parameters.

Once the scale and inequality parameters have been estimated, the income and inequality elasticities can be easily computed. Here, we follow the methodology suggested by Kakwani (1993) which relies on the analytical computation of the first order derivative of the chosen poverty measure with respect to its determinants. For the family of measures P_α, the growth elasticity of poverty η_μ is then:

(2)
$$\eta_{0,\mu} = -\frac{zf(z)}{P_0} \quad \alpha = 0,$$

(3)
$$\eta_{\alpha,\mu} = -\frac{\alpha(P_{\alpha-1} - P_\alpha)}{P_\alpha} \quad \alpha \neq 0.$$

The estimation of the inequality elasticity of poverty is less straightforward. The issue is that income distributions can change in various ways, hence yielding different values of the inequality elasticity of poverty. A solution consists of imposing a strong distributional hypothesis on the observed distributions (Quah, 2001; Bourguignon, 2003). A strong distributional hypothesis is met when the two following conditions are fulfilled: (i) the observed income distribution can be described by the chosen statistical distribution; (ii) the income distribution changes in such a way that the final distribution can also be described using the same functional form. This means that the initial and final distributions can both be approximated using, for instance, a lognormal distribution, and thereafter that the value of the inequality elasticity of poverty is unique.[11]

However, the drawback of this hypothesis is that it yields elasticities that cannot easily be compared when using different statistical distributions, because the estimated elasticities simultaneously depend on the characteristics of the used functional form and on the chosen redistribution pattern. For instance, moving from a lognormal distribution to another one does not entail the same redistribution process as a move from a gamma distribution to another one. This means that the choice of a particular statistical distribution is generally not neutral from a redistribution point of view and, as a consequence, may bias our assessment of the

[10]A noticeable exception is the Lorenz curve corresponding to the Pareto distribution.

[11]For example, the "natural" Gini elasticity of the headcount index under a strong lognormality assumption is:

$$\eta^*_{0,I} = \lambda\left(\frac{\log\left(\frac{z}{\mu}\right)}{\sigma} + \frac{\sigma}{2}\right)\left(\frac{\sigma}{2} - \frac{\log\left(\frac{z}{\mu}\right)}{\sigma}\right)\frac{I}{\sigma\sqrt{2}\varphi\left(\frac{\sigma}{\sqrt{2}}\right)},$$

were λ and φ represent the hazard rate and density function of the standard normal distribution.

relative contribution of growth and inequality changes to poverty reduction when a strong distributional assumption is used. Moreover, practical issues occur with strong distributional elasticities when considering functional forms with three or more parameters since inequality elasticities of poverty are not unique any more.

In order to compare inequality elasticities of poverty based on different functional forms, it is thus necessary to weaken the second condition of the strong distributional hypothesis and to replace it by a common unique redistribution rule. For the sake of simplicity, we can make use of the rule suggested by Kakwani (1993), such that:

$$L^*(p) = L(p) - \varepsilon(p - L(p)), \tag{4}$$

where ε indicates a proportional change in the Gini coefficient.[12] As noted in Araar and Duclos (2006), this transformation means that the final income is equal to the initial income plus ε times the difference between the initial income and the mean income ($y^* = y + \varepsilon(y - \mu)$). Consequently, this transformation of the Lorenz curve entails a Lorenz dominance relationship between L^* and L. So, for negative (positive) value of ε, the situation of the poorest member of the population never worsens (improves). From equation (4), Kakwani (1993) derives the following Gini elasticities of poverty:

$$\eta_{0,G} = (\mu - z)\frac{f(z)}{P_0} \quad \forall \alpha = 0, \tag{5}$$

$$\eta_{\alpha,G} = \alpha + \frac{\alpha(\mu - z)}{z}\frac{P_{\alpha-1}}{P_\alpha} \quad \forall \alpha \neq 0. \tag{6}$$

The relative importance of growth and inequality to poverty alleviation can easily be calculated on the basis of Kakwani's (1993) formulas. For the measures P_α, these relative contributions are:

$$\frac{\eta_{0,\mu}}{\eta_{0,G}} = \frac{z}{z - \mu} \quad \alpha = 0, \tag{7}$$

$$\frac{\eta_{\alpha,\mu}}{\eta_{\alpha,G}} = \frac{z(P_{\alpha-1} - P_\alpha)}{z(P_{\alpha-1} - P_\alpha) - \mu P_{\alpha-1}} \quad \forall \alpha \neq 0. \tag{8}$$

The case of the headcount index is particular as the ratio of the growth elasticity to the Gini elasticity obtained through Kakwani's transformation does not depend on the income distribution. Thus, it will be the same, whatever assumption is made on the shape of the observed income distribution. As *per-capita* income is the only relevant characteristic of the income distribution, equation (7) implies that growth is a more efficient lever of poverty reduction than pure

[12]It can easily be shown that ε can also be interpreted as the same proportional increase of every member of the S-Gini family of inequality indices as well as Aaberge's (2000) inequality measures.

redistribution if the ratio of the mean income to the poverty line is low.[13] On the contrary, the pursuit of redistribution objectives is a more effective tool for rich countries when considering the headcount index.

For $\alpha \neq 0$, distribution matters. As noted by Kakwani and Son (2004), the ratio of the growth and inequality elasticities is negative in most cases, i.e. $z < \mu$.[14] It can be easily shown that its absolute value decreases with mean income. So a pure redistribution objective becomes more and more attractive as *per-capita* income increases.[15]

2.2. *Ad Hoc Functional Forms for the Lorenz Curve*

In addition to the aforementioned "classical" statistical distributions, we can also use *ad hoc* functional forms for the Lorenz curve. Characterizing a distribution through the direct estimation of the Lorenz curve was first used by Kakwani and Podder (1973) and has become a common practice. These functional forms are generally employed for descriptive purposes. Yet, Datt and Ravallion (1992) suggested that they could be used to analyze poverty variations and Datt (1998) defines a methodology based on their use to estimate growth and inequality elasticities of poverty. These Lorenz curves are deemed *ad hoc* since they are generally not theoretically grounded—the only exception being Maddala and Singh (1977). This label may nevertheless be contested since most "classical" statistical distributions do not stem from any theoretical model of income formation.[16]

Due to their flexibility, these functional forms generally fit pretty well the data and the estimation of their parameters is often easier than most traditional statistical distributions. Nevertheless, the use of *ad hoc* Lorenz curves raises some problems. First, estimated parameters are more likely than "classical" distributions to yield curves that do not comply with validity conditions of a Lorenz curve (i.e. $L(0) = 0$, $L(1) = 1$ and $\partial^2 L(p)/\partial p^2 \leq 0$). Second, the underlying c.d.f. may not be defined for the value of the poverty line.[17] Third, these c.d.f. have sometimes no closed form. To estimate the value of the poverty measures and their elasticities, it is then necessary to use some known properties of the Lorenz curve, that is:

$$\left.\frac{\partial L(p)}{\partial p}\right|_{p=P_0} = \frac{z}{\mu}, \tag{9}$$

[13]Of course, one should have in mind that growth always occurs with some transformation of the relative distribution of incomes.

[14]The Gini elasticity of poverty is always positive if the poverty line is less than the mean income. In the opposite case ($z > \mu$), the elasticity will be negative if and only if $P_\alpha < \frac{z-\mu}{z} P_{\alpha-1}$. In pratice, such a peculiar situation is unlikely to happen.

[15]It deserves to be stressed that these results may not hold with redistribution processes that differ from the one assumed in equation (4).

[16]However, some authors tried to find some *ex-post* justifications for the use of existing statistical distributions in the context of income distribution (e.g. Parker, 1999, for the type II generalized beta distribution). From a historical perspective, it is interesting to note that the Pareto distribution was derived from empirical regularities and not from a formal theory of individual income determination.

[17]This feature is also shared by the Pareto distribution.

$$\left.\frac{\partial^2 L(p)}{\partial p^2}\right|_{p=P_0} = \frac{1}{\mu f(z)}. \tag{10}$$

In the present paper, we include the functional forms described by Kakwani and Podder (1973), Maddala and Singh (1977), Gaffney *et al.* (1980), Kakwani (1980), Arnold and Villaseñor (1989), Fernandez *et al.* (1991) and Chotikapanich (1993).[18] Among these curves, the one proposed by Kakwani (1980), also known as the beta Lorenz curve, and the one suggested by Arnold and Villaseñor (1989), also called the elliptical or the general quadratic Lorenz curve, are the most widely used. They are notably employed in POVCAL (Chen *et al.*, 2001), the World Bank's tool designed for the estimation of poverty and inequality measures.[19] These *ad-hoc* Lorenz curves as well as the corresponding c.d.f. are presented in Table 2.

It is worth noting that we confine our investigations to the comparison of parametric estimations of income distributions. Non-parametric estimations, especially kernel estimations, have become increasingly popular in the empirical literature (see, for instance, Sala-i-Martin, 2006, in the context of the world distribution of income). However, Minoiu (2006) has recently shown that kernel estimations often yield substantial errors for the estimation of poverty measures when applied on few points of the Lorenz curve as in the case of the present study.

3. Data and General Results

Data related to the relative income distribution are from the UNU–WIDER World Income Inequality Database (WIID), version 2.0b (UNU–WIDER, 2005). This database has the advantage of containing the income share that accrues to the different quantile of the population for a very large number of countries and a relatively large period. The estimation of the scale parameter of the "classical" statistical distributions has been realized using U.S.$ GDP *per capita* in 1996 PPP-adjusted terms from the Penn World Tables, version 6.1 (Aten *et al.*, 2002). The use of these national account aggregates presents some noticeable drawbacks (Deaton, 2005), but household surveys' mean value of income are rarely reported in the WIID. As our goal is to highlight the differences in the elasticities of poverty that uniquely stem from functional form choices, we can consider that biases in poverty estimations that may occur when using national accounts aggegates, can be ignored for the present study.

[18]The functional forms suggested by Basmann *et al.* (1990)—this form includes Kakwani and Podder (1973) as a limiting case—and by Castillo *et al.* (1999)—Gaffney *et al.* (1980) and Fernandez *et al.* (1991) are special cases of this parametric form—have not been retained for the present study. Although the parameters of the former specification can be estimated using linear estimators, the validity of the estimated Lorenz curve cannot easily be assessed. In the case of the latter functional form, estimations could not be considered as reliable as estimations were highly unstable. Due to the scarcity of information for each observed income distribution, the parametric approaches proposed by Ryu and Slottje (1996) and Holm (1993) could not be used in the present study.

[19]For an empirical investigation of POVCAL's performance, see Minoiu and Reddy (2007).

TABLE 2

AD-HOC LORENZ CURVES

Name	Lorenz Curve	Cumulative Distribution Function
Two-parameter distributions		
Chotikapanich (1993)	$L(p)=\frac{e^{\omega p}-1}{e^{\omega}-1}$	$F(y)=\frac{1}{\omega}\log\left(\frac{y(e^{\omega}-1)}{\omega\mu}\right)$
Three-parameter distributions		
Kakwani and Podder (1973)	$L(p)=p^{\beta}e^{-\rho(1-p)}$	$\frac{y}{\mu}=(\rho F(y)+\beta)F(y)^{\beta}e^{-\rho(1-F(y))}$
Gaffney *et al.* (1980)	$L(p)=\left(1-(1-p)^{\varphi}\right)^{\frac{1}{\zeta}}$	$\frac{y}{\mu}=\frac{\varphi}{\zeta}\left(1(1-F(y))^{\varphi}\right)^{\frac{1}{\zeta}-1}(1-F(y))^{\varphi-1}$
Fernandez *et al.* (1991)	$L(p)=p^{\vartheta}(1-(1-p)^{\varphi})$	$\frac{y}{\mu}=\varphi F(y)^{\vartheta}((1-F(y))^{\varphi-1}+\vartheta F(y)^{\vartheta-1}\left(1-(1-F(y))^{\varphi}\right)$
Four-parameter distributions		
Maddala and Singh (1977)	$L(p)=-\delta\varphi p+(1-\delta+\delta\varphi)p^{x}+\delta(1-(1-p)^{\varphi})$	$\frac{y}{\mu}=-\delta\varphi+\chi(1-\delta+\delta\varphi)F(y)^{\chi-1}+\delta\varphi(1-F(y))^{\varphi-1}$
Kakwani (1980)	$L(p)=p-\xi p^{\nu}(1-p)^{\upsilon}$	$\frac{y}{\mu}=1-\xi F(y)^{\nu}(1-F(y))^{\upsilon}\left(\frac{\nu}{F(y)}-\frac{\upsilon}{1-F(y)}\right)$
Arnold and Villaseñor (1989)	$L(p)=\frac{f(p^{2}-L(p))+gL(p)(p-1)+q(p-L(p))}{1-L(p)}$	$F(y)=-\frac{1}{2m}\left(n+r\left(g+2\frac{y}{\mu}\right)\right)\left(\sqrt{\left(g+2\frac{y}{\mu}\right)^{2}-m}\right)^{-1}$
		$w=-f-g-q-1,\ m=g^{2}-4f$
		$n=2fs-4q,\ r=\sqrt{n^{2}-4ms^{2}}$

