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BOUNDING LIFETIME INCOME USING A CROSS SECTION OF DATA

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Measures of income inequality based on current income are well known to overstate lifetime income inequality for two reasons: intracohort mobility and the shape of the age-earnings profile. Utilizing the concept of age equivalence scales along with varying assumptions concerning the extent of intracohort mobility, the method presented bounds lifetime income inequality using only cross-sectional data from the CPS, 1967–86. As a result, we are able to analyze changes in lifetime inequality over this period.

1. INTRODUCTION

It is well established that income inequality measures—such as the Gini coefficient—based only on current incomes may yield incorrect inferences concerning the degree of inequality for two main reasons. First, inequality estimates using cross-sectional data overestimate lifetime inequality in the face of significant intracohort mobility.[1] In other words, as individuals pass through their respective life cycles, their rankings change within their age cohort such that their age-income profiles cross. Paglin (1977, p. 527) states: "Mobility reduces the dispersion of lifetime incomes much below the annual income estimate . . . If this [an increase in the curvature of the age-income profile] is accompanied by increasing mobility within cohorts, then measures such as the weighted cohort Gini will not only overestimate inequality in a particular year but will underestimate the decline in

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[1]Buchinsky and Hunt (1999) and Arkes (1998) highlight the inequality-reducing effect of intracohort mobility. Gustaffson (1994) and Björklund (1993) document evidence of lower inequality in Sweden using longitudinal rather than cross-section data. de Fontenay *et al.* (2002), Maasoumi and Trede (2001), Cantó (2000), Bager-Sjögren and Klevmarken (1998), Bigard *et al.* (1998), among others present recent empirical studies measuring income mobility in various countries. See also Cowell (1977, 1985) and Shorrocks (1978) for a careful discussion of these notions.

inequality." In more recent work, Buchinsky and Hunt (1999, p. 351) state: "Lifetime wages will be more equally distributed than wages from any single year if individuals change position in the wage distribution over time." In addition, Gardiner and Hills (1999, p. F91) argue: "The existence of income mobility may moderate concerns about growing inequalities, especially if income mobility has increased."

Second, even if the age-income profiles of individuals do not cross, given the inverted U-shape of age-income profiles, one should not expect nor desire all individuals or households to have identical incomes. Paglin (1975) launched a historical debate, arguing that prefect income equality should be synonymous with all households *within* an age cohort having identical annual incomes. Inequality across age cohorts is deemed to be *functional*, or non-policy relevant. In Paglin's quest to measure "true" (or lifetime) income inequality, he subtracted a measure of between age cohort inequality from overall inequality using the familiar Gini coefficient to measure overall and between cohort inequality.

Utilizing the conventional Gini decomposition by subgroups of the population, we can examine in detail Paglin's suggested remedy. Consider a population of n income units and g age cohorts with n_i income units in the ith cohort. Let $\bar{x}$ represent the population mean income and $\bar{x}_i$ be the cohort mean. The overall Gini coefficient, G, is decomposable as

$$G = \sum_{i=1}^{g} \left(\frac{n_i}{n} \right) \left(\frac{n_i}{n} \frac{\bar{x}_i}{\bar{x}} \right) G_i + G_b + R \tag{1}$$

where G_i is the within-cohort Gini, G_b is the between-cohort Gini, and R is the residual representing the overlapping term. Paglin contends that between-group inequality, G_b, referred to as the Age-Gini, is equitable since it reflects solely the life cycle income profile faced by all individuals. Inequality in excess of between group inequality is *nonfunctional*, or policy relevant, and should be considered the true measure of income inequality. In other words, Paglin's solution replaces the 45° as the usual reference line in the calculation of the Gini coefficient with a new reference line, derived from the Lorenz curve of age cohort means. Paglin then proposed a new measure, denoted by the Paglin-Gini or P-Gini, as

$$G_p = G - G_b \tag{2}$$

The main criticism surrounding the P-Gini is the fact that netting out the between-group Gini, G_b, in equation (2) does not yield an estimate of the weighted sum of within-group Ginis. Because of the "messy" decomposability of the Gini coefficient, debate focused on whether the residual, R in equation (1), should be included in the within- or between-cohort Gini (Shorrocks, 1984). Paglin (1977) maintains that the residual is a part of within-group inequality, while Nelson (1977) and others have disagreed. The issue remains unresolved.

A second, and related, criticism of the P-Gini given by Johnson (1977) and others is that the P-Gini is not an accurate measure of lifetime income inequality in economies with significant intracohort mobility. Although Paglin (1977, p. 526) admits that the P-Gini "does not indicate actual differences in lifetime incomes," he contends that "no measure based on cross-section data for a particular year

can be used to measure actual lifetime income differences." Despite Paglin's caveat, Paglin (1975, p. 601) states that his measure of inequality "more closely approximates a measure of long-run interfamily inequality." However, the direction of the bias inherent in Paglin's estimate of long-run inequality is unknown. Paglin (1977, p. 526) writes that "the P-Gini will yield an estimate which is above or below the lifetime income inequality depending on the degree of cohort mobility."

