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PATH DEPENDENCE OF THE GENERAL PRICE LEVEL: A VALUE THEORETIC ANALYSIS

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The paper examines path dependence of price indexes, a problem that has become acute after international adoption of chain indexes for price and volume changes in the SNA93. Path dependence is an old issue of index numbers, and has been in the way of using chain indexes ever since their invention. The paper accepts chaining at its start, and tries to develop a sound deflation procedure from there. In this analysis path dependence is less an issue of appropriate formalization of a given concept, the traditional approach in index number theory, but rather a problem of developing an interpretation to a given and now worldwide recommended statistical practice. The key issue is found in the fact that money is not only the standard means of payment, concerning central banks, but also the standard of value in an economy, a feature that bears directly on the compilation of price indexes and national accounts. As a result, a chained weighted index is shown to be path independent, if it is interpreted as a change not of prices, but of the unit of account in which prices are expressed.

1. INTRODUCTION: STATING THE CASE

> I have been thinking a good deal on the subject lately but without much improvement. I see the same difficulties as before and am more confirmed than ever that strictly speaking there is not in nature any correct measure of value nor can any ingenuity suggest one, for what constitutes a correct measure for some things is a reason why it cannot be correct for others. (last sentence by David Ricardo, written to James Mill 1837 five days before his death)
>
> Value theory is not used to any significant degree as a tool of analysis in the other areas of economics. (Richard Ruggles in 1954 (1999a))

National accounts are primarily compiled in nominal terms, describing the economic situation in a specific country at a specific time. For comparisons between years, however, (or between months if inflation is surging), compilation in real terms is indispensable. Techniques vary between countries (and over time) so that harmonization must be achieved if the comparison of compilation results is to be meaningful. A milestone in this process of mutual adaptation has been marked by the appearance of the System of National Accounts, authorized by the major international economic organizations (SNA, 1993), and followed up on the European level by the European System of Accounts (ESA, 1995). Since then countries are recommended, or even obliged, to apply the chain index for determining aggregate price and volume changes.

Chain indexes have been practiced by some countries for some time; others have a different tradition, and must now convert their systems. Germany is a telling example. Laspeyres and Paasche were German statisticians, and ever since they

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published their first figures in the 1870s (Laspeyres, 1871; Paasche, 1874), Germany has faithfully adhered to the method designed by these scholars.[1] It is understandable that introducing chain indexes under such tradition meets with resistance. The "nonsense" of chain indexes is evoked against the "purity" of price change measured by the Laspeyres index (Von der Lippe, 2000), while, at the same time, unconventional theoretical solutions are sought to make the new method compatible with what one feels are important advantages of the traditional one (Hillinger, 1999). The arguments exchanged in this debate are not of national relevance only. They may deepen insight into the meaning of price and volume measures even in those countries that have comfortably accustomed themselves to chain indexes, accepting their characteristics at least as inevitable even if not as all welcome. In this paper one of the main arguments against the chain index is addressed, namely path dependence. Its meaning and causes are analyzed, and a way to arrive at path independence is shown, on the assumption, of course, that such quality of an index should be desirable at all.

As a brief, preliminary rationale at this point, to be discussed in depth later, path dependence means that the index of prices for a certain year depends not only on the variables of that year, but also on those attained earlier. It lacks thus uniqueness. Path independence is attained by the Laspeyres fixed base index, indeed, and lost by chaining. Hence the reluctance to change.

The specific concept under which we look at the issue is the function of money as a measuring rod of value. Money is the accounting unit of economic statistics, carrier of the general equivalent that remains the same no matter in which transaction it is being employed. Statement of this fact is more than trivial, as can be illustrated by the following paradox: In building national accounts, we make products of a different kind comparable and additive by measuring them in terms of their prices, i.e. the amount of money given in exchange for the product. In deflating the accounts we measure the rate of inflation by the change of the amount of products given in exchange for a unit of money. We measure products through money, and money through products. This is the vicious circle haunting Ricardo in the quote above searching for a "correct measure of value," and which forgotten task is being noted by Ruggles two centuries later.

By introducing explicitly the concept of money as a measuring rod of value into the concept of price, it will be possible to define an accounting unit that enables path independence to be maintained over time, even for chain-linked price and growth series.