To assess the robustness of our findings, we consider two poverty lines, namely the traditional \$1 and \$2 per person per day poverty lines.[20] As these poverty lines are not meaningful in the context of high-income countries, the sample has been reduced so as to keep observations for which GDP *per capita* is less than \$10,000. After dropping the observations for which the quality and the reference population were not satisfactory (see Appendix A), we get a sample of 1,132 distributions for 115 countries from 1960 to 2003.[21] For each distribution we can make use of 6 to 13 points—10 points on average—of the Lorenz curve to estimate the parameters of the different functional forms.[22]

The distributional parameters of some functional forms presented in the previous section, namely those defined by Pareto (Kakwani and Podder, 1973; Kakwani, 1980) and Arnold and Villaseñor (1989), have been estimated using ordinary least squares estimators.[23] However, the remaining parametrizations of the Lorenz curve cannot be linearized. Hence, the estimation of their parameters entails the use of non-linear least squares estimations. As the gradient vector could not be obtained for some functional forms, Newton-like algorithms could not be employed. To circumvent this problem, we have turned to the derivative-free algorithm suggested by Nelder and Mead (1965), known as the *downhill simplex*.[24]

Before looking to the different elasticity estimates, it is necessary to check whether validity conditions are met for each functional form and each observed distribution. As emphasized in the previous section, this procedure is particularly important in the case of the *ad-hoc* functional forms that can yield inconsistent values of the income density if the Lorenz curve is not properly fitted. It can be seen from Table 3 in which the percentage of consistent estimations are reported, that this step should not be skipped. While the validity conditions are systematically met with the "classical" statistical distributions—not surprisingly, the Pareto distribution is the exception—results are often disappointing when considering *ad-hoc* functional forms. In particular, it is worth noting that the curves defined by Kakwani (1980) and Arnold and Villaseñor (1989) yield consistent estimations only for a half of the observations in our sample. We argue that this result may cast serious doubt on the reliability of these functional forms for the estimation of poverty and inequality indices in POVCAL. As shown by the substantial variability of percentages with the value of the poverty line, it seems that a significant number of inconsistent estimations are due to corresponding density functions that are not defined for the value of z.

[20]Strictly speaking, the exact values are \$1.08 and \$2.16 in 1996 PPP-adjusted terms. The poverty line defined for the Millennium Development Goals is fixed for 1993 PPPs, but Penn World Tables 6.1 are based on 1996 values.

[21]It can be seen from Table 12 that our sample includes countries that would be considered as high-income countries according to the current World Bank's classification. To check for the influence of these observations, all the estimations and tests have also been realized on a reduced sample that does not include these observations. Results are qualitatively similar to those reported in the present paper.

[22]The points (0,0) and (1,1) have been added since some *ad-hoc* functional forms like Arnold and Villaseñor (1989) do not automatically respect the conditions $L(0) = 0$ and $L(1) = 1$.

[23]All the estimations, tests and plots have been realized with R (R Development Core Team, 2007). The corresponding scripts are available upon request.

[24]To assess the robustness of our estimations, estimations have also been performed using the *simulated annealing* method. For a review of non-linear model estimations and algorithm choices, see Greene (2000, chapter 5).

TABLE 3

Percentage of Consistent Estimations for Each Functional Form

Distribution	$z = \$1$	$z = \$2$
Pareto	27	37
Lognormal	100	100
Gamma	100	100
Weibull	100	100
Fisk	100	100
Singh–Maddala	100	100
Dagum	100	100
Beta 2	100	100
Chotikapanich (1993)	63	67
Kakwani and Podder (1973)	42	40
Gaffney *et al.* (1980)	98	99
Fernandez *et al.* (1991)	98	99
Kakwani (1980)	50	53
Maddala and Singh (1977)	83	87
Arnold and Villaseñor (1989)	41	48

Since inconsistent estimations of the Lorenz curves produce erroneous values for the growth and inequality elasticities of poverty—a common and striking result is the presence of positive values for the growth elasticity of poverty—it makes sense to compare the results between the sole functional forms that yield consistent elasticities for the whole sample of income distributions. However, Table 3 shows that the functional forms proposed by Maddala and Singh (1977), Gaffney *et al.* (1980) and Fernandez *et al.* (1991) yield satisfactory estimations for a large number of the observations of the sample. So as to include these specifications in our comparison of estimated elasticities, we consider a reduced sample for our analysis in this section and temporally leave aside the functional forms corresponding to the Pareto distribution and Kakwani and Podder (1973), Kakwani (1980), Arnold and Villaseñor (1989) and Chotikapanich (1993) Lorenz curves. The sample respectively shrinks to 951 (84 percent of total sample) and 985 (87 percent of the whole sample) observations when considering the \$1 and \$2 poverty lines.[25] Mean values of the growth and inequality elasticities are reported in Tables 4 and 5 for the functional forms that produce valid estimations for each observation of this reduced sample.[26] It illustrates the dependence of estimated elasticities on functional form choices. Differences are non-negligible from an economic point of view, in particular for the Gini elasticities of poverty. The contrast is striking when comparing the mean values of elasticities obtained with the lognormal and Weibull distributions: using the \$1 poverty line, the elasticities corresponding to the former functional form are on average three times larger than those stemming from the latter statistical distribution. Such differences are

[25]Student's *t*-tests do not lead to the rejection of the hypothesis that the reduced sample exhibits the same average characteristics (GDP *per capita* and Gini coefficient) as the whole sample.

[26]Estimated values for the whole sample with the sole adequate statistical distributions are presented in Table 3.

TABLE 4
Mean Values of Growth and Gini Elasticities of P_0, P_1 and P_2: Restricted Sample, U.S.$1 Poverty Line

	Growth Elasticity			Gini Elasticity		
Distribution	P_0	P_1	P_2	P_0	P_1	P_2
Lognormal	−3.52	−3.83	−4.08	38.28	48.88	59.2
	(0.18)	(0.2)	(0.19)	(3.13)	(3.54)	(3.62)
Gamma	−1.48	−1.56	−1.6	13.5	22.3	31.21
	(0.09)‡	(0.1)‡	(0.11)‡	(1.16)‡	(1.27)‡	(1.45)‡
Weibull	−1.18	−1.23	−1.25	10.05	18.78	27.28
	(0.06)‡	(0.06)‡	(0.06)‡	(0.65)‡	(0.82)‡	(1.11)‡
Fisk	−2.11	−2.23	−2.29	18.83	27.49	36.25
	(0.05)‡	(0.05)‡	(0.05)‡	(0.89)‡	(1.08)‡	(1.41)‡
Beta 2	−3.36	−3.68	−3.9	32.77	42.37	52.03
	(0.32)	(0.33)	(0.34)	(4.11)	(4.31)	(4.63)*
Singh–Maddala	−1.92	−2.04	−2.11	16.32	25.11	33.77
	(0.06)‡	(0.07)‡	(0.07)‡	(0.76)‡	(1.01)‡	(1.24)‡
Dagum	−1.96	−2.11	−2.18	16.04	24.9	33.55
	(0.11)‡	(0.12)‡	(0.13)‡	(1.06)‡	(1.26)‡	(1.5)‡
Gaffney *et al.* (1980)	−2.13	−2.13	−2.21	17.12	25.48	34.17
	(0.07)‡	(0.08)‡	(0.09)‡	(0.86)‡	(1.05)‡	(1.33)‡
Fernandez *et al.* (1991)	−1.85	−1.97	−2.03	15.47	24.1	32.8
	(0.08)‡	(0.09)‡	(0.1)‡	(0.84)‡	(1.03)‡	(1.26)‡
Maddala and Singh (1977)	−2.62	−2.84	−3	22.72	32.22	41.45
	(0.35)†	(0.36)*	(0.42)*	(2.85)‡	(3.65)†	(3.72)†
Mixed (*ssr*)	−3.89	−6.09	−7.28	38.49	60.77	98.11
	(3.72)	(8.14)	(12.01)	(43.58)	(102.39)	(138.03)
Mixed (*sea*)	−3.86	−6.78	−6.58	42.9	57.69	90.94
	(4.05)	(8.63)	(15.86)	(51.05)	(102.6)	(147.8)
Mixed (*wssr*)	−4.13	−7.23	−7.24	41.02	68.95	86.12
	(3.48)	(7.92)	(13.47)	(44.44)	(104.9)	(127.12)

Notes: Bootstrapped standard errors in parentheses. The symbols *, † and ‡respectively indicate that the mean value is significantly different from the one obtained under lognormality at the 10, 5 and 1% levels. The sample represents 84% of the whole sample. Mean value of income *per capita* is \$3,370 and average Gini coefficient is 0.41. For a definition of the series mixed (*ssr*), mixed (*sea*) and mixed (*wssr*), see Section 5.

economically significant and the use of these rival values of elasticities may result in diverging estimations if employed so as to predict future poverty levels.[27]

However, these differences may be due to the presence of extreme values for some functional forms. This issue is caused by the heterogenous behaviors of the different functional forms at the left tail of the income distribution, and the highly skewed distribution of estimated elasticities in our sample.[28] A way of dealing with this issue is to compare median values. These are presented for our reduced sample in Tables 6 and 7. We can note that the magnitude of median values is slightly lower than that of mean values, especially concerning the Gini elasticities of poverty, hence confirming that the distributions of growth and inequality elasticities of poverty are skewed toward zero. The differences between each

[27]Incidentally, one can note that, on average, Gini elasticities of P_α measures exhibit larger values than growth elasticities. This result contrasts with the traditional view of mean income growth as the most efficient vector of poverty alleviation. However, caution is needed when interpreting these figures since this result is to a certain extent due to the assumption made in Section 2.1 about the redistributive process that corresponds to observed inequality variations.

[28]For instance, the skewed shape of the distribution of growth elasticities can be explained by the fact that they are theoretically defined on the interval [0,+∞).

TABLE 5

Mean Values of Growth and Gini Elasticities of P_0, P_1 and P_2: Restricted Sample, U.S.$2 Poverty Line

	Growth Elasticity			Gini Elasticity		
Distribution	P_0	P_1	P_2	P_0	P_1	P_2
Lognormal	−2.71	−3.05	−3.3	15.37	20.94	26.35
	(0.13)	(0.13)	(0.14)	(1.09)	(1.17)	(1.26)
Gamma	−1.36	−1.48	−1.54	6.66	11.24	15.83
	(0.09)‡	(0.1)‡	(0.11)‡	(0.53)‡	(0.59)‡	(0.7)‡
Weibull	−1.08	−1.17	−1.21	4.83	9.37	13.9
	(0.06)‡	(0.07)‡	(0.07)‡	(0.34)‡	(0.4)‡	(0.5)‡
Fisk	−1.88	−2.05	−2.14	8.97	13.57	18.12
	(0.05)‡	(0.04)‡	(0.04)‡	(0.38)‡	(0.51)‡	(0.63)‡
Singh–Maddala	−1.69	−1.86	−1.95	7.78	12.33	16.98
	(0.06)‡	(0.06)‡	(0.07)‡	(0.38)‡	(0.47)‡	(0.57)‡
Dagum	−1.66	−1.83	−1.94	7.62	12.3	16.75
	(0.08)‡	(0.09)‡	(0.1)‡	(0.47)‡	(0.57)‡	(0.69)‡
Beta 2	−2.71	−3.04	−3.28	14.81	20.12	25.37
	(0.22)	(0.26)	(0.25)	(1.74)	(1.85)	(2.06)
Gaffney *et al.* (1980)	−1.89	−1.91	−2.02	8.37	12.71	17.22
	(0.06)‡	(0.07)‡	(0.07)‡	(0.37)‡	(0.5)‡	(0.6)‡
Fernandez *et al.* (1991)	−1.62	−1.8	−1.89	7.45	12.16	16.53
	(0.07)‡	(0.08)‡	(0.09)‡	(0.42)‡	(0.5)‡	(0.61)‡
Maddala and Singh (1977)	−2.11	−2.4	−2.6	10.3	15.35	20.41
	(0.26)†	(0.3)*	(0.35)	(1.54)†	(1.78)†	(2.03*
Mixte (*ssr*)	−2.36	−3.14	−3.53	11.11	19.91	25.43
	(4.39)	(4.01)	(7.55)	(4.66)	(11.18)	(25.88)
Mixte (*sea*)	−2.24	−3.7	−4.04	10.58	22.09	26.56
	(2.37)	(5.23)	(9.07)	(7.55)	(11.79)	(16.77)
Mixte (*wssr*)	−2.35	−3.23	−3.79	12.23	20.46	27.52
	(0.46)	(0.63)	(0.97)	(2.74)	(3.47)	(6.85)

Notes: Bootstrapped standard errors in parentheses. The symbols *, † and ‡respectively indicate that the mean value is significantly different from the one obtained under lognormality at the 10, 5 and 1% levels. The sample represents 87% of the whole sample. Mean value of income *per capita* is $3,523 and average Gini coefficient is 0.42. For a definition of the series mixed (*ssr*), mixed (*sea*) and mixed (*wssr*), see Section 5.

functional form are less pronounced than those observed with the mean values, but are still significant from an economic point of view.

