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ON THE EQUIVALENCE OF WEIGHTED COUNTRY-PRODUCT-DUMMY (CPD) METHOD AND THE RAO-SYSTEM FOR MULTILATERAL PRICE COMPARISONS

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The country-product-dummy (CPD) method, originally proposed in Summers (1973), has recently been revisited in its weighted formulation to handle a variety of data related situations (Rao and Timmer, 2000, 2003; Heravi *et al*., 2001; Rao, 2001; Aten and Menezes, 2002; Heston and Aten, 2002; Deaton *et al*., 2004). The CPD method is also increasingly being used in the context of hedonic modelling instead of its original purpose of filling holes in Summers (1973). However, the CPD method is seen, among practitioners, as a black box due to its regression formulation. The main objective of the paper is to establish equivalence of purchasing power parities and international prices derived from the application of the weighted-CPD method with those arising out of the Rao-system for multilateral comparisons. A major implication of this result is that the weighted-CPD method would then be a natural method of aggregation at all levels of aggregation within the context of international comparisons.

1. INTRODUCTION

Index number methods for international comparisons have assumed considerable importance in the literature. Aggregation of price and quantity data is the most crucial step in any comparisons of real gross domestic product across countries. In view of the particular emphasis placed on the aggregation issues, there have been many index number formulae proposed in the literature. Kravis *et al*. (1982) provide an excellent summary of the methods relevant for international comparisons. Given that the year 2005 is the benchmark year for the current round of world comparisons under the auspices of the International Comparison Program (ICP) at the World Bank, it is necessary to refocus on issues relating to aggregation methods. This paper may be considered in this vein.

Aggregation in the context of international comparisons is undertaken at two levels. First, price data on individual items are aggregated to yield purchasing power parities (PPPs) at the basic heading level. The basic heading level is generally considered the lowest level of disaggregation at which expenditure shares and weights are available. The second stage involves aggregation of basic heading level parities for the total economy gross domestic product.

The country-product-dummy (CPD) method, due to Summers (1973), is a procedure that has been in use for about three decades for aggregation of item

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level price data. The CPD method is particularly designed to handle data with missing prices. An alternative to the CPD method is the Eltetö-Köves-Szulc (EKS) method. In the more recent phases of the ICP, the EKS method formed the basis for aggregation below the basic heading level.[1] However, many aggregation methods have been proposed for purposes of aggregation above the basic heading level, including the Geary-Khamis method, the EKS method, the Rao multilateral system and the Ikle method. The Geary-Khamis method has been the principal aggregation method used for global comparisons due to its properties of transitivity, base invariance and additivity.[2] The EKS method is currently being used in deriving real income comparisons between OECD countries and within the European Union. The Rao-system is an expenditure share weighted logarithmic variation of the Geary-Khamis method, but has never been considered as a serious alternative to the Geary-Khamis method due to lack of an additive consistency property. The Rao system is fully described, along with its rationale and its properties, by Rao (1990).

In recent years there has been a resurgence of studies on the CPD method, including its use above basic heading level. Recent studies by Rao (2001, 2004) and Diewert (2002, 2004) describe an array of interesting properties of the CPD method when it is applied for aggregating price data at and above the basic heading level. Diewert (2002) shows how a number of well-known bilateral index number formulae can be derived using variations to the specification of the standard CPD model.

The main objective of this article is to show how the Rao-multilateral system can be derived using the weighted CPD method. In this respect the main contribution of the paper is to provide a multilateral generalization of the work of Diewert (2002) and also provide a direct link between a variant of the CPD method and a multilateral index number system used in aggregating data above the basic heading level. The equivalence result presented here is significant in that it, for the first time, provides a real link between the regression method and its resulting estimates of PPPs and international prices with those emanating from a variant of the Geary-Khamis method that defines the Rao-system.

The structure of the article is as follows. Section 2 outlines the CPD method and the weighted CPD method for aggregation above the basic heading level. Section 3 briefly describes the Rao-multilateral system and its relationship with the Geary-Khamis method. Section 4 provides a simple algebraic proof of the equivalence of the weighted CPD method and the Rao-multilateral system. The paper is concluded with some remarks in Section 5.

2. Country-Product-Dummy (CPD) Method

The country-product-dummy method was first proposed in Summers (1973) as a technique to fill gaps in price data at the item level and as a method to aggregate price data to provide purchasing power parities at the basic heading level. The

[1]Rao (2001) discusses the EKS method and its generalizations for aggregation both at the basic heading and above basic heading levels.

