A SIMPLIFIED MODEL FOR SOCIAL WELFARE ANALYSIS:
AN APPLICATION TO SPAIN, 1973-74 TO 1980-81

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Most of the literature on income distribution has concentrated on inequality. In this paper we introduce a concern for efficiency in a social welfare model. We propose a simple but useful specification which combines three features: (i) the selection of measurement instruments in the relative and the absolute case on the grounds of their properties for applied work; (ii) a procedure to make welfare comparisons across households with different needs, in a model in which equivalence scales depend only on household size; and (iii) the use of household specific statistical price indices to make intertemporal comparisons in real terms. The methodology is applied to the study of the role of prices and demographic effects in the evolution of the standard of living in Spain from 1973-74 to 1980-81.

INTRODUCTION

Most of the analytical and empirical literature on income distribution has concentrated on income inequality. In this paper we propose a simplified but convenient social welfare model which reflects also a concern for efficiency. The model is then applied to the evolution of the standard of living in Spain using two household budget surveys for 1973-74 and 1980-81. The main features of the approach are the following four.

(i) As Dutta and Esteban (1991) have shown, to express social or aggregate welfare in terms of only two statistics of the income distribution—the mean, and a measure of inequality—we need to specify the type of mean-invariance property we want our inequality indices to satisfy. This is politically important, since we know from the early discussion in Kolm (1976) that the choice of a mean-invariance class of inequality measures is not merely a technical matter, but a value laden question. In this paper, we consider two polar cases: the usual relative inequality concept, according to which a proportional change in all incomes leaves the level of inequality unchanged; and an absolute inequality concept, very seldom applied in empirical analysis, according to which inequality remains constant only if all individuals experience the same absolute income change.

(ii) We are interested in complete indicators which permit the decomposition of welfare changes into changes in the mean, and changes in either relative or absolute inequality. Which of these social evaluation functions (SEFs for short)

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Recent reports based on questionnaires indicate that people are by no means unanimous in their choice between relative, absolute or other intermediate notions of inequality. See, for instance, Amiel and Cowell (1992), and Ballano and Ruiz-Castillo (1994).
should we use in applied work? It turns out that the conditions usually required for
an admissible SEF plus an interesting decomposability property, lead to specific
functional forms: in the relative case, to a single member of the generalized entropy
family and, in the absolute case, to the Kolm–Pollak family indexed by a param-
eter representing degrees of aversion to inequality.

(iii) We assume that the only characteristic which gives rise to different needs
for social evaluation purposes is household size. The problem, of course, is that
to pool all households into a unique distribution we need a procedure to compare
non-income needs across household sizes. In the relative case, following Buhman
et al. (1988) and Coulter et al. (1992a, b), we use a parametric model of equiva-
ience scales which allows for different views about the importance of economies
of scale in consumption within the household. We extend the model to the absolute
case and establish the connection between the parametrization of economies of
scale in both cases.

(iv) To introduce the distributional role of changes in relative prices in inter-
temporal comparisons we use household specific statistical price indices. Since
statistical price indices provide only convenient bounds to the true cost-of-living
constructions, we show how our estimates also provide equally convenient bounds
to the change in the mean and inequality in real terms. We illustrate the advantages
of our procedure by comparing our results with those obtained by adopting the
usual assumption of a single inflation rate common to all households.

These measurement tools and methodological conventions are applied to
Spanish data from two large household budget samples, of about 24,000 observa-
tions each: the Encuestas de Presupuestos Familiares (EPF for short), collected in
1973–74 and 1980–81 by the Spanish Instituto Nacional de Estadistica with the
main purpose of estimating the base weights of the official system of Consumer
Price Indices. Like Slesnick (1991, 1993), we propose to identify a household
standard of living with current commodity consumption. We argue that, in our
case, this is better approximated by a measure of current total household expendit-
ures, net of expenditures on the acquisition of certain durables.

During this period, right after the first oil crisis and in the middle of a radical
political change in Spain, according to National Accounts data GNP grew at an
average annual rate of about 2.3 percent at constant prices of 1986, while accord-
ing to the Consumer Price Index there was a 322 percent inflation rate. In this
context, our main empirical conclusions are the following:

(i) Our estimates provide very good bounds for the real change in the mean
and relative inequality. Therefore, we can safely conclude that welfare for the
population as a whole has improved between 10–11 percent in the relative case.

(ii) When we apply the same inflation rate to all households in 1973–74,
estimates of welfare change are similar to those obtained with household specific
price indices. However, in this case we only capture the inequality change in
money terms. Given that relative prices in Spain have evolved during this period
in a pro-poor direction, the true improvement of inequality in real terms becomes
understated. As a consequence, too much of the real welfare improvement is
wrongly attributed to an increase in the mean.