In general, attempting to measure lifetime income inequality presents some of the same decomposition issues as measuring current income inequality at a point of time. A perusal of the decomposition formula given in (1) above makes this clear. It will always be difficult (at a point in time) to discern between cohort, time and age effects. We expect individuals, households and families to have less income when they are relatively young. Thus, a "young family" cohort is less of a policy issue than an "older family" cohort with respect to their marginal contribution to total inequality. We also expect cohorts to be mobile over time. A summary statistic like that given in (1) can control for within- and between-cohort effects, but at the cost of not being able to discern the "overlap" or interaction.

Given the shortcomings of the P-Gini and despite numerous efforts to amend the P-Gini, researchers appear to have settled for the only remaining option to measure lifetime inequality: wait for panel data sets which allow one to trace individuals over their lifetime and directly measure lifetime income.[2] This is the route taken in the recent work by Buchinsky and Hunt (1999) who utilize data over a 13 year period from the National Longitudinal Survey of Youth (NLSY), Arkes (1998) who use data from the Panel Study of Income Dynamics (PSID) during the 1970s and 1980s, and Björklund (1993) who utilizes 38 years of data on a relatively small group of Swedish men, among others. While this is a fruitful avenue for future research, it is limited by the availability of the data. Various panel data sets exist in only a handful of countries, and even where in existence, still do not capture the entire life cycle (as of yet).

The purpose of this paper is not to revise the P-Gini or to utilize partially completed panel data sets, but rather to bound lifetime income inequality using a cross-section of data. Clearly, such an undertaking requires fairly strong assumptions of the data. This paper explores only one of many possible sets of assumptions one could utilize to bound lifetime inequality, assumptions regarding the level of income mobility. Other possible avenues of bounding lifetime inequality include utilizing cross-sectional consumption expenditures to infer lifetime incomes assuming the applicability of life cycle consumption theories. Nonetheless, given varying assumptions about the degree of intracohort mobility and utilizing the logic of the Paglin critique of the usual Lorenz-based Gini coefficient, we estimate an upper and lower bound for the Gini coefficient of lifetime income inequality at a point in time. While our proposed measure may not be ideal, it does allow one to examine changes in the bounds over an extended period of time and will hopefully serve as motivation for others to explore alternative techniques to estimate such bounds. Moreover, companion studies on the level of mobility within countries may be incorporated in future work to tighten the bounds.

[2]See Formby *et al.* (1989) and the references for a partial list of attempts to revise the Paglin-Gini.

To perform such an exercise, the question turns to the issue of what constitutes the upper and lower bounds. The next section addresses this question. Section 3 discusses the data and provides the results using CPS data from 1967 to 1986 and compares the results with the typical Lorenz-Gini as well as the P-Gini. Section 4 contains some concluding remarks.

2. Framework

2.1. *Upper Bound*

To calculate the upper bound, we make the explicit assumption of no intracohort mobility. In other words, individuals are not permitted to change rankings within their age cohort as they move through the life cycle or, equivalently, the age-income profiles of individuals are parallel. As alluded to earlier, maintaining the assumption of no intracohort mobility constitutes an upper bound on lifetime income inequality since mobility will only serve to reduce lifetime inequality. This argument is also made in Lillard and Willis (1978, p. 986): "One extreme possibility is complete income stratification. In this case, knowledge of an individual's position in the income distribution in a cross-sectional survey in year t is a perfect predictor of his position in subsequent years."

Given the assumption of no mobility, we adjust for age differences by rescaling income across cohorts to make the incomes of all individuals directly comparable. To illustrate, consider two individuals, Mr. X and Ms. Y, where Mr. X is 30 and has an income of \$46,200 and Ms. Y is 50 and has an income of \$48,000. The mean income of all 30 year olds is \$42,000 and the mean income of all 50 year olds is \$50,000. According to Paglin's logic, a portion of the income differential is functional inequality due to the shape of the age-income profile and a portion is nonfunctional. By utilizing the concept of an age equivalence scale, we are able to remove the effects of age and compare incomes based solely on nonfunctional differences. Whereas the typical use of equivalence scales is to normalize household income controlling for differences in *consumption* patterns due to age, we utilize equivalence scales in the present context to normalize income controlling for differences in *earnings ability* due to age.

In the example, Mr. X is earning 10 percent more than the cohort mean for 30 year olds and Ms. Y is earning 4 percent less than cohort mean for 50 year olds. The assumption of no mobility is consistent with Mr. X earning 10 percent more than the mean of 50 year olds when he turns 50 years old (and Ms. Y earning 4 percent less than mean of 30 year olds when she was 30 years old). As a result, we can scale Mr. X's income up to Ms. Y's cohort by multiplying the mean income from Ms. Y's cohort by 1.1, or 110 percent. Thus, Mr. X's income is valued at $1.1 \times 50{,}000$, or \$55,000 on the scale of Ms. Y's cohort. Analogously, Ms. Y's income could be scaled down to Mr. X's cohort by multiplying Mr. X's cohort mean by 0.96, yielding an income of \$40,320. So, for the purposes of determining the upper bound for lifetime inequality, we compare the income bundles (46200, 40320) or (55000, 48000). For multiple age cohorts, the mean income of any particular cohort may be taken as the reference cohort. The result is robust to this choice.