It is worth stressing that these economic significant differences are often associated with statistical significant differences. For instance, mean-difference and median-difference tests show that differences are often significant at the 1 percent level when estimated values are compared with those obtained with the lognormal distribution.[29,30] A noticeable exception is the beta distribution of the

[29]Since elasticities have been derived from estimated parameters obtained from few points of the Lorenz curve, it was necessary to take account of prediction errors at this level when computing standard errors of mean and median values. As a consequence, a two stage bootstrap procedure has been employed. In the first stage, individual elasticities have been estimated on many samples with replacement of the points of the Lorenz curve. Then, the required statistics have been computed on samples with replacement of the available distributions. For technical reasons—the procedure implies the use of numerous large matrices—the first stage has been limited to 200 replications while 1,000 replications have been used for the second stage. These choices conform with recommandations made in Efron and Tibshirani (1993).

[30]Significance symbols reported in Tables 5 to 7 refer to non-parametric mean-difference and median-difference tests. In order to save space, we do not report test results with respect to each functional form. However, the small size of estimated standard errors is a strong indications that observed differences are generally statistically different.

TABLE 6

Median Values of Growth and Gini Elasticities of P_0, P_1 and P_2: Restricted Sample, U.S.$1 Poverty Line

Distribution	Growth Elasticity			Gini Elasticity		
	P_0	P_1	P_2	P_0	P_1	P_2
Lognormal	–2.72	–3.03	–3.27	18.67	28.7	37.91
	(0.11)	(0.11)	(0.11)	(1.5)	(1.82)	(2.26)
Gamma	–1.24	–1.32	–1.39	7.67	16.22	24.09
	(0.08)‡	(0.09)‡	(0.09)‡	(0.67)‡	(0.97)‡	(1.24)‡
Weibull	–1.13	–1.17	–1.2	7.29	15.26	22.92
	(0.06)‡	(0.06)‡	(0.06)‡	(0.55)‡	(0.82)‡	(1.16)‡
Fisk	–2.17	–2.22	–2.26	14.58	22.75	30.1
	(0.04)‡	(0.04)‡	(0.04)‡	(0.85)‡	(1.19)‡	(1.54)‡
Beta 2	–2.49	–2.7	–2.85	18.33	28.15	37.1
	(0.13)	(0.14)	(0.15)†	(1.21)	(1.57)	(1.83)
Singh–Maddala	–1.85	–1.93	–1.98	13.13	21.18	28.75
	(0.07)‡	(0.07)‡	(0.07)‡	(0.82)‡	(1.08)‡	(1.39)‡
Dagum	–1.68	–1.75	–1.82	12.07	20.51	28.82
	(0.1)‡	(0.11)‡	(0.11)‡	(0.89)‡	(1.2)‡	(1.5)‡
Gaffney *et al.* (1980)	–1.98	–1.97	–2.02	13.89	21.65	29.29
	(0.08)‡	(0.08)‡	(0.09)‡	(0.8)‡	(1.12)‡	(1.4)‡
Fernandez *et al.* (1991)	–1.72	–1.79	–1.84	12.23	20.13	28.15
	(0.09)‡	(0.09)‡	(0.1)‡	(0.87)‡	(1.16)‡	(1.38)‡
Maddala and Singh (1977)	–2.15	–2.32	–2.45	15.22	24.31	32.81
	(0.2)*	(0.22)†	(0.25)†	(1.64)*	(2.37)*	(2.02)*
Mixed (*ssr*)	–1.63	–2.3	–2.38	9.63	24.94	32.81
	(0.24)†	(0.23)*	(0.25)*	(2.44)†	(2.19)	(2.7)
Mixed (*sea*)	–1.7	–2.3	–2.38	10.97	25.03	32.93
	(0.23)†	(0.21)*	(0.25)*	(2.41)†	(2.21)	(2.66)
Mixed (*wssr*)	–1.94	–2.45	–2.54	13.58	26.12	34.28
	(0.24)†	(0.25)	(0.26)*	(2.51)*	(2.38)	(2.96)

Notes: Bootstrapped standard errors in parentheses. The symbols *, † and ‡respectively indicate that the median value is significantly different from the one obtained under lognormality at the 10, 5 and 1% levels. The sample represents 84% of the whole sample. Mean value of income *per capita* is $3,370 and average Gini coefficient is 0.41. For a definition of the series mixed (*ssr*), mixed (*sea*) and mixed (*wssr*), see Section 5.

second kind that yields estimations that are not significantly different from the corresponding lognormal elasticities of poverty.

Highlighting these significant differences between rival distributive functional forms is important since it points out the potential fragility of results based on distributive assumptions that may not be adequate to describe observed income distributions. In Tables 5 to 7, it is interesting to note that the lognormal distribution always provides the largest absolute mean and median values of both growth and Gini elasticities. On the contrary, lowest absolute values are obtained with the Weibull distribution. Thus, it seems that using the latter statistical distribution implies that more ambitious growth and redistribution objectives have to be reached than with the former so as to achieve a given poverty reduction target. Moreover, our results confirm that functional forms also influence how we appreciate the relative contribution of growth and redistribution to poverty alleviation. For instance, considering the mean values reported in Table 5 related to the measure P_2 with the $1 poverty line, a 1 percent decrease of the Gini coefficient (without growth) is on average 20 times more efficient that a 1 percent increase of mean income

TABLE 7
MEDIAN VALUES OF GROWTH AND GINI ELASTICITIES OF P_0, P_1 AND P_2: RESTRICTED SAMPLE, U.S.$2 POVERTY LINE

	Growth Elasticity			Gini Elasticity		
Distribution	P_0	P_1	P_2	P_0	P_1	P_2
Lognormal	–1.96	–2.31	–2.53	6.17	11.38	16.33
	(0.08)	(0.08)	(0.09)	(0.46)	(0.71)	(0.88)
Gamma	–1.09	–1.18	–1.25	3.35	7.75	11.98
	(0.08)‡	(0.08)‡	(0.09)‡	(0.32)‡	(0.44)‡	(0.62)‡
Weibull	–1.02	–1.09	–1.14	3.13	7.45	11.62
	(0.06)‡	(0.06)‡	(0.06)‡	(0.24)‡	(0.4)‡	(0.58)‡
Fisk	–1.94	–2.04	–2.1	6.23	10.67	14.83
	(0.04)	(0.04)‡	(0.04)‡	(0.4)	(0.57)†	(0.76)‡
Beta 2	–1.98	–2.22	–2.38	6.59	11.63	16.46
	(0.1)	(0.12)	(0.13)	(0.55)	(0.82)	(0.97)
Singh–Maddala	–1.67	–1.76	–1.83	5.55	9.82	14.01
	(0.07)‡	(0.07)‡	(0.08)‡	(0.43)†	(0.52)‡	(0.69)‡
Dagum	–1.52	–1.6	–1.67	5.08	9.55	13.95
	(0.09)‡	(0.09)‡	(0.11)‡	(0.4)‡	(0.51)‡	(0.74)‡
Gaffney *et al.* (1980)	–1.85	–1.78	–1.84	6.22	10.14	14.31
	(0.07)	(0.08)‡	(0.08)‡	(0.41)	(0.56)‡	(0.72)‡
Fernandez *et al.* (1991)	–1.55	–1.64	–1.7	5.21	9.6	13.64
	(0.08)‡	(0.09)‡	(0.1)‡	(0.42)‡	(0.54)‡	(0.69)‡
Maddala and Singh (1977)	–1.78	–1.96	–2.12	5.86	10.7	15.12
	(0.13)*	(0.15)*	(0.22)*	(0.49)	(0.75)	(1.02)*
Mixed (*ssr*)	–1.28	–1.9	–1.99	3.92	10.4	14.37
	(0.16)†	(0.15)*	(0.18)*	(0.44)‡	(0.65)	(0.92)*
Mixed (*sea*)	–1.32	–1.9	–1.99	4.04	10.27	14.4
	(0.17)‡	(0.14)*	(0.18)*	(0.45)‡	(0.66)	(1)†
Mixed (*wssr*)	–1.41	–1.98	–2.09	4.15	10.6	14.48
	(0.16)†	(0.14)	(0.17)	(0.45)†	(0.72)	(1.03)

Note: Bootstrapped standard errors in parentheses. The symbols *, † and ‡respectively indicate that the median value is significantly different from the one obtained under lognormality at the 10, 5 and 1% levels. The sample represents 87% of the whole sample. Mean value of income *per capita* is $3,523 and average Gini coefficient is 0.42. For a definition of the series mixed (*ssr*), mixed (*sea*) and mixed (*wssr*), see Section 5.

(without inequality change) using the Weibull distribution. On the contrary, the ratio is less than 15 to 1 under lognormality of the income distributions. As a consequence, the adoption of a specific functional form is also determinant for the appreciation of the pattern of growth with redistribution that could be considered as optimal in terms of poverty alleviation. In our case, elasticities based on the Weibull distribution would thus suggest to define more redistribution-oriented policies of poverty reduction than those based on lognormality.

4. WHICH FUNCTIONAL FORM IS THE "RIGHT" ONE?

In the previous section, we emphasized the crucial role of functional form choices, but this result is not sufficient until we can tell which distributive assumption provides the best approximations possible. For this purpose, we need to use some criterion to choose between the different functional forms the one that is the most appropriate for poverty analysis. In the next paragraphs, we will first perform a short review of goodness-of-fit criteria before investigating which of the aforementioned functional forms is the most adequate.

4.1. *Goodness-of-Fit Criteria*

Our analysis is based on the fundamental but reasonable assumption that the better is the approximation of the Lorenz curve, the closer are estimated elasticities to their real values. The direct implication is that the functional form that is supposed to yield the most satisfactory estimations, is the one that minimizes some function of Lorenz curve prediction errors. In this spirit, the traditional approach is to rely on the following standard statistics of goodness-of-fit:

$$ssr = \sum_{i=1}^{N} \left(L(p_i) - \hat{L}(p_i) \right)^2, \tag{11}$$

$$sae = \sum_{i=1}^{N} \left| L(p_i) - \hat{L}(p_i) \right|, \tag{12}$$

with $\hat{L}(p_i)$ denoting the estimated value of the Lorenz curve at p_i, and N being the number of available points. The main problem with these indices is that all prediction errors are considered and are given the same weight. In our particular case, this feature can be seen as conflicting with the most central axiom in poverty measurement, namely the focus axiom (Sen, 1976), which stipulates that income above the poverty line should not be considered. In other words, it does not matter if prediction errors occur for the non-poor quantiles of the population. The solution suggested by Datt (1998) is to modify the *ssr* criterion in the following manner:

$$pssr = \sum_{i=1}^{n} \left(L(p_i) - \hat{L}(p_i) \right)^2, \tag{13}$$

with n corresponding to the first population quantile such that $p_n \geqslant \hat{P}_0$. So, it consists of restricting the *ssr* criterion to prediction errors over the part of the Lorenz curve that correspond to poor individuals.

Nevertheless, Datt's suggestion is not appropriate here for both practical and theoretical reasons. First, this figure requires a sufficient number of Lorenz curve points so as to be computable. For the vast majority of the income distributions of our sample, data availability is limited to the share of total income by population deciles. If the estimated value of the headcount index is less than 10 percent for the chosen poverty line, the value of *pssr* is then zero. So it cannot be used to assess the adequacy of the different functional forms. Yet, the frequency of this result over the entire sample is about two thirds when considering the \$1 poverty line.[31] Second, in the case of the headcount index, we should not care about the quality of the fit for the points of the Lorenz curve below $\hat{P}_0$ since the headcount index is related to the Lorenz curve only at the corresponding quantile of the population (*cf.* relation (9) above).

[31]This frequency varies only marginally regardless of the statistical distribution used.

