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LIFE EXPECTANCY AND ECONOMIC WELFARE: THE EXAMPLE OF AFRICA IN THE 1990s

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A formulation for incorporating Life expectancy information into empirical economic welfare calculations is presented. In an application analyzing the economic progress of the African continent during the 1990s due consideration of life expectancy factors substantially modifies the conclusions drawn from standard welfare calculations.

INTRODUCTION

A potent argument for including life expectancy in economic welfare calculus is that the long term welfare of a society is probably best assessed by considering changes in the distribution of the lifetime wealth or utility of the individuals within the society. As Atkinson (1983) argues, examining the distribution of weekly, monthly or annual incomes, records different members of the society at various stages of their life cycle and engenders a spurious degree of inequality whereas the distribution of lifetime income is better able to catch "the distribution of life chances, as represented by a person's work career." Indeed, in a very basic sense, acknowledging Sen's arguments for extending well being comparisons to the space of capabilities (Sen, 1992) dictates that life expectancy, the length of time over which an individual has the capability of enjoying such things as he or she values, should be included in the calculation of their well being.

Clearly the unadulterated use of any flow measure will not account for the period of time over which it was enjoyed and equates the welfare of individuals enjoying the same consumption flow regardless of the span of time over which it exists. Notwithstanding these arguments, largely due the paucity of lifetime wealth or utility data, most empirical economic welfare comparisons have been conducted in the singular space of annual income or consumption flows forcing the presumption that factors distinguishing lifetime from periodic utility are constant across the populations under comparison. The impact of mortality on various aspects of the welfare calculus is being addressed in an emerging literature. When life expectancy is positively correlated with incomes the death of the poor paradoxically improves an income based welfare measure. Kanbur and Mukherjee (2004), by positing a normative length of lifetime, augment classic Foster Greer Thorbeke poverty indices with mortality effects in such a way that this paradox is

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surmounted. Cogneau and Grimm (2004) develop and implement a counterfactual analysis of income correlated mortality effects on income distributions and Klasen (2004) explores the gender based mortality gap in developing gender related indicators of well being. Here the impact of introducing life expectancy into the lifetime welfare calculus is examined.

Classic formulations of the Consumption Function under the Permanent Income Hypothesis readily establish the conditions under which there is an equivalency between comparing current consumption (or permanent income) patterns and lifetime utility (or wealth) patterns. Essentially given homogeneous preferences, constant time preference rates, interest rates and life expectancy across agents, flow and stock comparisons will be equivalent. Even if there is some small random variation in the "non-consumption" factors (or if life expectancy were positively correlated and rates of time preference were negatively correlated with incomes), the equivalency of current consumption and lifetime income or utility distributions still constitutes a reasonable approximation. However when the constancy assumption is no longer tenable, "current" and "lifetime" comparisons may well result in conflicting inferences regarding economic welfare.

With the exception of several African nations and a few former Soviet Socialist Republics, life expectancy has increased steadily throughout the world in recent years. In a 135 country panel sample employed in Anderson (2004), 27 countries suffered life expectancy declines over the 1990s; of those 21 were from the 38 African countries in the sample.[1] There is no greater continental life expectancy variation than in Africa where systematic health characteristic differences and major civil wars appear to distinguish declining and non-declining life expectancy groups in the 1990s.[2] The African decline in life expectancy is inexorably linked to the HIV/AIDS pandemic (Chapter 1, World Bank, 1999), not only because of its direct effects on mortality, but also because of the hospitable environment it presents to other opportunistic diseases such as tuberculosis, pneumonia, encephalitis and meningitis. It is a major killer of prime age adults (Murray and Lopez, 1996), second only to tuberculosis (the spread of which it is also partially responsible for) and entails substantial disability before death.

This study exploits a simple version of the Permanent Income Hypothesis to incorporate life expectancy into the welfare calculus and assesses the impact of relaxing the assumption of constant life expectancy across the individuals within a group when making economic welfare comparison. In the present context the individuals are representative agents of a sample of countries on the African con-

[1]The African countries sampled suffering a decline in life expectancy over the 1987–1999 period were Botswana, Burkino Faso, Burundi, Camaroon, Central African Republic, The Congo, Cote D'Ivoire, Ethiopia, Kenya, Lesotho, Malawi, Mali, Mozambique, Namibia, Rwanda, South Africa, Swaziland, Togo, Uganda, Zambia and Zimbabwe. Countries not suffering a decline were Angola, Benin, Chad, Equatorial Guinea, Gabon, Gambia, Ghana, Guinea, Guinea Bissau, Madagascar, Mauritius, Morocco, Niger, Nigeria, Senegal, Sierra Leone, Tunisia.