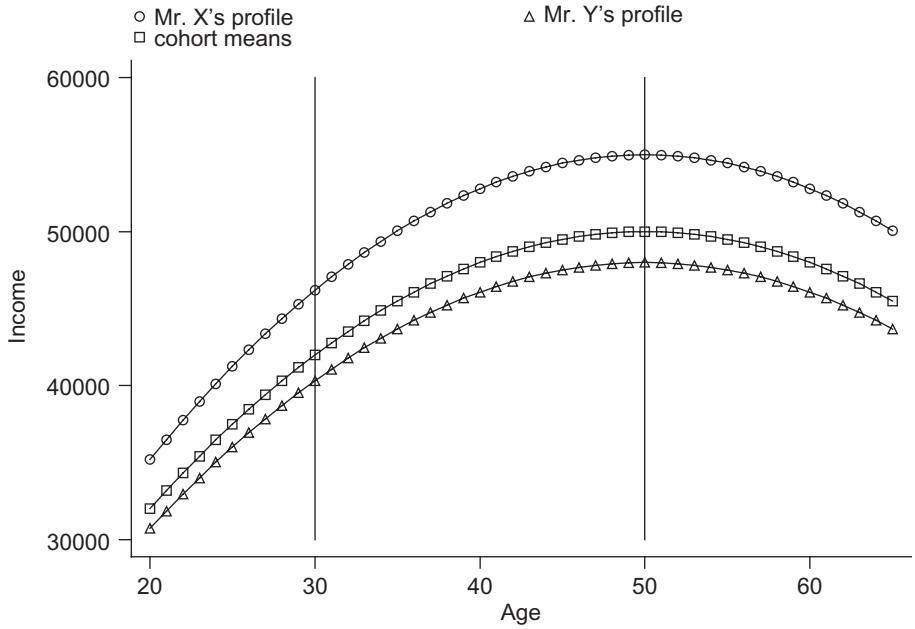

Figure 1. Mr. X's and Ms. Y's age-income profiles with no mobility

This method of adjusting for life cycle effects equalizes mean incomes across cohorts by scaling incomes by an appropriate age equivalence scale. Under the assumption of no mobility, individual *i*'s income is mapped into the reference cohort by

$$y_{isr} = \left(\frac{\mu_r}{\mu_s}\right) y_i \tag{3}$$

where y_{isr} is the equivalent income of individual i in cohort s scaled into the reference cohort r, μ_r is the cohort mean of the reference group, and μ_s is the mean income in cohort s. Thus, μ_r/μ_s is the age equivalence scale for all individuals in cohort s. Re-scaling each individual's income effectively removes between-group inequality under the assumption of no mobility. Figure 1 illustrates Mr. X's and Ms. Y's age-income profiles under no mobility as well as a hypothetical age-income profile of cohort means. The upper bound is constructed by projecting, either forward or backward, every individual to the same cohort using the hypothesized individual age-income profiles (under the assumption of no mobility) and then measuring inequality using the age-equivalent incomes.

2.2. *Lower Bound*

If the upper bound is obtained by assuming there is no intracohort mobility, what is the analogous assumption required to place a lower bound on income inequality? Lillard and Willis (1978, p. 986) state: "[A]t the opposite extreme from complete income stratification is complete income mobility in which an individ-

ual's probability of being in some discrete income class in a given period is independent of his prior income status." Gardiner and Hills (1999, p. F92) describe this "lottery model" of income determination as: "Each year a celestial income determination drum is twirled and, depending on the numbers which come up, each of us ends up randomly on Income Support or as a multi-millionaire. The next year we take our chances again" (see also Shorrocks, 1978).[3]

Following this logic, one can conceptualize complete or perfect intracohort mobility as each individual receiving an income in each period derived from a mean zero shock and the cohort mean income. The shocks for a given individual are uncorrelated across periods and shocks are uncorrelated across individuals within a period. In other words,

$$y_{ist} = \bar{y}_{st}(1+\delta_{it}) \tag{4}$$

where y_{ist} is individual i's income in cohort s at time t, $\bar{y}_{st}$ is the mean income of cohort s at time t, and δ_{it} is an individual- and period-specific mean zero *iid* shock.

Given this simple model of income determination, an individual's lifetime income (and ignoring any discount rate for simplicity), Y^*, is the sum of one's incomes over all T periods, where T is the number of years the individual works. As a result, individual differences in lifetime incomes are due to either luck or differences in T.[4] For simplicity, we focus only on the former and assume a constant T across individuals.

We can derive a measure of luck by answering the following question: Given an individual's sequence of draws, $\delta_i \equiv \{\delta_{i1}, \ldots, \delta_{iT}\}$, can we find a single value, $\bar{\delta}_i$, for which the individual would be indifferent between receiving the actual sequence of draws, δ_i, and receiving $\bar{\delta}_i$ in every period? If individuals are not credit constrained, then $\bar{\delta}_i$ is given by

$$\bar{\delta}_i = \frac{\Sigma_t \bar{y}_t \delta_{it}}{\Sigma_t \bar{y}_t} \tag{5}$$

where the cohort subscript s is suppressed (see the appendix for the derivation). Thus, all individual differences in lifetime income are captured by the single parameter $\bar{\delta}$.