So as to comply with these conflicting requirements, we propose a measure based on squared errors for which the weights decrease with the distance from the estimated value of the headcount index. To make comparisons possible between each functional form, the measure is normalized by the sum of weights. This normalization is also helpful since comparisons with the traditional *ssr* figure can be achieved. The suggested criterion is then:

$$(14) \qquad wssr = \frac{\sum_{i=1}^{N}\left(L(p_i)-\hat{L}(p_i)\right)^2\left(1-\left|p_i-\hat{P}_0\right|\right)^2}{\sum_{i=1}^{N}\left(1-\left|p_i-\hat{P}_0\right|\right)^2}.$$

A drawback of these different measures is that they focus on the sole precision aspect of the estimation. However, the use of more flexible functional forms may lead to an improvement of the fit that is not sufficient with respect to the loss of degrees of freedom, hence to the increase of standard errors. In order to compare non-nested models while penalizing for the addition of new parameters, we can use the Akaike and Schwartz information criteria (cf. Gujarati, 2004). These measures are respectively:

$$(15) \qquad aic = e^{2K/N}\,\frac{\sum_{i=1}^{N}\left(L(p_i)-\hat{L}(p_i)\right)^2}{N}$$

$$(16) \qquad bic = N^{K/N}\,\frac{\sum_{i=1}^{N}\left(L(p_i)-\hat{L}(p_i)\right)^2}{N}$$

where K is the number of estimated parameters.

4.2. *Comparison of Functional Forms Performances*

The results of the application of the *ssr*, *sae*, *wssr*, *aic* and *bic* criteria on our restricted sample with the \$1 poverty line are summarized in Table 8.[32] The top panel presents mean values of the different criteria for each functional form. To ease the reading and the comparison of the results, the value of each figure has been normalized by the smallest observed mean value over the sample. Thus, a value of one is attributed to the best performing functional and larger values indicate in which proportion the considered functional form yields less precise estimations according to the chosen criterion.

Our results show that the curve defined by Maddala and Singh (1977) is the most efficient form.[33] More generally, one can observe non-negligible gaps between

[32]The results for the sample corresponding to the \$2 poverty line are reported in Table 4 in the Appendix. The results are very close to those observed with the \$1 poverty line sample.

[33]Results corresponding to the whole sample are not reported here so as to save space. However, it is interesting to draw some comments on the results obtained when validity conditions are not taken into account. First, while the Maddala and Singh (1977) functional form is still the best performing one according to the *ssr* criterion, other criteria entail a marked preference for the curve suggested by Kakwani (1980). Second, the parametrizations proposed by Kakwani and Podder (1976) and Arnold and Villaseñor (1989) may sometimes yield consequent errors. This result is particularly surprising for the latter one since it is very flexible and its parameters can be obtained using ordinary least squares estimators. Our experience is that this phenomenon may be due to multicollinearity issues between the different regressors.

TABLE 8

Goodness-of-Fit, Mean and Median Values: Restricted Sample, U.S.$1 Poverty Line

Distribution	*ssr*	*sae*	*wssr*	*aic*	*bic*
Mean value					
Lognormal	9.69	3.39	10.58	6.47	6.26
Gamma	44.78	7.89	45.84	28.46	27.73
Weibull	45.45	8.57	48.59	29.21	28.42
Fisk	7.24	2.95	7.05	4.85	4.69
Singh–Maddala	1.41	1.30	1.58	1.17	1.15
Dagum	1.67	1.40	1.92	1.35	1.33
Beta 2	1.92	1.41	2.18	1.64	1.61
Gaffney *et al.* (1980)	1.40	1.27	1.55	1.14	1.12
Fernandez *et al.* (1991)	1.59	1.43	1.84	1.27	1.26
Maddala and Singh (1977)	1.00	1.00	1.00	1.00	1.00
Median value					
Lognormal	5.01	2.56	5.35	4.43	3.92
Gamma	34.54	6.86	34.73	27.03	25.15
Weibull	42.72	8.02	48.89	34.87	31.84
Fisk	4.18	2.21	4.31	3.19	2.95
Singh–Maddala	1.33	1.21	1.62	1.13	1.09
Dagum	1.40	1.26	1.71	1.21	1.17
Beta 2	1.23	1.17	1.25	1.05	1.02
Gaffney *et al.* (1980)	1.26	1.16	1.44	1.08	1.05
Fernandez *et al.* (1991)	1.47	1.32	1.84	1.26	1.21
Maddala and Singh (1977)	1.00	1.00	1.00	1.00	1.00

Notes: The sample represents 84% of the whole sample. Mean value of income *per capita* is $3,370 and average Gini coefficient is 0.41.

two-parameters on one hand and three- and four-parameter distributions on the other hand. It is interesting to note that all these results seem to be robust since rankings of the different functional forms are very stable whatever criterion is chosen.[34] The use of the *aic* and *bic* criteria confirms the superiority of three-parameter over two-parameter forms and of four-parameter over three-parameter forms, though the gain is reduced in this latter case. The gap is dramatically important when comparing the gamma and Weibull distributions with more flexible functional forms (e.g. errors are more than 45 times larger than with Maddala and Singh (1977) regarding the *wssr* criteria). This observation is not surprising since only one parameter is dedicated to the description of the shape of the relative distributions with two-parameter statistical distribution. This is clearly not sufficient considering the heterogeneity of observed income distributions. However, differences are also significant among two-parameter functional forms. Estimations are on average more precise with the lognormal and Fisk distributions than with the gamma and Weibull distributions. From a poverty point of view, this result deserves to be underlined since, for a given degree of inequality, the share of total income that is allocated to the bottom quantile of the population is smaller with the latter distributions than with the former.

The robustness of these results can be checked using the bottom part of Table 8 that reports the ratios of the median values for each criterion. Although

[34]More precisely, the ranking is exactly the same with respect to criteria based on the sum of squared errors and changes, but only marginally, with the *sae* criteria.

differences are less pronounced than with mean values, the main observations presented in the last paragraph still hold. However, it can be noted that the beta distribution of the second kind now perform slightly better than most three-parameter functional forms.

These results seem to suggest a relative stability of the ranking of the different functional forms tested in this study. However, we ought to be cautious before claiming that a particular functional form should unilaterally be preferred to the other ones. Yet, the average superiority of a statistical distribution does not mean that other functional forms systematically fit the data worse, and thus should be discarded. The Pareto distribution can be used on a very limited portion of our sample and generally yields poor description of observed income distributions, but it may perform very well in a few cases. In order to get a better assessment of the relative performance of each functional form—including those that were temporarily excluded from the analysis—the ranking of each one has been established for each observed income distribution of the complete sample using the different goodness-of-fit criteria. The frequency of first ranking as well as the value of the median rank are presented in Table 9.[35]

The frequencies reported in the first part of Table 9 confirm the previous results regarding the contrasting performances of the different functional forms. They also show that, except in some particular cases, each one of the tested parametrizations can be considered as the most relevant choice for the description of at least one income distribution. Nonetheless, two functional forms outperform others with respect to the *ssr*, *sae* and *wssr* criteria, namely the curves suggested by Maddala and Singh (1977) and Kakwani (1980). Indeed, these two parametrizations are the most appropriate for more than half (up to two thirds) of the observed income distributions. Their preponderance deserves all the more to be stressed since their use is constrained by the respect of validity conditions. For instance, Kakwani (1980) curve is the most performing one for about 30 percent of the observations, but yields consistent estimations with only half of the observed distributions of the sample. Nevertheless, the *aic* and *bic* criteria show that the precision gain does not fully compensate for the loss of degrees of freedom when using four-parameter functional forms. If one is concerned with the goodness-of-fit/precision tradeoff, these criteria indicate that the beta distribution of the second kind is a good candidate.[36] These results are confirmed by the estimations of the median values of the ranks presented in the bottom part of Table 9.

On the other side of the performance spectrum, one can note that the Pareto, gamma, Weibull, Fisk (Chotikapanich, 1993) and Kakwani and Podder (1973) distributions represent only a small part of the distributions of the sample regardless of the selection criteria. This result is relatively surprising for Kakwani and Podder's (1973) curve since it is more flexible than the other above-mentioned distributions. Table 9 indicates that the lognormal distribution performs significantly better than the other two-parameter distributions. However, the frequency of first ranking is only 3 percent according to the *wssr* criterion and the median rank is high. So lognormality of income distributions is clearly not the rule, but it

[35]Of course, the rankings have been realized among the sole consistent estimations.

[36]One needs to be cautious with this statement since the performances of the beta distribution of the second kind are very sensitive to the choice of poverty line when using the *wssr* criterion.

TABLE 9

Goodness-of-Fit: Frequency of First Ranking and Median Rank

Distribution	*ssr*	*sae*	$wssr_{\$1}$	$wssr_{\$2}$	*aic*	*bic*
First rank frequency (%)						
Pareto	0.0	0.0	0.0	0.1	0.1	0.1
Lognormal	1.1	1.9	3.3	2.5	5.8	7.2
Gamma	0.4	0.1	0.5	0.3	2.0	2.1
Weibull	0.0	0.1	0.2	0.2	0.5	0.7
Fisk	0.0	0.7	0.0	0.2	3.4	4.0
Singh–Maddala	3.2	3.8	5.1	4.9	6.9	6.7
Dagum	2.6	3.1	4.2	3.4	4.3	4.8
Beta 2	7.8	8.7	18.1	12.7	22.1	23.1
Chotikapanich (1993)	0.6	0.6	0.3	0.4	0.7	0.7
Kakwani and Podder (1973)	0.0	0.0	0.8	0.7	0.0	0.0
Gaffney *et al.* (1980)	4.4	5.7	5.4	5.1	9.2	9.1
Fernandez *et al.* (1991)	4.0	3.1	4.0	3.4	9.0	9.3
Maddala and Singh (1977)	35.6	32.8	31.2	30.5	14.9	13.6
Kakwani (1980)	31.2	29.6	18.1	25.4	12.9	11.0
Arnold and Villaseñor (1989)	9.3	9.9	8.8	10.3	8.0	7.7
Median rank						
Pareto	12	12	12	12	12	12
Lognormal	9	9	8	9	9	9
Gamma	10	10	10	11	10	10
Weibull	10	10	10	11	10	10
Fisk	8	8	8	8	8	8
Singh–Maddala	5	5	5	5	4	4
Dagum	5	5	5	6	5	5
Beta 2	4	4	3	4	3	3
Chotikapanich (1993)	12	12	11	12	12	12
Kakwani and Podder (1973)	13	13	10	12	13	13
Gaffney *et al.* (1980)	4	4	4	4	4	4
Fernandez *et al.* (1991)	6	6	6	6	6	6
Maddala and Singh (1977)	2	2	2	2	4	4
Kakwani (1980)	1	1	2	2	3	3
Arnold and Villaseñor (1989)	3	3	3	3	4	4

Note: $wssr_{\$1}$ and $wssr_{\$2}$ respectively correspond to the \$1 and \$2 poverty lines.

can be the exception.[37] However, since this functional form yields growth and inequality elasticities of poverty that, on average, differ significantly from the one obtained with most three- and four-parameter parametrizations from both economic and statistical points of view, it is legitimate to question the relevance of this assumption for the estimation of the growth and inequality elasticities of poverty.

Before investigating the lognormal case, it is interesting to note that our results only partially support those of prior studies that compared the performance of some of the functional forms used in this paper. First, it is important to note that such studies are rather scarce for large samples of income distributions. To our knowledge, our study is the most comprehensive one that has been realized considering both the number of observed income distributions and the number of

[37]It is interesting to note that the lognormal is the most adequate for a very heterogenous set of countries. For instance, in our sample the exceptions are Belarus (1998), Central African Republic (1992), El Salvador (1977), Gambia (1992), Guatemala (1987), Honduras (1987, 1990, 1998), Israel (1992), Latvia (1996), Mexico (1984), Moldova (1997), Nigeria (1980), Romania (1994), South Africa (1993), Thailand (1986, 1990) and Tunisia (1965) using the *ssr* criterion.

functional forms. Using 82 observed distribution at various years for 23 developed and middle-income countries, Bandourian *et al.* (2002) observed that the Weibull and Dagum were the best-fitting models for the two- and three-parameter distribution family, when opposed to the gamma, lognormal, generalized gamma, beta 1, beta 2 and Singh–Maddala distributions. On the contrary, our results suggest that the Weibull distribution is a poor choice and that more precise descriptions can be obtained using the Fisk and lognormal distributions.[38] In the same spirit, the Dagum distribution is outperformed by the beta distribution of the second kind as well as the Gaffney *et al.* (1980) curve, although its use may be cumbersome since its c.d.f. has no closed form.

