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MULTILATERAL COMPARISONS OF PRODUCTIVITY, TERMS-OF-TRADE AND FACTOR ACCUMULATION

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This paper proposes a new approach for multilateral comparisons using index numbers. The new approach combines two recently-proposed innovative techniques to examine differences among economies at various levels. The Minimum Spanning Tree algorithm, based on the idea of minimizing substitution bias of bilateral comparisons, provides a possible ordering for panel data. Making use of the suggested ordering, bilateral Törnqvist price and quantity indexes are calculated and multilateral indexes are obtained by chaining. An index-number based approach is then used to decompose the differences in GDP at the bilateral level. Different sources that contribute to the differences in GDP are considered: productivity differences, terms of trade differences, factor endowments differences and domestic output price differences. The newly formed indexes are base-invariant which provides strong support for using the technique for multilateral comparisons. An illustration of the technique using data from China and four OECD countries is included.

1. INTRODUCTION

This paper aims to use the combination of two innovative approaches, which are backed up by strong theoretic foundations, to create a new way for performing international comparisons using index numbers. In particular, we propose a new method to measure the differences in gross domestic product (GDP) between countries over time and the extent to which these can be explained by differences in inputs, prices and productivity.

This paper examines a wide range of technical issues, which have long been of interest in the literature of multilateral comparisons. These include methods for the linking of bilateral comparisons, the decomposition of GDP differences between countries over time and the conversion of national currency values to a common unit.

In recent years, there has been rapid growth in interest in the methodological problems relating to multilateral comparisons (Caves, Christensen, and Diewert, 1982; Selvanathan and Prasada Rao, 1992; Dowrick and Quiggin, 1994, 1997; Nuxoll, 1994; Pilat and Prasada Rao, 1996; Hill, 1997; Neary, 1997). Multilateral comparisons involve the comparisons of, for example, productivity, price or quantity of more than two observations simultaneously. These comparisons can be done in two ways. The first is to construct bilateral indexes which are intransitive

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and then link them by chaining to obtain transitivity, whereas the second way is to use multilateral indexes which are transitive by construction.[1] In this paper, we are interested in dealing with the first case and a feature of the paper is the introduction of a new way for ordering panel data.

Some examples of the bilateral indexes are the Laspeyres, Paasche, translog/Törnqvist and the Fisher indexes. The Fisher and Törnqvist indexes are "superlative".[2] The Fisher has several advantages over other bilateral indexes. One well-known advantage is that it is relatively free of substitution bias. Since it is constructed to lie between Paasche and Laspeyres purchasing power parities (quantity indexes), which are thought to be the lower and upper bounds of the true indexes, once the Paasche and Laspeyres spread (PLS) shrinks, the Fisher index is close to the true index.[3] Törnqvist indexes have been found to be similar in magnitudes to the Fisher indexes (Diewert, 1976, 1992). Both of them are geometric means of indexes based on base and current period share weighted averages of price relatives. In this paper, we use the Törnqvist bilateral index as the building block for multilateral comparisons, as the proposed decomposition technique is based on Törnqvist indexes.

For multilateral comparisons, after the bilateral indexes are constructed, linking is carried out. One serious problem associated with the use of bilateral indexes is in regard to the ordering of observations before chaining takes place. In a time-series context, it is common to chain bilateral indexes chronologically as adjacent periods tend to have similar patterns of price and consumption.[4] It is the ordering of cross-section data that requires some thought. Some criterion needs to be set. At present, there appears to be an absence of methods suggested for application to the problem of ordering in the context of panel data. By making use of the PLS,[5] the Kruskal's MST algorithm provides a possible criterion (Hill, 1999b).

In this paper, we first apply the Kruskal's MST algorithm (Hill, 1999a, 1999b) in the context of panel data to give a possible ordering for linking bilateral comparisons. Then, we apply Fox's (1999) relative GDP-decomposition technique (which is based on a technique proposed by Diewert and Morrison (1986); see also Kohli (1990) and Fox and Kohli (1998)) at each of the bilateral comparison levels. Contributions of domestic price, terms of trade (TOT), labor and capital quantity to GDP[6] can then be obtained. Prior to the above steps, national currencies are converted to a common unit using PPPs.

[1]Note that the indexes obtained from the first way are different from true multilateral indexes, as they still depend on the order by which the bilateral comparisons are linked. True multilateralization implies that all the original bilateral indexes are replaced by new indexes and these new indexes are all transitive.

[2]Diewert (1976) has defined an index number which is exact for a flexible function as "superlative." A flexible function can provide a second order approximation to an arbitrarily twice-differentiable linearly homogeneous function at a point.

[3]Refer to Diewert (1992) and Hill (1999a) for arguments for the use of Fisher index for binary comparisons.

[4]See Diewert (1996) for arguments for chronological chaining of time series data, and see Hill (1999b) for the justification of this using Kruskal's Minimum Spanning Tree (MST) algorithm.

[5]The PLS is the numerical difference of these two indexes. It reflects the variability in relative quantities and prices, and the correlation between, for example, two countries.

[6]Note that we are using nominal GDP in this paper because this allows us to assess the contribution of price differences to differences in GDP. In other words, if we use real GDP, the contribution of price differences could not be assessed.

An application using data on five countries over the period 1986 to 1992 is considered. The countries in the sample are China, Japan, the U.S., Canada and Australia.

The paper is structured as follows. Section 2 describes the method used to calculate purchasing power parities (PPPs), the methodology of Kruskal's Spanning Tree Method and Fox's decomposition technique for panel data. Section 3 describes the data set. Section 4 discusses the results, and Section 5 concludes.

2. Methodology

This section provides a brief summary of the methodology used in this paper. Before the introduction of the new approach for multilateral comparison, technical issues regarding the conversion of national currency values are discussed.

Calculation of PPPs Using the Geary-Khamis Method

To facilitate the comparison of GDP between countries, we have to convert the national currencies of each country in each year to a common unit. There are at least two ways we can do this. Academic economists favor the use of PPPs, while exchange rates are also commonly used. As is widely known, exchange rates may not adequately reflect real price differences between countries, so we focus on converting the national currency values using PPPs. Regarding the calculation of PPPs, we make use of a variation of the Geary-Khamis (GK) method. The GK method[7] originated from Geary (1958), and was modified by Khamis (1967, 1969, 1970, 1972).