(iii) During this period, dominated by the first oil crisis and other adverse
economic circumstances in Spain, there is a moderate increase in mean household
expenditures in real terms of about 4-7 percent. As we reported in Ruiz-Castillo (1995a), real inequality decreased between 15-20 percent according to the relative inequality index we use in this paper. One way to appreciate the magnitude of such change, is provided by the following unusual finding: absolute inequality for the total population decreased for all values of the equivalence scale parameter. We estimate a welfare increase in the absolute case which ranges from 38,000 to 55,000 pesetas, or from 27,000 to 48,000 pesetas, depending on the value of the aversion to inequality parameter.

(iv) There are considerable variations among subgroups in the partition by household size. In particular, there are exceptions to the generalized improvement in inequality: as the parameter reflecting the aversion to inequality increases there are two household sizes for which absolute inequality increases. By the use of decomposable measurement instruments we are able to understand how results at the household level get translated to the population as a whole. In both the relative and the absolute case most of our results are rather robust to changes in the parameters which reflect the generosity of the equivalence scales.

The rest of the paper is organized in four sections and a brief statistical Appendix. The first section presents the measurement framework, which includes the parametrization of equivalence scales in the relative and the absolute case, the social evaluation functions, and the nature of our approximation to social welfare change using statistical price indices. The second section is devoted to the measurement of a household standard of living. The third section contains the empirical results, while the final section offers some concluding remarks on the potential implications of our results for similar studies in other countries.

I. THE MEASUREMENT FRAMEWORK

I.1. Interhousehold Welfare Comparisons

Suppose we have a population of \( h = 1, \ldots, H \) households which can differ in a single dimensional variable—say, income—representing its standard of living, \( x^h \), and/or a vector of household characteristics. In this paper, households of the same size are assumed to have the same needs and, therefore, their incomes are directly comparable. Consequently, we believe that it is important to investigate separately each of the subgroups in the basic partition by household size. However, social evaluation within subgroups need not yield unanimous results. Moreover, it is always convenient to extract conclusions for the population as a whole. Therefore, we need a procedure to establish inter-household welfare comparisons. This is, of course, the role played by equivalence scales.

We assume that larger households have greater needs, but also greater opportunities to achieve economies of scale in consumption. Denote household size by \( s^h \) and, for each household \( h \), define adjusted income in the relative case by

\[
y^h(\Theta) = x^h / (s^h)^{\Theta}, \quad \Theta \in [0, 1].
\]

Assume there are \( m = 1, \ldots, M \) household sizes. If we think of a single adult as the reference household, the expression \( 1/m^\Theta \) can be interpreted as the number of equivalent adults in a household of size \( m \). Thus, the greater is \( \Theta \), the greater
the number of equivalent adults for each household or, in other words, the smaller
the economies of scale. When \( \Theta = 0 \) and economies of scale are assumed to be
infinite, adjusted income coincides with unadjusted household income; while if
\( \Theta = 1 \) and economies of scale are completely ruled out, then adjusted income
equals per capita household income.

In the absolute case, for each household \( h \) of size \( m \), define adjusted income by
\[
y^h(\lambda^m) = x^h - \lambda^m(m-1).
\]
The parameter \( \lambda^m \) can be interpreted as the cost of a reference adult when house-
hold size is \( m \). Thus, for each \( m \) economies of scale vary inversely with \( \lambda^m \).

Let \( x^m \) be the vector of original incomes for households of size \( m \). Notice
that, if \( I(\cdot) \) is any scale invariant index of relative inequality, then we have
\[
I(y^m(\Theta)) = I(x^m/(\Theta^\Theta)) = I(x^m), \quad m = 1, \ldots, M.
\]
Similarly, if \( A(\cdot) \) is any translation invariant index of absolute inequality, then
we have
\[
A(y^m(\lambda)) = A(x^m - \lambda^m(m-1)) = A(x^m), \quad m = 1, \ldots, M.
\]
Thus, the two models share the convenient property that, within each ethically
homogeneous subgroup, the adjustment process does not alter the underlying
inequality: the inequality of adjusted income is equal to the inequality of original
income.