[2]Rao (1997) provides a description of these properties.

basic rationale underlying the CPD technique is that the observed price of a commodity in a given country is the product of three components. The first is the country effect representing the general price level in the country, the second is the commodity effect which measures the relative price level of a commodity and the third component is a random disturbance term. Rao (2001) provides further insights into the CPD method and the generality of the method to encompass a number of other methods. This included the fact that the CPD method can be considered as a special case of some of the hedonic models used in the literature (Kravis *et al.*, 1982).

The following notation is used throughout the paper. Let p_{ij}, q_{ij} and w_{ij} represent, respectively, price, quantity and value share of i-th commodity in j-th country. In this paper we examine the problem of aggregation from the basic heading level upwards. Therefore, the prices refer to PPPs at the basic heading level and quantities refer to real value aggregates. Without loss of generality, we assume that prices, quantities (real values) and value shares are all positive.[3]

The basic premise underlying the CPD method is that the observed price of a commodity, say i, in a given country, say j, is the product of the purchasing power parity of currency of j-th country, and the relative price of i-th commodity, both defined using a numeraire currency or a numeraire commodity. Let π^* and η_j^* represent relative price and purchasing power. Then

$$p_{ij} = \pi_i^* \eta_j^* u_{ij}^*$$

which can be re-written in logarithmic form as:

$$\ln p_{ij} = \pi_i + \eta_j + u_{ij}$$

where u_{ij} is a random disturbance term. This model can be estimated using a simple regression framework with dummy variables for commodities and countries in the following model, for $i = 1,2, \ldots N$ and $j = 1,2, \ldots, M$.

$$\ln p_{ij} = \pi_1 D_1 + \pi_2 D_2 + \ldots + \pi_N D_N + \eta_1 D_1^* + \eta_2 D_2^* + \ldots + \eta_M D_M^* + u_{ij} \tag{1}$$

where $\mathbf{D_i}$ is the i-th commodity dummy variable, taking value equal to 1 for commodity i and zero for all other commodities; $\mathbf{D_j^*}$ is the j-th country dummy variable, taking value equal to 1 for all observations for country j and zero for all other countries. The random disturbance term u_{ij} is assumed to be independently and identically distributed with zero mean and variance σ^2. In addition, these random disturbances are assumed to follow a normal distribution. The assumption of independently distributed disturbances is relaxed in recent work by Rao (2004) where the disturbances are assumed to be correlated across countries.

The CPD technique simply regresses the logarithm of observed price of commodities from different countries on the country and commodity (or product) dummy variables to yield estimates of π_i's and η_j's. Typically, regressions used here are unweighted since there are no expenditure weights available at the item level within the ICP.

[3]This assumption can be dropped without affecting the results. However, the assumption of positive prices and values and value shares simplifies the proofs.

In view of the specification of the model in (1) and due to the presence of perfect multicollinearity, least squares estimates of the $(N + M)$ parameters, π_i and η_j, can be obtained only after selecting one of the commodity prices, π_i, or one of the country currencies, η_j, as a numeraire. Since exponential of η_j can be considered as the purchasing power parity of currency of country j, setting one of the PPP_j's equal to 1 is the same as setting the corresponding η_j to be equal to zero. In this case, PPPs of all the currencies and the international prices of commodities are all measured relative to the numeraire currency unit, and thus expressed in the currency units of the numeraire country.

Weighted CPD Method

In this section, we briefly describe the weighted version of the CPD method. The use of weights becomes necessary and relevant due to a number of reasons. Firstly, price data are usually collected on the basis of a sampling design. The use of survey designs implies a certain weighting pattern reflecting the design as well as non-response errors. Secondly, for aggregation below the basic heading level it may become necessary to account for differential coverage both in terms of the number of products that may be considered representative and those that are non-representative but are also priced. Rao (2001) provides a range of weighting patterns for aggregation below the basic heading level. Thirdly, for aggregation above the basic heading level there are expenditure shares available from the national accounts or from the household expenditure surveys. Finally, it may also become necessary to account for the structure of the disturbances in the model. Rao (2001), Aten and Menezes (2002) and Deaton *et al*. (2004) provide examples of weighting patterns and how these can be applied in making price level comparisons.