In order to utilize this parameter to obtain the lower bound on income inequality, our strategy is to simulate draws of $\bar{\delta}$ for each individual given assumptions about its distribution (discussed below), compute lifetime incomes, Y^*, given the draws for $\bar{\delta}$, and then compute the Gini coefficient for the simulated lifetime incomes. Inequality measures based on the simulated lifetime incomes reflect inequality due to random variation only; the effects of age and imperfect mobility are completely removed. In addition, changes in the lower bound over time will arise only as a result of changes in the distribution of $\bar{\delta}$.

A few comments are necessary, however. First, in order to simulate draws of $\bar{\delta}$ for each individual, we need to make an assumption about the distribution of

[3]The terminology of "perfect mobility" was originally developed by Prais (1955).

[4]Luck, here, refers to an individual enjoying an average δ over one's lifetime greater than zero or an average δ close to zero, but with positive shocks occuring in periods with high mean income. Gardiner and Hills (1999) also refer to "luck" in their description of the "lottery model" of income determination.

the shocks from each period. The usual assumption when estimating wage functions is log normality; however, in this case we are dealing with total income, not wage earnings. In addition, under the assumption of log normality, while one can derive the mean and variance of $\bar{\delta}$, the distribution is unknown. While methods are available to sample from log concave densities, we simplify matters by assuming $\delta \sim iid\ N(0,\sigma^2)$, which, as shown in the appendix, implies that

$$\bar{\delta} \sim N\left(0, \sigma^2 \frac{\Sigma_t \bar{y}_t^2}{(\Sigma_t \bar{y}_t)^2}\right) \tag{6}$$

The variance of δ, σ^2, is estimated non-parametrically.[5] From (4)

$$\mathrm{Var}(\delta) = \sigma^2 = \mathrm{Var}\left(\frac{y_{ist}}{\bar{y}_{st}}\right) \tag{7}$$

Thus, σ^2 may be directly estimated from the data and does not hinge on the assumption of normality.

Second, when computing the variance in (6), we assume that each cohort has the same mean income level when they reach the same age. In other words, in the example used previously, Mr. X's cohort will also have a mean income of $50,000 when they reach 50. This assumption allows us to ignore growth in mean income levels over time. As Paglin (1975, p. 602) states: "[I]n a dynamic economy with annual growth in real per capita income of 2 percent, there will be very large differences in lifetime incomes of older workers and young workers entering the labor force." Since growth will only exacerbate measures of lifetime income inequality, comparing lifetime incomes under the assumption of zero growth reinforces the notion of a lower bound. Essentially, this process computes an age equivalence scale for lifetime income under the assumption of perfect mobility by re-scaling all individuals to being born in the same year.

Figure 2 illustrates this case. Mr. X's income in each period is simulated by drawing a normally distributed multiplicative shock in each period (with mean 1 and a standard deviation of 0.2) and multiplying it by the cohort mean in each period as given in (4). Next, $\bar{\delta}$ is computed and a smoothed income profile is calculated as well as his lifetime income, Y^*. Under the assumption of well-functioning credit markets, Mr. X is indifferent between the smoothed income profile and the vector of volatile (or random) incomes. Interestingly, despite the extreme volatility of Mr. X's "actual" simulated incomes, $\bar{\delta} = -0.02$, barely different to zero. Consequently, his smoothed income profile differs very little from the age-income profile of cohort means. Finally, the lower bound for inequality is measured using the lifetime income of Mr. X and all other individuals; abstracting from inequality due to imperfect intracohort mobility, economic growth, and age differences. Consequently, under the lower bound, all differences between lifetime incomes are due solely to stochastic variation.

[5]While the assumption of Gaussian shocks does permit possible negative income values, this is not an issue in the empirical application in the following section. However, since inequality measures are based on income—not wages—this approach is sufficiently general to accommodate negative income values should they occur in the data.

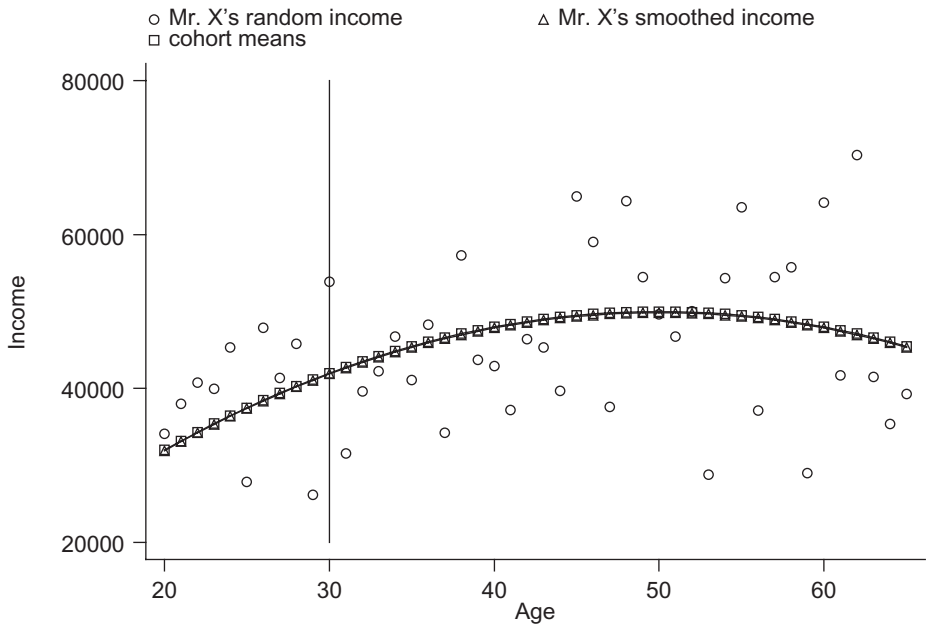

Figure 2. Mr. X's random and smoothed income under perfect mobility ($\bar{\delta} = -0.02$)