In this paper, due to the fact that our price data are expressed in index form, with the quantity series represented by constant price series (1985 level) and prices calculated as current price series/quantity, rather than in prices and physical quantities for sets of goods and services as originally propounded by the GK method, a slight variation of the GK method had to be made[8]:

$$\overline{P}_i = \sum_{j=1}^{M} \frac{P_{ij}}{\hat{P}\hat{P}\hat{P}_j} \left[\frac{Q_{ij}}{\sum_{j=1}^{M} Q_{ij}} \right] \tag{1}$$

[7]The original purpose of the GK method was "to obtain unique global exchange rates, or PPPs and average prices in a uniform currency to enable the calculation of different types of international indices, and to aggregate over different commodities and countries for any meaningful economic flow . . ." (Khamis, 1984). The following shows the two linear equations which form the basis of the GK system (i) $P_i = \sum_{j=1}^{M} \frac{P_{ij}}{PPP_j} \left[\frac{Q_{ij}}{\sum_{j=1}^{M} Q_{ij}} \right]$ (ii) $PPP_j = \frac{\sum_{i=1}^{N} P_{ij} Q_{ij}}{\sum_{i=1}^{N} P_i Q_{ij}}$ where i represents different categories of goods and j represents different countries. The first equation calculates the international prices for each category of goods, which reflects relative category values, whereas the second equation calculates the country PPPs, which depict relative country price levels. These equations are estimated simultaneously. The GK method in this present form is applicable to data pertaining to one single year.

[8]See Khamis (1984) for other possible variants of the Geary equations.

$$\hat{P}\hat{P}\hat{P}_j = \frac{\sum_{i=1}^{N} P_{ij} Q_{ij}}{\sum_{i=1}^{N} \overline{P}_i Q_{ij}} \quad (2)$$

where P represents price index and Q represents quantity index, i represents different components of GDP and j represents different observations (a particular country at a particular year). Hence P_{ij} represents price index of a particular component of GDP for a country at a particular year, and similarly for Q_{ij}.

$\hat{P}\hat{P}\hat{P}_j$ calculates the observation (each observation refers to a particular country over a particular time period) PPPs, which represents the relative observation price levels (to U.S. 1985 level) while $\overline{P}_i$ represents international prices for each component (i.e. consumption, investment, export and imports) of GDP, which reflects relative component values. The numerator of equation (1) is GDP, which is calculated using the expenditure approach: consumption + investment + export − imports.

In this paper, equations (1) and (2) are applied to panel data and the resulting PPPs are used to convert the price data. The PPPs relative to U.S. 1985 are shown in Table 1. Once the conversion has been done, the MST algorithm can be used to generate the ordering of observations for comparison.

The Spanning Tree Method

As mentioned in the previous section, we make use of the MST algorithm to obtain an ordering for the panel data. By using the idea of minimizing the substitution bias,[9] we ensure that similar observations are linked together. A spanning tree is a connected graph which links a set of vertices (periods or countries) together in such a way that there is only one path between any pair of vertices. A spanning tree with K vertices should have $K - 1$ edges. By connecting the edge of two vertices, a bilateral index number comparison can be obtained. Multilateral indexes can then be obtained through chaining of these bilateral indexes.

There are K^{K-2} different possible spanning trees in the case of K vertices. Since there are five countries in our data set, each covering the period 1986–92, there are 35 vertices and 34 edges for each of the 35^{35-2} spanning trees.

The Minimum Spanning Tree

The MST is the one which represents the "best possible" way of linking the vertices. Kruskal's algorithm uses the criterion that the linked vertices should have the smallest summed $K - 1$ PLS bilateral indexes.[10] Since Paasche and Laspeyres indexes provide the lower and upper bounds on the true indexes, the MST indi-

[9]Substitution bias arises when we construct an index using quantity or price vector of one particular observation as weights. In this case, the fact that there are corresponding relative changes in price or quantity is not taken into account. The size of the substitution bias depends on the size of substitution elasticities and the magnitude of relative price changes. The PLS provides a measure of substitution bias.

[10]For the logic behind this criterion, see Hill (1999a).

rectly implies that the generated multilateral indexes are the closest on average to the true indexes.

One justification for the use of the spanning tree method is that it lets the data decide the spanning tree structure, instead of imposing it arbitrarily. It provides a contribution to the index number literature via the ordering of observations, by which the resulting multilateral comparisons are the least sensitive to the bilateral index formula used.

Decomposition of GDP in a Panel Context

Once an ordering of the observations (each observation represents a country in a particular year) has been decided by the MST, bilateral comparisons can be calculated and the decomposition of GDP can then be carried out.

The technique used for the decomposition originated from Diewert and Morrison (1986) and was subsequently developed by Kohli (1990) and Fox (1999). The former two papers consider the decomposition of the GDP growth of a country in a time-series context, whereas the latter considers the decomposition in a panel context. In this paper, we extend the technique to decomposing differences in GDPs for panel data. This approach has several advantages: (1) It does not ignore the fact that many countries are open economies, rather than closed economies. Effects of TOT movements on economic growth are explicitly included. (2) It makes it possible to work with relatively short time periods. (3) It allows the assessment of the contribution of increases in domestic output prices to nominal GDP growth. (4) With the assumption that the technology can be approximated by a translog GDP function, the decomposition of GDP can be obtained by using the data alone, without the knowledge on the technology.