Typically, application of the unweighted CPD involves the minimization of residual sum of squares where each observation is equally weighted. However, when a model is fitted, especially in the context of index number construction, it is important that the fitted model tracks more important price observations more closely than for those items which are not considered relevant. Let w_{ij} reflect a measure of importance attached to tracking the i-th item (or basic heading PPP) price in j-th country. The most common measure of such importance is the expenditure share of the commodity in a given country. Then the weighted CPD method involves minimization of the weighted residual sum of squares:

$$WRSS = \sum_{j=1}^{M}\sum_{i=1}^{N} w_{ij}(\ln p_{ij} - \pi_1\mathbf{D_1} - \pi_2\mathbf{D_2} - \ldots - \pi_N\mathbf{D_N} - \eta_1\mathbf{D_1^*} - \eta_2\mathbf{D_2^*} - \ldots - \eta_M\mathbf{D_M^*})^2 \quad (2)$$

where w_{ij} is the weight attached to ij-th price observation.[4]

[4]The use of weights in hedonic regressions has been comprehensively discussed in Silver (2002). Since the CPD model can be considered as a special case of hedonic regression models where the only quality characteristics are the country and the product identifiers, results reported by Silver (2002) are very relevant here. The choice between quantity and expenditure share weights was discussed at length and the general conclusion is that expenditure share weights are the most appropriate set of weights in the case of hedonic regressions. This conclusion by Silver (2002) is highly relevant in the present context where the weighted CPD method using expenditure share weights is shown to be equivalent to the Rao-system.

Use of equation (2) is equivalent to applying ordinary least squares to the following transformed model where each variable is multiplied by the square root of w_{ij}. Thus, we have a standard linear regression model of the form

$$Y_{ij} = \pi_1 \mathbf{X}_1 + \pi_2 \mathbf{X}_2 + \ldots + \pi_N \mathbf{X}_\mathbf{N} + \eta_1 \mathbf{Z}_\mathbf{1} + \eta_2 \mathbf{Z}_\mathbf{2} + \ldots + \eta_M \mathbf{Z}_\mathbf{M} + v_{ij} \tag{3}$$

where:

$$Y_{ij} = \sqrt{w_{ij}} \ln p_{ij}; \quad \mathbf{X}_\mathbf{i} = \sqrt{w_{ij}} \mathbf{D}_\mathbf{i}; \quad \text{and} \quad \mathbf{Z}_\mathbf{j} = \sqrt{w_{ij}} \mathbf{D}_\mathbf{j}^*.$$

Two points are worth noting here. Firstly, the regression model used in (3) resembles some of the models used under stochastic approach to index numbers described in Clements and Izan (1987) and Selvanathan (1989). However, the model specified here *differs* from their approach in that the disturbance terms are not assumed to be heteroscedastic. The weighted regressions used here are justified along on the lines of standard principles of index number construction that require greater weighting for more important price observations. This approach is more in line with Theil's (1967) approach and provides a point of departure from the general stochastic approach. Secondly, application of (3) is similar to the M-estimators used in the estimation of parameters using weights matrices within the least squares approach (Davidson and Mackinnon, 1993, pp. 587–96).

As the main purpose of this short paper is to examine the link between the weighted CPD method and the Rao-system (discussed below), the weights in the weighted CPD model above are taken to represent expenditure or value shares.

3. Rao-System for Multilateral Comparisons

Now we turn to the second of the two methods whose equivalence is the subject matter for this short paper. The Rao-system, proposed originally in a brief form in Rao (1972), is presented in a more detailed form with all its mathematical and statistical properties in Rao (1990). The Rao-system is a variation of the Geary-Khamis method cast in a set of log-linear equations. The origins of the system can be attributed to the stochastic formulation of the Geary-Khamis system described by Khamis (1984). The system is based on the same conceptual framework as that of the Geary-Khamis system. It uses the twin concepts of purchasing power parities of currencies (PPP_j, $j = 1,2, \ldots, M$) and international average prices of commodities (P_i, $i = 1,2, \ldots, N$) that are central to the Geary Khamis method. Geary (1958), Khamis (1972) and Kravis *et al.* (1982) provide an excellent exposition of the Geary-Khamis method.

The basic equations that defined the Rao-system are:

$$PPP_j = \prod_{i=1}^{N} \left(\frac{p_{ij}}{P_i} \right)^{w_{ij}} \quad \text{for } j = 1,2, \ldots M, \quad \text{and} \tag{4}$$

$$P_i = \prod_{j=1}^{M} \left(\frac{p_{ij}}{PPP_j} \right)^{\frac{w_{ij}}{\sum_{j=1}^{M} w_{ij}}} \quad \text{for } i = 1,2, \ldots, N \tag{5}$$

where $w_{ij} = \dfrac{p_{ij}q_{ij}}{\sum_{i=1}^{N} p_{ij}q_{ij}}$ is the expenditure or value share of i-th commodity in j-th country.[5] Since the Rao-system, just as in the case of the Geary-Khamis system, is used for purposes of aggregation above the basic heading level, the expenditure share data are available. It is possible that in some countries the expenditure shares may be zero for some expenditure categories or basic headings.