[2]Health Expenditure and Risk Factor Data from the World Development Indicator series on 36 of the 38 countries sampled (information for Equatorial Guinea and Swaziland was unavailable) at the end of the 1990s indicate statistically significant higher average health expenditure/GDP shares (3.635% versus 1.185%) and incidences of HIV (19.42% versus 5.92%) and tuberculosis (0.141% versus 0.080%) in countries with declining life expectancy. In 1994, 1 million Tutsis were killed by rival Hutus in Rwanda (14.5% of the population).

tinent. The question addressed is, have changes in life expectancy occurred in such a manner as to compromise inferences regarding the progress of welfare drawn from consumption data alone? The aforementioned data for 38 African nations in the last decade of the 20th century is used to examine the issue. In Section 1 the theoretical background for assuming the irrelevancy of life expectancy considerations is examined and formulations admitting its inclusion in calculations are developed and discussed. Section 2 reports the results of empirical welfare comparisons, excluding and including life expectancy as a factor. Conclusions are drawn in Section 3. The results indicate significant differences in the inferences drawn from comparisons which include life expectancy as a variable factor compared to inferences drawn from exercises where it is excluded from the calculus. Public intervention in the AIDS pandemic is usually rationalized on grounds of amelioration market failure due to informational asymmetries (Over, 1992); here support for intervention is garnered by demonstrating empirically that declining life expectancy has engendered a statistically significant decline in welfare in the African continent.

1. The Role of Life Expectancy in Welfare and Inequality Calculations

Assuming (for simplicity) no bequests or inheritances, popular intertemporal theories of consumer behavior (Modigliani and Brumberg, 1954; Friedman, 1957) have agents maximizing the present value of lifetime utility U subject to the present value of lifetime wealth W. Utility may be defined over a lifetime T as:

$$U = \int_0^T U(C(t))e^{-r^*t}\,dt$$

where $U(C)$ is an instantaneous utility function which is assumed parametrically constant over $0 < t < T$ and r^* is an individual's time preference discount rate which is also assumed constant. Lifetime wealth may be defined as:

$$W = \int_0^T Y(t)e^{-rt}\,dt$$

where $Y(t)$ is the individual's instantaneous income at time t and r is the interest rate (assumed constant) at which agents can freely borrow and lend. These theories, which usually assume a constant relative risk aversion form of $U(C(t))$, exhibit a consumption smoothing property (Browning and Lusardi, 1996) resulting in a consumption process of the form:

$$C(t) = e^{\frac{(r-r^*)t}{\zeta}} C_0 = e^{gt}C_0 \quad t = 0, 1, \ldots, T$$

where g, the long run consumption growth rate implied by r, r^* and the coefficient of relative risk aversion ζ, in turn implies that wealth may be written as:

$$W = \int_0^T C_0 e^{gt}\,dt = \frac{C_0}{g}(e^{gT} - 1) \tag{1}$$

The consumption smoothing property at the heart of the permanent income hypothesis presents a strong argument for employing the consumption variate in empirical studies of welfare issues since it more accurately proxies the welfare or lifetime wealth of an agent than an income variate which is redolent with transitory components. Furthermore if life expectancy, long run interest rates, time preferences and inter-temporal substitution elasticities are assumed constant across agents, wealth becomes proportionate to consumption and their distributions are equivalent for comparison purposes (for example, the distribution of the logarithm of wealth is equal to a location shifted distribution of the logarithm of consumption).

Though variations in agents' life spans are generally ignored in calculations, they may impinge upon economic welfare calculations considerably if they are not constant across agents and over time, especially if they vary systematically for some reason. Note that the partial derivatives of wealth with respect to initial consumption and life expectancy are respectively $(e^{gT} - 1)/g$ and C_0e^{gT} and the corresponding elasticities are 1 and $(gTe^{gT})/(e^{gT} - 1)$, which for plausible values of g and T can be substantially different from 0. In a similar fashion, by employing an instantaneous indirect utility function specification that underlays Working–Leser type Engel curves with constant relative prices and time preference discount rates and zero growth, an expression for lifetime utility may be obtained as:

(1a) $$U = \int_0^T (a + b\ln C_0)e^{-r^*t}dt = (a + b\ln C_0)\frac{(e^{-r^*T} - 1)}{-r^*}$$

with similar partials and elasticities with respect to $(a + b\ln C)$ and T as above with $-r^*$ replacing g. Thus with constant long run interest rates and life expectancy across agents, welfare may be reasonably approximated by a linear function of the logarithm of consumption.