The only remaining question is the following. The unit interval provides a
natural range of variation for the parameter \( \Theta \) in the relative case, but how do
we fix \( \lambda^m \) for each \( m \) in the absolute case? The following procedure permits us to
establish a connection between the parametrization of equivalence scales in the
two cases. Let \( y^m(\Theta) \) and \( y^m(\lambda^m) \) be the adjusted income vectors for households
of size \( m \) in the relative and the absolute case, respectively, and let \( \mu(\cdot) \) denote
the mean of any distribution. Given \( \Theta \), we choose \( \lambda^m(\Theta) \) for each \( m \) so that the
mean of both vectors is the same, that is, so that
\[
\mu(y^m(\lambda^m)) = \mu(y^m(\Theta)).
\]
It is easy to see that this condition implies:
\[
\lambda^m(\Theta) = [\mu(y^m(\Theta))(\Theta^\Theta - 1)]/(m-1) = [\mu(x^m)(\Theta^\Theta - 1)]/[(m-1)m^\Theta].
\]
Thus, the greater \( \Theta \) is, the greatest is \( \lambda^m \) and the smaller are the economies of
scale.

1.2. Admissible Social Evaluation Functions

A SEF is a real valued function \( W \) defined in the space \( R^m \) of adjusted
incomes, with the interpretation that for each income distribution \( y = (y^1, \ldots, y^M) \), \( W(y) \) provides the "social" or, simply, the aggregate welfare from a
normative point of view. Let us assume that our SEFs satisfy the requirements
discovered by Dutta and Esteban (1991) for expressing welfare as a function of
the mean and an index of relative or absolute inequality. In addition, let us adopt
a multiplicative or an absolute trade off between the mean and inequality in the relative and the absolute case, respectively. However, which SEFs within these classes should we use in applied work? The following property leads us to an appropriate selection.

Suppose that we have two islands where income is equally distributed but whose means are different. If they now form a single entity, there will be no within-island inequality but there would be inequality between them. In income inequality theory we search for additively separable measures capable of expressing this intuition. In our context, for any partition we are interested in expressing social welfare for the population as the sum of two terms: a weighted average of welfare within the subgroups, with weights equal to demographic shares, minus a term which penalizes the inequality between subgroups. In this case, we say that the SEF is additively decomposable.

Let \( \mu^* \) be the distribution in which each household is assigned the mean income of the subgroup to which it belongs in the partition by household size, \( \mu(x^m) \). Let \( I_1(\cdot) \) be the first index of relative inequality originally suggested by Theil:

\[
I_1(x) = (1/H)\left[\sum(x^h/\mu(x)) \ln (x^h/\mu(x))\right].
\]

Consider SEFs which can be expressed as the product of the mean and a term equal to one minus a relative inequality index of the Generalized Entropy class. Ruiz-Castillo (1995b) shows that the only SEF among them with the property of additive decomposability with demographic weights, is the following:

\[
W(x) = \mu(x)(1 - I_1(x)) = \sum_m [H^m/H] W_T(x^m) - \mu(x) I_1(\mu^*),
\]

where \( H^m \) is the number of households of size \( m \), so that \( \sum_m H^m = H \). Thus, social welfare is seen to be a weighted average of the welfare within each subgroup with weights equal to demographic shares, minus the between-group inequality weighted by the population mean.

In the absolute case, Blackorby et al. (1981) show that analogous requirements lead to the Kolm–Pollak family of SEFs:

\[
KP_\gamma(x) = - \left[1/\gamma\right] \ln \left[\left(1/H\right)\Sigma_h e^{-\gamma x_h}\right], \quad \gamma > 0,
\]

where \( \gamma \) is interpreted as an aversion to inequality parameter: as \( \gamma \) increases, social indifference curves show increasing curvature until, in the limit, only the poorest household income matters. The absolute inequality index associated with \( W_\gamma \) is

\[
A_\gamma(x) = \left[1/\gamma\right] \ln \left[\left(1/H\right)\Sigma_h e^{\gamma(x^h - x^m)}\right].
\]

Let us denote by \( \xi^* \) the distribution in which each household is assigned the equally distributed equivalent income of the subgroup to which it belongs, \( \xi(x^m) \). Then

\[
A_\gamma(x) = \sum_m [H^m/H] A_\gamma(x^m) + A_\gamma(\xi^*),
\]

so that

\[
KP_\gamma(x) = \mu(x) - A_\gamma(x) = \sum_m [H^m/H] W_T(x^m) - A_\gamma(\xi^*).
\]
Thus, social welfare is equal to the mean minus the Kolm-Pollak absolute inequality index. On the other hand, social welfare is a weighted average of the welfare within each subgroup with weights equal to demographic shares, minus the inequality between the subgroups.²