Concerning *ad-hoc* Lorenz curves, comparisons are as rare as with "classical" statistical distributions and samples are generally small. For instance, Schader and Schmid (1994) use 16 series of grouped data for the former Federal Republic of Germany between 1950 and 1988. Their results show that the curves suggested by Kakwani and Podder (1976) and Kakwani (1980) outperformed the lognormal, Singh–Maddala and Dagum distributions as well as the curves defined by Kakwani and Podder (1973), Gaffney *et al.* (1980), Gupta (1984), Arnold and Villaseñor (1989), Basmann *et al.* (1990) and Fernandez *et al.* (1991). Cheong (2002) realizes a similar study using American data for the period 1977–83. He concludes that the Gaffney *et al.* (1980) and Kakwani (1980) curves should be prefered to the one proposed by Kakwani and Podder (1976), Fernandez *et al.* (1991) and Chotikapanich (1993). In the present study, our results tend to favor the parametrizations proposed by Maddala and Singh (1977) and Kakwani (1980) among *ad-hoc* functional forms. However, since the latter yields numerous inconsistent estimations, it may be wise to turn to the former.

The main conclusion that can be drawn from the results presented in this section as well as from the comparison with previous studies is that it is undoubtedly difficult to rank functional forms with respect to their performance, even if greater flexibility is generally associated with better fit. This prudence is reinforced by the time-instability of the performance of the different functional forms. In the previous paragraphs, we have noted that no functional form seems well-suited to deal with the heterogeneity of the income distributions included in our sample. The next step is to ask whether this variability of performance is due to differences between intrinsic characteristics of the countries in the sample. In other words, if a functional form is relevant to describe the income distribution in a given country, should we be confident about its use for the estimation of the distribution at other points of time?[39] To answer this question, we have decomposed the variance of each goodness-of-fit criterion in their within- and between-country components for each functional form. The share of the between component of the total variance is reported in Table 10 for the restricted sample corresponding to the $1 poverty line. The results of this decomposition are quite surprising since they show that the most important part of the performance instability of the functional forms is due to within-country changes in the income distributions. Differences can be

[38]It is important to have in mind that the two samples do not overlap since our sample is mainly composed of low-income countries. Another issue is that Bandourian *et al.* (2002) assess the quality using prediction errors of the density function while we focus on predictions errors of the Lorenz curve.

[39]We would like to thank an anonymous referee for suggesting an investigation of this question.

TABLE 10

Between-Country Share of Total Variance of the Different Goodness-of-Fit Criteria: Restricted Sample, \$1 Poverty Line

Distribution	*ssr*	*sea*	*wssr*	*aic*	*bic*
Lognormal	23.0	28.4	17.2	19.6	20.0
Gamma	26.7	33.2	22.3	25.2	25.5
Weibull	27.6	32.8	23.8	26.0	26.3
Fisk	27.8	30.0	33.0	27.3	27.5
Beta 2	19.9	24.7	16.4	15.5	16.0
Singh–Maddala	20.2	25.6	19.7	16.7	17.2
Dagum	22.1	25.3	18.6	19.2	19.7
Gaffney *et al.* (1980)	22.1	24.5	18.4	19.2	19.7
Fernandez *et al.* (1991)	22.9	27.1	19.4	19.3	19.8
Maddala and Singh (1977)	19.0	27.3	18.0	14.8	15.3

noted between the various functional forms and criteria, but at least two thirds of the observed heterogeneity cannot be explained by cross-country fixed effects.

Thus, the promotion of any particular functional form for the description of observed income distributions is clearly tricky and a pragmatic position is undoubtedly preferable.[40] Indeed, we argue that it may be suitable not to impose a common functional form for every observed income distribution, but to choose the most adequate for each one of these distributions. This solution is adopted in the next section to investigate the drawbacks of a blind use of the lognormal distribution to compute growth and inequality elasticities of poverty.

5. The Lognormal Case

In a recent paper, Lopez and Servèn (2006), using the Dollar and Kraay (2002) database, have concluded that the lognormality assumption cannot be rejected for the estimation of income distributions in cross-section analysis.[41] Our objective in this section is to investigate whether the lognormality assumption yields acceptable estimations of the growth and inequality elasticities of poverty defined in Section 2.1. Indeed, it does not matter that more flexible functional

[40]Here, it may be interesting to draw a parallel with Thurow's (1970) famous assertion that "God is not a beta distribution." In other words, statistical distributions are not used to explain the shape of observed income distribution but just to describe them. Consequently, only practical reasons may justify the use of the same functional form to estimate a heterogenous set of income distributions.

[41]The assumption is tested by regressing the observed income share of a given quintile of population by a constant and the income share of the corresponding theoretical quantile under lognormality. Lopez and Servèn (2006) conclude that lognormality cannot be rejected since the two coefficients are jointly not different from the couple (0,1). The empirical illustration in Bourguignon (2003) can also be seen as a lognormality test. The author test consists of regressing the observed relative variation of the headcount index by the sum of the products of the rate of growth of its determinants, namely the mean income and the Gini coefficient, by the corresponding elasticities under strong lognormality assumption. In this case, lognormality is not rejected since both coefficients are not significantly different from unity.

However, these tests only show that lognormality is *on average* a valid assumption. The limit is that the distributions of the sample may on average exhibit characteristics of lognormality without any of them being properly described by the lognormal distribution. This may explain why our results are consistent with the findings of the aforementioned studies, while apparently at odds with them.

forms fit better observed income distributions if the corresponding estimated elasticities do not significantly differ from those obtained under lognormality.

The comparison of mean and median values realized in Section 3 suggests that the use of lognormality may lead to an overestimation of the growth and inequality elasticities. However, this finding is questionable since we have shown in the preceding section that no single functional form should be employed to describe such a heterogenous sample of income distributions as ours. Thus, from our point of view, a more adequate approach consists of creating mixed series that include, for each observed income distribution, the value of the elasticity that corresponds to the best performing functional form according to the chosen goodness-of-fit criterion.[42] Here, these series are only based on the *ssr*, *sae* and *wssr* criteria, the latter being our benchmark for reasons explained earlier. The composition of these series is indicated by the first three columns of the top panel of Table 9. So, they are mainly constituted by values obtained through the beta distribution of the second kind and the curves proposed by Maddala and Singh (1977), Kakwani (1980) and Arnold and Villaseñor (1989).

The mean and median values of these mixed series are reported at the bottom of Tables 4 to 7 (see also Table 13 for the whole sample). The comparison of mean values suggests that the lognormal assumption generally entails an underestimation of the magnitude of the elasticities whatever poverty line and poverty measure are chosen. However, differences are not significant at the 10 percent confidence level when considering mean values, but the results are driven by the presence of few very large values in the mixed series. On the contrary, the converse phenomenon can be observed with respect to median differences. Here, we observe that lognormality tends to overestimate what we suppose to be the real magnitude of the growth and inequality elasticities of poverty. From a statistical point of view, median differences are most of the time significantly different from zero for the growth elasticities of poverty, but the picture is more uncertain with the Gini elasticities of poverty: while the mixed series inequality elasticities of the headcount index are significantly lower than the one computed under lognormality, the statistical significance vanishes once the poverty gap and the squared poverty gap are used. Nevertheless, the difference exceeds 2 points whatever measure and poverty line is considered.

Of course, misestimation is doubtless a matter of concern, but we can imagine that this bias may easily be corrected if lognormal elasticities are highly correlated with their real values. To test this assumption, we have plotted in Figures 1 and 2 the couple of values stemming from the lognormal and mixed-*wssr* elasticity series for each observed income distribution from the sample. The plots help to appreciate how weak are the links between the two series, in particular for the growth elasticities. On each subfigure, the correlation coefficients have been reported. They clearly show that the correlation between lognormal and the mixed series based on the *wssr* criterion is rather low, ranging from 0.3 to 0.6.[43] Interestingly, one can observe that correlations are weaker for the growth elasticities of poverty, but that they increase significantly with the poverty line (without being large

[42]As a robustness test, we have also constructed series corresponding to second-rank estimations according to the different criteria. Results for these second-best series, not reported here, do not differ from those obtained with best-fitting series.

[43]The same values are observed with the mixed series based on the *ssr*, *sae*, *aic* and *bic* statistics.

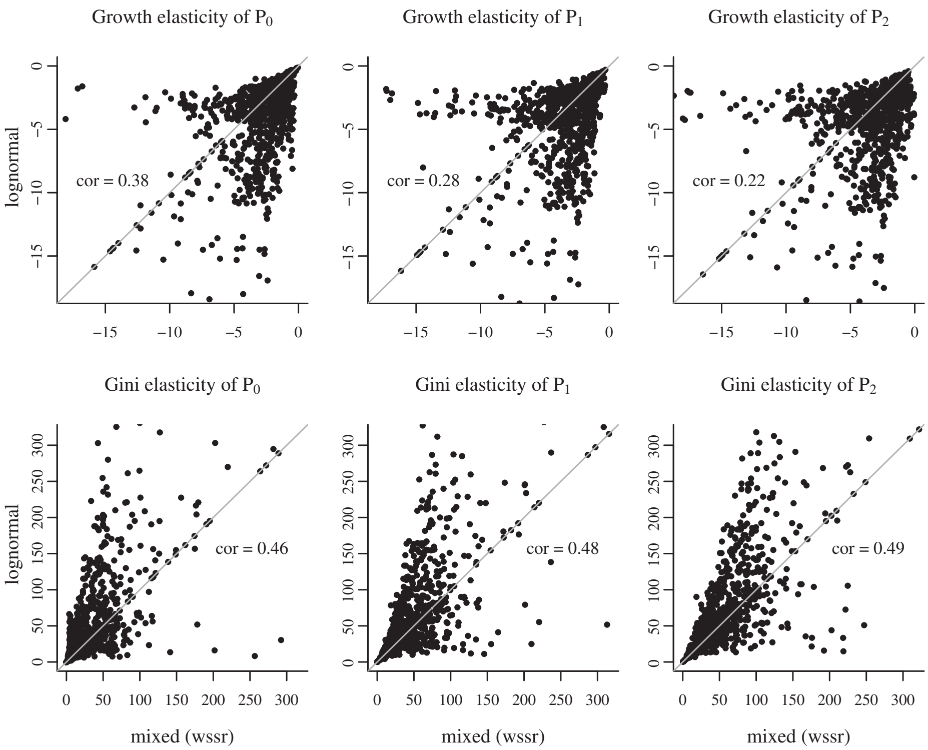

Figure 1. Comparison of the Estimated Elasticities of Poverty between Lognormal Distribution and *wssr* Series, U.S.$1 Poverty Lines

enough). In general, these results indicate that simulations based on the lognormality assumption should be considered with extreme caution.[44]

Finally, even if the lognormal distribution is not adequate for the prediction of individual growth and inequality elasticities of poverty, we may think that it can be used to assess the relative effectiveness of pure growth and pure distributional objectives to achieve poverty alleviation. In Section 2.1 (equation 7), we have noted that the ratio of the growth to Gini elasticities is independent of the relative distribution of income under assumption (4) when the headcount index is considered.[45] However, this is not the case for the other members of the family of poverty measures P_α. In order to check whether the lognormal assumption biases policy recommendations in favor of growth or distributional objectives, we can use the ratio of the lognormal elasticities ratio to the one corresponding to mixed series, that is, $\Upsilon = \dfrac{\eta^L_{\alpha,\mu} / \eta^L_{\alpha,I}}{\eta^M_{\alpha,\mu} / \eta^M_{\alpha,I}}$, where the exponents L and M denote elasticities obtained within the lognormal and mixed series respectively. A value greater (lower) than unity for this policy bias ratio Υ indicates that the lognormal assumption bias politics toward the growth (inequality reduction) objective.

[44]Such risks are perfectly illustrated in CGE models by Boccanfuso *et al.* (2008).
[45]Of course, this result does not hold if another redistribution process is assumed.