Welfare Comparisons

The welfare of a society depends upon the distribution of some social felicity functional $H(U)$ (or $H(W)$) of lifetime utility (or wealth) among its constituent agents. Following Atkinson (1970, 1987), Kolm (1976) and Foster and Shorrocks (1988), welfare states can be ordered by contemplating the expected gain from moving from one distribution of utilities to another. The difference in the expected value of an $H(U)$ with the properties $(-1)^{j-1}\partial^j H/\partial U^j > 0$ $j = 1, \ldots, I$ for some $I > 0$ based upon moving from distribution function $G(U)$ to $F(U)$ each defined on $[a,b]$ is:

$$E_F(H(U)) - E_G(H(U)) = \int_a^b H(U)(dF - dG)$$

A necessary and sufficient condition for this to be positive for a given I is:

(2) $$\int_a^U (F_{i-1}(z) - G_{i-1}(z))dz \le 0 \quad \textit{for all } U$$

with strict inequality holding for some U and where, letting $f(U) = F_0(U)$, $F_i(U)$ is defined recursively as:

$$F_i(U)\int_a^U F_{i-1}(z)dz: \quad (U \le b, i \ge 1)$$

and $G_i(U)$ is defined similarly. When (2) is satisfied $f(U)$ is said to stochastically dominate $g(U)$ at order i. Though the ordering is not complete it is unambiguous and, given the properties of $H(U)$, facilitates orderings of unobservable distributions of $H(U)$ via the observable distributions of U. In terms of an underlying social welfare functional $H(U)$, First Order Stochastic Dominance relates to an ordering of social preferences based upon monotonic utilitarian social welfare functions, Second Order to $H(U)$'s that express a Daltonian social preference for mean preserving progressive transfers (Dalton, 1920) and Third Order to $H(U)$'s that express a social preference for mean preserving progressive transfers at lower utility levels.

Tests for these conditions have proliferated in the literature in recent years (McFadden, 1989; Anderson, 1996; Davidson and Duclos, 2000; Barrett and Donald, 2003); essentially the first two employ a sequence of inequality tests using techniques outlined in Wolak or Stoline and Ury (Stoline and Ury, 1979; Wolak, 1989), the latter two employ modifications of the Kolmogorov–Smirnov two sample test. In all cases the empirical density function or functions of it are employed as analogues of $F_i(U)$ $I = 1, \ldots, I$. Given a random sample U_j, $j = 1, \ldots, n$ the empirical density function, denoted $F^e(U,U_j)$, is defined as:

$$F^e(U,U_j) = \frac{1}{n}\sum_{j=1}^{n} I(U_j \le U)$$

where $I(U_i \le U)$ is an indicator function equal to 1 when its argument is true and equal to 0 when it is false. In the present context the data cannot be viewed as randomly sampled, but rather they constitute a stratified sample so that one element from each of the populations of n agent types is randomly sampled where the types have weights $\gamma_j j = 1, \ldots, n$ these weights are such that:

$$\gamma_j > 0, \quad j = 1, \ldots, n \; together \; with \sum_{j=1}^{n} \gamma_j = n.$$

In this case the empirical density function becomes:

$$F^e(U,U_j) = \frac{1}{n}\sum_{j=1}^{n} \gamma_j I(U_j \le U)$$

and the estimator functions for higher order integrals will be weighted accordingly. Put another way, the weight of each country's representative agent in the welfare function should be proportionate to that country's population size so that each individual on the African continent has the same weight in the welfare function and the same notional probability of being sampled.

If inequality rather than welfare is of interest, resort can be made to considering Lorenz dominance relationships which are equivalent to second order dominance comparisons between distributions with equal means (Foster and Shorrocks, 1988). Again the orderings are not complete but they are unambiguous and are equivalent to social welfare orderings when distribution means are genuinely equal. Complete orderings can be obtained from the comparison of Gini coefficients which are related to Lorenz comparisons through the well known relationship between the Gini coefficient and the area beneath the Lorenz curve.[3] Clearly employing these indices on consumption data alone ignores the effect of life expectancy variation in the calculations. However, a multivariate version of the Gini index can be employed to combine the effects consumption levels and life expectancy. The multivariate Gini is constructed by following the interpretation of the standard Gini as the average mean normalized difference between all points in one dimensional space and simply calculating the average mean normalized distance between all points in K dimensional space (details of the sample weighted version are provided in Appendix 1 and Anderson (2004)).