The Rao-system, as is the case with the Geary-Khamis system, is a simultaneous equation system with $(M + N)$ equations in as many unknowns. Rao (1990) provides a proof of the existence and uniqueness (up to a factor of proportionality) of the solution for the unknown parities and international prices. Among one of the principal characteristics, the weighting system employed here is invariant to the size of the country, unlike the Geary-Khamis system, and its use of value shares for weights is consistent with price index number literature. For example, the Tornqvist index as well as Theil's (1973, 1974) variants are all based on expenditure-share based weighting systems. However, the Rao-system, due to its log-linear specification, does not result in additively consistent purchasing power parities and international prices.

4. Equivalence of the Weighted-CPD and the Rao-System for Multilateral Comparisons

The main purpose of this section is to establish the equivalence of the weighted CPD method described in Section 2 and the Rao-system in Section 3. Such an equivalence is likely to enhance the practical importance of both methods involved. The proof of the equivalence is based on establishing that the normal equations of the least squares method applied to the model in (3) coincide with equations (4) and (5) that define the Rao system.

Let

$$\pi = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \cdot \\ \pi_N \end{bmatrix} \quad \text{and} \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \cdot \\ \eta_M \end{bmatrix}$$

be the vectors of international prices and purchasing power parities in logarithmic form. Correspondingly let

$$\mathbf{X} = [\mathbf{X_1} \quad \mathbf{X_2} \ldots \mathbf{X_N}] \quad \text{and} \quad \mathbf{Z} = [\mathbf{Z_1} \quad \mathbf{Z_2} \ldots \mathbf{Z_M}]$$

where $\mathbf{X_i} = \sqrt{w_{ij}}\,\mathbf{D_i}$; and $\mathbf{Z_j} = \sqrt{w_{ij}}\,\mathbf{D_j^*}$.

Using the notation established, the weighted CPD model can be written as

$$Y = x\pi + Z\eta + V \qquad (6)$$

Application of the least squares procedure to estimate the parameter vectors of the model, η and π, leads to the following system of normal equations. If $\hat{\eta}$

[5] As w_{ij}'s represent value shares, for any given country j these shares add up to unity over all the commodities.

and $\hat{\pi}$ denote the least squares estimators of η and π, then the normal equations can be written as

(7)
$$\begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z \end{bmatrix} \begin{bmatrix} \hat{\pi} \\ \hat{\eta} \end{bmatrix} = \begin{bmatrix} X'Y \\ Z'Y \end{bmatrix}$$

Solutions to the normal equations yields the least squares estimates of π and η, which can then be used in deriving *PPP*'*s* and P_i'*s* for multilateral comparisons.

From equation (7), we have two systems of linear equations of the form:

(8)
$$\mathbf{X'X}\hat{\pi} + \mathbf{X'Z}\hat{\eta} = \mathbf{X'Y}, \quad \text{and}$$

(9)
$$\mathbf{Z'X}\hat{\pi} + \mathbf{Z'Z}\hat{\eta} = \mathbf{Z'Y}$$

Solving (8) and (9) in the form of an interdependent system, we have, respectively the following two equations.

(10)
$$\hat{\pi} = (\mathbf{X'X})^{-1}\mathbf{X'}[\mathbf{Y} - \mathbf{Z}\hat{\eta}], \quad \text{and}$$

(11)
$$\hat{\eta} = (\mathbf{Z'Z})^{-1}\mathbf{Z'}[\mathbf{Y} - \mathbf{X}\hat{\pi}]$$

Since $\mathbf{X}$ and $\mathbf{Y}$ are matrices whose elements are in the form of $\mathbf{X}_i = \sqrt{w_{ij}}\mathbf{D_i}$; and $\mathbf{Z_j} = \sqrt{w_{ij}}\mathbf{D_j^*}$ where $\mathbf{D_i}$ and $\mathbf{D_j^*}$ are respectively the product and country dummy variables. Using this specialized structure and the fact that w_{ij} sum to unity over all the commodities for any country j, and after some algebraic manipulation, equations (10) and (11) can be simplified and expressed as