Taking into account our definitions of adjusted income, in the relative case we have

$$W(y(\Theta)) = \sum_m [H'/H] W_1(x''_m/m^\Theta) - \mu(y(\Theta)) I_1(\mu^*(\Theta)), \quad \Theta \in [0, 1],$$

where $\mu^*(\Theta)$ is the distribution in which each household is assigned the mean income of the subgroup to which it belongs in the partition by household size, $\mu(y''(\Theta))$. In the absolute case, let $\lambda(\Theta) = (\lambda^1(\Theta), \ldots, \lambda^M(\Theta))$ be the vector of equivalence scale parameters fixed as in equation (1). Then we have

$$KP_r(y(\lambda(\Theta))) = \sum_m [H''/H] KP_r(x'')$$

$$- \sum_m [H''/H] \lambda''(\Theta)(m-1) - \Lambda_r(\xi^*(\lambda(\Theta))),$$

where $\xi^*(\lambda(\Theta))$ is the distribution in which each household is assigned the equally distributed equivalent income of the subgroup to which it belongs, $\xi(y''(\Theta))$.

In welfare economics we are mostly interested in personal welfare, rather than in household welfare. Following standard practice, we can extend the SEF domain to distributions in which each household adjusted income is weighted by household size or, in other words, in which each person is assigned the adjusted income of the household to which she belongs. The above formulas for $W$ and $KP_r$ can be easily transformed for this case: demographic shares, $H''/H$, as well as expressions $\mu^*(\Theta)$ and $\xi^*(\lambda(\Theta))$, must be replaced by their counterparts in the distribution of persons.

1.3. The Nature of Our Approximation in the Presence of the Substitution Bias of Statistical Price Indices

Omitting here any reference to the parameter $\Theta$ to simplify the notation, let $y_1 = (y'_1, \ldots, y'_n)$ and $y_2 = (y'_1, \ldots, y''_n)$ be the vector of adjusted household expenditures in the two situations under comparison. Suppose we want to compare $y_1$ and $y_2$ in real terms at prices of situation 2, $p_2$. Let $y''_{12}$ be household $h$'s adjusted income in situation 1 expressed at prices $p_2$. Ideally we would compute $y''_{12} = y'_1 L(p_2, p_1; u'_1)$, where $L(p_2, p_1; u'_1)$ is a true cost-of-living index of the Laspeyres type and $u'_1$ is the utility level achieved by household $h$ in situation 1. Similarly, to compare $y_1$ and $y_2$ in real terms at prices of situation 1, $p_1$, we would use for each household the expression $y''_{21} = y''_2 / P(p_2, p_1; u''_2)$, where $P(p_2, p_1; u''_2)$ is a true cost-of-living index of the Paasche type and $u''_2$ is the utility level achieved by household $h$ in situation 2.

In this favourable case, how would the classical number index problem manifest itself? To answer this question, we must review the formulas for social welfare change we use in the sequel. Recall that in the relative case, for example, social

²Of course, both in the relative and the absolute case the property of decomposability is essential for the study of any other partition. For an application to partitions by geographic and socioeconomic characteristics, which will not be treated here, see Ruiz-Castillo (1995c).
welfare at any period \( t = 1, 2 \) is equal to

\[ W(y_t) = \mu(y_t)[1 - I_1(y_t)]. \]

To evaluate the social welfare change at constant prices \( p_2 \), we compare the distributions \( y_2 \) and \( y_{12} \) by means of the expression

\[ \Delta W(p_2) = \Delta \mu(p_2) \Delta E(p_2), \]

where

\[ \Delta \mu(p_2) = \mu(y_2)/\mu(y_{12}), \]

\[ \Delta E(p_2) = [1 - I_1(y_2)]/[1 - I_1(y_{12})], \]

and

\[ \Delta W(p_2) = W(y_2)/W(y_{12}). \]

Equation (2) measures the real change in the mean. Equation (3) reflects the change in real inequality. It is greater (smaller) than one as real inequality decreases (increases) in period 2 relative to period 1. Equation (4) measures the change in real welfare. Let us denote by \( \Delta W(p_1) \), \( \Delta \mu(p_1) \), and \( \Delta E(p_1) \) the corresponding expressions for the social evaluation problem at prices \( p_1 \), which involves the comparison of distributions \( y_{1} \) and \( y_1 \). Notice that there are no a priori reasons for \( \Delta \mu_1(p_2) \) or \( \Delta E(p_2) \) to be greater or smaller than \( \Delta \mu(p_1) \) or \( \Delta E(p_1) \), respectively. Hence, nothing can be said on theoretical grounds about the relationship between \( \Delta W(p_2) \) and \( \Delta W(p_1) \). Nevertheless, in an empirical situation one hopes that these two magnitudes are close to each other.