2. The Example of Africa in the 1990s

The impact of incorporating life expectancy into the welfare calculus in the various comparison techniques is examined by employing a per capita measure of consumption and life expectancy in a "national representative agent" interpretation of (1) or (1a) above. However three caveats regarding the implications of the approximations implicit in (1) and (1a) are in order before proceeding.

(1) The model employed here is the simplest version of the permanent income hypothesis; when individual time preference rates and inter-temporal transfer rates are equal it predicts complete income smoothing, that is, consumption is constant through the lifetime. This is clearly an approximation to reality. The evidence with respect to individuals and households is that this is not so, largely because of incomplete insurance and financial markets resulting in individuals' consumption profiles that tend to be slightly humped (see Blundell *et al.*, 2004, and references therein).

(2) Although the model is very much an "individualistic" one, here it is implemented across countries in a representative agent context and, Blanchard (1985) notes, economies confront cross-sectional constraints beyond the life-cycle constraints of its individuals. In addition the analysis based upon a representative agent model works when the population is stable in the sense that the age structure does not change over time, but this is an unlikely proposition in the African context. A more realistic population model (Bommier and Lee, 2003) would demand a more complex representation.

(3) Related to (2) is a question about the measure of life expectancy employed. These are usually based on age specific mortality rates in a given year and reflect the life conditions of preceding cohorts. They would only represent the life expectancy of the current cohort if that cohort experienced the same life conditions. In the African context, with the onset of the AIDS

[3]Closely related to the Gini is the polarization index developed in Esteban and Ray (1994) which permits the distinction of polarization and inequality characteristics within distributions.

TABLE 1

Summary Statistics for the 38 Countries Listed in Footnote 1

	1990	1992	1995	1997	1999
ln GDP per capita					
Population weighted mean	7.1469	7.1032	7.1024	7.1008	7.1034
Population weighted std dev	3.8765	3.8419	3.8462	3.8332	3.8317
Maximum value	9.1532	9.0604	9.0631	9.0669	9.0304
Minimum value	6.2893	6.1478	6.3023	6.1896	6.0410
Growth per capita GDP					
Population weighted mean	–0.0002	–0.0198	–0.0005	0.0019	0.0033
Population weighted std dev	0.0320	0.0415	0.0351	0.0294	0.0443
Maximum value	0.0729	0.0771	0.0776	0.4134	0.2399
Minimum value	–0.0920	–0.0991	–0.1210	–0.1148	–0.1824
Life expectancy					
Population weighted mean	50.9193	50.0648	50.3088	49.7314	47.894
Population weighted std dev	28.8035	29.1709	28.5000	28.0522	26.831
Maximum value	68.0840	69.3610	70.8873	71.9049	72.530
Minimum value	35.1937	34.1146	35.9746	37.2146	37.415
Growth in life expectancy					
Population weighted mean	0.0014	0.0011	–0.0048	–0.0052	–0.0191
Population weighted std dev	0.0106	0.0117	0.0140	0.0148	0.0234
Maximum value	0.0158	0.0152	0.0312	0.0290	0.0074
Minimum value	–0.0613	–0.0723	–0.0425	–0.0476	–0.1097
Lifetime utility (based upon [1a] assuming $a = 0$, $b = 1$ and $r^* = 0.015$)					
Population weighted mean	256.6684	255.7887	253.8871	253.3236	247.2527
Population weighted std dev	45.5922	47.7290	45.9464	45.7303	44.4127
Maximum value	369.1869	368.9974	371.2438	377.1235	381.4169
Minimum value	189.2637	179.5329	178.4715	176.5190	172.9688

pandemic, this is most unlikely to be the case, and as such the life expectancy measures employed would represent upward biased estimates of true life expectancy rates at the dates they are employed.

Data from the World Bank World Development Indicator series on per capita purchasing power parity GDP in constant 1995 $US together with Population Size and Life Expectancy were collected for 38 African countries for 1987, 1990, 1992, 1995, 1997 and 1999 (Footnote 1 contains the list of countries in the sample). GDP per capita growth rates were calculated as the annual average over years since the preceding observation. Sample weighting was based upon relative population size each year. In the sample of countries and data period under consideration, life expectancy by nation has ranged from 34.11 to 72.53. Table 1 presents summary statistics for this sample of countries together with those for a lifetime utility function combining ln(per capita GDP) and life expectancy in the form of [1a] assuming $a = 0$, $b = 1$ and $r^* = 0.015$. Values of r^*—the real risk free rate of time preference—between 0.005 and 0.03, whilst obviously altering the location and scale of the wealth calculations, did not alter the substance of the following orderings and rankings, increasing r^* above 0.04 resulted in a loss of discriminatory power in the stochastic dominance results reported in Table 2. Values of $a > 0$ increase the influence of life expectancy relative to the impact of ln(per capita GDP).