The approach to decomposing GDP differences has a solid theoretical foundation. Following theorem 1 in Diewert and Morrison (1986), the geometric mean of the Laspeyres and Paasche-type productivity indexes is set up to measure the impact on GDP that has occurred solely due to the change in technology. The Laspeyres-type productivity index, $R^{L}_{t,t-1}$ is shown in equation (3) and the Paasche-type productivity index is shown in equation (4).

$$R^{L}{}_{t,t-1} = \frac{\pi(p_{t-1}, x_{t-1}, t)}{\pi(p_{t-1}, x_{t-1}, t-1)} \tag{3}$$

$$R^{P}{}_{t,t-1} = \frac{\pi(p_{t}, x_{t}, t)}{\pi(p_{t}, x_{t}, t-1)} \tag{4}$$

π represents the GDP function, $\pi = \pi(p_t, x_t, t) = \max_{yt}\{ p'_j y_t \colon (y_t, x_t) \in T_t \}$ and T_t is the production possibility set at time t. P and x stand for prices and quantities respectively.

According to Diewert and Morrison (1986), if the GDP function has a translog form as shown in equation (5), the geometric mean of the Paasche and Laspeyres-type productivity indexes is exactly equal to a Törnqvist output index divided by a Törnqvist input index, which is shown in equation (6). In this case, a complete decomposition of GDP growth can be obtained. See equation (7).

$$\ln\pi = \alpha_0 + \Sigma\alpha_i \ln p_i + \Sigma\beta_j \ln x_j + {}^1/_2\, \Sigma\Sigma\gamma_{ih} \ln p_i \ln p_h + {}^1/_2\, \Sigma\Sigma\phi_{jk} \ln x_j \ln x_k$$
$$+ \Sigma\Sigma\delta_{ij} \ln p_i \ln x_j + \Sigma\delta_{iT} \ln p_i t + \Sigma\phi_{jT} \ln x_j t + \beta_T t + {}^1/_2\, \phi_{TT} t^2 \tag{5}$$
$$i,h \in \{X,M,I,G,C\};\ j,k \in \{L,K\}$$

where $\Sigma\alpha_i = 1$, $\Sigma\beta_j = 1$, $\gamma_{ih} = \gamma_{hi}$, $\phi_{jk} = \phi_{kj}$, $\Sigma\gamma_{ih} = 0$, $\Sigma\phi_{jk} = 0$, $\Sigma_i\delta_{ij} = 0$, $\Sigma_j\ \delta_{ij} = 0$, $\Sigma\delta_{iT} = 0$ and $\Sigma\phi_{jT} = 0$. X, M, I, G, C, L, K are export, import, investment, government purchases, consumption goods, labor and capital respectively. Labor and capital are exogenously given inputs. Imports are treated as a negative output according to the standard definition of GDP (Burgess, 1974; Kohli, 1978).

$$R_{t,t-1} = \frac{\Gamma_{t,t-1}}{P_{t,t-1} \cdot X_{t,t-1}} \tag{6}$$

where:

$\Gamma_{t,t-1}$ is (1 plus) the rate of increase in nominal GDP between times $t - 1$ and t
$P_{t,t-1}$ is Törnqvist output price index
$X_{t,t-1}$ is Törnqvist fixed input quantity index

R measures the effect of change in technology (i.e. TFP change) on GDP between t and $t - 1$. Rearrange equation (6):

$$\Gamma_{t,t-1} = R_{t,t-1} \cdot P_{t,t-1} \cdot X_{t,t-1} \tag{7}$$

Equations (3) to (7) are specifically designed for decomposing GDP growth for time series data; however, in this paper, besides decomposing GDP growth within countries, we are also interested in decomposing differences in GDP between countries over time. In other words, we want to apply the technique to panel data. To achieve this aim, an extension of the Diewert and Morrison (1986) decomposing technique is needed and it is discussed as follows.

Modification of the Diewert and Morrison (1986) technique

In the case of two observations (a and b, each represents a particular country at a particular time), the geometric mean of the Paasche and Laspeyres-type productivity indexes of them, denoted by $R^P{}_{a,b}$ and $R^L{}_{a,b}$ is equal to $R = (R^P{}_{a,b} R^L{}_{a,b})^{1/2} = \dfrac{\Gamma}{P \cdot X}$. Differences in GDP between these two observations can be decomposed into differences in productivity, prices and quantities, i.e. $\Gamma = R \cdot P \cdot X$, where Γ is the ratio of GDP between the two observations in comparison. It is equal to 1 if GDP of the two observations in comparison is the same. If it is greater than 1, then the GDP of the observation is greater than that of the base observation. Similarly, if it is smaller than 1, the GDP of the observation is smaller than that of the base observation. R is the productivity index, P is the price index and X is the quantity index. Note that the formulae here are constructed in the same way as those in equation (6). The only difference is that they are constructed to be applied to panel data here, whereas those in equation (6) are constructed for time series data.

$$\Gamma = \frac{P_b X_b}{P_a X_a} \tag{8}$$

$$(9) \qquad P = \exp\left[\sum_{n=1}^{N} {}^1/_2 (S_{na} + S_{nb}) \ln \frac{P_{nb}}{P_{na}}\right]$$

$$(10) \qquad X = \exp\left[\sum_{m=1}^{M} {}^1/_2 (S_{ma} + S_{mb}) \ln \frac{P_{mb}}{P_{ma}}\right]$$

where S_{na}, S_{ma} and S_{nb}, S_{mb} are the shares of price and quantity in GDP for observations a and b.

The price index can be further decomposed into TOT adjustment indexes, i.e. $A_{a,b}$ and the non-trade good price indexes, i.e. $P_{Na,b}$. Similarly, the input quantity indexes can be further decomposed into specific indexes for labor and capital respectively, i.e. $X_{ja,b}$. The productivity index is treated as a residual. Each of these effects is calculated as follows:

$$(11) \qquad A_{a,b} = \exp\left\{ {}^1/_2 (S_{Ma} + S_{Mb}) \ln \frac{P_{mb}}{P_{ma}} + {}^1/_2 (S_{Xa} + S_{Xb}) \ln \frac{P_{xb}}{P_{xa}} \right\}$$

where S_{Ma} and S_{Mb} are the shares of imports in GDP for observations a and b and S_{Xa} and S_{Xb} are the shares of exports in GDP for observations a and b. Since import is treated as a negative output, S_{Ma} and S_{Mb} have negative magnitudes. Note that each observation represents a particular country at a particular time period.

