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PURCHASING POWER PARITIES FOR INDUSTRY COMPARISONS USING WEIGHTED ELTETO–KOVES–SZULC (EKS) METHODS

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This study has three main objectives. First, it develops a generalization of the commonly used EKS method to multilateral price comparisons. It is shown that the EKS system can be generalized so that weights can be attached to each of the link comparisons used in the EKS computations. These weights can account for differing levels of reliability of the underlying binary comparisons. Second, various reliability measures and corresponding weighting schemes are presented and their merits discussed. Finally, these new methods are applied to an international data set of manufacturing prices from the ICOP project. Although theoretically superior, it appears that the empirical impact of the weighted EKS method is generally small compared to the unweighted EKS. It is also found that this impact is larger when it is applied at lower levels of aggregation. Finally, the importance of using sector specific PPPs in assessing relative levels of manufacturing productivity is indicated.

1. INTRODUCTION

International comparisons of productivity levels by industry of origin are a key measure of economic performance next to comparisons of per capita income and other economy-wide aggregate measures. Many international comparisons of output and productivity by sector rely on the use of the exchange rate or GDP PPPs derived from the expenditure side in the International Comparisons Project (ICP) for the conversion of national currency values to a common unit (see for example, Bernard and Jones, 1996; Dollar and Wolff, 1993). However, while comparisons for the total economy can be made using an expenditure approach, it raises problems for comparisons by industry of origin (agriculture, industry, and services). This requires a subjective allocation of PPPs to individual industries as, for example, in Harrigan (1999). In addition, as expenditure represents not only the production value of the industry in question but also the added value of industries further down the chain, these PPPs require adjustment for taxes and trade

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and transport margins. While these margins can be "peeled off" as done by, for example, Jorgenson, Kuroda and Nishimizu (1987) and Jorgenson and Kuroda (1990) for Japan *vis-à-vis* the U.S., and by Lee and Tang (2000) for Canada *vis-à-vis* the U.S., this does not solve all problems. Firstly, at industry level, expenditure PPPs also need to be adjusted to exclude the relative prices of imported goods and include the relative prices of exported goods. Secondly, and most importantly, expenditure PPPs exclude price ratios for intermediate products, which account for a substantial part of output in manufacturing and services.

Since 1983, the International Comparisons of Output and Productivity (ICOP) project at the University of Groningen has been engaged in research on comparisons of sectoral purchasing power parities, real output and productivity from the production side of the economy. To date, the ICOP project has undertaken a large number of bilateral comparisons spanning several benchmark years ranging from 1975 to 1997, and including more than 30 countries. The main focus of the ICOP work has been on the comparisons of manufacturing output and productivity, but increasingly attention is being paid to service industries (see Maddison and van Ark, 2002 and van Ark and Timmer, 2001 for recent overviews of ICOP work). The most common feature of all the ICOP studies is the binary nature of the comparisons. However, since each comparison involves only the pair of countries under consideration, the totality of ICOP comparisons lacks the internal consistency (or transitivity) between all possible direct and indirect comparisons. This lack of transitivity among ICOP comparisons to date has limited empirical analysis of productivity and convergence studies involving large sets of countries.

In an earlier contribution to the *Review*, Pilat and Rao (1996) reported on a first attempt to construct consistent multilateral comparisons on the basis of ICOP data for the manufacturing sector. Using a small set of countries, Pilat and Rao constructed multilateral comparisons using branch-level PPPs derived from several binary comparison exercises. Though the Pilat and Rao paper represents a major effort to construct transitive multilateral comparisons, their attempt was only partial in that the sectoral comparisons were based on non-transitive branch-level comparisons. In this paper we show that transitivity for comparisons below the branch level can also be achieved. This overcomes the main criticism against the Pilat and Rao exercise. Hence it is possible to develop a fully multilateralized data set on producer prices in manufacturing industries which can be used for international productivity comparisons.

A second, more general, contribution of this study is to examine the feasibility of incorporating explicitly some measures of reliability of binary comparisons, such as reflected in the number of product matches made, by the Paasche–Laspeyres spread or by indices of price similarity, into the construction of transitive multilateral methods using recent major developments in the area of index number methods for international comparisons. In particular, attention will be focused on developing reliability measures which can be used in a weighted Elteto–Koves–Szulc (EKS) procedure. The EKS system has attracted renewed attention after its adoption by the OECD and Eurostat in the beginning of the 1990s as an addition to the traditional Geary-Khamis method in compiling international comparisons of GDP. Although this paper provides empirical implementations of the weighted EKS method using the ICOP-database, the suggestions for improvement of the unweighted EKS method are equally valid for the ICP work. Where necessary, differences and similarities between the ICOP and ICP databases are indicated.

The paper is organized as follows. In Section 2 the ICOP manufacturing data base is presented and previous multilateralisation attempts are discussed. The weighted EKS method is presented in Section 3 using the stochastic approach to index number construction. In the generalized system various weights can be applied to the underlying binary comparisons. These weights should reflect the reliability of the binaries. In Section 4 various reliability measures are presented and discussed. Using the example of the food manufacturing branch as our illustration throughout the paper, the differences between the alternative weighting schemes are shown. In Section 5 the weighted EKS system is applied at the branch and total manufacturing levels. Concluding remarks are given in the final section.

2. ICOP Manufacturing Data Base and Multilateralization

The ICOP data base aims at providing international output and productivity comparisons using the industry-of-origin approach. In this approach industry-specific conversion factors are derived on the basis of relative producer prices. To this end, use is made of the manufacturing census which is held frequently in most countries. The census usually provides detailed information on ex-factory output values (excluding taxes and subsidies) and quantities for a large number of detailed products. By dividing outputs by quantities, unit values are derived. These unit values can be considered as an average price, averaged throughout the year for all producers and across a group of nearly similar products. Subsequently, broadly defined products with similar characteristics are matched, for example ladies' shoes, cigarettes, cheese and car tires. So far, ICOP comparisons have been made on a bilateral basis, usually taking the U.S. as the base country. For each matched product, the ratio of the unit values in both countries is taken. This unit value ratio (UVR) indicates the relative producer price of the matched product in the two countries. Product UVRs are used to derive an aggregate UVR for manufacturing branches[1] and total manufacturing based on a particular weighting scheme using gross value of output or value added. The reader is referred to Maddison and van Ark (1988), van Ark (1993) and van Ark and Timmer (2001) for extensive descriptions of the ICOP methodology.

In contrast to standard ICOP research where three steps are used, we use a two-step aggregation procedure in this study. The first step involves the aggregation from product level to branch level. This is somewhat similar to the first step involved in the International Comparison Project (ICP) where item level prices are aggregated in order to compute PPPs at basic heading levels.[2] A major difference between ICP and ICOP is that in the ICOP work quantity and value data are available at the product level. In contrast, in the ICP value and value share data are

[1]In this paper, manufacturing branches refer to 2- or 3-digit ISIC industries within manufacturing. The total manufacturing industry is referred to as the manufacturing sector.

[2]A basic heading is a level of aggregation above which it is possible to assign weights in the form of expenditures or expenditure shares.

available only at the basic heading level. This extra information will be used in the aggregation process. The second step is the aggregation from branch level to the manufacturing sector level. At this level we have price data, in the form of PPPs derived through aggregation below the branch level, and data on the total value of output in manufacturing branches. This is comparable to the second step in ICP from basic heading level to major expenditure categories and total GDP. A particular feature of the ICOP data base is its bilateral approach. This means that ICOP does not work with a pre-specified product list as is used in the ICP. Instead, in each binary comparison it works with as many products as feasible, depending on data availability. This implies that the product-list may be very different between different sets of binaries. This has the important advantage that country characteristicity is maintained as much as possible. On the other hand, it prohibits the direct use of multilateral methods. Multilateral comparisons are expected to satisfy an important index number property, namely base-country invariance. Within ICOP, comparisons between countries A and B can only be made through binaries with the U.S. (star comparisons), and therefore, the resulting comparisons are clearly not base-invariant.