2. Setting the Stage: Elements of Standard Theory

2.1. *Measurement of Inflation*

If money is the accounting unit of national accounts, its value must be established before one enters into an analysis of the accounts. The procedure is known

[1]As an illustration, Germany adheres to the fixed base principle so much that it recalculates growth rates for early years all the way back every time a change of base year is undertaken. So the argument there has always been chaining, the only difference being that whether it is applied every five years or every single year, does not apply here.

to the general public as the measurement of inflation. Speaking more technically, it is the determination of the change of the general price level. Such measurement has not always been accepted in economics. In the individualistic view established through the "marginalist revolution" notions such as an "objective value" of money, or "general" purchasing power, were considered bad method and not well received. Money had its individual value to whoever it was owned by, and no general measure could make sense, by the very definition of value. As a consequence, Richard Ruggles wrote in 1968 that "the concept of a price index as a measure of the level of prices no longer has significant support among economists" (1999b, p. 187).

Today the situation differs. The development of macroeconomics and its accompanying politics mollified the pure individualistic view of economics. Managing a government budget, designing a central bank policy, or debating a social security system cannot possibly be performed on an individualistic hypothesis. It must address average situations, use average prices, and find an average value of money, based on surveys and rules of statistics, which explicitly aim at ignoring individual differences. They pave the way from a subjective to an objective concept of value, valid for every member of a society in the same way. Two commodity baskets have been singled out in practice to serve as standards of measurement for the value of money: aggregate private consumption, on the one hand, and more broadly, aggregate gross domestic product, on the other. Even if no single individual spends his or her money in the proportions of these baskets they are widely trusted as the general standard of value of a national currency.

Let $w_i(t)$ be the nominal value of transactions collected in class i, $i = 1, \ldots,$ n of the standard commodity basket at time t, as established in the national accounts. In order to deflate them prices are needed fitting to this classification. Time series of price data are produced regularly by statistical offices. Distinguishing between pure price changes and price changes that represent quality change, they separate the monetary component, the change in purchasing power of money on the side of the buyer, from the real component of a value change pertaining to the underlying commodity offered by the seller. Let $p_i(t)$ be the time series of prices for product i, $i = 1, \ldots, n$. It is customary to write time series in discrete form, because statistical observation of infinitesimal time intervals approaching zero is impossible. But for the moment differential treatment under the assumption of continuous functions in time is the better presentation, because it enables thorough and transparent investigation of path dependence within the algebraic framework of functional analysis.

The time series $p_i(t)$, $i = 1, \ldots, n$ appears to be like what we know as a price in everyday economics, but it is not. It is merely an index of prices in the sense that its absolute value is undetermined, figuring merely as a pragmatic convention, which is often indicated by an abbreviation like (1995 = 100) in a statistical table. When about every five years the series are rebased, each time series i, $i = 1, \ldots, n$ is multiplied by a factor different for each series. Consequently, price indexes are not comparable between goods or markets. For example, to compare the index for product 1 with that of product 2 in the year 2000, one standing at 110, the other at 114, inferring that the latter prices are higher, is meaningless, because at the next rebasing, in the year 2000, both will be standing at 100 and thus be equal. What the time series are made for is not comparison of prices, but of price changes between products, relative changes to be precise. So instead of working with $p_i(t)$ we work with its logarithmic derivative:

$$\widehat{p}_i(t) = \frac{dp_i(t)}{dt} \times \frac{1}{p_i(t)}, \quad i = 1, \dots, n. \tag{2.1}$$

These derivatives represent the relative dwindling (or sometimes increasing) purchasing power of money in a particular product market i. The arithmetic average of these changes weighted with their expenditure shares yields the rate of inflation. If w_i is the nominal expenditure on product group i and w the total expenditure on all product groups of the commodity basket the share x_i of products i within that basket is given by:

$$x_i = \frac{w_i}{w}, \quad i = 1, \dots, n \tag{2.2}$$

so that:

$$\sum_{i=1}^{n} x_i = 1. \tag{2.3}$$

These variables also depend on time, of course. $\widehat{p}(t)$, the overall rate of change of the general price level, is then defined by:

$$\widehat{p}(t) = \sum_{i=1}^{n} x_i(t)\widehat{p}_i(t) = \sum_{i=1}^{n} x_i(t)\frac{dp_i(t)}{dt} \times \frac{1}{p_i(t)}. \tag{2.4}$$