$$(12) \qquad X_{ja,b} = \exp\left\{ {}^1/_2 (S_{ja} + S_{jb}) \ln \frac{X_{jb}}{X_{ja}} \right\}, \quad j \in \{L, K\}$$

where S_{ja} and S_{jb} are the shares of factors (i.e. labor and capital) in GDP for observations a and b. Note that for the calculation of quantity indexes, there is a need to convert the quantity between the two observations in comparison into common units.[11]

$$(13) \qquad P_{Na,b} = \exp\left\{ {}^1/_2 (S_{Ia} + S_{Ib}) \ln \frac{P_{Ib}}{P_{Ia}} + {}^1/_2 (S_{Ga} + S_{Gb}) \ln \frac{P_{Gb}}{P_{Ga}} + {}^1/_2 (S_{Ca} + S_{Cb}) \ln \frac{P_{cb}}{P_{ca}} \right\}$$

where S_{Ia}, S_{Ib}, S_{Ga}, S_{Gb}, S_{Ca} and S_{Cb} are the shares of investment, government and consumption in GDP for observations a and b respectively.

$A_{a,b}$ measures the deviation in TOT of observation b from that of observation a, $X_{ja,b}$ measures the deviation in factor j of observation b from that of observation a, and $P_{Na,b}$ measures the deviation in domestic prices of observation b from that of observation a.

Following Kohli (1990), by using equations (8) to (13), a complete decomposition of differences in GDP between two observations in the translog case can be obtained (note that it is the observed GDP differences based on the data directly):

$$(14) \qquad \Gamma_{a,b} = R_{a,b} \cdot A_{a,b} \cdot X_{La,b} \cdot X_{Ka,b} \cdot P_{Na,b}$$

[11]See Section A in the Appendix for the justification of this adjustment.

$\Gamma_{a,b}$ represents the difference in nominal GDP between observations a and b and $R_{a,b}$ represents the productivity deviation between observations a and b and it is treated as a residual.

The above formulae are applied to each of the $(K-1)$ direct bilateral comparison between two observations indicated by the MST. Any decomposition for indirect comparison between two observations can be calculated as follows:

(15)
$$\begin{aligned}\Gamma_{a,b}\cdot\Gamma_{b,c} &= (R_{a,b}\cdot P_{a,b}\cdot X_{a,b})(R_{b,c}\cdot P_{b,c}\cdot X_{b,c})\\ &= (R_{a,b}\cdot R_{b,c})(P_{a,b}\cdot P_{b,c})(X_{a,b}\cdot X_{b,c})\end{aligned}$$

$\Gamma_{a,b}$ represents the ratio of GDP between observations a and b, $\Gamma_{b,c}$ represents the ratio of GDP between observations b and c, and the multiplication of the two ratios gives the ratio of GDP between observations a and c, i.e. $\Gamma_{a,c}$. R, P and X represent the bilateral Törnqvist productivity, price and quantity indexes between two observations. $(R_{a,b}\cdot R_{b,c})$, $(P_{a,b}\cdot P_{b,c})$ and $(X_{a,b}\cdot X_{b,c})$ measure the deviations in productivity, prices and quantity of observation c to observation a respectively. Note that $(R_{a,b}\cdot R_{b,c})$, $(P_{a,b}\cdot P_{b,c})$ and $(X_{a,b}\cdot X_{b,c})$ are not equal to the direct comparisons, i.e. $R_{a,c}$, $P_{a,c}$ and $X_{a,c}$ between observations a and c.

The above algorithm can be generalized to include more than one intermediate observation. The number of intermediate observations between the comparison of two observations is determined by the structure of the MST.

To obtain multilateral comparison for all observations, each observation's GDP is compared to a common base observation's GDP, and the decomposition procedure is carried out using the above steps. An advantage of this method is that the ratios of GDP among all observations are the same, regardless of the base observation chosen. Similarly, the ratios of the resulting decomposition components among observations are base invariant as well. This means that the indexes being calculated are unaffected by the choice of base observation once the ordering of the linking has been fixed. With all the components of the base observation equalling one, the resulting decomposition components can then be compared relatively across observations.

Note that the resulting decomposition indexes may change if new observations are added or old observations are removed, as the number of intermediate comparisons for some indirect comparisons may increase or decrease.

3. Data

The methods described in Section 2 are applied to annual data for four OECD countries (U.S., Japan, Canada and Australia), and China. The data set includes aggregate prices and quantities for consumption (government and non-government), investment, exports, imports, labor and capital for the years 1986–92. The sum of the value of the first four variables equals aggregate GDP (where imports are taken to be a negative output).

Annual data of current and constant prices for the OECD countries are taken from Fox (1997), which provides a description of the method used for constructing the OECD data set. Most of the raw data in Fox (1997) are taken from the OECD National Accounts. For annual data of China, current and constant prices of the expenditure components are taken from Table 1 and Table 2 of World Bank

TABLE 1

PSEUDO-PPPS

	Australia	Canada	Japan	U.S.	China
1985	0.6866	0.7200	0.0042	1.0000	0.3411
1986	0.7376	0.7298	0.0044	1.0265	0.3646
1987	0.7882	0.7613	0.0043	1.0587	0.3883
1988	0.8556	0.7882	0.0043	1.1027	0.4336
1989	0.9073	0.8224	0.0044	1.1500	0.4714
1990	0.9427	0.8521	0.0044	1.1981	0.5054
1991	0.9617	0.8666	0.0046	1.2400	0.5389
1992	0.9727	0.8714	0.0046	1.2732	0.5774

Note: The pseudo-PPPs are expressed in terms of \$US and relative to U.S. 1985. For each country in the sample, the PPPs have been increasing over time which indicates that there is a need to pay more to buy the same amount of products than in the past.

(1997a). Current prices for indirect taxes and subsidies are drawn from Table 1 of World Bank (1997b) as well. Regarding the data for labor, total employment and total labor income are used. These two series are taken and calculated respectively from the Appendix table of Hu and Khan (1997).[12] The capital income series can be constructed by multiplying (1 − labor income share) by the national income series. To obtain the quantity of capital services, we use the procedure of Kohli (1982).[13]

Certain adjustments have been made to the two data sets, so that they are as comparable as possible.[14] The current-price series were treated as values and the constant-price series were treated as quantities. The price series were then derived by taking value/quantity. All of the original variables are expressed in national currencies.