Pilat and Rao (1996) made an important step to tackle this problem for comparisons of manufacturing output and productivity. They applied various multilateral indices to Fisher UVRs at the manufacturing branch level to arrive at base-invariant UVRs for total manufacturing. This was not completely satisfactory because these Fisher UVRs at the branch level were derived in binary comparisons with the U.S. and hence were neither transitive nor base-invariant. Hence they were not “truly” multilateral. To tackle this fundamental problem, a different approach has to be taken and UVRs have to be built up right from the product level.

Rao, Selvanathan and Pilat (1995) applied this approach to two major manufacturing branches (food manufacturing and chemicals, petroleum and coal products), using a set of countries for the benchmark year 1975. The set included Brazil, Mexico, Korea, Japan, the U.K. and U.S. The chosen branches are characterized by a large number of relatively homogeneous products. For each branch they drew up a list of products (containing respectively 67 and 61 products) for which data were available in at least two of the six countries. Subsequently, they applied various multilateral systems to the product level data (Geary-Khamis and Theil-Tornqvist with coverage adjustment) to generate transitive and base-invariant PPPs at the branch level. In this study we follow a similar approach to derive multilateral manufacturing PPPs for the benchmark year 1987. The countries covered in this study are Australia, Canada, Germany, Indonesia, Japan, South Korea, Taiwan[3] and the United States. This set covers a wide range of countries at different levels of development and from different regions. As such it provides a good data set to test various multilateral methods. A new feature of this study compared to the original Rao, Selvanathan and Pilat (1995) study is the attempt to derive multilateral indices for all manufacturing branches instead of only two. Using

[3]The Taiwanese census is available for 1986 and not for 1987. Hence Taiwanese quantities refer to the year 1986. Binary UVRs for 1986 are applied to the 1987 U.S. unit values to arrive at 1987 Taiwanese unit values (see Timmer, 1998).

TABLE 1

NUMBER OF PRODUCTS BY BRANCH AND COUNTRY FOR WHICH DATA ARE AVAILABLE IN MULTILATERAL DATA SET, 1987

	U.S.	Australia	Canada	Germany	Indonesia	Japan	South Korea	Taiwan
Food, beverages and tobacco	52	28	33	29	22	17	29	11
Textile mill products	20	11	6	9	8	12	7	4
Wearing apparel	24	16	16	16	11	5	5	11
Leather products	11	5	6	6	6	4	9	7
Wood products	8	3	4	4	7	1	4	5
Paper, printing & publishing	10	2	4	5	6	5	2	6
Chemical products	45	13	28	13	18	27	35	12
Rubber and plastic products	7	1	5	2	3	5	5	2
Non-metal. mineral products	10	6	5	7	3	9	5	5
Basic & fabr. metal products	30	7	8	21	9	16	20	8
Mach. & transport equipment	13	5	4	4	5	7	9	2
Electrical mach. and equip.	26	7	2	15	9	16	12	14
Total manufacturing	256	103	121	131	107	124	142	87
Number of matches in original binary comparisons	–	178	200	271	214	193	190	119

Source: Based on matching tables from binary comparisons with the U.S. Australia from Pilat *et al.* (1993), Canada from De Jong (1996), Germany from ICOP/LCRA estimates (1996), Indonesia from Szirmai (1994), Japan and South Korea from Pilat (1994) and Taiwan from Timmer (1998).

these results, we are able to derive "more meaningful" multilateral PPPs for aggregate manufacturing.

On the basis of the original binary comparisons, we drew up a list of 256 manufacturing products for which data on prices and quantities are available for at least three countries.[4] Table 1 shows the number of products per branch and per country for which data have been included in our multilateral data set.

In Table 2 we show the prices available for the food, beverages and tobacco manufacturing branch. This branch will be used as an example throughout this paper.[5] The main point to note here is that prices are recorded for all the commodities in the U.S., but only for a subset in the case of other countries. From Table 2 it is also clear that binary price comparisons between countries can be made only on the basis of price (and quantity) data for commodities that are common to both countries. It can be seen from the table that some comparisons may be based only on a handful of commodities. Hence the number of commodities for which price data are available for any pair of countries differs across binaries. A similar situation arises in the case of ICP. This indicates that comparisons between certain pairs of countries are weaker or less reliable than some

[4]See Rao and Timmer (2000) for full details on the preparation of this data set and a complete list of product prices and quantities.

[5]Data for the other eleven branches are available upon request from the authors.

TABLE 2

Unit Values in the Food Branch in Eight Countries, in National Currencies per Quantity Unit, 1987

		Quantity unit	U.S. (US$)	Australia (A$)	Canada (Can$)	Germany (Dm)	Indonesia (Rupiah)	Japan (Yen)	Korea (Won)	Taiwan (NT$)
1	Bacon	kg	2.52		3.42				3,575	
2	Beef tallow	kg	0.31					85.0	358	
3	Beer	litre	0.63	0.74	1.17	1.35	1,093	296.5	279	14.5
4	Butter	kg	3.15	2.23	5.24	7.33	3,127	1,183.2	3,055	
5	Candy not containing chocolate	kg	2.72				2,163		2,081	
6	Canned meat	kg	2.58	3.35		4.49		617.3	3,878	
7	Cattle feeds	kg	0.16	0.19		0.36				
8	Cheese	kg	2.96			5.19		659.6	6,007	
9	Chewing gum	kg	5.27		12.22	8.47			2,644	
10	Chocolate	kg	4.46		5.21	9.12	3,785		2,342	137.4
11	Cigarettes	piece	0.03	0.02	0.02		26	3.0	20	0.4
12	Cocoa butter	kg	4.75			12.51	4,871			
13	Chicken feed	kg	0.16	0.24	0.26	0.43				
14	Concentrated milk	kg	0.84	0.89	1.66				1,932	
15	Dog and cat food	kg	0.70		0.86	1.52				
16	Dry whole milk	kg	2.46	1.87		5.24				
17	Fluid milk	kg	0.42	0.40	0.74	0.74	440		523	36.4
18	Frankfurter	kg	2.44		3.38		2,206			
19	Gin	litre	1.59			2.44		349.1	2,261	
20	Glucose syrup	kg	0.16	0.45					305	
21	Grape wines	litre	0.85	1.36	2.28	3.97				
22	Ham	kg	3.38		3.23				4,623	
23	Ice cream	litre	0.83	0.99	1.47		2,630		770	62.1
24	Ice milk	litre	0.62	1.37			390			
25	Instant coffee	kg	16.31	17.38					10,223	
26	Jams	kg	1.57	1.87		2.94				
27	Malt	kg	0.18	0.27		0.72				
28	Margarine	kg	1.14		1.41		556	277.4	837	
29	Milk powder	kg	1.86				4,369	537.9	3,113	161.4
30	Molasses	kg	0.05	0.05		0.19	65			
31	Natural cheese	kg	2.94	2.18	4.33					
32	Non-fat dry milk	kg	1.76	1.32		4.03				
33	Pig feeds	kg	0.21	0.24	0.26	0.39				
34	Redried tobacco	kg	4.56		6.49		2,699		2,969	
35	Refined sugar	kg	0.54		0.60	1.33	475	197.5	303	18.1
36	Rice milled	kg	0.24				360	317.5	702	10.8
37	Roasted coffee	kg	5.19	12.07	7.47		3,247		4,624	
38	Rum	litre	1.63			3.43			2,082	
39	Sausages	kg	2.89		3.73	8.26			1,830	
40	Semolina	kg	0.19	0.41	0.46					
41	Shortening oils	kg	0.64	1.11				176.4		
42	Soy bean oil	kg	0.39		0.58			101.6	530	
43	Soybean meal	kg	0.21		0.33					9.9
44	Starches	kg	0.18	0.54	0.44			74.7	348	
45	Tea	kg	6.62		7.29			1,666.2		110.8
46	Wheat flour	kg	0.19	0.35	0.42	0.63	261	102.6	208	16.6
47	Whiskey	litre	2.88		4.92	6.03		1,455.4	9,929	
48	Yogurt	kg	1.39	1.41	3.15	2.16	2,002			
49	Young chickens	kg	1.15	2.44	2.33	3.35	4,462			
50	Beef	kg	2.54		3.44	4.16	3,616			
51	Cocoa powder	kg	2.24		3.37	4.10	3,517			
52	Turkeys	kg	1.37	3.00	2.62	4.74				