3. Empirical Illustration

To illustrate the computation of the upper and lower bounds on lifetime inequality, we obtained the identical data set used in Formby *et al.* (1989) from the authors. The data come from the U.S. Current Population Survey (CPS) and span the years 1967–86. The use of the data from Formby *et al.* is chosen simply to permit comparison of our results to the P-Gini estimates presented in the literature. The data exclude single member households as well as those with negative income. Income is measured annually, as total pre-tax household income. For further details, refer to Formby *et al*. The results, using 1, 5, and 10 year age intervals to compute the cohort means, are presented in Table 1.

Column 3 reports the typical Lorenz-Gini. As is well known, inequality—as measured by the Lorenz-Gini—increased significantly over the range of the data, with the Gini increasing by nearly 10.7 percent between 1967 and 1986. Columns 4 through 6 report the P-Gini using the three different age intervals to define a cohort. As the interval increases, a greater share of inequality is considered within group inequality and the P-Gini rises. The results from the P-Gini coefficients indicate that approximately two-thirds of income inequality, as measured by the Lorenz-Gini, is nonfunctional; in other words, constitutes "true" inequality. In addition, the P-Gini (column 4) increased by 12.1 percent over the range of the data.

Columns 7 through 12 report the upper and lower bounds calculated using the same three age intervals. As with the P-Gini, both the upper and lower bounds increase as the interval increases, with the lower bound being more sensitive to the

TABLE 1

TREND OF INEQUALITY IN THE UNITED STATES: COMPARISONS OF P-GINI WITH BOUNDED LIFE CYCLE ADJUSTED GINI, 1967–86

						Bounds for Life Cycle Adjusted Gini[b]					
			P-Gini[a]			1 Year		5 Year		10 Year	
Year	Obs	Gini[a]	1 year	5 year	10 year	Upper	Lower	Upper	Lower	Upper	Lower
1967	37407	0.345	0.231	0.233	0.237	0.327	0.046	0.328	0.098	0.331	0.137
1968	37948	0.341	0.230	0.232	0.238	0.322	0.044	0.323	0.093	0.326	0.127
1969	36168	0.344	0.226	0.227	0.234	0.324	0.046	0.325	0.099	0.328	0.137
1970	36650	0.349	0.229	0.232	0.237	0.329	0.047	0.330	0.101	0.333	0.136
1971	35416	0.351	0.227	0.230	0.235	0.328	0.046	0.329	0.101	0.333	0.139
1972	34913	0.352	0.226	0.229	0.233	0.329	0.045	0.331	0.099	0.334	0.137
1973	34472	0.351	0.223	0.226	0.230	0.327	0.044	0.329	0.094	0.331	0.134
1974	33664	0.348	0.220	0.222	0.227	0.323	0.042	0.325	0.091	0.328	0.128
1975	35670	0.349	0.229	0.232	0.237	0.328	0.043	0.330	0.092	0.332	0.128
1976	42218	0.350	0.229	0.231	0.236	0.328	0.043	0.330	0.092	0.335	0.130
1977	41079	0.352	0.229	0.231	0.236	0.331	0.042	0.332	0.092	0.336	0.129
1978	40960	0.353	0.232	0.234	0.238	0.332	0.042	0.334	0.091	0.334	0.129
1979	48273	0.352	0.227	0.230	0.235	0.330	0.042	0.332	0.092	0.335	0.127
1980	48148	0.353	0.230	0.232	0.236	0.331	0.041	0.332	0.091	0.346	0.126
1981	43328	0.363	0.243	0.246	0.250	0.343	0.043	0.344	0.093	0.357	0.132
1982	43346	0.372	0.254	0.258	0.262	0.354	0.045	0.355	0.099	0.357	0.138
1983	43009	0.373	0.250	0.253	0.259	0.353	0.045	0.354	0.096	0.357	0.136
1984	43360	0.379	0.261	0.263	0.270	0.361	0.046	0.362	0.102	0.365	0.143
1985	42288	0.381	0.260	0.262	0.269	0.362	0.047	0.363	0.102	0.366	0.143
1986	41923	0.382	0.259	0.262	0.268	0.364	0.046	0.365	0.102	0.367	0.143

Notes:
[a]Gini and P-Gini measures are identical to those reported in Formby *et al.* (1989).
[b]See text for details.

level of disaggregation.[6] Using the one year age interval (columns 7 and 8), we find that if there were no intracohort mobility (or "complete income stratification" in the terminology of Lillard and Willis (1978)), inequality as measured by the Lorenz-Gini would overstate lifetime inequality by roughly 5 percent. Thus, 5 percent of overall inequality is due simply to the inverted U-shape of the age-earnings profile. If the economy were characterized by perfect mobility (or "complete income mobility" in the words of Lillard and Willis (1978)), then the Lorenz-Gini would overstate "true" lifetime inequality by approximately 650 to 750 percent.