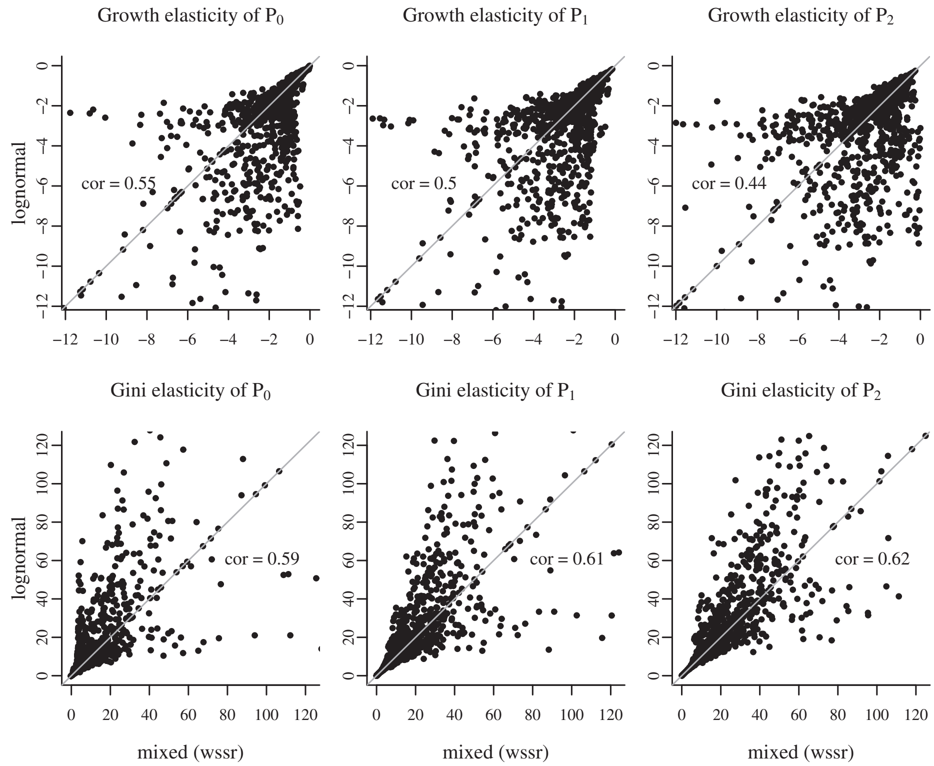

Figure 2. Comparison of the Estimated Elasticities of Poverty between Lognormal Distribution and *wssr* Series, U.S.$2 Poverty Lines

Figure 3 presents a Gaussian kernel density estimation of the density of the ratio Υ for the measures P_1 and P_2. On average (vertical plain lines), it seems that the use of the lognormal distribution yields a pro-growth bias, in particular for the measure P_2, but this result is mainly due to the presence of extreme values of the ratio as shown by the median values (vertical dashed lines). However, the values of Υ are heavily spread around the value of one. For instance, we can notice that the relative importance of the growth elasticity can be overestimated or underestimated in excess of 50 percent under lognormality.

This large variance may be explained by the presence of middle-income countries in which poverty rates are generally very low. As the various functional forms used in the mixed series behave very differently at the bottom quantiles of the population, we can assume that the lognormal distribution is a more adequate choice when the sample is shrunk to low-income countries. To test this hypothesis, we have performed a non-parametric estimation of the mean of this ratio conditional to the level of *per-capita* income.[46] The output is reported in Figure 4 and confirms only partially that assumption. We can observe that the growth effect tends to be overestimated comparing to the inequality effect as

[46]Estimations are realized with a Gaussian kernel. The value of the bandwidth is chosen using the cross-validation procedure.

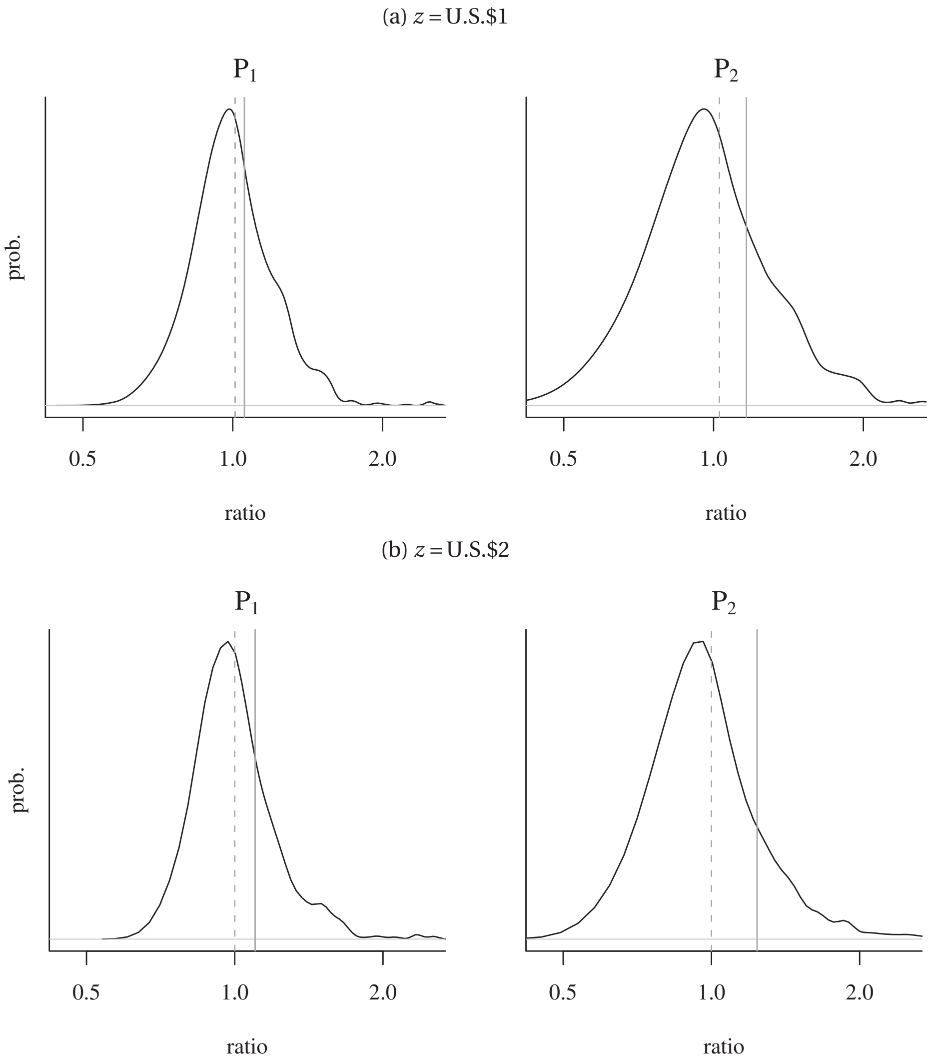

Figure 3. Gaussian Kernel Density of the Policy Bias Ratio for P_1 and P_2 (lognormal against *wssr* mixed series)

Note: Vertical solid and dashed lines respectively correspond to mean and median values. On each subfigure, the optimal value of the bandwidth has been chosen using the cross-validation technique.

per-capita income exceeds a level between \$1,500 and \$2,500. However, it should be noted that the converse phenomenon often occurs for some intervals under \$1,000 mean income value. All these results are statistically significant at the 5 percent level.

Finally, we have tried to see whether the variability of the ratio Y could also be attributed to varying performances of the lognormality assumption with the degree of inequality. Results of a non-parametric estimation of the mean of our

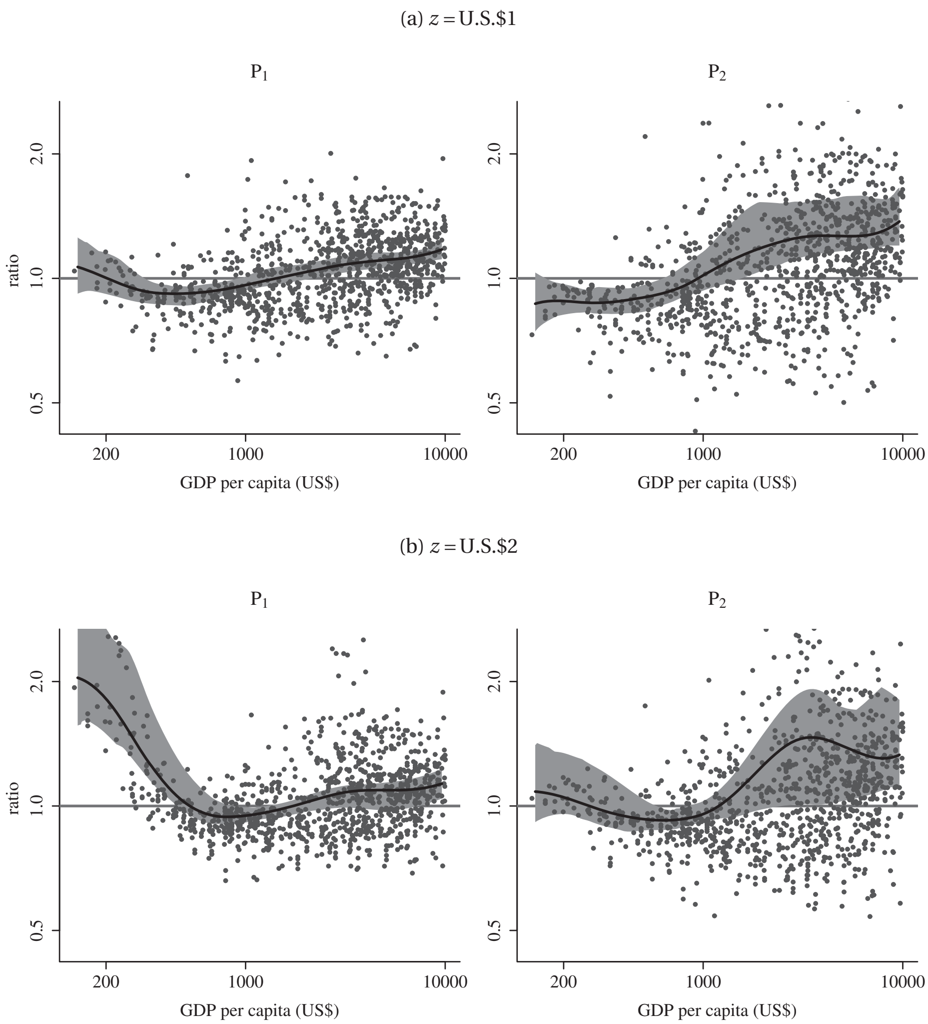

Figure 4. Sensitivity of the Policy Bias Ratio to GDP *per capita* for P_1 and P_2 (lognormal against *wssr* mixed series): non-parametric regression

Note: The thick curve corresponds to the Gaussian kernel estimation of the conditional mean of the politicy bias ratio Υ. The shaded area represents the 95% confidence interval of this conditional mean using a bootstrap procedure (1,000 replications, optimal bandwidth determined with a cross-validation procedure).

policy bias ratio conditional to the initial value of the Gini index are reported in Figure 5. They show no significant bias for highly unequal countries, but we can note that the relative effect of growth is overestimated with respect to inequality reduction under the lognormality hypothesis when the Gini coefficient is less than a threshold of 0.42–0.45. As more than 60 percent of our sample includes distributions which exhibit a Gini index of less than 0.45, this result cannot be regarded as trivial.

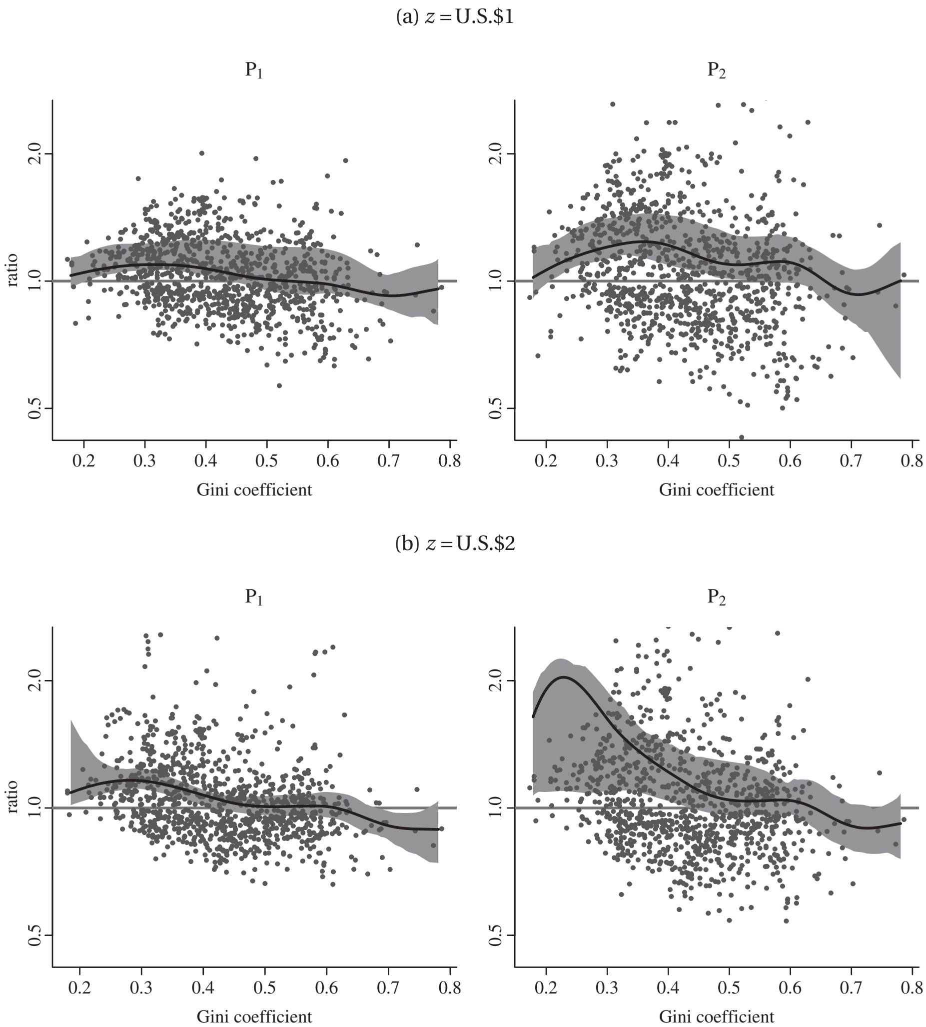

Figure 5. Gini Index vs Policy Bias Ratio for P_1 and P_2 (lognormal against mixed-series *wssr*): non-parametric regression

Note: The thick curve corresponds to the Gaussian kernel estimation of the conditional mean of the political bias ratio Υ. The shaded area represents the 95% confidence interval of this conditional mean using a bootstrap procedure (1,000 replications, see note 29).