Source: Rao and Timmer (2000), Appendix Table III.

others. One way to measure this is by looking at the number of common items for which prices are available in both countries. An attempt is made in this study to incorporate this information into the construction of multilateral index numbers.

3. The Weighted EKS Method

A. *Preliminaries and Notation*

Let p_{ij}^{b} and q_{ij}^{b} represent the unit value and the production quantity of the *i*-th matched product in *j*-th country ($i = 1, 2, \ldots, N$ and $j = 1, \ldots, M$). Superscript b refers to branch b, b varying in general from 1 to B, but in the present case we have a total of 12 branches in the manufacturing sector comparisons. We note that prices and quantities are positive whenever they are observed in a certain country. It is possible that the table of prices and quantities has blank entries as shown in Table 2.

In this study we focus mainly on the construction of price index numbers. Quantity indices can be derived indirectly using the value ratios. Let I_{jk} ($j, k = 1, \ldots, M$) represent the price index number for country k with country j as the base. Since prices in these countries are expressed in national currencies, I_{jk} can be interpreted as a measure of the purchasing power parity between currency k and j and denoted by PPP_{jk}. If PPPs are all expressed with respect to a base currency (currency of a numeraire or reference country), we may simply denote the parities by PPP_j ($j = 1, 2, \ldots M$). In such cases it is important to indicate the numeraire currency, in our case US$.

The matrix of all pairwise comparisons can be written as:

$$I_{MxM} = \begin{bmatrix} I_{11} & I_{12} & \cdot & \cdot & I_{1M} \\ I_{21} & I_{22} & \cdot & \cdot & I_{2M} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ I_{M1} & \cdot & \cdot & \cdot & I_{MM} \end{bmatrix} \tag{1}$$

We note that $I_{ij} = 1$ for all j and if the index satisfies country reversal test: $I_{jk} \times I_{kj} = 1$ for all j and k then $I_{kj} = 1/I_{jk}$ in the above matrix.

The problem is one of combining the price and quantity data to construct a matrix of price comparisons. For this purpose it is possible to use a range of standard index number formulae. In this paper we focus in particular on those methods that satisfy the "transitivity" property.

An index number formula I_{jk} is said to satisfy the transitivity property if and only if for all choices of j, k and ℓ ($j, k, \ell = 1, 2, \ldots, M$), the index satisfies

$$I_{jk} = I_{j\ell} \cdot I_{\ell k} \tag{2}$$

Equation (2) requires that the formula should be such that the application of the formula to make a direct comparison I_{jk} should result in the same measure as an indirect comparison between j and k through a link country ℓ. Note that the transitivity property ensures internal consistency of the index numbers in the matrix given in (1).

A further point of relevance is stated in the following result:

Result 1: an index number formula I satisfies transitivity property in (2) if and only if there exist M positive real numbers $\Pi_1, \Pi_2, \ldots, \Pi_M$, such that

$$(3) \qquad I_{kj} = \frac{\Pi_k}{\Pi_j} \quad \text{for all } j \text{ and } k$$

Proof of this statement is straightforward (see Rao and Banerjee, 1984). This result is quite important since it shows that when transitivity property is satisfied, all we need to measure are M real numbers $\Pi_1, \Pi_2, \ldots, \Pi_M$, and then all the necessary indices in (1) can be calculated using these M numbers, thus reducing the dimensions of the problem involved. The numbers in (3) can be given a simple interpretation, with Π_j representing the general price level in country j.

B. *Elteto–Koves–Szulc (EKS) Method*

In this section we consider the standard Elteto–Koves–Szulc (EKS) formulation (see Hill, 1982; Kravis *et al.*, 1982; Rao, Maddison and Lee, 2002) and some new variants of the EKS method proposed in this study. The Elteto–Koves–Szulc (EKS) method, proposed by Elteto and Koves (1964) and Szulc (1964), is designed to construct transitive multilateral comparisons from a matrix of binary/pairwise comparisons derived using a formula which does not satisfy the transitivity property. The EKS method in its original form uses the binary Fisher PPPs (F_{jk}: $j, k = 1, \ldots M$) as the starting point.

The computational form for the EKS index is given by

$$(4) \qquad EKS_{jk} = \prod_{l=1}^{M} [F_{jl} \cdot F_{lk}]^{1/M}$$

The formula defines the EKS index as an *unweighted* geometric average of the linked (or chained) comparisons between countries j and k using each of the countries in the comparisons as a link.

The EKS method in (4) produces comparisons which are transitive. In addition these indices also satisfy the important least squares property that indices in (4) deviate the least from the pairwise Fisher binary comparisons.[6] This property is in line with the property of characteristicity espoused in Drechsler (1973). Since Fisher index is considered to be ideal and possesses a number of desirable properties (see Diewert, 1992), the EKS method has a certain appeal since it preserves the Fisher indices to the extent possible, while constructing multilateral index numbers. However, a major problem with the EKS formula is that it gives equal weights to all linked comparisons $[F_{jl} \cdot F_{lk}]$, effectively assuming that they are of equal reliability. Following Rao (1997), it can be argued that in practice it is possible to show that some link comparisons are intrinsically more reliable than others. For example in the present study, we find that some pairwise Fisher indices are based on price data for many commodities while in other cases comparisons are based on prices for only one or two items. It is desirable to take this information into account when constructing the EKS multilateral indices. We outline a

[6]A formal proof of this is given in Rao and Baneerjee (1984).

new method that explicitly accounts for reliability measures using different criteria to consider different measures of reliability.