Following the rules of differential calculus we arrive at a measure of change within a finite interval of time (0, t) by forming the integral:

$$p(t) = p_0 \exp\left(\sum_{i=1}^{n} \int_0^t x_i(\tau)\frac{dp_i(\tau)}{d\tau} \times \frac{1}{p_i(\tau)} d\tau \right). \tag{2.5}$$

This is called the “Divisia index” in index number theory, but it is not really an index in the sense of an operational concept, because as a variable defined in differential terms it does not lend itself directly to observation. Serving rather as a theoretical tool of analysis of change in time, it does not stand in competition to any actual index used in price statistics, which are necessarily based on finite and discrete time intervals. It represents the mathematical cornerstone to which all chain indexes defined in discrete time converge as the underlying time interval approaches zero. Equation (2.5) is thus our theoretical definition of the measure of inflation, and its inverse measures the devaluation of money as unit of measurement in the national accounts, to which appropriate approximations in discrete time must be found at a later stage.

2.2. *Path Dependence*

The Divisia formula is known to be path dependent, a flaw so serious that it has impeded full acceptance by the profession until today. Path dependence is a formal characteristic referring to a definite integral of a function of more than one variable. It means that the integral with respect to one variable depends on the values the other variables assume as the first variable grows from its lower to its upper bound of integration. For example, if you have two variables p and q, the value b of the integral

$$b(t) = \int_0^t q(\tau)\frac{dp}{d\tau}d\tau \tag{2.6}$$

depends on the values the variable q assumes as p moves between $p(0)$ and $p(t)$, the "path" of integration of the variables $q(\tau), p(\tau)$. The consequence, unwarranted in statistical application, is that the integral does not lead to a unique function of the interval limits $p(0)$, $q(0)$, and $p(t)$, $q(t)$. In particular, by choosing an appropriate function of $q(\tau)$ the integral over dp can be made to differ from zero even if p and q both return to their original starting point.

For example, a sinoidal movement of price and quantity about an equilibrium (q_e, p_e), such as

$$\begin{aligned} q &= q_e + \cos t \\ p &= p_e + \sin t \end{aligned} \tag{2.7}$$

begins and ends with $q = q_e + 1$ and $p = p_e$ over a time interval of $(0, 2\pi)$. Yet, inserting (2.7) in (2.6) yields:

$$b(2\pi) = \int_0^{2\pi} (q_e + \cos\tau)\cos\tau d\tau = \pi, \tag{2.8}$$

suggesting an increase of the "price level" b by an amount of π at each cycle. This is the famous circularity test which to fail chain indexes are generally reproached for.

One solution is to make q independent of time and thus take it out of the integral. This is the traditional Laspeyres approach ("constant quantities"). It is a simple, but also crude treatment to the problem, valid as an approximation of first order, but as experience has shown, it cannot apply to longer stretches of time over which the constancy assumption is obviously counterfactual. The example also shows, by the way, that path dependence is not a matter of aggregation of more than one product, but due to the weighting attached to the aggregating indexes. Unweighted indexes are naturally path independent.

Generally speaking, an integral of a given differential is path dependent if the differential is not complete. A complete (total) differential is a differential to which an integrating function exists. In this understanding the expressions

$$du = qdp \tag{2.9}$$

and

$$dv = pdq \tag{2.10}$$

are incomplete, because no state functions exist from which they may be derived. Hence the corresponding integrals are path dependent. In contrast, the expression

$$dw = pdq + qdp \tag{2.11}$$

is a total differential, which integrates to the function $w = pq$ independent of the path of integration. Whether or not an integrating function exists can be ascertained by forming the cross derivatives of the given differential. To a given differential

$$dw = f(p,q)dq + g(p,q)dp \tag{2.12}$$

a function, called state function $w(p, q)$, exists integrating the differential dw path independently, if and only if the cross-derivatives are equal, namely:

$$\frac{\partial f(p,q)}{\partial q} = \frac{\partial g(p,q)}{\partial p}. \tag{2.13}$$

The resulting integral is called state or potential function, because it represents a unique state of the described aggregate. Applying the rule to the example above we find that condition (2.13) holds for the differential (2.11) with $f_p = g_q = 1$, but not for differentials (2.9) and (2.10) where the cross derivatives amount to 0 and 1 respectively. The expression (2.5) for the general price level is path dependent, because there is no function $f(x_i, p_i)$ from which the differential (2.4) could be derived.