TABLE 2

STOCHASTIC DOMINANCE RESULTS

Variable	Comparison Years			
	'90(A) v '92(B)	'92(A) v '95(B)	'95(A) v '97(B)	'97(A) v '99(B)
Life expectancy	B > A 1st order	No decision	A > B 2nd order	A > B 1st order
lnGDP	No decision	A > B 1st order	No decision	B > A 2nd order
Lifetime utility	A > B 2nd order	A > B 1st order	No decision	A > B 1st order

"A > B *i*'th order" denotes the relevant distribution in year A dominates the corresponding distribution in year B at the *i*'th order of dominance implying that distribution A is socially preferred to that of B at the appropriate order.

Average ln(per capita GDP) changes very little through time; indeed growth rates of per capita GDP are not significantly different from 0 in any time period except for 1990–1992 (which shows a deterioration). This should be seen in the context of a decline in the variation of the ln(per capita GDP) measure and a population growth which has (with the exception of the effect of the Hutu-Tutsi conflict) been steady throughout the decade with an average of 2.28 percent per annum, a maximum of 3.07 percent (The Gambia) and a minimum of 1.05 percent (Mauritius). All of which suggests that the level of economic welfare in Africa based upon a pure per capita GDP measure would appear to be stable over the period and more evenly distributed. Essentially population growth has been slowest in the poorest and the richest nations which has engendered this stabilizing and equalizing effect. On the other hand life expectancy has steadily deteriorated with significantly negative growth rates in the last three observation periods. The correlation between ln per capita GDP and life expectancy appears to be diminishing (respectively 0.6974, 0.6887, 0.6863, 0.6146 and 0.4337 for the observation years in the 1990s), but is not completely out of line with corresponding statistics for the world.[4] Per capita lifetime utility shows a steady but insignificant decline over the period. So it may be deduced that on average, while the population was growing per capita GDP levels were being sustained, though the period over which that flow of income was being enjoyed was diminishing.

The results of stochastic dominance comparisons based upon a 0.05 critical region, details of which are in Appendix 2, are reported in Table 2. Employing life expectancy alone as a welfare indicator results in recording an improvement from 1990 to 1992 and successive deterioration from 1995 to 1999. On the other hand the logarithm of per capita GDP alone records a deterioration in the 1992 to 1995 period and an improvement in the 1997 to 1999 period. The "Lifetime Utility" measure, which in effect combines both GDP and Life Expectancy considerations, records declines in welfare in all periods except the 1995 to 1997 comparison during which time the progress of welfare is indeterminate. The three comparison instruments thus generate quite distinct stories regarding the progress of economic welfare.

[4]Data for 135 countries over the same period yields corresponding statistics ranging from 0.8825 to 0.8260 (see Anderson, 2004). Note that this is the correlation between the log of income and the level life expectancy; it would be larger if the correlation was between variables both in either levels or logs. Furthermore these are comparisons of cross country aggregates rather than within country individual comparisons which yield somewhat stronger mortality-income co-variability.

TABLE 3*

INCOME DIFFERENCES FOR INCREASING AND DECREASING LIFE EXPECTANCY GROUPS

	Years				
	1990	1992	1995	1997	1999
Life expectancy means and std deviations, Decreased life expectancy group	50.3556 6.9165	50.1035 7.5398	48.1569 5.6120	46.7428 4.6119	44.4061 3.4790
Life expectancy means and std deviations, Increased life expectancy group	51.5619 6.8867	52.1624 7.0982	52.7135 7.3136	53.0750 7.4677	51.7817 8.0862
ln(per capita GDP) means and std deviations, Decreased life expectancy group	7.1682 0.9856	7.0883 0.9856	7.1264 0.9465	7.1472 0.9306	7.1352 0.9183
ln(per capita GDP) means and std deviations, Increased life expectancy group	7.1227 0.5444	7.1203 0.5600	7.0756 0.5814	7.0489 0.6067	7.0679 0.6421
Difference in per capita GDP growth rates means, "t" value and $P(T > t \mid \mu_u - \mu_l \leq 0)$	0.0345 3.5234 0.0006	0.0370 3.0409 0.0022	−0.0235 −2.0822 0.9778	−0.0247 −1.2083 0.8826	0.0156 0.8415 0.2028

*All means and standard deviations are population weighted estimates as are the elements used to calculate the growth rate difference statistics.