(12)
$$\hat{\pi}_i = \sum_{j=1}^{M} \frac{w_{ij}}{\sum_{j=1}^{M} w_{ij}} (\ln p_{ij} - \hat{\eta}_j) \quad \text{for } i = 1,2,\ldots,N; \quad \text{and}$$

(13)
$$\hat{\eta}_j = \sum_{i=1}^{M} w_{ij}(\ln p_{ij} - \hat{\pi}_i) \quad \text{for } i = 1,2,\ldots,M.$$

Derivation of equations (12) and (13) is fairly straightforward since these are derived by minimizing the weighted residual sum of squares expression given in equation (2). The normal equations from the weighted least squares, given in equations (8) and (9) can be derived by considering the first order conditions for minimization of:

$$\min_{\pi,\eta} \sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}(\ln p_{ij} - \pi_i - \eta_j)^2$$

with respect to $\pi_i (i = 1,2,\ldots, N)$ and $\eta_j (j = 1,2,\ldots M)$. The resulting normal equations are identical to equations (12) and (13).

Now if we let $P_i = exp(\hat{\pi}_i)$ and $PPP_j = (\hat{\eta}_j)$ and express equations (12) and (13) in their exponential form, we have the following systems of equations:

$$P_i = \prod_{j=1}^{M} \left(\frac{p_{ij}}{PPP_j} \right)^{\frac{w_{ij}}{\sum_{j=1}^{M} w_{ij}}} \text{and}$$

$$PPP_j = \prod_{i=1}^{N} \left(\frac{p_{ij}}{P_i} \right)^{w_{ij}}.$$

It can be seen that these equations are identical to the equation system (3) and (4) that defines the Rao-system for multilateral comparisons. It is important to note here that $exp(\hat{\eta}_j)$ is not an unbiased estimator of $exp(\eta)$. It is customary to use Goldberger's adjustment in deriving unbiased estimators. Rao and Selvanathan (1992) describe a method for the generation of minimum variance unbiased estimator of $exp(\eta)$.

The correspondence between weighted CPD and the Rao-system established here applies equally to cases with and without gaps in the data as long as there are solutions for the normal equations of the weighted least squares approach in equations (14) and (15). The conditions for the existence of a solution that is unique up to a factor of proportionality are discussed fully in Rao (1990).

This result shows that the purchasing power parities and international prices that are derived from the weighted version of the CPD model and the application of weighted least squares where each squared residual is weighted by the corresponding expenditure share, w_{ij}, coincide with the parities and international prices derived from the Rao-system of equations.

5. Implications and Concluding Remarks

The principal finding reported in this article has several implications for the International Comparison Program (ICP), especially with respect to the choice of an appropriate aggregation method for international comparisons of real gross domestic product. The equivalence result is quite surprising since the two methods, the generalized CPD and the Rao-systems, have quite distinct underlying conceptual frameworks.

The result is also significant for a number of other reasons. First, the result provides an interpretation for the econometric estimates of the parameters of the CPD model within the standard framework of international prices and purchasing power parities just like the Geary-Khamis system. Second, if the unweighted CPD method is used for purposes of aggregating price data below the basic heading level, then the interpretation according to the weighted CPD method makes it a natural choice for aggregation above the basic heading level. Such a choice provides a unifying econometric framework for the estimation of PPPs at all levels of aggregation. Third, since the properties of the Rao-system are well established, the PPPs and international prices from weighted CPD can be associated with the same properties. Of some relevance here is the property of the Rao-system which states that in the case of binary comparisons (with $M = 2$) the Rao-system provides a price index that is very similar to the Tornqvist index which is known to be a superlative index satisfying several economic theoretic and axiomatic properties (see Diewert, 1976; Caves *et al.*, 1982a, 1982b). In fact, the Rao-binary index is a weighted geometric average of price relatives with weights that are symmetric means of expenditure shares in the two countries or time periods involved.

The result established in this article provides a basis for further statistical refinements in the estimation of the purchasing power parities using more sophisticated econometric methods to handle various data related issues, including the possibility of making adjustments for quality differences (hedonic methods) and in incorporating spatial structures implicit in the price data. A statistical framework for the estimation of purchasing power parities makes it possible to incorporate sampling weights to price data and it can also provide measures of reliability in the form of standard errors. Thus the result paves the way for integrating the conceptual framework of the Geary-Khamis type with that of the statistical modeling of price data implicit in the CPD model. These developments have the potential to take international comparison work to a different level of sophistication.

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