An incomplete differential can be made complete by extending it through some other function of the variables in question. Application of the so-called integrating factor defines a new differential to which an integrating function exists and can be computed independently of path. This is a second possible approach, more sophisticated than the Laspeyres method of simply ignoring variation of one variable. It also leads to well known index number formulae. Purely formal experiments, however, do not suffice. Formally, the index number problem has been resolved by committing all nations to the chain index. Room for debate is not there but in the area of concepts. What is worth putting into question is not the mathematical form of the index, but the conceptual premises underlying its operation. This is an important matter of value theory, not well covered in the literature of today, but interesting in its bearing on our problem, as we hope to show now.

3. New Elements of Value Theory

3.1. *The Meaning of a General Price Index*

The riddle occupying Ricardo when writing his last letter may be cast into the question of what it is that changes with the price of a commodity: is the change a signal of revaluation of the commodity, or of devaluation of the countervailing currency unit? Questions of such a kind have not been solved in index number theory; "they have merely been buried" (Ruggles, 1999b, p. 188). We try to unbury what may be found.

The first observation regarding Ricardo's question is simple logic. The two phenomena are mutually exclusive. You can have either a revaluation on the commodity side or a devaluation on the money side, but you cannot attribute it to both at the same time. You must decide. The second observation concerns the paradox mentioned above of circularity in measurement between the value of commodities and the value of money. Using definition (2.4) for the rate of change of the general price level we can form two extreme cases for illustration. Imagine all prices p_i rise by a common rate r, then this will most likely be interpreted as a devaluation of money. Imagine the opposite that only one price p_i rises ceteris paribus, then this will be interpreted as a revaluation of the corresponding commodity. Thus we rephrase our paradox: When one price changes this means revaluation at constant money value, when all of them change this means money devaluation at constant commodity value. Reality lying somewhere in between, where does it belong? We also must decide.

Fortunately we have not much choice. Economic practice, governments and central banks in particular, have decided already. Price indexes when employed in measuring the rate of inflation are interpreted as measuring change of the currency vis-à-vis the product group in question, not the other way around. The interpretation is supported by the statistical methodology followed in establishing price series. Care is taken to isolate what is called the "pure" price change out of the observed price change, which may be a compound change incorporating a quality change of a product in addition to the pure price change. "Quality" refers to any factor influencing the price of the product on the demand or on the supply side of the market. In as much as an observed price change measures a change in such quality, it refers to the product; in as much as it is purified of such effects, it measures the change in purchasing power of the circulating currency (Reich, 2001). How can this conceptual verification of the empirical content of a statistical price index be used to rectify a formal fault?

3.2. *Interconnecting the Prices*

There are several means to complete a differential and thus make its integral path independent. One way to complete the differential (2.4) is to follow the example (2.9)–(2.11) and write in analogy:

$$\widehat{q}(t) = \sum_{i=1}^{n} x_i \widehat{q}_i(t) \tag{3.1}$$

with:

$$q_i(t) = \frac{w_i(t)}{p_i(t)} \tag{3.2}$$

for the volume of the product class i. Adding (2.14) and (3.1) yields:

$$\widehat{p}(t) + \widehat{q}(t) = \sum_{i=1}^{n} x_i(\widehat{p}_i + \widehat{q}_i) = \sum_{i=1}^{n} x_i \widehat{w} = \widehat{w}_i. \tag{3.3}$$

This is the well known fact that the product of price index and quantity index yields the nominal value index. This amendment to definition (2.4) will not aid our purpose.

Coming from the opposite direction and postulating that the general price level should be a level in the strict sense, namely a path independent state function of the underlying price observations we can supplement the traditional definition of price index change (2.4) by writing:

$$\widehat{p}(t) = \sum_{i=1}^{n} x_i \widehat{p}_i + \sum_{i=1}^{n} \dot{x}_i \log p_i . \tag{3.4}$$

This is a total differential implying the function

$$\log p = \sum_{i=1}^{n} x_i \log p_i \tag{3.5}$$

as its integral, as can be proven by differentiation. The corresponding price index is:

$$p(t) = \prod_{i=1}^{n} p_i(t)^{x_i(t)}, \tag{3.6}$$

which leads to a Tornqvist type index.