In order to generalize the EKS method to incorporate weights to various linked comparisons involved in equation (4), it is necessary to look at the EKS method from a different angle. Suppose we wish to derive a set of index numbers I_{jk} which are transitive and minimize the log-distance from the Fisher indices, then we

(5a)
$$\text{minimize} \quad \sum_j \sum_k (\ln I_{jk} - \ln F_{jk})^2$$
$$\text{subject to} \quad I_{jk} = I_{jl} \cdot I_{lk} \quad \forall\, j, k, l$$

Using the result stated above on transitive index numbers, the above problem can be restated as one finding $\Pi_1, \Pi_2, \ldots, \Pi_M$, which minimizes

(5b)
$$\sum_j \sum_k (\Pi_k - \Pi_j - \ln F_{jk})^2$$

Then the required index I_{jk} can be shown to be equal to the ratio $\exp(\hat{\Pi}_k)/\exp(\hat{\Pi}_j)$ where ($\wedge$) shows that these are solutions to the minimization problem. After some simple algebraic manipulation it can be shown that the EKS index is related to the solution above as:

(5c)
$$EKS_{jk} = \frac{\exp(\hat{\Pi}_k)}{\exp(\hat{\Pi}_j)} = \exp(\hat{\Pi}_k - \hat{\Pi}_j)$$

Considering further equation (5b), it is evident that $\hat{\Pi}$'s are the ordinary least squares estimators of Π's (which are the best linear unbiased estimators) in the following model specification:

(6)
$$\ln F_{jk} = \Pi_k - \Pi_j + u_{jk}$$
$$\text{with} \quad E(u_{jk}) = 0 \quad \text{and} \quad v(u_{jk}) = \sigma^2$$

Given the model specification in (6), it is possible to discriminate between different pairwise comparisons using some indicators of reliability. This can be achieved using the following model:

(7)
$$\ln F_{jk} = \Pi_k - \Pi_j + u_{jk}$$
$$\text{with} \quad E(u_{jk}) = 0 \quad \text{and} \quad v(u_{jk}) = \frac{\sigma^2}{w_{jk}}$$

where w_{jk} is a measure of reliability. If w_{jk} is large we consider that particular Fisher index, F_{jk}, to be reliable. Modified EKS indices can be obtained by applying generalized least squares to (7) or ordinary least squares to the following transformed equation:

(8)
$$\sqrt{w_{jk}}\ln F_{jk} = \sqrt{w_{jk}}\,\Pi_k - \sqrt{w_{jk}}\,\Pi_j + u^*_{jk}$$
$$\text{with} \quad E(u^*_{jk}) = 0 \quad \text{and} \quad v(u^*_{jk}) = \sigma^2 \quad \forall\, j, k = 1, \ldots M, j \neq k$$

Given the general structure underlying the process of according weights to different linked comparisons, it is necessary to specify the matrix weights to make the method operational. In the next section we consider various sets of weights for aggregation.

4. Weighting Schemes for the Weighted EKS System

A. *Introduction*

The general idea in weighted EKS is to use information on the reliability of the underlying binaries for the weighting matrix. Given the nature of the generalizations involved, it is possible to arrive at a number of alternative specifications of the matrix of weights based on how one may wish to measure reliability. We present a number of measures which have been proposed in the literature and make suggestions for new measures.

In this paper we make a distinction between two types of measures of reliability. The first is based on statistical measures of reliability. From a sampling perspective, binary comparisons based on a small number of matched products priced in both countries would be less reliabile. Similarly, if the products matched and used in a binary comparison cover only a small proportion of the total output size of the manufacturing sector (or a branch) in the two countries involved then the products considered may not be representative of the whole sector and hence any comparison based on price data for these products would be less reliable. Alternatively, reliability can also be considered from an index number theoretical perspective. It is generally known that if price relatives (price ratios) for various products are similar then the underlying price index would be the same irrespective of which index number formula one may choose to use. It can be argued that binary PPPs which are more sensitive to the choice of a particular bilateral index number formula should be assigned a lower weight. In this paper we make use of three measures of reliability reflecting this concept of reliability. These are: Hill (1999) distance measure based on the Laspeyres–Paasche spread and the price and quantity similarity indices.

B. *Statistical Measures of Reliability*

Weights Based on Number of Matches

These weights are defined using the number of items that are common to a given pair of countries. A comparison between two countries for a given branch is considered to be more reliable if it is based on more matches. Let n_{jk} be the number of common products between j and k and n^* the total number of items in the branch (according to our pre-specified list described in Section 2), then we specify:

$$
\begin{aligned}
&w_{jk} = \frac{n_{jk}}{n^*} \quad \forall j,k \quad j \neq k \\
&w_{jk} = 0 \qquad j = k
\end{aligned}
\tag{9}
$$

This measure is constrained from 0 to 1. It reaches its upperbound when all items on the list are priced in both countries and is zero when no matches were made. We put a – on the diagonal as the Fisher index will be 1 by definition and

TABLE 3

Weights Based on Number of Product Matches for All Binary Comparisons, Food Manufacturing, 1987

	U.S.	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
U.S.	–	0.538	0.635	0.558	0.423	0.327	0.558	0.212
Australia	0.538	–	0.327	0.327	0.212	0.135	0.231	0.096
Canada	0.635	0.327	–	0.346	0.308	0.192	0.365	0.173
Germany	0.558	0.327	0.346	–	0.231	0.154	0.250	0.096
Indonesia	0.423	0.212	0.308	0.231	–	0.154	0.269	0.173
Japan	0.327	0.135	0.192	0.154	0.154	–	0.288	0.135
Korea	0.558	0.231	0.365	0.250	0.269	0.288	–	0.173
Taiwan	0.212	0.096	0.173	0.096	0.173	0.135	0.173	–

Source: See Table 2.

TABLE 4

Average Coverage Ratios for Binary Comparisons, Food Manufacturing, 1987

	U.S.	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
U.S.	–	0.27	0.38	0.32	0.43	0.20	0.32	0.19
Australia	0.27	–	0.24	0.22	0.40	0.12	0.23	0.14
Canada	0.38	0.24	–	0.27	0.42	0.21	0.29	0.17
Germany	0.32	0.22	0.27	–	0.19	0.15	0.20	0.14
Indonesia	0.43	0.40	0.42	0.19	–	0.37	0.46	0.38
Japan	0.20	0.12	0.21	0.15	0.37	–	0.24	0.18
Korea	0.32	0.23	0.29	0.20	0.46	0.24	–	0.25
Taiwan	0.19	0.14	0.17	0.14	0.38	0.18	0.25	–

Source: see Tables 1 and 2.

is not meaningful.[7] Table 3 provides a matrix of weights for the food, beverages and tobacco branch. It shows that, for example, for the Canada–U.S. binary comparison prices of 33 products were used out of a total of 52 items (33/52 = 0.635), whereas for the Germany–Taiwan comparison prices of only five products have been used. Consequently, the Canada–U.S. comparison gets a bigger weight in the generalized EKS formula. Note that the table is symmetric.

Weights Based on Coverage Ratios

The second matrix considered for weighting purposes is the matrix of coverage ratios, following Rao, Selvanathan and Pilat (1995). For each country j and branch k, the coverage ratio c_{jk}, is defined as the ratio of the matched output value (output for which price information is available) to branch k output. For each pair of countries j and k, we define the weight w_{jk} as the average of the coverage ratios in countries j and k based on the products matched between countries j and k. The coverage ratios range from 0 to 1 and higher ratios imply greater reliability of the comparison. Hence they have a higher weight in the weighted EKS procedure. Table 4 shows the weight matrix based on the coverage ratio measure.[8]

[7]For pairs of countries for which no common commodities could be found, Laspeyres and Paasche indices were derived through a link involving the U.S. Consequently, a zero weight was assigned.

[8]The coverage ratios in Table 4, constructed using our multilateral data set, are lower than the coverage ratios reported in the respective ICOP binary comparisons. This is mainly because we had to drop quite a number of matches which appeared in only one binary comparison (see Section 2).

C. *Reliability Measures Reflecting Index Uncertainty*

Weights Based on Economic Distance

Selvanathan and Rao (1992) use the economic distance between two countries as a measure of the reliability of a price comparison. Economic distance was measured as the absolute log difference in per capita income. This implies that the actual price difference exhibits more variation if the countries are at different levels of development. This measure is rather crude as it delivers one weight for all aggregation levels. It will not be considered further here. Other measures are more specific and use information contained in the price data itself.