6. Concluding Remarks

The consequent diversity of observed income distribution shapes and *per-capita* incomes across the world logically results in heterogenous responses of poverty to growth and inequality variations. In order to take this heterogeneity into account when formulating anti-poverty programs for some countries or groups of countries, the best solution is to use formulas like the one proposed by Kakwani (1993) to compute growth and inequality elasticities for each income distribution, each measure and each poverty line. However, the fact remains that it is still difficult to get empirical income distribution data for a large set of developing countries directly from household surveys. As a consequence, most studies (Bourguignon, 2003; Epaulard, 2003; Kalwij and Verschoor, 2005) rely on so-called secondary datasets, like the one used in this paper, combined with some distributional assumption whose relevance is generally not questioned.

Two questions are of interest. Which distributional assumption is the most adequate for the estimation of growth and inequality elasticities of poverty when the information concerning the relative distribution of income is incomplete? What are the risks associated with the use of a functional form that does not properly fit available data?

Our results show that no clear and definite answer can be given to the first question. Of course, some functional forms yield much more precise estimations than less flexible ones, but none of the rival forms that have been tested in the present study performs better than the others for a major part of the distributions in our sample. Pragmatism is surely the solution and authors should be advised not to impose systematically the same distributional assumption to the observed distributions used in their cross-country or panel studies, but to choose the best performing form among a defined pool of functional forms using some adequate goodness-of-fit criterion. Some may argue that we should turn to more flexible functional forms like the generalized beta distribution McDonald and Xu (1995) or Bernstein polynomial function sequences Ryu and Slottje (1996). However, most functional form entails the use of non-linear estimators that often cannot provide efficient estimations in the presence of local extrema. Moreover, the most flexible forms employed in our study perfectly illustrate how difficult it can be to comply with the whole set of validity conditions that should be met in the context of poverty analysis.

Concerning the second question, we have also shown that the use of a rigid functional form yields poor estimations of the growth and inequality elasticities of poverty in most cases. Here we choose to focus on the statistical distribution that is the most widely used to describe income distributions, i.e. the lognormal distribution. Our findings imply that the results of most cross-country poverty studies based on the lognormal assumption should be considered with caution. Indeed, lognormal growth and inequality elasticities generally tend to overestimate the values corresponding to each observed income distribution. Moreover, the use of the lognormal assumption does not allow for a good appreciation of the relative merit of growth and redistribution objectives in terms of poverty alleviation. For instance, we have found that it generally entails a bias in favor of a mean-income growth objective in middle-income or not much unequal countries.

Finally, one can be surprised at economists' intensive use of the lognormal distribution, although many more flexible functional forms have been proposed in the literature. Because of its empirical weaknesses, shall this statistical distribution be relegated to the historical section of inequality and poverty textbooks? Our point of view is that we may still employ the lognormal distribution in economic models so as to illustrate the distributive effects of any policy measure or market imperfection for instance. However, it would be wise to rule it out from empirical studies in favor of more flexible statistical distributions.

Appendix A: Description of the Sample

The World Income Inequality Database (WIID) is a secondary dataset based on many datasets of within-countries income inequality, such as Deininger and Squire (1996).[47] Its 2.0b version includes information from 1867 to 2005 for 153 countries. About one half of the observations, that is 2,850 observations, contain information on the share of total income by quantile of population. Our sample has been constructed using observations from this set that comply with the following two conditions:

- The original survey should be designed so that representativity is ensured by the geographical coverage of the survey. Rural or urban surveys have been consequently removed.
- The original survey should be designed so that representativity is ensured by the population coverage of the survey. In particular we chose to discard surveys that only consider the working age population.

In a few cases, data that seem not to be reliable have been removed. The final sample contains 1,132 observations. It is important to note that the comparability of the observation is limited since they are not all based on the same income definition and unit of analysis (cf. Table 11). This is particularly true for consumption data that generally underestimate income inequalities but are more reliable measures of well-being (Atkinson and Bourguignon, 2000; Deaton and Zaidi, 2002). It could be interesting to reduce the sample to the sole expenditure or income observations, but it may bias our results since these characteristics are not randomly distributed. For instance, data from Latin American countries are generally based on income series whereas African data are essentially based on consumption. The geographic and period distribution of the observations is reported in Table 12.

TABLE 11

Composition of the Sample by Unit of Analysis and Income Definition

Category	Frequency
Income definition	
Consumption	278
Gross income	373
Net income	356
Others[a]	125
Unit of analysis	
Person	737
Household/family	395

Note: [a]Income is either gross or net, but the survey is insufficiently documented.

[47]For a discussion about the drawbacks of these secondary datasets, see Atkinson and Brandolini (2001).

TABLE 12

Distribution of the Data by Period and Region

Period	EAP	LAC	NA	MENA	SA	SSA	EECA	WE	Total
1960–64	23	27	0	2	15	8	1	18	94
1965–69	28	21	8	3	20	16	2	22	120
1970–74	35	32	11	2	17	1	2	35	135
1975–79	27	25	7	4	11	7	1	28	110
1980–84	28	26	0	1	9	6	1	25	96
1985–89	28	48	0	5	18	27	10	7	143
1990–94	34	63	0	4	6	56	29	0	192
1995–99	23	79	0	8	8	36	63	0	217
2000–05	2	14	0	1	3	3	2	0	25
Total	228	335	26	30	107	160	111	135	1,132

Note: EAP: East Asia and Pacific; LAC: Latin America and Caribbean; NA: North America; MENA: Middle East and North Africa; SA: South Asia; SSA: Sub-Saharan Africa; EECA: East Europe and Central Asia; WE: West Europe.

Appendix B: Additional Tables and Figures

TABLE 13

Mean Values of Growth and Gini Elasticities of P_0, P_1 and P_2: Whole Sample

	Growth Elasticity			Gini Elasticity		
Distribution	P_0	P_1	P_2	P_0	P_1	P_2
\$1 poverty line						
Lognormal	–3.68	–4.02	–4.25	43.97	55.02	65.2
	(0.12)	(0.12)	(0.12)	(2.17)*	(2.36)*	(2.76)*
Gamma	–1.52	–1.59	–1.63	15.2	24.3	33.09
	(0.08)‡	(0.08)‡	(0.08)‡	(0.92)‡	(1.02)‡	(1.2)‡
Weibull	–1.18	–1.23	–1.25	10.69	19.61	28.42
	(0.05)‡	(0.05)‡	(0.05)‡	(0.54)‡	(0.68)‡	(0.83)‡
Fisk	–2.11	–2.22	–2.28	19.88	28.79	37.64
	(0.03)‡	(0.03)‡	(0.03)‡	(0.62)‡	(0.83)‡	(1.03)‡
Singh–Maddala	–1.93	–2.04	–2.29	17.54	26.23	38.43
	(0.08)‡	(0.07)‡	(1.01)*	(0.91)‡	(0.95)‡	(23.53)
Dagum	–2.35	–2.53	–2.57	23.72	33.3	42.52
	(0.75)	(0.81)	(0.75)	(14.82)	(13.61)	(13.59)
Beta 2	–4.27	–4.56	–4.78	52.19	62.83	72.26
	(0.52)	(0.51)	(0.53)	(8.37)	(8.62)	(9.25)
Mixed (*ssr*)	–3.93	–6.15	–5.78	41.82	66.12	101.54
	(3.71)	(6.55)	(13.18)	(39.76)	(76.15)	(104.94)
Mixed (*sea*)	–3.99	–5.65	–6.74	37.7	74.25	97.86
	(3.76)	(8.91)	(10.64)	(42.76)	(74.64)	(106.65)
Mixed (*wssr*)	–4.37	–6.61	–7.31	48.09	71.89	104.63
	(3.76)	(7.28)	(11.21)	(36.4)	(90.56)	(108.21)
\$2 poverty line						
Lognormal	–2.62	–2.96	–3.21	15.27	20.66	25.79
	(0.09)	(0.09)	(0.1)	(0.84)	(0.91)	(1.04)
Gamma	–1.32	–1.43	–1.49	6.46	11.04	15.44
	(0.07)‡	(0.08)‡	(0.09)‡	(0.45)‡	(0.5)‡	(0.56)‡
Weibull	–1.05	–1.14	–1.18	4.69	9.07	13.49
	(0.05)‡	(0.05)‡	(0.06)‡	(0.27)‡	(0.32)‡	(0.41)‡
Fisk	–1.81	–1.98	–2.08	8.62	13.11	17.56
	(0.04)‡	(0.04)‡	(0.04)‡	(0.31)‡	(0.41)‡	(0.51)‡

TABLE 13 (*continued*)

Distribution	Growth Elasticity P_0	Growth Elasticity P_1	Growth Elasticity P_2	Gini Elasticity P_0	Gini Elasticity P_1	Gini Elasticity P_2
Singh–Maddala	−1.63	−1.79	−2.12	7.58	12.03	18.25
	(0.07)‡	(0.08)‡	(0.95)*	(0.43)‡	(0.47)‡	(8.47)*
Dagum	−1.7	−1.84	−1.96	7.97	12.39	16.95
	(0.21)‡	(0.22)‡	(0.22)‡	(1.43)‡	(1.57)‡	(1.35)‡
Beta 2	−2.76	−3.1	−3.31	15.93	21.43	26.27
	(0.27)	(0.29)	(0.3)	(2.28)	(2.28)	(2.47)
Mixed (*ssr*)	−2.2	−3.29	−4.62	11.33	19.08	28.44
	(1.58)	(2.77)	(8.27)	(5.26)	(7.49)	(19.58)
Mixed (*sea*)	−2.22	−3.09	−3.99	10.83	18.7	29.3
	(1.66)	(5.39)	(6.76)	(4.32)*	(8.64)	(9.1)
Mixed (*wssr*)	−2.39	−3.18	−3.92	12.48	20.06	26.86
	(0.39)	(0.8)	(1.46)	(2.31)	(4.45)	(10.12)

Notes: Standard errors in parentheses. Symbols *, † et ‡ indicate that the value is significantly different at the 10, 5 and 1% levels from the estimated mean value under lognormality. For a definition of the series mixed (*ssr*), mixed (*sea*) and mixed (*wssr*), see Section 5.

TABLE 14

Goodness-of-Fit, Mean and Median Values: Restricted Sample, U.S.$2 Poverty Line

Distribution	*ssr*	*sae*	*wssr*	*aic*	*bic*
Mean value					
Lognormal	9.58	3.35	11.49	6.42	6.21
Gamma	44.58	7.84	49.28	28.45	27.68
Weibull	45.62	8.57	51.20	29.42	28.58
Fisk	7.13	2.91	7.26	4.79	4.63
Singh–Maddala	1.42	1.30	1.57	1.18	1.15
Dagum	1.67	1.41	1.94	1.35	1.33
Beta 2	1.91	1.40	2.47	1.63	1.60
Gaffney *et al.* 1980)	1.41	1.27	1.57	1.14	1.13
Fernandez *et al.* 1991)	1.60	1.44	1.81	1.29	1.27
Maddala and Singh (1977)	1.00	1.00	1.00	1.00	1.00
Median value					
Lognormal	4.97	2.56	5.53	4.37	3.87
Gamma	34.33	6.86	35.94	27.16	25.24
Weibull	42.91	8.06	50.25	35.24	32.13
Fisk	4.02	2.18	4.19	3.10	2.84
Singh–Maddala	1.33	1.21	1.53	1.15	1.10
Dagum	1.41	1.26	1.74	1.23	1.19
Beta 2	1.23	1.18	1.34	1.06	1.02
Gaffney *et al.* 1980)	1.26	1.16	1.45	1.09	1.06
Fernandez *et al.* (1991)	1.48	1.32	1.82	1.28	1.23
Maddala and Singh (1977)	1.00	1.00	1.00	1.00	1.00

Notes: the sample represents 87% of the whole sample. Mean value of income *per capita* is $3,523 and average Gini coefficient is 0.42.