Kernel estimates[5] in Figures 1, 2 and 3 of the respective Life Expectancy, per capita lnGDP and Lifetime Utility distributions for the years 1990, 1992, 1995 and 1999 highlight these results, with easily discernable shifts in the Life Expectancy and Lifetime Utility distributions over time standing out against the comparative stability of the distribution of the GDP measure. To examine further the effects of diminishing life expectancy, the sample of countries was split into those who suffered a decline in life expectancy over the period of the sample and those that did not. This results in 21 countries in the former category and 17 in the latter. Table 3 presents statistics which surprisingly suggest that the significantly different life expectancy experiences in the two groups have had little impact on per capita GDP which is not significantly different for the two groups (presumably reflecting the lack of correlation between ln per capita GDP and life expectancy observed earlier). Note however that the standard deviation in the two groups is significantly different (upper tail probabilities of the variance ration test range from 0.0080 in 1990 to 0.0694 in 1999), indicating greater heterogeneity in the decreasing life expectancy group. There is some evidence of differential growth rates, but this is in both directions and in the early part of the sample when the life expectancy experiences were not that different.

If the focus of attention is the inequality aspect of welfare, then the progress of an inequality indicator for the variables needs to be examined. Three inequality indices are explored, a standard GINI coefficient, a GINI coefficient adjusted for the differing population sizes of the various countries, and a polarization index introduced by Esteban and Ray (1994). In addition, multivariate versions of these statistics (details are provided in Appendix 1) which weight ln(per capita GDP) and life expectancy equally are considered. The results are reported in Table 4. With respect to the GDP measure a steady trend of increasing inequality emerges from

[5]An Epanechnikov kernel with a fixed bandwidth h = 1.06(min(std dev, interquartile range)/1.34)/$n^{0.2}$ was used following Silverman (1986).