Weights Based on Hill's Distance Function

Hill (1999) provides a formal measure of reliability in terms of sensitivity based on the Laspeyres–Paasche spread (LPS) and discusses various properties of this measure. The distance between two countries j and k (d_{jk}) is measured for all j and k by

$$d_{jk} = \left| \ln\left(\frac{L_{jk}}{P_{jk}} \right) \right| \tag{10}$$

where L_{jk} and P_{jk} respectively refer to the Laspeyres and Paache (price/quantity) index numbers. Since a large value of d_{jk} represents a larger spread between the Laspeyres and Paasche indices, we postulate that the weights needed for our weighted EKS method are inversely proportional to the distance function. Thus, for all j and k ($j \neq k$)

$$w_{jk} = \frac{1}{d_{jk}} \tag{11}$$

If only one item was matched, the weight is assigned a value of zero.

Table 5 shows the weight matrix based on the Hill distance measure for the food, beverages and tobacco manufacturing branch. The table shows that, for example, binaries of Canada, Germany and Japan with the U.S. get a much higher weight than the comparisons of Korea and Indonesia with the U.S. Due to the definition of the distance given above this table is also symmetric.[9]

This weighting scheme is not without problems due to its singularity when the ratio of the Laspeyres to the Paasche index goes to one and the weights go to infinity. This is illustrated by the weight for the Australia–Korea binary which (inci-

[9]Serguei Sergueev, in a private communication, raised issues with the use of the Paasche–Laspeyres spread mainly based on the symmetric treatment of these two indices within the definition of the Hill's distance function. Following on from economic theory of producer behavior and construction of producer price index numbers (Fisher and Shell (1972) provide a good exposition), a case can be made for an asymmetric treatment where a condition that Paasche is greater than Laspeyres is imposed. Thus cases where this inequality is violated may be considered inadmissible. But such asymmetric treatment requires the following assumptions. Since Paasche and Laspeyres can provide bounds for the true producer price index only when the underlying production technology is homothetic, we need to impose homotheticity on production technology. Even under this assumption, it is necessary to further assume that producers are revenue maximizers. In our case, we are considering the whole manufacturing sector and it is not possible to identify any revenue maximizing behavior except at the firm level.

TABLE 5

Weights Based on Hill's Distance Function for All Binary Comparisons, Food Manufacturing, 1987

	U.S.	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
U.S.	–	9.37	54.29	43.35	2.36	79.65	3.65	4.20
Australia	9.37	–	11.61	26.92	11.93	4.70	1,496.83	2.68
Canada	54.29	11.61	–	118.66	28.66	12.18	6.29	7.42
Germany	43.35	26.92	118.66	–	1.92	17.05	3.74	14.33
Indonesia	2.36	11.93	28.66	1.92	–	7.65	7.56	9.65
Japan	79.65	4.70	12.18	17.05	7.65	–	6.16	10.27
Korea	3.65	1,496.83	6.29	3.74	7.56	6.16	–	5.30
Taiwan	4.20	2.68	7.42	14.33	9.65	10.27	5.30	–

Source: Table 2.

dentally) is by far the largest of all. Its Laspeyres–Paasche spread is very close to 1, although 12 matches have been made.[10]

Weights Based on Similarity Indices

The preceding specification of the weights is largely driven by the fact that binary comparisons between countries which are dissimilar (in terms of price and/or quantity structures) are intrinsically less reliable, and, therefore, less emphasis needs to be placed on the preservation of such binary comparisons. If capturing similarity in price or quantity structures is the main purpose, it seems natural to obtain measures of price or quantity similarity and use them directly in the computation of weighted EKS indices. This may be accomplished through the use of the Bortkiewicz (1924) decomposition of the Laspeyres–Paasche spread. He decomposed this spread into three components: variability in the price and quantity ratios and the strength of the correlation between the price and quantity ratios over time or across countries. This suggests a direct focus on similarity in price and quantity structures. Similarly, Allen and Diewert (1981) proposed to look at the standard deviations of the price and quantity indices.

Van Ark, Monnikhof and Timmer (1999) provide a decomposition of the spread into the different components along these lines for many binary ICOP comparisons. However, weights based on variability measures suffer from the same disadvantage as the weights based on Hill's measure: they are unbounded. An alternative is provided by the measures proposed by Van Ark, Monnikhof and Timmer (1999) which is a variant of the similarity indices used in ICP work (see Kravis, Heston, and Summers, 1982, pp. 348 ff.). The latter index was proposed to measure price similarity in the context of regionalization. Van Ark, Monnikhof and Timmer (1999) modify this index and introduce quantity weights to make the indices unit invariant. They proposed the following price and a quantity similarity index:

[10]As remarked by one referee the small number of countries in the present exercise limits the number of indirect comparisons which can be made. The use of these weights would be more meaningful in an exercise with a larger set of countries with a wider and more gradual variety of Paasche–Laspeyres spreads.

TABLE 6

WEIGHTS BASED ON PRICE SIMILARITY INDEX FOR ALL BINARY COMPARISONS, FOOD MANUFACTURING, 1987

	U.S.	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
U.S.	–	0.93	0.97	0.98	0.65	0.81	0.92	0.81
Australia	0.93	–	0.97	0.99	0.95	0.94	0.91	0.86
Canada	0.97	0.97	–	0.97	0.60	0.96	0.88	0.87
Germany	0.98	0.99	0.97	–	0.58	0.98	0.82	0.86
Indonesia	0.65	0.95	0.60	0.58	–	0.86	0.97	0.89
Japan	0.81	0.94	0.96	0.98	0.86	–	0.75	0.85
Korea	0.92	0.91	0.88	0.82	0.97	0.75	–	0.84
Taiwan	0.81	0.86	0.87	0.86	0.89	0.85	0.84	–

Source: See Table 2.

(12)
$$SP^{XU(U)} = \frac{\sum_{i=1}^{m}(p_i^X q_i^U)(p_i^U q_i^U)}{\sqrt{\sum_{i=1}^{m}(p_i^X q_i^U)^2 \sum_{i=1}^{m}(p_i^U q_i^U)^2}} \quad \text{and} \quad SP^{XU(X)} = \frac{\sum_{i=1}^{m}(p_i^X q_i^X)(p_i^U q_i^X)}{\sqrt{\sum_{i=1}^{m}(p_i^X q_i^X)^2 \sum_{i=1}^{m}(p_i^U q_i^X)^2}}$$

with $SP^{XU(U)}$ as the price similarity index between countries X and U, using quantities of country U as weights, and $SP^{XU(X)}$ as the price similarity index between countries X and U, using quantities of country X as weights. In the same way price weights are applied in the quantity similarity indexes:

(13)
$$SQ^{XU(U)} = \frac{\sum_{i=1}^{m}(p_i^U q_i^X)(p_i^U q_i^U)}{\sqrt{\sum_{i=1}^{m}(p_i^U q_i^X)^2 \sum_{i=1}^{m}(p_i^U q_i^U)^2}} \quad \text{and} \quad SQ^{XU(X)} = \frac{\sum_{i=1}^{m}(p_i^X q_i^X)(p_i^X q_i^U)}{\sqrt{\sum_{i=1}^{m}(p_i^X q_i^X)^2 \sum_{i=1}^{m}(p_i^X q_i^U)^2}}$$

with $SQ^{XU(U)}$ as the quantity similarity index between countries X and U, using prices of country U as weights, and $SP^{XU(X)}$ as the quantity similarity index between countries X and U, using prices of country X as weights.