These methods have been used in index number theory in search for an ideal index, but have not succeeded. There is a third possibility for getting rid of path dependence. A differential may be made complete by transforming independent into dependent variables. For example, if in the case of differential (2.9) we had an economic law making the variable q (quantity) dependent on the variable p (price) we would have:

$$du = q(p)dp \tag{3.7}$$

which is uniquely integrable even if the integral does not exist as an explicit function. The law determines the path. But we have no such law. We have from economic theory a law that the quantity demanded decreases with expected price, and the quantity supplied increases, their combination allowing for movement of realized prices in any direction. We have from statistical observation the stylized fact called the Laspeyres effect, which means that the Paasche index usually turns out lower than the Laspeyres index due to a negative correlation between price and volume changes. But that is not connection enough to make a formal function.

The solution is to re-interpret the variables in (2.4). Given condition (2.3) as part of the definition of the index we take the variables $\widehat{p}_i$ not as n independent variables, but as statistical realizations of one variable $\widehat{p}$ obeying to the frequency distribution $x(\widehat{p})$. The share x_i expressing the frequency of $\widehat{p}$ assuming the value $\widehat{p}_i$. Equation (2.4) may then be read as the expected value of that distribution.

The formal justification for the interpretation is supplied by the variable i on which both x_i and $\widehat{p}_i$ depend. It has no particular meaning except as a numbering of product classes. Its only purpose is to secure consistent matching of prices collected in price statistics with expenditure shares compiled in the national accounts. The order of the series i is arbitrary, any permutation is equally admissable for compilation purposes. Thus from

$$\widehat{p}(i) = \widehat{p}_i \tag{3.8}$$

we deduce the inverse function

$$i = i(\widehat{p}) \tag{3.9}$$

so that

(3.10) $$x_i = x(i) = x(i(\hat{p})) = x(\hat{p})$$

obtains. Applying (3.10) to (2.4) yields:

(3.11) $$\hat{p}(t) = \sum_{i=1}^{n} x_i(t)\hat{p}_i(t) = \int_{-\infty}^{\infty} x(\hat{p}, t)\hat{p}d\hat{p}.$$

Formally, equation (3.11) is nothing but a rewrite of (2.4), but it expresses a different meaning. Now we have not n independent variables of product price, but only one of money purchasing power. The rate of inflation is compiled as the mean of a statistical distribution reflecting the fact that a specific currency unit may be spent anywhere within the product basket in its quality as the general means of payment. The rate of inflation $\hat{p}$ changes as the distribution $x(\hat{p}, t)$ changes over time, and the overall price level is now determined uniquely by integrating the time dependent distribution over time,

(3.12) $$p(t) = \int_{0}^{t}\int_{-\infty}^{+\infty} x(\hat{p}, \tau)\hat{p}d\hat{p}d\tau.$$

Path dependence has become time dependence.

As an application of the conceptual shift in perspective, see Table 1 and Figure 1. The first shows a typical price table, namely the changes observed in Germany between 1995 and 1996 for 12 product classes of household consumption. The second shows how this information is transformed into a frequency distribution of changes of only one variable, money purchasing power. The weights are freed of their commodity specification and ordered according to size of the corresponding price change, which yields a frequency distribution of the change. Thus the price change of 2.4 percent is disconnected from its classes "housing" and "transport," and entered as a joint probability of (0.274 + 0.139) = 0.413 of a general purchasing power change of this magnitude in Figure 1. Surely, the distribution changes over time and with it its average, but its path in time is uniquely determined, because it is the only variable in the expression.

TABLE 1

PRICE CHANGES OF HOUSEHOLD CONSUMPTION EXPENDITURE IN GERMANY 1995–96

Number	Product Group	Weight	Price Change (percent/year)
1	Food	131	0.6
2	Alcoholic beverages	42	0.8
3	Clothing and footwear	69	0.7
4	Housing	274	2.4
5	Furnishings	70	0.7
6	Health	34	1.5
7	Transport	139	2.4
8	Communication	23	0.9
9	Leisure	104	0.4
10	Education	7	3.7
11	Hotels	46	1.1
12	Miscellaneous	61	0.5
	Total	1,000	$\hat{p}$ = 1.4

Source: Statistisches Jahrbuch der Bundesrepublik Deutschland 1996.