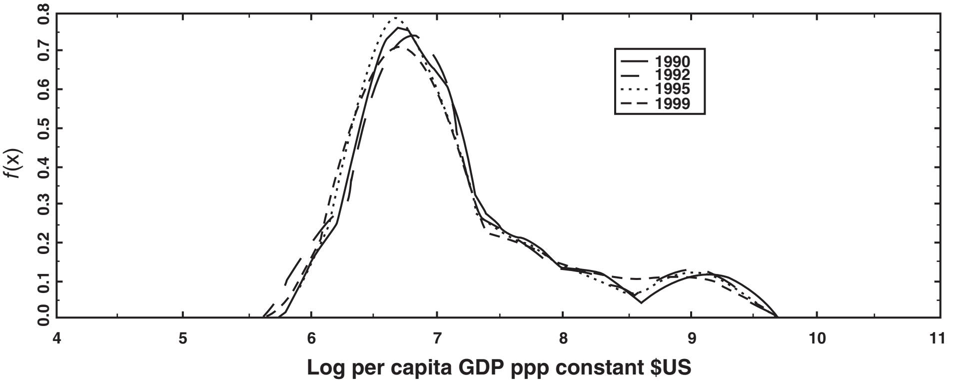

Figure 1. Population Weighted Log per capita GDP

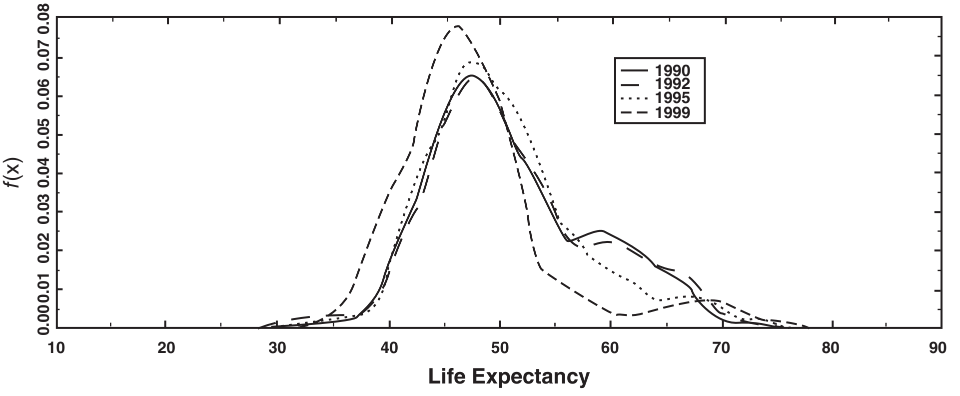

Figure 2. Population Weighted African Life Expectancy

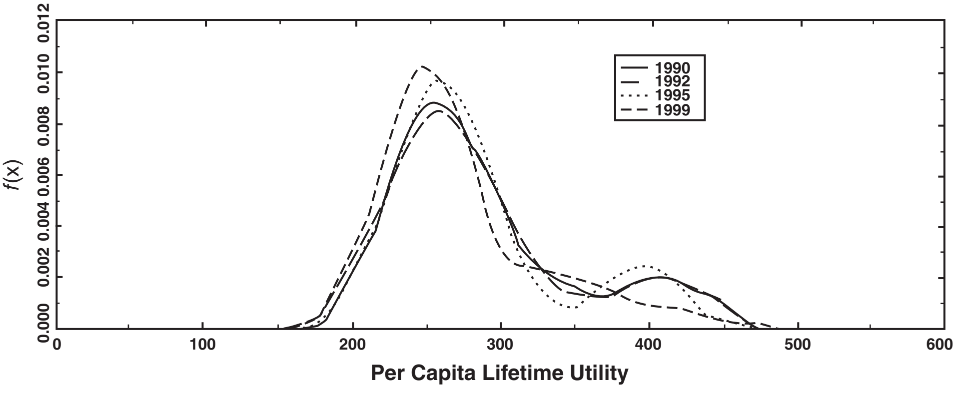

Figure 3. Population Weighted Per Capita Lifetime Utility

TABLE 4

INEQUALITY AND POLARIZATION INDICES

	ln(per capita GDP)			Life Expectancy			Wealth			Multivariate*		
Year	GINI	GINW	GINP	GINI	GINW	GINP	GINI	GINW	GINP	GINI	GINW	GINP
1990	0.0530	0.0558	0.1645	0.0741	0.0733	0.2046	0.0964	0.1010	0.2925	0.1402	0.1383	0.4410
1992	0.0540	0.0571	0.1690	0.0804	0.0782	0.2165	0.1019	0.1049	0.3029	0.1724	0.1617	0.4792
1995	0.0555	0.0548	0.1601	0.0734	0.0720	0.1995	0.0982	0.0973	0.2786	0.1280	0.1145	0.3360
1997	0.0583	0.0543	0.1569	0.0732	0.0714	0.1968	0.0980	0.0933	0.2639	0.1504	0.0956	0.2746
1999	0.0610	0.0552	0.1588	0.0783	0.0718	0.1879	0.0971	0.0921	0.2570	0.1248	0.0897	0.2322

*Results in this column relate to multivariate versions of the statistics introduced in Appendix 1 which weight GDP and life expectancy equally.

GINI corresponds to the standard Gini coefficient.

GINW corresponds to the Gini coefficient having applied sample weights to the observations.

GINP corresponds to the Esteban and Ray (1994) polarization statistic with a polarization parameter of 1 (values of 0.5 and 1.5 did not change the orderings of the results).

the standard CINI index which disappears when a population weighting scheme is used. Evidently countries which have systematically increased or decreased their per capita incomes over the sample period have relatively small populations, causing their impact to be diluted in the population weighted calculation. The polarization index indicates a decline in polarization after an initial increase. Hence increasing inequality and declining polarization are observed simultaneously, but the relative magnitudes of the changes are very small, reflecting the close proximity of the distributions in Figure 1. The life expectancy measure on the other hand indicates no discernable trend in the standard GINI, whilst the population weighted and polarization versions exhibit a distinct "inverted U" shaped pattern, indicating increasing and then decreasing inequality and polarization over the sample period. This "depolarization" effect is easily seen in Figure 2 with the disappearance of the upper mode in the 1990/1992 distributions by the end of the decade. This same "inverted U" pattern is observed in all three inequality statistics relating to the wealth variable, and in the population weighted and polarization versions of the multivariate indices with the "depolarization" effect is even more evident in Figure 3. This is no doubt related to the fact that some of the wealthier African countries (for example, South Africa and Botswana) have been hardest hit by the AIDS pandemic (World Bank, 1999). Again with respect to inequality indices the GDP based measures tell a different story from those based upon a combination of consumption and life expectancy influences.