The geometric average of the Laspeyres and Paasche type similarity indices in (12) and (13) is used as the weight required in the generalized EKS method. In contrast to the previous measure, similarity indices have a natural upper and lower-bound, the index being 0 in the case of completely dissimilar structures, and 1 in the case of complete similarity. In Tables 6 and 7 we provide the weights based on the price and quantity similarity indices for the food manufacturing branch.[11]

As expected, price similarities between OECD countries are generally higher than between OECD countries and less developed countries. Especially the price structure in Indonesia is rather different. The same is true for similarity in production quantity structure. Consequently, weights for binaries within the OECD are close to one and higher than for other binaries. Comparing the weights in both tables, it can be inferred that in general price structures are more similar than

[11]The similarity index is only useful in the case of two or more matched products. Binaries with only one or no matches, have been assigned a weight of 0.

TABLE 7

Weights Based on Quantity Similarity Index for All Binary Comparisons, Food Manufacturing, 1987

	U.S.	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
U.S.	–	0.89	0.94	0.78	0.66	0.88	0.68	0.75
Australia	0.89	–	0.95	0.87	0.37	0.89	0.74	0.61
Canada	0.94	0.95	–	0.84	0.56	0.92	0.70	0.66
Germany	0.78	0.87	0.84	–	0.25	0.85	0.64	0.66
Indonesia	0.66	0.37	0.56	0.25	–	0.69	0.90	0.89
Japan	0.88	0.89	0.92	0.85	0.69	–	0.87	0.89
Korea	0.68	0.74	0.70	0.64	0.90	0.87	–	0.87
Taiwan	0.75	0.61	0.66	0.66	0.89	0.89	0.87	–

Source: See Table 2.

quantity structures, not only for OECD countries but also for the less developed countries.

5. Empirical Implementation of the Weighted EKS Method

A. *Manufacturing Branch Level Results*

The various weighting schemes can be used at different levels of aggregation, depending on the type of data which are available. Weights based on number of matches require only information about the number of matches in each binary, whereas the other weights require both price and quantity data. So within ICP, the first weighting scheme can only be used from aggregation from basic heading level onwards as below basic heading level no quantity data are available. Within ICOP quantity data are available at all levels of aggregation. Hence we can apply all weighting schemes described above in the aggregation from product to branch level in order to calculate transitive multilateral PPPs. Table 8 presents the results for the food, beverages and tobacco manufacturing branch.

The standard ICOP PPPs refer to the Fisher PPPs which are not transitive. These are given for reference in column 1. Pilat and Rao (1996) used the Fisher PPPs as an input into aggregation at a higher level. The table presents a choice of six alternative methods which are transitive: unweighted EKS and five variants of the weighted EKS. The transitive unweighted EKS PPPs are rather different from the binary Fisher as follows from a comparison of columns 1 and 2 in the table. Here we focus on the importance of using weighted EKS rather than unweighted. In the last column we provide the maximum spread between the various weighted EKS variants and unweighted EKS. It follows that using weighted EKS can lead to both downward and upward adjustments of the multilateral PPPs. The latter is the case for Australia, Canada and Indonesia (up to 3.3 percent), while the former is the case for the other countries, down to 3.3 percent in the case of Germany. Surprisingly, these adjustments are rather small, considering the fact that the data set includes both advanced and less developed countries.

Looking at particular variants of the weighted EKS, one sees that the one based on the number of matches in column 3 is rather close to the unweighted EKS. One would expect that for countries with a large number of items for which

TABLE 8

MULTILATERAL PPPS FOR FOOD MANUFACTURING, 1987 (IN NATIONAL CURRENCY PER US$)

	Fisher	Unweighted EKS	Weighted EKS: Number of Matches	Weighted EKS: Coverage Ratios	Weighted EKS: Hill's Distance	Weighted EKS: Price Similarity Index	Weighted EKS: Quantity Similarity Index	Max Difference Between Weighted EKS and Unweighted EKS
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
U.S.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
Australia	1.20	1.06	1.09	1.06	1.07	1.07	1.08	2.8%
Canada	1.47	1.48	1.49	1.47	1.48	1.49	1.48	0.7%
Germany	2.20	2.13	2.17	2.14	2.06	2.11	2.09	−3.3%
Indonesia	1215.9	1295.0	1285.2	1304.2	1338.3	1315.9	1316.3	3.3%
Japan	307.2	327.1	321.4	320.5	321.0	330.1	328.7	−1.9%
S Korea	904.7	913.0	917.1	914.3	903.5	916.4	921.1	−1.0%
Taiwan	35.4	35.8	35.6	35.2	35.2	36.0	35.5	−1.7%

Source: Product data from Table 2. EKS using equation (4) and weighted EKS using equation (8) with weights from Tables 3 to 7.

prices are available, the weighted EKS would be closer to the binary Fisher than the unweighted EKS. This expectation is not always borne out. For example Germany has a high number of priced items (see Table 1) and the product weighted EKS is closer to the Fisher than the unweighted one as expected. However, Taiwan has a low number of product matches, but nevertheless the weighted EKS is closer to the original binary Fisher than the unweighted EKS. Similarly, the Hill's distance weighted EKS in column 5 generates some surprising result for Germany as the PPP is pulled even further away from the original Fisher, which is not expected looking at the weights in Table 5.

From a theoretical perspective weighted EKS parities are to be preferred above the unweighted version. The choice for a transitive multilateral index is then between a particular set of weights. There is no a priori reason to prefer one specification against the other. Ideally we would have liked to incorporate several measures of reliability into a single model, which is left for further research. It is also possible to take a weighted or unweighted geometric average of various columns in Table 8 as a compromise, where weights may reflect researchers' subjective ranking of the weighting specifications.

From the results for the food manufacturing branch one could conclude that the empirical impact of using weighted EKS rather than its unweighted version is only limited. Furthermore, by comparing the columns for weighted EKS another conclusion can be that the choice of a particular weighting scheme does not really matter.

However, these conclusions are not valid for all manufacturing branches. Results for the other branches show that differences between the various EKS variants can be substantial. For example, for the chemical products branch and the basic and fabricated metal products branch differences of more than 15 percent are found between the unweighted EKS and a weighted variant. The biggest differences are found for developing countries (Indonesia, South Korea and Taiwan). In most cases it is the weighting scheme based on Hill's distance which generates "extreme" results. This can be attributed to the unbounded property of this measure referred to in the previous section. But using other "bounded" weighted EKS variants even for developed countries (Australia, Canada and Germany), differences of more than 5 percent are occasionally found.[12]

B. *Total Manufacturing Sector Comparisons*

While technically the problem of aggregation is the same whether it is below or above the manufacturing branch level, the main difference is in the type of data available for this purpose. At branch level we have price data, in the form of PPPs derived through aggregation below branch level, and data on the total value of manufacturing output in the sector. The quantity information is implicit in the data. This means that we have a table of price and quantity information with no missing entries. Within ICP, the Geary–Khamis method has been the main aggregation procedure in this situation and used in all the phases of ICP until now. This is mainly because of its additive or matrix consistency property. Kravis *et al*. (1982) provide an excellent discussion about the choice of the methodology and it will not be repeated here. However, in the more recent times, the OECD and EUROSTAT comparisons of GDP are being compiled using the EKS method. This shift towards the use of EKS system is mostly due to the "characteristicity" property associated with the EKS method. In this section we will present aggregate estimates for manufacturing price relatives using EKS, showing results for both the unweighted and weighted versions. Again we can apply various weighting matrices. Here we used the same set of weights as for aggregation to branch level, except for the weighting matrix based on the number of matches which consists of ones for all binaries due to the complete table of prices and quantities at this level.