References

Aaberge, Rolf, "Characterizations of Lorenz Curves and Income Distributions," *Social Choice and Welfare*, 17, 639–53, 2000.

Aitchison, John and James A. C. Brown, *The Lognormal Distribution*, Cambridge University Press, 1957.

Araar, Abdelkrim and Jean-Yves Duclos, *Poverty and Equity: Measurement, Policy, and Estimation with DAD*, Springer, 2006.

Arnold, Barry and José Villaseñor, "Elliptical Lorenz Curves," *Journal of Econometrics*, 40, 327–38, 1989.

Aten, Bettina, Alan Heston, and Robert Summers, *Penn World Table Version 6.1*, Center for International Comparisons at the University of Pennsylvania (CICUP), October 2002, http://pwt.econ.upenn.edu.

Atkinson, Anthony and François Bourguignon, "Income Distribution and Economics," in Anthony Atkinson and François Bourguignon (eds), *Handbook of Income Distribution*, Handbooks in Economics, Vol. 1, North Holland, 1–58, 2000.

Atkinson, Anthony and Andrea Brandolini, "Promise and Pitfalls in the Use of 'Secondary' Datasets: Income Inequality in OECD Countries as a Case Study," *Journal of Economic Literature*, 39, 771–99, 2001.

Bandourian, Ripsy, James McDonald, and Robert Turley, "A Comparison of Parametric Models of Income Distribution across Countries and over Time," Luxembourg Income Study Working Paper 305, 2002.

Basmann, R., K. Hayes, J. Johnson, and D. Slottje, "A General Functional Formfor Approximating the Lorenz Curve," *Journal of Econometrics*, 43, 77–90, 1990.

Besley, Timothy and Robin Burgess, "Halving Global Poverty," *Journal of Economic Perspectives*, 17(3), 3–22, 2003.

Bhalla, S., "Poor Results and Poorer Policy: A Comparative Analysis of Estimates of Global Inequality and Poverty," *CESifo Economic Studies*, 50, 85–132, 2004.

Boccanfuso, Dorothée, Bernard Decaluwé, and Luc Savard, "Poverty, Income Distribution and CGE Modeling: Does the Functional Form of Distribution Matter?" *Journal of Economic Inequality*, 6, 149–84, 2008.

Bourguignon, François, "The Growth Elasticity of Poverty Reduction: Explaining Heterogeneity across Countries and Time Periods," in Theo Eicher and Stephen Turnovsky (eds), *Inequality and Growth: Theory and Policy Implications*, MIT Press, 3–26, 2003.

Castillo, Enrique, José-Maria Sarabia, and Daniel Slottje, "An Ordered Family of Lorenz Curves," *Journal of Econometrics*, 91, 43–60, 1999.

Chen, Shaohua, Gaurav Datt, and Martin Ravallion, *POVCAL, A Program for Calculating Poverty Measures from Grouped Data*, World Bank, 2001, http://www.worldbank.org/resarh/povmonitor/software.htm.

Chenery, Hollis, Montek S. Ahluwalia, Clive L. G. Bell, John H. Duloy, and Richard Jolly, *Redistribution with Growth: Policies to Improve Income Distribution in Developing Countries in the Context of Economic Growth*, Oxford University Press, 1974.

Cheong, Kwang S., "An Empirical Comparison of Alternative Functional Forms for the Lorenz Curve," *Applied Economic Letters*, 9, 171–6, 2002.

Chotikapanich, Duangkamon, "A Comparison of Alternative Functional Formfor the Lorenz Curve," *Economic Letters*, 41, 129–38, 1993.

Collier, Paul and David Dollar, "Can the World Cut Poverty in Half? How Policy Reform and Effective Aid Can Meet International Development Goals," *World Development*, 29, 1787–802, 2001.

Dagum, Camilo, "A New Model of Personal Income Distribution: Specification and Estimation," *Économie Appliquée*, 30, 413–37, 1977.

Datt, Gaurav, "Computational Tools for Poverty Measurement and Analysis," Discussion Paper 50, FNCD, 1998.

Datt, Gaurav and Martin Ravallion, "Growth and Redistribution Components of Changes in Poverty Measures: A Decomposition with Applications to Brazil and India in the 1980s," *Journal of Development Economics*, 38, 275–95, 1992.

Deaton, Angus, "Measuring Poverty in a Growing World (or Measuring Growth in a Poor World)," *Review of Economics and Statistics*, 87, 1–19, 2005.

Deaton, Angus and Salman Zaidi, "Guidelines for Constructing Consumption Aggregates for Welfare Analysis," Living Standards Measurement Study Working Paper 135, World Bank, 2002.

Deininger, K. and L. Squire, "A New Dataset Measuring Income Inequality," *World Bank Economic Review*, 10, 565–91, 1996.

Dollar, David and Aart Kraay, "Growth is Good for the Poor," *Journal of Economic Growth*, 7, 195–225, 2002.
Efron, Bradley and Robert Tibshirani, *An Introduction to the Bootstrap*, Monographs on Statistics and Applied Probability, Chapman & Hall/CRC, 1993.
Epaulard, Anne, "Macroeconomic Performance and Poverty Reduction," Working Paper 03/72, IMF, 2003.
Fernandez, A., A. Garcia, M. Ladoux, G. Martin, and P. Ortega, "A New Functional Form for Approximating the Lorenz Curve," *Review of Income and Wealth*, 37, 447–52, 1991.
Foster, James, Joel Greer, and Erik Thorbecke, "A Class of Decomposable Poverty Measures," *Econometrica*, 52, 761–6, 1984.
Gaffney, J., Anthony Koo, Norman Obst, and Robert Rasche, "Functional Forms for Estimating the Lorenz Curve," *Econometrica—Notes and Comments*, 48, 1061–2, 1980.
Greene, W., *Econometric Analysis*, edition 4, Prentice Hall, 2000.
Gugerty, Mary Kay and Michael Roemer, "Does Economic Growth Reduce Poverty?" CAER II Discussion Paper 5, Harvard Institute for International Development, 1997.
Gujarati, Damodar, *Économétrie*, Ouvertures Économiques, De Boeck, 2004.
Gupta, Manash, "Functional Form for Estimating the Lorenz Curve," *Econometrica—Notes and Comments*, 52, 1313–14, 1984.
Heltberg, Rasmus, "The Growth Elasticity of Poverty," in Rolph van der Hoeven and Anthony Shorrocks (eds), *Growth, Inequality, and Poverty: Prospects for Pro-Poor Economic Development*, UNU–WIDER Studies in Development Economics, Oxford University Press, 81–91, 2004.
Holm, J., "Maximum Entropy Lorenz Curves," *Journal of Econometrics*, 59, 377–89, 1993.
Jenkins, Stephen J., "Distributionally-Sensitive Indices and the GB2 Income Distribution," *Review of Income and Wealth*, 55, 392–8, 2009.
Kakwani, Nanak, "On a Class of Poverty Measures," *Econometrica*, 48, 437–46, 1980.
———, "Poverty and Economic Growth with Application to Côte d'Ivoire," *Review of Income and Wealth*, 39, 121–39, 1993.
Kakwani, Nanak and Nripesh Podder, "On the Estimation of Lorenz Curves from Grouped Observations," *International Economic Review*, 14, 278–92, 1973.
———, "Efficient Estimation of the Lorenz Curve and Associated Inequality Measures from Grouped Observations," *Econometrica*, 44, 137–48, 1976.
Kakwani, Nanak and Hyun Son, "Economic Growth and Poverty Reduction: Initial Conditions Matter," Working Paper 2, International Poverty Centre, 2004.
Kalwij, Adiaan and Arjan Verschoor, "A Decomposition of Poverty Trends across Regions: The Role of Variation in the Income and Inequality Elasticities of Poverty," Working Paper 36, UNU–WIDER, 2005.
Kanbur, Ravi and Nora Lustig, "Why is Inequality back on the Agenda?" in Boris Pleskovic and Joseph Stiglitz (eds), *Annual World Bank Conference on Development Economics, 1999*, World Bank, 2000, 285–306.
Kleiber, Christian and Samuel Kotz, *Statistical Size Distributions in Economics and Actuarial Sciences*, Wiley, 2003.
Li, Hongyi, Lyn Squire, and Heng-fu Zou, "Explaining International and Intertemporal Variations in Income Inequality," *The Economic Journal*, 108(446), 26–43, 1998.
Lopez, Humberto and Luis Servèn, "A Normal Relationship? Poverty, Growth and Inequality," World Bank Policy ResearchWorking Paper Series 3814, World Bank, 2006.
Maddala, G. S. and S. K. Singh, "A Function for Size Distribution of Incomes," *Econometrica*, 44, 963–70, 1976.
———, "A Flexible Functional Formfor Lorenz Curves," *Économie Appliquée*, 30, 481–6, 1977.
McDonald, James, "Some Generalized Functions for the Size Distribution of Income," *Econometrica*, 52, 647–63, 1984.
McDonald, James and Y. Xu, "A Generalisation of the Beta Distribution with Applications," *Journal of Econometrics*, 66, 133–52, 1995 (erratum: *Journal of Econometrics*, 69, 427–8).
McKay, A., "Poverty Reduction Through Economic Growth: Some Issues," *Journal of International Development*, 9, 665–73, 1997.
Metcalf, Ch., "The Size Distribution of Personal Income During the Business Cycle," *American Economic Review*, 59, 657–68, 1969.
Minoiu, Camelia, "Poverty Analysis Based on Kernel Density Estimates from Grouped Data," Job Market Paper, Institute for Social and Economic Research and Policy, Columbia University, 2006.
Minoiu, Camelia and Sanjay Reddy, "The Assessment of Poverty and Inequality Through Parametric Estimation of Lorenz Curves," Working Paper 07-02, Institute for Social and Economic Research and Policy, 2007.

Mount, T. and A. Salem, "A Convenient Descriptive Model of Income Distribution: The Gamma Density," *Econometrica*, 42, 1115–28, 1974.
Nelder, J. and R. Mead, "A Simplex Method for Function Minimization," *Computer Journal*, 7, 308–13, 1965.
Parker, S., "The Generalized Beta as aModel for the Distribution of Earnings," *Economic Letters*, 62, 197–200, 1999.
Quah, Danny, "Some Simple Arithmetic on How Income Inequality and Economic Growth Matter," Discussion Paper, LSE, Economics Department, 2001.
R Development Core Team, *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna, Austria, 2007, www.r-projet.org.
Ravallion, Martin, "Growth, Inequality and Poverty: Looking Beyond Averages," *World Development*, 29, 1803–15, 2001.
———, "Inequality is Bad for the Poor," World Bank Policy Research Working Paper Series 3677, World Bank, 2005.
Ryu, H. and D. Slottje, "Two Flexible Functional Form Approaches for Approximating the Lorenz Curve," *Journal of Econometrics*, 72, 251–74, 1996.
Sala-i-Martin, Xavier, "The World Distribution of Income: Falling Poverty and . . . Convergence, Period," *Quarterly Journal of Economics*, 121, 351–97, 2006.
Schader, Martin and Friedrich Schmid, "Fitting Parametric Lorenz Curves to Grouped Income Distributions—A Critical Note," *Empirical Economics*, 19, 361–70, 1994.
Sen, Amartya, "Poverty: An Ordinal Approach toMeasurement," *Econometrica*, 44, 219–31, 1976.
Stacy, E., "A Generalization of the Gamma Distribution," *Annals of Mathematical Statistics*, 33, 1187–92, 1962.
Thurow, L., "Analyzing the American Income Distribution," *American Economic Review, Papers and Proceedings*, 60, 261–9, 1970.
UNU–WIDER, *World Income Inequality Database: User Guide and Data Sources*, 2005, http://www.wider.unu.edu/wiid/wiid.htm.
World Bank, *A Better Investment Climate for Everyone*, World Development Report, Oxford University Press, 2005.
Zheng, Buhong, "Aggregate Poverty Measures," *Journal of Economic Surveys*, 11, 123–62, 1997.