To carry on the above program, we need to estimate a complete demand system in order to compute the true cost-of-living indices. In this paper, we propose to approximate the index \( L(p_2, p_1; u_h) \) for each \( h \) by its upper bound \( L(p_2, p_1; w_h) \), where \( w_h \) is the vector of total expenditure commodity shares of household \( h \) in situation 1. Similarly, we propose to approximate the index \( P(p_2, p_1; u_h^2) \) by its lower bound \( P(p_2, p_1; w_h^2) \), where \( w_h^2 \) is the vector of total expenditure commodity shares of household \( h \) in situation 2. Let us denote our estimates of \( y_{12}^h \) and \( y_{21}^h \) by \( z_{12}^h = y_{12}^h L(p_2, p_1; w_h^1) \) and \( z_{21}^h = y_{21}^h / P(p_2, p_1; w_h^2) \), respectively. The question is: which is the nature of our approximations to the true changes in the mean, inequality and welfare in real terms?

It should be clear that, because of the substitution bias of our household specific price indices, for each \( h \) our constructions \( z_{12}^h \) and \( z_{21}^h \) overestimate the true ones, \( y_{12}^h \) and \( y_{21}^h \), respectively. Two important consequences follow from here. In the first place, taking into account equation (2), our estimates for the real change in the mean at prices \( p_2[p_1] \) provide a lower (upper) bound for their true value, \( \Delta \mu(p_2) [\Delta \mu(p_1)] \). In the second place, let us adopt the reasonable assumption that the substitution bias is greater for the rich than the poor. Assume also that the change in relative prices from \( p_1 \) to \( p_2 \) is less damaging to the poor than to the rich, as we know to be the case for Spain in this period. Then, following the argument given in Ruiz-Castillo (1995b), it can be shown that our estimates at \( p_2[p_1] \) for the expressions \( \Delta E(p_2) [\Delta E(p_1)] \) provide an upper (lower) bound for the true constructions.
Therefore, nothing definite can be said about the relationship between our estimates and the true values for $\Delta W(p_2)$ and $\Delta W(p_1)$. However, in any empirical situation one would like to obtain that $\Delta \mu(p_2) \leq \Delta \mu(p_1)$ and $\Delta E(p_2) \geq \Delta E(p_1)$, in the hope that the true changes in the mean and in relative inequality lie between these limits. In this case, our estimates for $\Delta W(p_2)$ and $\Delta W(p_1)$ have a good chance of being close to each other.3

II. The Measurement of the Standard of Living

Our data comes from two large budget surveys collected in 1973–74 and 1980–81. They consist of 24,151 and 23,707 observations, representative of a population of approximately 9 and 10 million households, respectively, occupying residential housing in all of Spain except the northern African cities of Ceuta and Melilla.

The EPFs are spread out uniformly over a period of 52 weeks. All household members of 14 or more years of age are supposed to record all expenditures which take place during a sample week. Then, in depth interviews are conducted to register past expenditures over reference periods beyond a week and up to a year. From that information, the INE estimates annual household total expenditures.4

On the other hand, a maximum of four income recipients are asked about the income earned from different sources during the year prior to the sample week. Therefore, household expenditures and household income are not estimated for the same period.

Given the nature of our data, we have several reasons for choosing household expenditures rather than household income to approximate a household standard of living. (i) There is a general presumption that current expenditure is a better proxy of permanent income than current income which includes more volatile transitory components. (ii) Although the EPF’s include valuable information on income perceived by a maximum of four household members, the surveys are primarily designed to measure household expenditure with the purpose of estimating the Consumer Price Index weighting system. Therefore, we expect the INE to devote more care and attention to the expenditure side. (iii) Several individuals might be inclined to underreport income. For instance, those working in the underground economy, the self-employed, professionals of all sorts, or people working in the agricultural sector. However, none of them are particularly prone to misreport their expenditures. Therefore, we expect that expenditures for those individuals are better measured than income. On the other hand, we expect respondents to report equally well their expenditures on goods and services acquired in either the underground or the regular economy. Therefore, the activities of both demanders and suppliers of the underground economy are better captured through the expenditure side. (iv) It turns out that INE’s estimates of total expenditures for more than 60 percent of households are greater than household income.5

3Of course, an entirely analogous problem must be faced in the absolute case.
4By taking into account the available information on bulk purchases, Peña and Ruiz-Castillo (1998) improved upon INE’s original estimates of annual food and drinks expenditures. Our measure of household total expenditures includes the corresponding correction.
5This is in agreement with results in Sanz (1996) showing a loss close to 23 percent when income information in the EPF’s is compared with National Accounts data.
Moreover, contrary to all expectations, there is evidence showing less total income inequality than total expenditure inequality. In our opinion, these facts need some explanation before income data can be comfortably used.