The P-Gini lies in between the upper and lower bounds, although closer to the upper bound (using either the one or five year age intervals). Using the one year age interval, the P-Gini is approximately 40 percent below the upper bound and 450 percent above the lower bound. Equivalently, considering the interval marked by the lower and upper bounds, the P-Gini is roughly two-thirds of the way toward the upper bound. Thus, the P-Gini implicitly allows for some degree of intracohort mobility, although it is far from "perfect" mobility. However, it is not obvious from construction of the P-Gini how mobility is incorporated into calculations of the P-Gini. Thus, without specific research on the actual degree of intracohort mobility, there is no reason a priori to expect the P-Gini to be a more

[6]The reason for this follows directly from (7). The variance of δ increases as the interval widens since there is greater deviation between individual incomes and cohort means.

accurate measure of lifetime inequality than any other value in between the two bounds.

Buchinsky and Hunt (1999), however, do analyze the degree of mobility using data from the NLSY and conclude that inequality is reduced by 12 percent to 26 percent when within group mobility over a four year time horizon is incorporated. While this only represents mobility over a four-year span and not lifetime mobility, reducing the Lorenz-Gini in column 3 by these percentages yields a measure of inequality which falls within the upper and lower bounds, but significantly closer to the upper bound. For example, in 1986, reducing the Lorenz-Gini by 12 percent (26 percent) gives a "lifetime" Gini coefficient of 0.336 (0.283).[7] One would expect these results to fall near the upper bound since the actual Gini is only scaled by a measure of mobility over a small portion of an individual's life cycle, implicitly assuming no mobility over the remainder. Thus, Buchinsky and Hunt's results provide some indication of the accuracy of the bounds, while also illustrating the fact that using incomplete panel data to assess lifetime inequality may not yield a complete picture.

Besides, as a comparison to the P-Gini, computing the bounds on lifetime income inequality also enables us to examine changes in the bounds over the range of the data. Using the one-year age interval results, the upper bound increases from 0.327 to 0.364 (11.3 percent) from 1967 to 1986; however, the lower bound remains unchanged. Given the large sample size in the CPS (column 2), the lower bound will change from one year to the next if there is a change in the variance of $\bar{\delta}$ in (6). Thus, an unchanging lower bound is consistent with little to no change in the distribution of this parameter between 1967 and 1986.[8] With the five- or ten-year age intervals, the lower bound does increase over the range of the data; by 4.1 percent with the five-year age interval and 4.4 percent with the ten-year age interval.

With the one-year age interval, the rising upper bound and stationary lower bound indicates a wider interval for lifetime inequality (with a difference in the Gini coefficients of 0.281 in 1967 and 0.318 in 1986). In addition, the upper bound is 7.1 times greater than the lower bound in 1967, increasing to 7.9 times greater in 1986. Even with the five- and ten-year age interval results, we also find the interval for lifetime income inequality to be widening despite the fact that the lower bound now increases over time as well. With the five-year (ten-year) age interval, the difference in the Gini coefficients is 0.230 (0.194) in 1967 and 0.263 (0.224) in 1986. Equivalently, with the five-year (ten-year) age interval, the upper bound is 3.3 (2.4) times larger than the lower bound in 1967 and 3.6 (2.6) times greater in 1986.

To more explicitly compare the distributions of actual income as well as adjusted incomes from year to year, Table 2 presents the results from Wilcoxon (1945) rank sum tests and Kolmogorov–Smirnov tests (Kolmogorov, 1933; Smirnov, 1939) for the equality of distributions. Wilcoxon rank sum tests are based

[7]Jenkins (2000) shows that the Gini coefficient in England based on incomes averaged over a six year period is approximately 12 percent less than the Gini coefficient based only on cross-sectional income.

[8]Note, even if the true distribution of δ is log normal, rather than normal, this bias should not impact comparisons over time, only the value of the lower bound computed in any given period.

TABLE 2

TESTS FOR ANNUAL CHANGES IN THE DISTRIBUTION OF ACTUAL AND LIFE CYCLE ADJUSTED INCOMES, 1967–86[a]