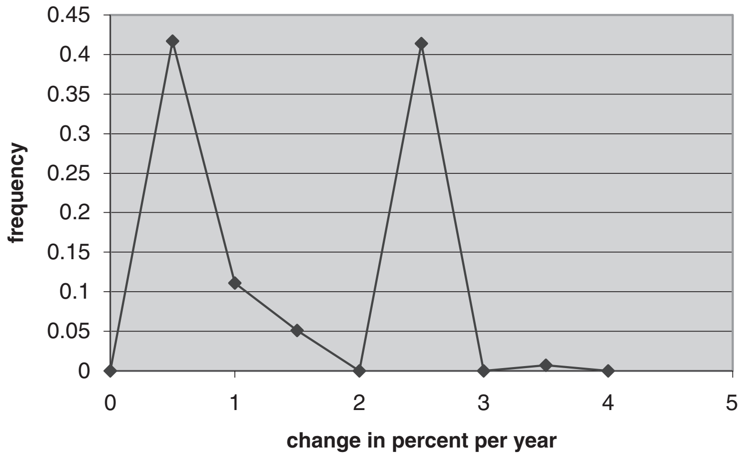

Figure 1. Relative Frequency of Purchasing Power Changes

4. Conclusions and Perspectives

4.1. *Chain Indexes*

The infinitesimal time intervals used above for defining the theoretical variable of purchasing power change are not measurable in practice. Statistical indexes, therefore, are defined in terms of finite time intervals. The optimal length of such interval is an interesting problem in itself. It must not be too short so as to absorb shocks and chaotic movements and represent an equilibrium price, but it ought also not to be too long in order to show the movement in time of this very equilibrium. At any rate, depending on which particular index number formula is used for its links, a chain index represents an approximation of first (Laspeyres index) or second (Fisher index) order to the Divisia function.

For a discrete chain index, path independence is defined as follows (Eichhorn and Voeller, 1976). Given a function $P(q^1, p^1, q^3, p^3)$ for defining a price index between periods 1 and 3, the index is path independent, if and only if, it complies with the condition

$$P(q^1, p^1, q^3, p^3) = P(q^1, p^1, q^2, p^2) \times P(q^2, p^2, q^3, p^3) \tag{4.1}$$

for any value of the intermediate variables (q^2, p^2). It is called the circular test in this tradition. The plausibility argument behind is the following. Imagine two countries A and B starting with the same values of prices and quantities in period 1 and ending at the same values of these variables in period 3. Should not the resulting measure of inflation be the same, too?

The answer to the question is: "no," the reason being that in (4.1) the variables p^t are not uniquely identified. They are measured in currency A in country A and currency B in country B, which currencies may undergo different changes in purchasing power, despite the identical nominal values shown. If, instead, equation (4.1) is interpreted not as a sampling of prices of commodities, but of expended currency units of purchasing power $p(t)$ it reads

(4.2) $$\int_{t^1}^{t^2} dp + \int_{t^2}^{t^3} dp = \int_{t^1}^{t^3} dp$$

and is time dependent, but path independent by construction.

The difference in meaning of the general price index between prices of commodities and purchasing power of a currency unit may be illustrated by means of analogy. Take the measurement of the average velocity of cars on a road. Given n cars $i = 1, \ldots, n$, you would define their average velocity v as the unweighted sum of the individual velocities v_i divided by the number of cars, $v = \frac{1}{n}\sum_{i=1}^{n} v_i$. If, however, you count the number of people m_i in each car i, and calculate an average using these weights, $u = \frac{1}{m}\sum_{i=1}^{n} m_i v_i$ you change the meaning of the resulting average. Now it is not the average velocity of cars, but of the people traveling in them that is being expressed in the number. Both figures are observable, both have a meaning, both can be used side by side, once they have clearly been distinguished.

Similarly it makes a difference in meaning whether you want to measure prices of commodities or purchasing power of money. In the first instance, prices are attached to commodities without regard to their weight. The sampling takes place in the n-dimensional (non-Euclidian) space of commodities. A corresponding average gives each commodity equal weight and is then defined as:

(4.3) $$P(t) = \sqrt[n]{\prod_{i=1}^{n} p_i(t)}.$$

This index is path independent, because it is defined as an integrated function already. If, however, you choose a weighted index, using expenditures of a certain sector of the economy as weights, the meaning of the index changes implicitly and inevitably, turning it into a measure of purchasing power of the circulating currency. And with this interpretation the weighted index is also path independent, because the variables in equation (4.1) are not independent any more, as elaborated above.