Note that since the price series are in index format, with the price of each variable for each country in 1985 set to 1, a problem arises when constructing the MST.[15] Due to this, we start off with the second observation, i.e. use 1986 as the first year of our sample.

4. RESULTS

The psuedo-PPPs are used as the conversion factors for the price data, so that every country's currencies are now expressed in common units. The set of price data is applied to the construction of the MST and the decomposition part of this paper respectively.

Structure of the Minimum Spanning Tree

The MST for China, Japan, Australia, Canada and the U.S., with price data adjusted by PPPs, from 1986 to 1992 is depicted in Figure 1. The MST has 34

[12]The labor income is calculated by multiplying column 1 (national income) and column 6 (labor income share) from the table.

[13]See Section B in the Appendix for calculation of the capital series.

[14]See Section C in the Appendix for details of adjustments on the data set.

[15]See Section D in the Appendix for explanation of possible problems for the construction of the MST when data are expressed as index format in this case.

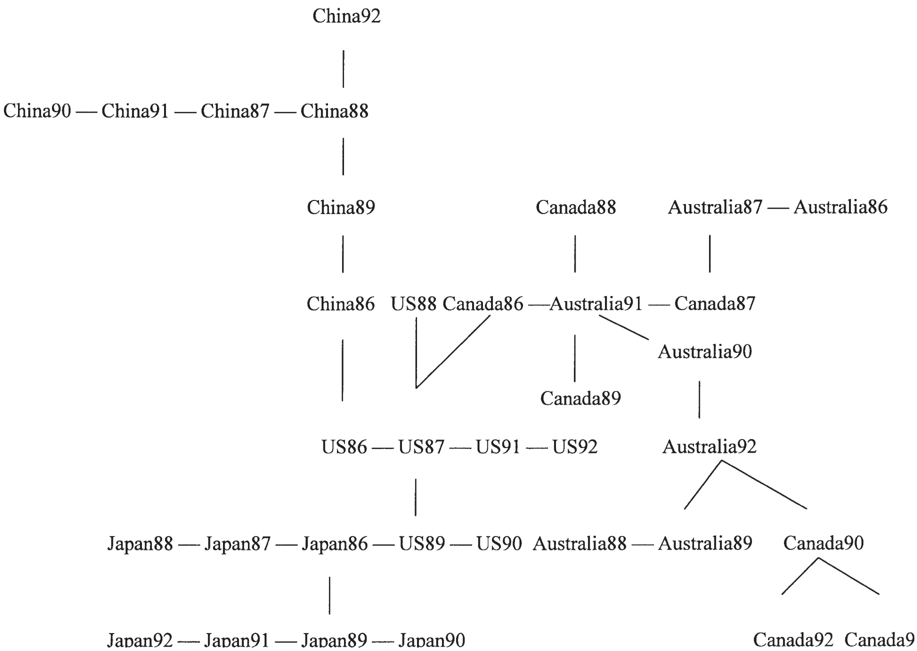

Figure 1. Minimum Spanning Tree (Prices Adjusted by PPPs)

edges. Generally, each country forms a cluster in the MST. This is logical as it implies that each country is more similar to itself in different years than it is to other countries in any year of the sample.[16] For Japan, China and the U.S., most of the adjacent years are connected together, like the setting of "string" spanning trees.[17] The symptom of star spanning trees[18] can also be found. For example, Australia91 acts like the center of a star spanning tree, which is connected to several observations for Canada, and US87 is also connected with five observations. A detailed discussion of the sensitivity analysis of the structure of the MST is provided in the Appendix.

Results of Decomposition

After the construction of the MST, we could then assess the differences in the sources of GDP across countries over time. We are particularly interested to look at the differences in GDPs between countries over time and the extent to which they could be explained by differences in inputs and other factors.

The results for decomposing the differences in GDP between two observations (based on the direct bilateral comparisons indicated by Figure 1) are normalized to have China90 act as the base observation. This means that we are

[16]Hill (1999b) obtains similar results in time series comparisons.
[17]See Figure 2 for the graph of a string spanning tree.
[18]See Figure 3 for the graph of a star spanning tree.

Figure 2. A String Spanning Tree

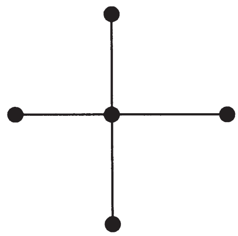

Figure 3. A Star Spanning Tree

looking at the relative comparisons among all observations, with China90 acting as the base for all comparisons. These results are shown in Table 2.[19]

In Table 2, the last column shows the ratio of GDP between the two observations.[20] Column 1 shows the difference in productivity between two observations. Column 2 and column 3 show the differences in labor and capital utilization, respectively, calculated using equation (12). Column 4 shows the difference in TOT,[21] which is calculated using equation (11) and column 5 shows the difference in domestic prices, calculated by using equation (13). Each of these components is calculated using the Törnqvist bilateral indexes.

If the nominal GDP ratio is greater than 1, any component that has a magnitude greater than 1 indicates that it contributes positively to the larger GDP in comparison. On the other hand, if the component has a magnitude smaller than 1, it indicates that it is shaving off GDP from the observation with larger GDP. The same concept applies to the case of a nominal GDP ratio smaller than 1.

Looking specifically at the results of China first, we found that labor, capital and domestic price are the main driving forces of GDP over the sample years. Contributions from each of these factors to the difference in GDPs is found to be positive for the entire period. Our results are supported by the fact that yearly growth rates of labor and capital quantity have been positive during the sample period. For labor quantity, most of the increases came from increased employment in the newly rising township and village enterprises in the mid-80s and increased labor share of GDP is recorded. For capital, most of the capital increase resulted from possible overinvestment in capital by the SOEs due to the soft budget constraints that they were facing. See Sachs and Woo (1997) for references.