3. Conclusions

A simple application of the Permanent Income Hypothesis has provided a means of introducing life expectancy considerations into the calculus of economic welfare comparisons. In the case of Africa, significantly different inferences regarding the progress of economic welfare are drawn from exercises which include life expectancy as a consideration as opposed to those which assume it to be irrelevant. Specifically, when life expectancy is assumed constant across agents, so that welfare differences are measured by distributional differences in consumption alone, welfare diminishes and then improves and no significant progress or deterioration over the whole period may be inferred. When life expectancy is included in the calculations a substantial decline in welfare is recorded. Similarly when inequality issues are addressed, different stories emerge dependent upon across agent life span assumptions. When life expectancy is ignored there is little apparent change in inequality indices; when it is included a "depolarization" and diminishing inequality trend is revealed Undoubtedly this is because of the dramatic changes that have taken place in Africa with respect to life expectancy and the fact that it does not appear to impinge on the maintenance of per capita GDP levels, at least in the short run. Clearly when average life spans remain constant, or when they change in harmony with consumption levels so that some function of the consumption variate alone remains an adequate proxy for lifetime utility or wealth, current empirical approaches remain appropriate. However, when life expectancy changes systematically and out of harmony with consumption patterns, as it has in the tragedy that has been Africa in the 1990s, ignoring it in welfare calculations will result in misleading inferences.

Appendix 1: A Sample Weighted Multivariate Gini Coefficient

Let the value of the k'th of K characteristics of the i'th individual be x_{ik} (in the current case the characteristics would be the logarithm of consumption and life expectancy). Suppose that sampling is stratified so that one from each of the populations of n agent types is randomly sampled and the types have weights $\gamma_j\ j = 1, \ldots, n$ where the population weights are such that:

$$\gamma_j > 0, \quad j = 1, \ldots, n \textit{ together with} \sum_{j=1}^{n} \gamma_j = n.$$

Then the sample mean value of the k'th factor be written as:

$$\bar{x}_k = \frac{\sum_{i=1}^{n} \gamma_i x_{ik}}{n}$$

and a population weighted multivariate Gini (GINIMPW) may be written as:

$$\text{GINIMPW} = \frac{1}{2K^{0.5}\left(\sum_{i=1}^{n} \gamma_i\right)^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_i^{1+\theta} \gamma_j \sqrt{\sum_{k=1}^{K} \left(\frac{x_{ik} - x_{jk}}{\bar{x}_k}\right)^2}$$

where the θ, $(0 \leq \theta < 1.6)$ is a polarization parameter (Esteban and Ray, 1994) indicating the degree of polarization reflected in the index ($\theta = 0$ corresponds to a standard multivariate Gini), so that a population weighted Gini is GINIMPW with $K = 1$ and $\theta = 0$ and the standard Gini is GINIMPW with $\gamma_i = 1$ for all I, $K = 1$ and $\theta = 0$ (see Anderson, 2004).

Appendix 2

	Comparison Years							
	'90(A) v '92(B)		'92(A) v '95(B)		'95(A) v '97(B)		'97(A) v '99(B)	
Variable	P(B > A)	P(A > B)	P(B > A)	P(A > B)	P(B > A)	P(A > B)	P(B > A)	P(A > B)
Life expectancy	0.9857	0.0321	0.4943 0.0000 0.0000	1.0000 0.0000 0.0000	0.2241 0.0001	0.1119 0.5317	0.0000	1.0000
ln(per capita GDP)	0.9562 0.1721 0.1927	0.9998 0.5177 0.4997	0.0002	0.9969 0.2580 0.4819	0.9994 0.1264 0.2627	0.9882 0.2110	0.9552 0.0436	0.9863
Lifetime utility	0.2678 0.0230	0.9997 0.5309	0.0081	0.5723	0.9108 0.2263 0.1438	0.9897 0.0926 0.5166	0.0003	1.0000

The probabilities P(B > A) (P(A > B)) reported in the above are those indicated for the Wolak (1989) multiple inequality criteria under the null hypothesis B > A (A > B). In each cell the first pair correspond to the 1st order dominance comparison, the second to the 2nd order dominance comparison and the third to the 3rd order dominance comparison. The comparisons are based upon the technique outlined in Anderson (1996) employing 10 equiprobable cells. To establish i'th order dominance of B over A the dominance of B over A must not be rejected and the dominance of A over B must be rejected at the i'th order. Since for $j > I$, i'th order dominance implies j'th order dominance orders 1, 2 and 3 are considered in succession, the process stopping when a dominance order has been established. Employing a 0.05 critical region results in the decisions reported in Table 2.

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