Interpreting the general price index as change of purchasing power is not a new idea. In fact, it is from such general usage that the legitimacy is drawn here to do so for individual price indexes as well. From it follows a second advantage of this theory besides path independence. Accepting that the general inflation is measured as a statistical distribution enables application of the standard tools at hand in this area. The moments of the distribution may be calculated, and they contain interesting information. From Table 1, for example, the mean yields the rate of inflation ($\widehat{p} = 1.4$). In addition, the standard deviation may be derived, which amounts to $s = 0.88$, in this case. Both figures taken together raise the question of how meaningful it is to speak of one rate of inflation only. The bi-modal character of the distribution, which is clearly visible from Figure 1, suggests rather that there are two populations underneath, responding differently to monetary policy measures.

Finally, there is room for refining the technique of price collection itself. If one is interested in a general measure of purchasing power only, stratified sampling over commodities may do the job quicker and cheaper than the full price statistics. Ways for doing so have been suggested by Nancy Ruggles (1999) for international comparisons that may equally well be applied to national surveys.

4.2. *National Accounting*

The concept of general price level as determining the standard of value in an economy has implications for the national accounts, because they depend on the standard as their unit of account. In chapter XVI on volume and price measures the SNA93 opens a door for clarification of the issue. There it distinguishes between "volume" and "real value." Volume is the result of dividing a specific price index into its corresponding nominal transactions value. Real value is achieved if you apply the general price level instead. We will briefly venture in this direction.

In the SNA, real values are defined for income flows only (para. 16.146ff). But there is no reason why the concept might not be extended to product flows as well, since it is the same money paying for products and for income. If its value depreciates in one area it does so equally in others, as long as it serves as the unique measure of value throughout the economy. Thus one may define the real value ω_i (as opposed to nominal value w_i) of an expenditure group i by:

$$\omega_i(t) = \frac{w_i(t)}{p(t)}. \tag{4.4}$$

Real variables are directly comparable over time, because the elementary unit against which they are measured is now constant if only by means of construction. Real values are additive over product groups. They inherit this characteristic from their parent nominal values. Thus real values incorporate both additivity over time and over product groups. They are the ideal accounting variables, connecting income flows and product flows in a common framework consistent over time. The statistical operation is simple, which is dividing all nominal accounts by the general price level across the board (uniform deflation).

Yet, simple as it is, the deflation procedure cannot stop there. Clearly, for all income transactions, which by definition, do not incorporate an underlying commodity structure, applying the general deflator is the only appropriate deflation method. But product aggregates carry a commodity structure of specific volumes and prices, which is useful information worth revealing. Therefore it is convenient to introduce the notion of "real price" π_i, which yields a price system measured in constant money units, comparable over time, in contrast to nominal prices p_i, namely:

$$\pi_i(t) = \frac{p_i(t)}{p(t)}. \tag{4.5}$$

Real prices are relative prices. They show the price movement relative to the general price level. The relativity shows in the fact that summed over all products the sum of the movements vanishes for the standard commodity basket. It follows from equations (2.4) and (4.5) that:

$$\sum_{i=1}^{n} x_i \hat{\pi}_i = \sum_{i=1}^{n} x_i (\hat{p}_i - \hat{p}) = 0. \tag{4.6}$$

The real price change of the commodity standard is zero. Its real price is 1 always, by definition.

Is there a meaningful distinction between volume and real value? As Neubauer established in his 1978 *Allgemeines Statistisches Archiv* article the difference is crucial in interpreting the national accounts and double deflation in particular. If you pursue a volume oriented double deflation you arrive at an analysis of production. If you do a simple, inflation oriented recalculation you arrive at real values being similar to nominal values except for the change in money value. These are two different methods of compiling value added yielding different results with different meanings behind, which are both interesting. Thus it makes sense to visualize the concept of real value as being different from volume, and to separate the relative price movement out of the nominal price movement, as that includes change of the very accounting unit in which prices are measured, and not of the commodity price itself. In fact it does not only make sense, it seems necessary, once the incomparability of nominal prices and nominal values over time is being realized.

For the national accounts, realizing that their unit of account is not constant over time might introduce caution into the direct juxtaposition of nominal aggregates for different years in one table. It would clarify the issue if at least the price level of each year would be shown in the head of the table so that users can be aware of the variability of the accounting unit, and be given the possibility to derive real values, that are comparable over time. The analysis of product transactions over time in form of the bisection into volume and price will also profit, because the two components can be made additive if properly defined. But that is another topic.

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