The positive contributions from price effects are supported by the fact of increasing price levels that resulted from the decontrolling of prices in China

[19]Note that the choice of the base observation is arbitrary and the general results would not be affected if other observations are chosen as the base.

[20]Note that each of the bilateral comparisons are decided by the structure of the MST.

[21]Diewert and Morrison (1986) argued that in an open economy, an improvement in the TOT will lead to an increase in domestic production for any given amount of inputs, so that it has the effect similar to an increase in TFP, whereas a deterioration in the TOT will decrease domestic production, having an effect similar to a decrease in TFP.

TABLE 2

MULTILATERAL COMPARISONS, WITH CHINA 1990 AS THE BASE (CALCULATED USING PRICE DATA ADJUSTED BY PSEUDO-PPPS)

	Productivity	Labor Quantity	Capital Quantity	TOT	Domestic Price	Nominal GDP ratio
China1986	1.3239	0.8118	0.6936	1.0009	0.5317	0.3967
China1987	1.3041	0.8580	0.7423	0.9941	0.6000	0.4955
China1988	1.2060	0.9091	0.8432	0.9900	0.7544	0.6905
China1989	1.0828	0.9541	0.9314	0.9882	0.8942	0.8503
China1990	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
China1991	0.9778	1.0516	1.0620	1.0037	1.1339	1.2427
China1992	0.9809	1.0809	1.1737	0.9906	1.3310	1.6407
USA1986	1.3420	4.3858	1.4679	1.0274	0.4826	4.2836
USA1987	1.3061	4.5806	1.4900	1.0208	0.5167	4.7022
USA1988	1.2709	4.8043	1.5167	1.0186	0.5611	5.2931
USA1989	1.2289	5.0094	1.5453	1.0171	0.6110	5.9114
USA1990	1.1789	5.1617	1.5722	1.0135	0.6654	6.4519
USA1991	1.1381	5.2328	1.5888	1.0163	0.7104	6.8318
USA1992	1.1212	5.3905	1.6057	1.0171	0.7501	7.4043
Japan1986	1.3377	2.0988	1.0433	1.0450	0.4786	1.4650
Japan1987	1.3802	2.1012	1.0536	1.0427	0.4665	1.4861
Japan1988	1.4260	2.1388	1.0685	1.0416	0.4609	1.5644
Japan1989	1.4277	2.1913	1.0934	1.0402	0.4827	1.7173
Japan1990	1.4456	2.2255	1.1163	1.0318	0.4988	1.8483
Japan1991	1.4291	2.2764	1.1446	1.0360	0.5287	2.0398
Japan1992	1.4181	2.2748	1.1677	1.0392	0.5475	2.1433
Canada1986	1.3567	0.7702	0.6840	1.0250	0.4765	0.3491
Canada1987	1.3080	0.8112	0.7028	1.0324	0.5156	0.3969
Canada1988	1.2713	0.8516	0.7203	1.0381	0.5515	0.4465
Canada1989	1.2186	0.8851	0.7409	1.0432	0.5985	0.4989
Canada1990	1.1644	0.9047	0.7561	1.0396	0.6462	0.5351
Canada1991	1.1391	0.8980	0.7669	1.0355	0.6725	0.5462
Canada1992	1.1322	0.9025	0.7736	1.0315	0.6835	0.5573
Australia1986	1.2680	0.5003	0.5339	1.0156	0.5332	0.1834
Australia1987	1.2043	0.5328	0.5505	1.0271	0.6033	0.2189
Australia1988	1.0054	0.7940	0.6921	1.0776	0.4493	0.2675
Australia1989	0.9374	0.8488	0.7137	1.0710	0.5087	0.3094
Australia1990	0.9506	0.6466	0.6037	1.0409	0.8574	0.3312
Australia1991	0.9621	0.6412	0.6099	1.0355	0.8967	0.3493
Australia1992	0.8959	0.8851	0.7409	1.0432	0.5985	0.3668

(which started in the mid-1980s). Regarding the insignificant effect of TOT, it reflects the fact that China's economy is less open than one may expect from just looking at the trade-ratios. Possible reasons for this lack of openness include institutional factors (e.g. nontariff barriers to trade) which restrict the impact of trade on the Chinese economy. As pointed out by Naughton (1996), restrictions on import trade may indirectly hamper the efforts of Chinese enterprises to export as well. This is because a lot of export activities required processing imported materials, so limited access to imports may limit a domestic firm's ability to respond quickly to changing market demand.

Productivity effects are large compared to the four OECD countries; however, they are insignificant contributors to GDP differences. Since the TFP difference acts as a residual in the model, it could reflect a number of factors other than difference in pure technical progress, e.g. improvement of quality of inputs, the possibility of knowledge spillover, structural changes and so on.

Comparing China's results with those of the OECD countries, we could find that the U.S. and Japan have particularly larger labor and capital effects. This could be the result of the larger magnitude of the labor and capital inputs, which is evidenced from the raw data. Also, for both countries, the TOT effects are relatively large. This reflects the fact that there was increasing openness of both the U.S. and Japanese economies during the sample period. For Japan, after having a less open market for 40 years, regular progress in trade has occurred since the 1950s and it has become the world's leading exporter of manufactured goods in the 1980s. For the U.S., it has been actively engaged in trade during the sample period, which was evidenced by the increasing percentage of exports to GDP and relatively stable percentage of imports to GDP. The TOT has been improving since the 1990s as the trade balance was generally improving since the early 1990s.

Increasing openness (which led to indirect exchange of technology know-how) and increased R&D have led to large productivity effects for Japan and the U.S. during the sample period. For the U.S., our results of declining productivity are consistent with the general finding of slowdown of productivity growth since 1973. Technological slowdown and declining of quality of education in the U.S. over the 1980s and 1990s may be the main reasons for productivity slowdown. For Japan, productivity was found to be improving; this was mainly the result of Japanese innovations in assembly lines and other manufacturing operations during the 1980s.