In addition to the above reasons to prefer the EPF measure of household expenditures, we agree with Slesnick (1991, 1993) that, ideally, we should identify the standard of living with commodity consumption. Lacking information on leisure and public goods consumption, our starting point must be household total expenditures as an approximation to household consumption of private goods and services.

The EPF has a rather wide concept of total expenditure, including expenditures on items not covered by the Consumer Price Index (like funeral articles; contributions to non-profit institutions; gambling expenditures; fines; hunting, fishing and other fees), as well as a number of imputations for home production, wages in kind and subsidized meals at work. To avoid double counting, transfers to other households or to household members absent from home are excluded.

Our experience with the 1980–81 EPF indicates that discontinuous household expenditures on some durables, whose occurrence may distort heavily the total, are best considered investment rather than consumption. These refer to current acquisitions of cars, motorcycles and other means of private transportation, as well as house repairs financed by either tenants or owner-occupiers. Life and housing insurance premiums are excluded on the same grounds. Thus, our estimate of household current consumption equals total household expenditures, net of these investment items.

Ideally, we should include an estimate of the consumption services currently provided by these investment flows as well as by the stock of household durables acquired in the past. We do this for housing—without doubt the more important household durable—since the INE includes a market rental value for owner-occupied housing, as well as for the rest of the stock which is neither rented nor owned by the household occupying it. Such rental values are estimated by the owner or the occupying household, respectively.

III. Empirical Results

III.1 Changes in Real Terms Within the Partition by Household Size

a. The Change in Mean Household Expenditures

We begin by investigating the role of prices in inequality and welfare comparisons within each subgroup in the partition by household size. Our data were collected from July 1973 to June 1974 (situation 1), and from April 1980 to March 1981 (situation 2). Since we have information on the quarter in which each household was interviewed in the 1980–81 EPF, we choose \( p_2 \) = winter of 1981. Since this is not the case for the 1973–74 EPF, we choose \( p_1 = (1/2)p_{73} + (1/2)p_{74} \). In what follows, we denote by \( z_t \) and \( z_{12} \) the 1973–74 expenditure

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6See Ayala et al. (1993) and Del Rio and Ruiz-Castillo (1996).
8Statistical price indices were constructed using a 58 commodity breakdown for which the INE publishes monthly price data at the national level. For a detailed analysis of this topic, see Sastre (1998).
TABLE 1
NUMBER OF PERSONS AND MEAN HOUSEHOLD EXPENDITURES AT CONSTANT PRICES IN THE PARTITION BY HOUSEHOLD SIZE

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<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>2.2</td>
<td>2.1</td>
<td>92,810</td>
<td>110,564</td>
</tr>
<tr>
<td>2</td>
<td>10.9</td>
<td>11.4</td>
<td>168,029</td>
<td>187,062</td>
</tr>
<tr>
<td>3</td>
<td>15.7</td>
<td>15.1</td>
<td>238,221</td>
<td>255,562</td>
</tr>
<tr>
<td>4</td>
<td>23.8</td>
<td>25.5</td>
<td>281,734</td>
<td>301,520</td>
</tr>
<tr>
<td>5</td>
<td>19.8</td>
<td>20.1</td>
<td>312,423</td>
<td>327,372</td>
</tr>
<tr>
<td>6</td>
<td>13.2</td>
<td>12.5</td>
<td>336,611</td>
<td>347,418</td>
</tr>
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<td>7</td>
<td>6.9</td>
<td>6.8</td>
<td>364,756</td>
<td>382,492</td>
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<tr>
<td>8 and more</td>
<td>7.5</td>
<td>6.5</td>
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<tr>
<td>All</td>
<td>100.0</td>
<td>100.0</td>
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distribution at $p_1$ and $p_2$ prices, respectively; and by $z_1$ and $z_2$ the 1980–81 expenditure distribution at those prices. The percentage distribution by household size in the two surveys, as well as the mean household expenditures at prices $p_1$ and $p_2$ are shown in Table 1.\textsuperscript{9}

There is little change in the frequency distributions by household size in this relatively short period of time. Consequently, mean household size slightly declines from 4.61 to 4.53 persons per household. However, there are important changes in mean household expenditures which are summarized in Table 2.