One can take the various results at branch level as input into the calculation of the EKS PPPs for total manufacturing. In Table 9 we use branch data generated by weighted EKS using the number of matches as weight, such as given in column 3 in Table 8 for food manufacturing The table shows the results of applying various weighted aggregation schemes to this data. The entries in Table 9 are scaled by the results from using unweighted EKS both below and above branch level. So in column 2 one can see that the impact of using branch data generated by weighted EKS on the results for total manufacturing ranges from –2.1 percent in the case of Taiwan to +1.6 percent in the case of South Korea. It also shows that the impact of using weighted EKS for aggregation to total manufacturing compared to using unweighted aggregation is only small. This may also, in part, be due to the limited number of countries included in the study. Comparing columns 2 to 6 reveals that the difference is never larger than 1 percent. The main conclusion from this table can be that the use of weighted EKS has more impact when used at a low level of aggregation.

[12]Results for the remaining branches can be found in Rao and Timmer (2000) and are available upon request from the authors.

TABLE 9

Effect of Using Weighted EKS Method on PPPs for Total Manufacturing, 1987

Method Below Branch / Method Above Branch	Unweighted EKS / Unweighted EKS (1)	Weighted EKS, Using the Number of Matches as Weight: Unweighted EKS (2)	WEKS Coverage Ratios (3)	WEKS Hill's Distance (4)	WEKS Price Similarity Index (5)	WEKS Quantity Similarity Index (6)
Australia	100	101.2	101.6	101.1	101.2	101.1
Canada	100	100.6	100.7	100.5	100.7	100.6
Germany	100	100.1	100.2	99.1	100.1	100.2
Indonesia	100	101.3	101.8	101.0	101.3	101.2
Japan	100	99.7	100.1	99.7	99.7	99.4
South Korea	100	101.6	102.5	100.6	101.6	101.3
Taiwan	100	97.9	98.8	98.8	97.9	97.7

Source: See Table 8.

Note: Various weighting schemes applied on branch data which are generated by weighted EKS using the number of matches as weight. Entries scaled by the results from using unweighted EKS both below and above branch level given in column 1.

6. Concluding Remarks

In this study we have considered the problem of constructing consistent multilateral comparisons using the existing ICOP database. In contrast to the earlier study undertaken by Pilat and Rao (1996), considerable emphasis is placed on the construction of transitive multilateral comparisons below the branch level. The feasibility of deriving a truly multilateral manufacturing price data set for a broad set of countries was demonstrated. This opens the way for using sectoral prices for empirical analysis of productivity and convergence involving large sets of countries. In Table 10 the importance of using sector specific PPPs in international comparisons is indicated. The first four columns show various currency converters which have been used, such as the exchange rate and the ICP GDP PPP. Comparing with the unit value ratios from ICOP (either in the binary form or the multilateral ones) it follows that especially the use of GDP PPPs in manufacturing comparisons can be highly misleading. This is particularly true for less developed countries in which the low GDP PPP is mainly determined by the low relative prices in services, not so much in manufacturing. The use of GDP PPPs in, for example, labor productivity comparisons leads to highly overstated relative performance of less developed countries such as Indonesia, South Korea and Taiwan. This is shown in the last columns of Table 10. In 1996, relative value added per hour worked in South Korea was 47 percent of the U.S. when using GDP PPPs. However, using manufacturing UVRs, this level appears to be much lower at about 32 percent. Japan on the other hand appears to perform much better when using manufacturing UVRs rather than GDP PPPs as services are relatively expensive in this country compared to the U.S.

The second contribution of this paper is the development of the weighted EKS method. The general idea in weighted EKS is to use information on the reliability of the underlying binaries for the weighting matrix. From a theoretical perspective, our preference is for the use of a weighted EKS method in the place of

TABLE 10

Comparison of Labor Productivity Using Alternative PPPs, Total Manufacturing, 1987 (National Currency per US$)

	PPPs				Relative Labor Productivity Level[a] 1996		
	Exchange Rate	GDP PPP	Binary UVR	Multilateral UVR[b]	Using GDP PPP	Using Binary UVR	Using Multilateral UVR
U.S.	1.00	1.00	1.00	1.00	100.0	100.0	100.0
Australia	1.43	1.22	1.49	1.42	57.8	47.3	49.6
Canada	1.33	1.21	1.33	1.34	85.1	77.4	76.8
Germany[c]	1.80	2.23	2.21	2.22	83.8	84.6	84.2
Indonesia[d]	1644	417	1200	1259	32.2	11.2	10.7
Japan	144.6	209.2	181.5	184.1	72.2	83.2	82.0
South Korea	823.0	474.3	699.6	688.0	46.8	31.7	32.2
Taiwan	31.9	22.7	29.60	31.1	36.9	28.3	26.9

Notes:

[a]Gross value added per hour worked put at comparable basis using PPPs.

[b]Weighted EKS using number of matched products below branch level and weighted EKS using price similarity above branch level.

[c]German value added excludes publishing industry and output in establishments with less than 20 employees.

[d]Indonesia excludes output in establishments with less than 20 employees. Labor productivity refers to value added per worker and is for the year 1995.

Source: Exchange rate and GDP PPP from PWT 5.6. Taiwan updated from 1985 PPP given in Yotopoulos and Lin (1993). Original Fisher binary unit value ratios (UVR) for Australia from Pilat *et al.* (1993), Canada from De Jong (1996), Germany from van Ark (1993), Indonesia and Taiwan from Timmer (2000), South Korea and Japan from Pilat (1994). Multilateral manufacturing unit value ratios (UVR) from this study. Relative labor productivity levels are updates from original binary studies.

the standard EKS method. The price-quantity data compiled for the multilateral exercise here from the ICOP project suggest that the binary comparisons differ in their reliability. A similar situation holds for the ICP. The weighted EKS takes into account this information. Various measures of reliability and their corresponding weighting matrices have been discussed. These include weights based on number of matches, output covered, Hill's distance function and on price and quantity similarity indices. Since each matrix leads to a different set of PPPs, it is necessary to choose the most appropriate method and the PPPs resulting from it. At this stage, we have not been able to incorporate the reliability measures simultaneously in deriving our weighted EKS indices. It would require some *a priori* weighting when introduced into the covariance structure of the disturbances discussed in Section 3. Further work in this area is needed.

Empirical implementation of the weighted EKS showed two main issues. First, we found that the choice of a particular aggregation method below the branch level is more important than the choice of a particular aggregation method above branch level. Results are more sensitive to the choice of the former than to the choice of the latter. Second, it appears that the effect of using weighted EKS compared to unweighted EKS was usually small. This is true for most branches and especially for total manufacturing. Extreme values were mainly found for the weight matrices which use unbounded weights such as those based on Hill's distance measure. Therefore use of unbounded weights is not recommended.

In conclusion, the paper has illustrated a number of extensions and generalizations of the commonly used EKS method in the context of constructing consistent multilateral comparisons of manufacturing sector prices at the branch level, as well as at the sector as a whole. The weighting patterns described here are designed to adequately handle data related issues such as the coverage ratios and the number of items sampled, as well as weights defined using various measures of reliability of the binary comparisons which provide the building blocks for transitive PPPs at the branch and sectoral level. Some of the weighting schemes devised here can also be appropriate in the case of international comparisons on the expenditure side within the ICP. It is necessary to account for data related features adequately in the process of deriving meaningful and reliable measures of PPPs for international comparisons of real income, output and productivity.