For Australia and Canada, similar to the case of China, labor, capital and domestic price are the main driving forces of GDP over the sample years. Larger labor and capital effects could be supported by reasons similar to those discussed above (e.g. large quantity and positive growth rates over the sample period). For price effects, during the 1980s, both Australia and Canada had high inflation rates; this explains the resulted large price effects. The result of high inflation rates was that imports increased and the price competitiveness of exports fell, which confirmed the insignificant TOT effect.

Although productivity is relatively high for Canada (over the whole sample period) and Australia (in earlier years), it is not a significant contributor to GDP. For Australia, the results show that productivity started to become a contributor to GDP since 1989. This could possibly be explained by the lagged response to increased levels of innovation during the late 1980s. Such an argument is supported by evidence that a marked increase in business expenditure on R&D did occur in Australia during the late 1980s.

5. Conclusion

This paper uses the combination of two different newly developed approaches to assess the impacts of the terms of trade (TOT), factors and prices in contributing to differences in GDP across countries and over time. Alternative uses and aspects of the application of these approaches have been proposed and discussed. Special attempts have been made to take account of the nature of the available data.

The combination of Hill's (1999a, 1999b) spanning tree methodology and the Fox (1999) decomposing technique forms a new technique for multilateral comparisons. The former uses the Paasche-Laspeyres spread (PLS) index to provide a possible way of linking, whereas the latter decomposes GDP differences among countries over time.

Hill's MST approach is extended to provide a way of ordering panel data. The structure of the resulting MST shows that there is a general tendency for observations of the same countries to form a cluster by themselves. The Fox (1999) decomposing technique is then used to decompose the differences of GDP among observations. In this paper, the combination of these two methods is applied to panel data and the properties of this newly formulated method in a panel context has been evaluated.

The resulting multilateral indexes, which are formed using the linking up of direct bilateral indexes indicated by the structure of the MST are base-invariant. This provides strong support for using the newly formulated method for multilateral comparisons as it can give a cardinal scaling of observations that is independent of the choice of the base observation.

Nevertheless, the resulting decomposition indexes may change if new observations are added or old observations are removed when constructing the MST, as the number of intermediate comparisons for some indirect comparisons may increase or decrease, depending on the structure of the MST.

Certain problems on the construction of the MST and multilateral indexes with data expressed in index format have been discussed in this paper as well. Pseudo-PPPs are used as the conversion factor for national data. Calculation of the pseudo-PPPs has been carried out using the basics of the Geary-Khamis (GK) method. To take account of the nature and type of the available data, variants of the GK equations have been proposed. Such an alternative may have different economic implications. Indeed, the accuracy and usefulness of the application of this GK variation needs furthered research.

An empirical application for illustration of the technique has been done on China and four OECD countries. Results showed that China is similar to Australia and Canada regarding the main factors affecting GDP (i.e. labour, capital and domestic price), but it has relatively weaker TOT effects. For Japan and the U.S., the TOT, labor and capital effects are significant factors contributing to GDP. These results conform to our expectations.

Appendix

A. *Justification of Adjustment of Quantity in the Calculation of Translog Quantity Indexes*

When calculating the quantity indexes for the decomposition of GDP differences between observations, there is a need to convert the quantities of each observation into a common unit. This can be done by multiplying the quantity of each observation, i.e. X_{ja} and X_{jb} in equation (12) by the relevant conversion factors (E).

$$X_{ja,b} = \exp\left\{{}^1/_2(S_{la}+S_{lb})\ln\left[\frac{X_{lb}}{X_{la}}\cdot E\right] + {}^1/_2(S_{ka}+S_{kb})\ln\left[\frac{X_{kb}}{X_{ka}}\cdot E\right]\right\}, j \in \{L,K\}$$

$$= \exp\left\{{}^1/_2(S_{la}+S_{lb})\left\{\ln\left[\frac{X_{lb}}{X_{la}}\right]+\ln E\right\}\right\}\cdot \exp\left\{{}^1/_2(S_{ka}+S_{kb})\left\{\ln\left[\frac{X_{kb}}{X_{ka}}\right]+\ln E\right\}\right\}$$

$$= \frac{X_{lb}}{X_{la}}^{{}^1/_2(S_{la}+S_{lb})} \cdot E^{{}^1/_2(S_{la}+S_{lb})} \cdot \frac{X_{kb}}{X_{ka}}^{{}^1/_2(S_{la}+S_{lb})} \cdot E^{{}^1/_2(S_{ka}+S_{kb})}$$

$$= \frac{X_{lb}}{X_{la}}^{{}^1/_2(S_{la}+S_{lb})} \cdot \frac{X_{kb}}{X_{ka}}^{{}^1/_2(S_{la}+S_{lb})} \cdot E^{{}^1/_2(S_{ka}+S_{kb}+S_{la}+S_{lb})}$$

(A1) $$= \frac{X_{lb}}{X_{la}}^{{}^1/_2(S_{la}+S_{lb})} \cdot \frac{X_{kb}}{X_{ka}}^{{}^1/_2(S_{la}+S_{lb})} \cdot E^1$$

$S_{la} + S_{lb} + S_{ka} + S_{kb} = 2$ as $S_{la} + S_{ka}$ and $S_{lb} + S_{kb}$ are both equal to one.

Similar to the quantity index, the price index in equation (13) can also be rewritten as:

(A2) $$P_{a,b} = \frac{P_{zb}}{P_{za}}^{{}^1/_2(S_{za}+S_{zb})} \cdot E^1 \quad z \in \{c,i,g,x,m\}$$

c, i, g, x and m stand for private consumption, investment, government consumption, export and import, respectively.

Recall that before conversion to a common unit,

(A3) $$\Gamma = \frac{\mathrm{GDP}_a}{\mathrm{GDP}_b} = R\cdot P\cdot X$$

where Γ is the GDP ratio of two countries, with each GDP expressed in national currencies. R represents the productivity index, P represents the price index and X represents the quantity index.