There are two facts to be emphasized. In the first place, our estimates for the lower and upper bounds for the true changes in the mean of $p_2$ and $p_1$ are very close together for every household size. This means that, in spite of the substitution bias of the statistical price indices, we can be reasonably confident about the quality of our approximation to the change in the mean at constant prices. In the

\textbf{TABLE 2}
CHANGE IN THE MEAN HOUSEHOLD EXPENDITURES AT CONSTANT PRICES IN THE PARTITION BY HOUSEHOLD SIZE: 1973-74 TO 1980-81

<table>
<thead>
<tr>
<th>Household Size</th>
<th>Mean Increase in Real Terms, in % At $p_2 =$ Winter of 1981</th>
<th>At $p_1 =$ Average of 1973 and 1974</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.0</td>
<td>19.1</td>
</tr>
<tr>
<td>2</td>
<td>9.9</td>
<td>11.3</td>
</tr>
<tr>
<td>3</td>
<td>6.7</td>
<td>7.3</td>
</tr>
<tr>
<td>4</td>
<td>6.6</td>
<td>7.0</td>
</tr>
<tr>
<td>5</td>
<td>4.3</td>
<td>4.8</td>
</tr>
<tr>
<td>6</td>
<td>2.4</td>
<td>3.2</td>
</tr>
<tr>
<td>7</td>
<td>4.4</td>
<td>4.9</td>
</tr>
</tbody>
</table>

\textsuperscript{9}All results make use of the information on sampling weighting factors provided by INE, so that our estimates are always blown up estimates for the total population.

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second place, although all households experience some increase in the mean, the improvement is inversely related to household size. Single person and two person households—representing about 28 percent of all households and 13 percent of all persons—have 18 and 10 percent increases. The important group of 3 and 4 person households—representing about 40 percent of both households and persons—have a 7 percent increase. Finally, large households of 5 to 7 persons—representing 26 percent of all households and 40 percent of all persons—experience a small increase of 2–5 percent.

b. Welfare Change in the Relative Case

Social welfare is a function of efficiency and distributional considerations. In the relative case, for any period \( t \) our SEF expresses a multiplicative trade off between these two forces

\[
W(z_t) = \mu(z_t)E(z_t),
\]

where \( E(z_t) = (1 - I_1(z_t)) \) and \( I_1(\cdot) \) is the first inequality index suggested by Theil. At \( p_2 \) for instance, the greater the ratio \( E(z_2)/E(z_{12}) \), the greater the reduction in real inequality measured by \( I_1(\cdot) \). In Table 3 we present the percentage change in \( E(\cdot) \) from 1973–74 to 1980–81 at prices \( p_1 \) and \( p_2 \), as well as the final change in relative welfare.

<table>
<thead>
<tr>
<th>Household Size</th>
<th>Percentage Change in ( E(\cdot) ), in %**</th>
<th>Percentage Change in Welfare, in %**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>At ( p_1 )</td>
<td>At ( p_2 )</td>
</tr>
<tr>
<td>1</td>
<td>12.5</td>
<td>15.7</td>
</tr>
<tr>
<td>2</td>
<td>8.4</td>
<td>10.0</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>4</td>
<td>3.0</td>
<td>3.4</td>
</tr>
<tr>
<td>5</td>
<td>4.2</td>
<td>4.7</td>
</tr>
<tr>
<td>6</td>
<td>5.9</td>
<td>6.7</td>
</tr>
<tr>
<td>7</td>
<td>4.3</td>
<td>4.7</td>
</tr>
</tbody>
</table>

* Percentage change in \( E(\cdot) \), in %, at \( p_2 = 100 \{E(z_2) - E(z_{12})\}/E(z_{12}) \). Similarly at \( p_1 \), \( E(\cdot) = 1 - I_1(\cdot) \), where \( I_1(\cdot) \) = Theil inequality index.

** Percentage change in welfare, in %, at \( p_2 = 100 \{W(z_2) - W(z_{12})\}/W(z_{12}) \). Similarly at \( p_1 \).

Notice that our estimates for the lower and upper bounds for the true changes in \( E(\cdot) \) at \( p_2 \) and \( p_1 \) are very close together for every household size. As far as the differences across household sizes, the improvement in real inequality is particularly strong among single person and two person households. The remaining subgroups experience a modest improvement in \( E(\cdot) \) which ranges from 2 to 6 percent, increasing monotonically with household size.