			Life Cycle Adjusted Income[b]			
	Actual Income		Upper Bound		Lower Bound	
Years	Wilcoxon Test[c]	K-S Test[d]	Wilcoxon Test	K-S Test	Wilcoxon Test	K-S Test
1967 v. 1968	–10.44 (p = 0.00)	0.04 (p = 0.00)	3.38 (p = 0.00)	0.02 (p = 0.00)	–108.43 (p = 0.00)	0.33 (p = 0.00)
1968 v. 1969	–8.44 (p = 0.00)	0.03 (p = 0.00)	–20.01 (p = 0.00)	0.07 (p = 0.00)	–92.87 (p = 0.00)	0.29 (p = 0.00)
1969 v. 1970	1.00 (p = 0.32)	0.01 (p = 0.00)	4.09 (p = 0.00)	0.02 (p = 0.00)	–0.15 (p = 0.88)	0.01 (p = 0.31)
1970 v. 1971	1.86 (p = 0.06)	0.02 (p = 0.00)	–8.74 (p = 0.00)	0.03 (p = 0.00)	72.79 (p = 0.00)	0.24 (p = 0.00)
1971 v. 1972	–8.03 (p = 0.00)	0.03 (p = 0.00)	–10.45 (p = 0.00)	0.04 (p = 0.00)	–104.94 (p = 0.00)	0.33 (p = 0.00)
1972 v. 1973	–4.08 (p = 0.00)	0.02 (p = 0.00)	11.95 (p = 0.00)	0.04 (p = 0.00)	–42.43 (p = 0.00)	0.13 (p = 0.00)
1973 v. 1974	2.56 (p = 0.01)	0.02 (p = 0.00)	–9.76 (p = 0.00)	0.04 (p = 0.00)	–1.56 (p = 0.12)	0.02 (p = 0.00)
1974 v. 1975	6.14 (p = 0.00)	0.02 (p = 0.00)	14.12 (p = 0.00)	0.05 (p = 0.00)	69.00 (p = 0.00)	0.22 (p = 0.00)
1975 v. 1976	–3.76 (p = 0.00)	0.02 (p = 0.00)	–8.51 (p = 0.00)	0.03 (p = 0.00)	–60.74 (p = 0.00)	0.18 (p = 0.00)
1976 v. 1977	–3.47 (p = 0.00)	0.02 (p = 0.00)	–3.52 (p = 0.00)	0.02 (p = 0.00)	–48.04 (p = 0.00)	0.14 (p = 0.00)
1977 v. 1978	–4.25 (p = 0.00)	0.02 (p = 0.00)	–4.28 (p = 0.00)	0.02 (p = 0.00)	–61.72 (p = 0.00)	0.18 (p = 0.00)
1978 v. 1979	–4.17 (p = 0.00)	0.02 (p = 0.00)	–7.49 (p = 0.00)	0.02 (p = 0.00)	9.05 (p = 0.00)	0.03 (p = 0.00)
1979 v. 1980	2.61 (p = 0.01)	0.01 (p = 0.00)	1.60 (p = 0.11)	0.01 (p = 0.07)	13.82 (p = 0.00)	0.04 (p = 0.00)
1980 v. 1981	2.05 (p = 0.04)	0.01 (p = 0.00)	5.86 (p = 0.00)	0.02 (p = 0.00)	–31.78 (p = 0.00)	0.09 (p = 0.00)
1981 v. 1982	3.47 (p = 0.00)	0.02 (p = 0.00)	12.85 (p = 0.00)	0.04 (p = 0.00)	17.97 (p = 0.00)	0.05 (p = 0.00)
1982 v. 1983	–1.29 (p = 0.20)	0.01 (p = 0.02)	–19.55 (p = 0.00)	0.06 (p = 0.00)	14.08 (p = 0.00)	0.04 (p = 0.00)
1983 v. 1984	–8.81 (p = 0.00)	0.03 (p = 0.00)	–0.84 (p = 0.40)	0.01 (p = 0.31)	–148.68 (p = 0.00)	0.44 (p = 0.00)
1984 v. 1985	–4.29 (p = 0.00)	0.02 (p = 0.00)	–7.00 (p = 0.00)	0.02 (p = 0.00)	–30.23 (p = 0.00)	0.09 (p = 0.00)
1985 v. 1986	–4.44 (p = 0.00)	0.02 (p = 0.00)	–11.42 (p = 0.00)	0.04 (p = 0.00)	–26.09 (p = 0.00)	0.07 (p = 0.00)

Notes:

[a]Incomes adjusted for inflation using the annual GDP price deflator.

[b]Adjusted incomes based on one-year age intervals.

[c]Results list the z-value for the Wilcoxon rank sum tests and the associated probability values.

[d]Results list the D-value for the K-S tests and the associated probability values.

on a statistic which is derived by combining two years of data, ranking the data in ascending order, and summing the ranks from either one of the years.[9] The difference between the sum of the ranks and the expected value of the sum of the ranks under the null hypothesis of no difference between the distributions forms the basis of the test statistic. Kolmogorov–Smirnov tests, on the other hand, compare the empirical cumulative distribution functions from any two years and use the maximum difference between the two distribution functions as the basis for the test statistic. As a result, tests of this latter type are not particularly responsive to differences in the tails of the distributions.

As the results in Table 2 indicate, rarely do we fail to reject the equality of the distributions across any pair of adjacent years. Using the actual income and the Wilcoxon rank sum tests (column 2), we cannot reject the null hypothesis (at the 5 percent level of significance) of no change in the distribution of (real) income from the preceding year in 1970, 1971, and 1983. We never fail to reject the null using the Kolmogorov–Smirnov tests (column 3). Using the age-equivalent income assuming no intracohort mobility (i.e., the upper bound; columns 4 and 5), we only fail to reject the null hypothesis of no change from the preceding year in 1980 and 1984. This result is robust to the choice of test. Finally, using the age-equivalent income under the assumption of perfect mobility (i.e., the lower bound; columns 6 and 7), only in 1970 and 1974 do we fail to reject the null hypothesis of no change from the preceding year, although we do not reject the null in 1974 using the Kolmogorov–Smirnov test.