Then, we use the conversion factor, E to convert the GDP of both countries into a common currency. Correspondingly, there is a need to adjust each component in the RHS of equation (A3) using the conversion factor:

(A4) $$\Gamma_1 = \frac{\mathrm{GDP}_b}{\mathrm{GDP}_a}\cdot E = (R\cdot P\cdot X)\cdot E$$

According to equations (A1) and (A2), equation (A4) can be re-written as:

(A5) $$\Gamma_1 = \frac{\mathrm{GDP}_b}{\mathrm{GDP}_a}\cdot E = (R/E\cdot E)\cdot(P\cdot E)\cdot(X\cdot E)$$

where Γ_1 is the adjusted GDP ratio, $(P\cdot E)$ and $(X\cdot E)$ are the price index and quantity index adjusted by conversion factor respectively. We have shown in equations (A1) and (A2) that after conversion, the price index and quantity index become $(P\cdot E)$ and $(X\cdot E)$ respectively. $(R/E\cdot E)$ acts as the residual, which counteracts the effects of the conversion factors in $(P\cdot E)$ and $(X\cdot E)$, so that $\frac{\mathrm{GDP}_b}{\mathrm{GDP}_a}$ is equal to $(R/E \cdot E) \cdot (P \cdot E) \cdot (X \cdot E)$. So it is clear that besides converting the prices of countries in comparison when calculating the price index, there is also a need to convert the quantities of countries when calculating the quantity indexes as well.

B. *Calculation of the Capital Series*

To obtain the quantity of capital services, we used the method used by Kohli (1982) and Fox and Kohli (1998). Two assumptions are made:

(a) The flow of capital services is proportional to the beginning-of-period capital stock and this stock can be obtained by accumulating real gross investment subject to a constant depreciation rate, δ.

(A6) $$X_{kt} = (1-\delta)X_{kt-1} + Y_{it-1} \quad t = 1985-92$$

which can be written as:

(A7) $$X_{kt} = Y_{it-1} + (1-\delta)Y_{it-2} + (1-\delta)^2 Y_{it-3} + \ldots, \quad t = -\infty, \ldots, 1992$$

(b) Prior to some period $T \leq 1985$, real investment grew at a constant rate, γ

(A8) $$Y_{it} = Y_{it-1}(1+\gamma) \quad t = -\infty, \ldots, T$$

By making use of equations (A7) and (A8) valued at time T, X_{kt} at 1985 can be obtained by:

(A9) $$\begin{aligned} X_{kt} &= \frac{Y_{it}}{1+\gamma}\left[1 + \frac{1-\delta}{1+\gamma} + \left(\frac{1-\delta}{1+\gamma}\right)^2 + \ldots\right] \\ &= \frac{Y_{it}}{1+\gamma}\left[1/1 - \frac{1-\delta}{1+\gamma}\right] \\ &= \frac{Y_{it}}{\delta+\gamma} \end{aligned}$$

Since there is no corresponding official data on real investment before the year 1985, γ is calculated as the within sample average rate of growth of real investment from 1985 to 1992, which is equal to 10.52 percent. δ is set to 5 percent. The resulting 1985 real capital stock is found to be 1701.44 billions of 1990 yuan.

C. *Adjustment to Enhance the Consistency of the Two Data Sets*

Certain adjustments have been done to make the OECD and China data sets comparable. First, both data sets are normalized as that quantities are in 1985 domestic currency. Second, due to the method of derivation of the capital series,[22] the share of labor and share of capital of GDP add up to one for each country in the OECD data set. To be consistent with the OECD data set, the labor share and capital share of national income for China is taken as proxy for the labor and capital shares of GDP for China. The values of labor and capital are calculated

[22]See p. 8 in Fox (1997) for the calculation of the capital series for the OECD data set.

as the product of GDP and the respective labor and capital shares. Quantity of capital is calculated using the procedure of Kohli (1982). A description of the method has been provided in part B of the Appendix.

D. *Problem Associated with Data Expressed in Index Format when Constructing the Minimum Spanning Tree*

If the data are expressed in index format, with one year acting as the base year, i.e. the price for each variable in that year is equal to 1, then there is a problem when constructing the MSTs. For example, if 1985 is treated as the base year, when more than one country is considered, the 1985 entry for one country against another in the PLS indexes will be equal to 1. Taking the log of the PLS matrix, the 1985 entry of one country against another country will be equal to 0. This has the effect of avoiding comparisons between these two countries in 1985 when using the Spanning Tree algorithm, i.e. countries in 1985 will never be linked together in a spanning tree if their price series are indexes based in 1985. This is the same as setting a restriction to avoid comparison between the countries in 1985, which is not what we intend to do. This problem exists as long as the data are expressed in index format.

E. *Sensitivity Analysis of the Structure of the Minimum Spanning Tree*

In recent years, international comparisons have moved to global samples consisting of larger numbers of countries over a longer span of time. In order to test the suitability and sensitivity of the MST method for larger dimensions, we experimented with the construction of an MST which includes only two countries with two years of data, then progressively increased the number of countries and years. In addition, the effects of increasing observations for more than one year at a time were also examined. It was found that the structure of the MST for the original observations is robust in both cases, i.e. the original observations are still connected to the same observations when a larger sample is used.[23] Similar results have been obtained for the reverse case of beginning with a large sample and progressively decreasing the sample size.[24] Hence, we can conclude that when viewed from the perspective of clusters of countries, the MST is in our context stable when data are added or deleted.

[23]For example, we started off with two observations for two countries, China86, China87, US86, and US87. Initially, China86 was connected to China87 and US86 and US86 was connected to US87. When China88 and US88 were added, the former was connected to China86 and China87, whereas the latter was connected to US87. Similarly, when China88 and China89 and US88 and US89 were added at the same time, China observations were clustered together, whereas U.S. observations were clustered together. Similar results are obtained when more observations are added.

[24]From Figure 1, 14 of 35 observations are connected to only one other country in the MST. If the observation deleted from the comparison is one of these countries, e.g. Australia86, then the structure of the MST of the remaining 21 observations would be the same as before. However, when the observation deleted is one of the 21 observations connected to two or more other countries, e.g. US91, then the structure of the MST is changed. However, we observe that observations of the same countries still form clusters by themselves for the latter case.

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