References

Allen, R. C. and W. E. Diewert, "Direct versus Implicit Superlative Index Number Formulae," *Review of Economics and Statistics*, 63, 430–5, 1981.

Bernard, A. B. and C. I. Jones, "Comparing Apples to Oranges: Productivity Convergence and Measurement Across Industries and Countries," *American Economic Review*, 86(5), 1216–38, 1996.

Bortkiewicz, L. von, "Zweck und Struktur einer Preisindexzahl," *Nordisk Statistik Tidskrift*, 3, 1924.

De Jong, G., "Canada's Post-war Manufacturing Performance: A Comparison with the United States," Research memorandum GD-32, Groningen Growth and Development Center, Groningen, 1996.

Diewert, W. E., "Fisher Ideal Output, Input and Productivity Indexes Revisited," *Journal of Productivity Analysis*, 3, 211–48, 1992.

Dollar, D. and E. N. Wolff, *Competitiveness, Convergence, and International Specialization,* MIT Press, Cambridge, MA, 1993.

Drechsler, L., "Weighting of Index Numbers in Multilateral Comparisons," *Review of Income and Wealth*, March 1973.

Elteto, O. and P. Koves, "On an Index Computation Problem in International Comparisons" (in Hungarian), *Statistztikai Szemle*, 42, 507–18, 1964.

Fisher, F. M. and K. Shell, *The Economic Theory of Price Indices: Two Essays on the Effects of Taste, Quality, and Technological Change*, Academic Press, New York, 1972.

Harrigan, James, "Estimation of Cross-Country Differences in Industry Production Functions," *Journal of International Economics*, 47(2), 267–93, 1999.

Hill, R. J., "Comparative Price Levels Across Countries Using Minimum-Spanning Trees," *The Review of Economics and Statistics*, 81(1), 135–42, 1999.

Hill, T. P., *Multilateral Measurements of Purchasing Power and Real GDP*, Eurostat, Luxembourg, 1982.

Jorgenson, D. W. and M. Kuroda, "Productivity and International Competitiveness in Japan and the United States, 1960–1985," in C. R. Hulten (ed.), *Productivity in the U. S. and Japan*, Studies in Income and Wealth, 51, University of Chicago Press, Chicago, 1990.

Jorgenson, D. W., M. Kuroda and M. Nishimizu, "Japan–U.S. Industry-level Productivity Comparisons, 1960–1979," *Journal of the Japanese and International Economics*, 1, 1–30, 1987.

Kravis, I. B., A. Heston, and R. Summers, *World Product and Income*, Johns Hopkins, Baltimore, MD, 1982.

Lee, F. C. and J. Tang, "Methodological Issues in Comparing Productivity Levels and International Competitiveness in Canada and the United States," paper presented at AEA meetings, Boston, 7–9 January 2000.

Maddison, A. and D. S. Prasada Rao, "A Generalized Approach to International Comparisons of Agricultural Output and Productivity," Research Memorandum GD-27, Groningen Growth and Development Centre, Groningen, 1996.

Maddison, A. and B. van Ark, *Comparisons of Real Output in Manufacturing*, Policy, Planning and Research Working Papers WPS5, World Bank, Washington DC, 1988.

———, "The International Comparison of Real Product and Productivity," in A. Maddison, D. S. Prasada Rao and W. F. Shepherd (eds), *The Asian Economies in the Twentieth Century*, Edward Elgar, Aldershot, U.K. and Brookfield, U.S., 5–26, 2002.

Pilat, D., *The Economics of Rapid Growth. The Experience of Japan and Korea*, Edward Elgar, Aldershot, U.K. and Brookfield, U.S., 1994.

Pilat, D. and D. S. Prasada Rao, "A Multilateral Approach to International Comparisons of Real Output, Productivity and Purchasing Power Parities in Manufacturing," Research Memorandum, no. 440, Faculty of Economics, University of Groningen, 1991.
———, "Multilateral Comparisons of Output, Productivity and Purchasing Power Parities in Manufacturing," *Review of Income and Wealth*, 42(2), 113–30, 1996.
Pilat, D., D. S. Prasada Rao, and W. F. Shepherd, "Australia and United States Manufacturing: A Comparison of Real Output, Productivity Levels and Purchasing Power, 1970–1989," *COPPAA Series #1*, Centre for the Study of Australia-Asia Relations, Griffith University, Brisbane, 1993.
Rao, D. S. Prasada, "Aggregation Methods for International Comparison of Purchasing Power Parities and Real Income: Analytical Issues and Some Recent Developments," *Bulletin of International Statistical Institute*, LVII, 197–200, 1997.
Rao, D. S. Prasada and K. S. Baneerjee, "A Multilateral System of Index Numbers Based on Factorial Approach," *Statistiche Hefte*, 27, 297–312, 1984.
Rao, D. S. Prasada, A. Maddison and B. Lee, "International Comparison of Farm Sector Performance: Methodological Options and Empirical Findings for Asia-Pacific Economies, 1900–1994," in A. Maddison, D. S. Prasada Rao and W. F. Shepherd (eds), *The Asian Economies in the Twentieth Century*, 27–56, 2002.
Rao, D. S. Prasada, E. A. Selvanathan and D. Pilat, "Generalized Theil–Tornqvist Indices with Applications to International Comparisons of Prices and Real Output," *Review of Economics and Statistics*, 352–60, 1995.
Rao, D. S. Prasada and M. P. Timmer, "Multilateralisation of Manufacturing Sector Comparisons: Issues, Methods and Empirical Results," Research Memorandum GD-47, Groningen Growth and Development Centre, July 2000.
Selvanathan, E. A. and Prasada Rao, D. S., "An Econometric Approach to the Construction of Generalized Theil–Tornqvist Indices for Multilateral Comparisons," *Journal of Econometrics*, 54, 335–46, 1992.
Szirmai, A., "Real Output and Labour Productivity in Indonesian Manufacturing, 1975–90," *Bulletin of Indonesian Economic Studies*, 30(2), 49–90, 1994.
Szulc, B., "Index Numbers of Multilateral Regional Comparisons" (in Polish), *Przeglad Statystyczny*, 3, 239–54, 1964.
Timmer, M. P., "On the Reliability of Unit Value Ratios in International Comparisons," Research memorandum GD-31, Groningen Growth and Development Centre, Groningen, 1996.
———, "Catch Up Patterns in Newly Industrializing Countries. An International Comparison of Manufacturing Productivity in Taiwan, 1961–1993," Research memorandum GD-40, Groningen Growth and Development Centre, Groningen, 1998.
———, *The Dynamics of Asian Manufacturing. A Comparative Perspective in the Late Twentieth Century*, Edward Elgar, Cheltenham, 2000.
van Ark, B., *International Comparisons of Output and Productivity*, Monograph series no. 1, Groningen Growth and Development Center, Groningen, 1993.
van Ark, B. and M. P. Timmer, "PPPs and International Productivity Comparisons: Bottlenecks and New Directions," Paper presented at the Joint World Bank–OECD Seminar on Purchasing Power Parities: Real Advances in Methods and Applications, Washington DC, 30 January–1 February, 2001.
van Ark, B., E. J. Monnikhof, and M. P. Timmer, "Prices, Quantities and Productivity in Industry: A Study of Transition Economies in a Comparative Perspective," in R. Lipsey and A. Heston (eds), *International and Interarea Comparisons of Prices, Income and Output*, NBER, Chicago University Press, 327–64, 1999.
Yotopoulos, P. A. and J. Lin, "Purchasing Power Parities for Taiwan," *Journal of Economic Development*, 18(1), 7–51, 1993.