The implications of these tests are that the distributions of adjusted (real) incomes—controlling for age and fixing the degree of mobility at either of the two extremes—are no less stable over time than the distributions of actual (real) incomes. In the end, we reject the hypothesis of no distributional change as often with the adjusted incomes as we do with the actual incomes. This indicates that the changes which occurred from year to year in the distribution of actual incomes are due to reasons other than changes in the age distribution or changes in the degree of intracohort mobility.

4. Conclusion

It is well known that inequality measures based on cross sectional income do not adequately reflect lifetime income inequality for two main reasons. First, intracohort mobility will reduce lifetime inequality below inequality in any one year. Second, as Paglin (1975) argued, the shape of age-income profile implies that individuals of varying age should not expect to have identical income at a given point in time. Given this knowledge, there have been two approaches on how to more accurately measure "true" inequality: devise a more appropriate cross sectional inequality measure (e.g., the P-Gini and its descendants) or utilize existing, but incomplete, panel data sets.

The purpose of this paper is to salvage the use of comprehensive cross sectional data, such as the CPS, while still being able to say as much as we can about

[9]To make the data comparable over time, the GDP deflator is used to control for inflation.

lifetime income inequality in a given year. However, instead of attempting to correct the P-Gini or introduce a new cross sectional measure of inequality, we bound lifetime income inequality at a point in time by varying our assumptions about the degree of intracohort mobility. The lower bound assumes "perfect" or "complete" mobility (the "lottery model"), while the upper bound assumes no intracohort mobility. In addition, we also heed Paglin's criticism of the typical Gini coefficient when calculating the bounds by utilizing the concept of an age equivalence scale to remove differences due to individuals being in different stages of their respective life cycles.

The results indicate a rather large and increasing difference between the upper and lower bound, thus emphasizing the increasingly important role of within-group mobility in determining lifetime inequality. Thus, more research is called for so that we may better understand the income trajectories of individuals. In addition, we find that the correction suggested by Paglin, the P-Gini, and inequality adjusted for the effects of four-year mobility obtained by Buchinsky and Hunt (1999) lie in this interval, but closer to the upper bound. Finally, while the upper bound has increased over time (along with the Lorenz- and P-Gini), the lower bound has remained relatively stable. This implies that the cost (in terms of higher inequality) of intracohort immobility increased from the 1960s to the 1980s.

Appendix

Derivation of $\bar{\delta}$

Income in any period t for individual i is given by

$$y_{it} = \bar{y}_t(1+\delta_{it}) \tag{A1}$$

Lifetime income, Y^*, is simply the sum of income over one's lifetime (ignoring any discount rate):

$$Y_i^* = \sum_{t=1}^{T} y_{it} \tag{A2}$$

For each individual, $\bar{\delta}_i$ is defined as the constant shock in every period which leaves lifetime income unchanged. In other words,

$$Y_i^* = \sum_{t=1}^{T} \bar{y}_t(1+\delta_{it}) = \sum_{t=1}^{T} \bar{y}_t(1+\bar{\delta}_i) \tag{A3}$$

Re-arranging terms,

$$\bar{\delta}_i = \frac{\Sigma_{t=1}^{T}\bar{y}_t(1+\delta_{it}) - \Sigma_{t=1}^{T}\bar{y}_t}{\Sigma_{t=1}^{T}\bar{y}_t}$$

$$= \frac{\Sigma_{t=1}^{T}\bar{y}_t\delta_{it}}{\Sigma_{t=1}^{T}\bar{y}_t}$$

which is equation (5) in the paper.

Distribution of $\bar{\delta}$

To derive the distribution of $\bar{\delta}$, re-write equation (5) as

$$\bar{\delta}_i = \frac{1}{\Sigma_{t=1}^{T} \bar{y}_t} (\bar{y}_1 \delta_{i1} + \ldots + \bar{y}_T \delta_{iT}) \tag{A4}$$

Assuming $\delta \sim iid\ N(0,\sigma^2)$, then

$$E[\bar{\delta}_i] = \frac{1}{\Sigma_{t=1}^{T} \bar{y}_t} (\bar{y}_1 E[\delta_{i1}] + \ldots + \bar{y}_T E[\delta_{iT}]) = 0 \tag{A5}$$

the variance of $\bar{\delta}$, Var($\bar{\delta}_i$), is

$$\begin{aligned} \mathrm{Var}(\bar{\delta}_i) &= \left(\frac{1}{\Sigma_{t=1}^{T} \bar{y}_t} \right)^2 (\bar{y}_1^2 \mathrm{Var}(\delta_{i1}) + \ldots + \bar{y}_T^2 \mathrm{Var}(\delta_{iT})) \\ &= \left(\frac{1}{\Sigma_{t=1}^{T} \bar{y}_t} \right)^2 \sum_{t=1}^{T} \bar{y}_t^2 \sigma^2 \\ &= \sigma^2 \frac{\Sigma_{t=1}^{T} \bar{y}_t^2}{(\Sigma_{t=1}^{T} \bar{y}_t)^2} \end{aligned}$$

and $\bar{\delta}$ has a normal distribution since it is the sum of normally distributed variables.

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