Given the good approximations obtained to the changes in the mean and inequality in real terms, it is not surprising that the welfare changes we estimate are very robust to the choice of the reference price vector. At any rate, single
person and two person households experience an increase of 35 and 21 percent in real relative welfare, respectively, while the remaining subgroups present only a 9-10 percent increase.

c. Welfare Change in the Absolute Case

In the absolute case, for any period $t$ the Kolm-Pollak SEF expresses an additive trade off between the mean and inequality

$$KP_r(z_t) = \mu(z_t) - A_r(z_t).$$

The welfare change in real terms is seen to be equal to the change in the mean less the change in absolute inequality. At prices $p_2$, for example

$$\Delta KP_r(p_2) = \Delta \mu(p_2) + \Delta A_r(p_2),$$

where

$$\Delta \mu_2(p_2) = \mu(z_2) - \mu(z_{12}),$$

$$\Delta A_r(p_2) = A_r(z_{12}) - A_r(z_2),$$

and

$$\Delta KP_r(p_2) = KP_r(z_2) - KP_r(z_{12}).$$

A positive (negative) sign for $\Delta A_r(\cdot)$ corresponds to a decrease (increase) of absolute inequality during the period 1973-74 to 1980-81.

We have experimented with several values of the inequality aversion parameter $\gamma$. In Table 4 we present our estimates of inequality and welfare change at prices $p_2$ for $\gamma = 5 \times 10^{-7}$ and $\gamma = 1.75 \times 10^{-6}$.

<table>
<thead>
<tr>
<th>Household size</th>
<th>$\gamma = 5 \times 10^{-7}$</th>
<th>$\gamma = 1.75 \times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
<td>-1.2</td>
</tr>
<tr>
<td>2</td>
<td>14.8</td>
<td>7.0</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>-1.5</td>
</tr>
<tr>
<td>4</td>
<td>7.9</td>
<td>3.6</td>
</tr>
<tr>
<td>5</td>
<td>16.7</td>
<td>10.2</td>
</tr>
<tr>
<td>6</td>
<td>24.3</td>
<td>15.9</td>
</tr>
<tr>
<td>7</td>
<td>15.4</td>
<td>8.4</td>
</tr>
</tbody>
</table>

To interpret the results, we must take into account that, maintaining relative inequality constant, any change in the mean causes absolute inequality to vary in the same direction. Thus, large increases in the mean for small households pushes down changes in absolute inequality. On the other hand, maintaining the mean

10 In the only previous empirical study we know of with complete indicators of absolute inequality from this family, Blackorby et al. (1981) choose values of $\gamma$ equal from $5 \times 10^{-8}$ to $5 \times 10^{-4}$ for distributions expressed in Canadian dollars.
constant, a change in relative inequality causes a change in the same direction in absolute inequality.

The main result is that, given the general improvement in relative inequality, most subgroups experiment an improvement in absolute inequality. Such an improvement appears to be greater for larger households whose mean increases are smaller (see Table 2). It should be noticed that, as the aversion to inequality increases, the percentage inequality change from 1973–74 to 1980–81 generally decreases, becoming negative for single person and three person households. The reason for this is that single person households experience large mean increases, while three person households have the smallest improvement in relative inequality (see Table 3).

In comparison to the relative case, percentage changes in absolute welfare are somewhat smaller for all household sizes. However, the pattern across subgroups is maintained: single person and two person households experience approximately a 19 or 13 percent improvement, while the remaining subgroups exhibit only a 5–7 or a 7–9 percent increase depending on the choice of the aversion to inequality parameter.\textsuperscript{11}

III. 2. Welfare Change in the Population as a Whole

We have seen that, during this period, there are important differences in the social evaluation of households of different sizes. How do these differences get aggregated at the population level? The answer depends necessarily on the way household size is taken into account in the definition of adjusted household expenditure.

a. The Relative Case

Recall that, in the relative case, adjusted expenditure for household $h$ is defined by

$$y^h(\Theta) = \frac{x^h}{(s^h)^\Theta}, \quad \Theta \in [0, 1].$$

Therefore, the mean of the adjusted expenditure distribution is a decreasing function of $\Theta$. This is of course what we observe in Table A in the Appendix. On the other hand, we found in Ruiz-Castillo (1995a) that relative inequality follows a $U$ pattern with $\Theta$, which gets translated into an inverted $U$ pattern for $E(\cdot)$ in the same Table.\textsuperscript{12} Due to the dominant influence of the mean, in both surveys relative welfare turns out to be decreasing with $\Theta$ at both $p_2$ and $p_1$. What about the changes from 1973–74 to 1980–81? We study the changes in the mean, inequality and welfare at prices $p_2$, for instance, by means of the expression

$$\Delta W(p_2) = \Delta \mu(p_2) \Delta E(p_2),$$

\textsuperscript{11}It should be noticed that, for several household sizes, our estimates for the lower and upper bounds for the true changes in $A(\cdot)$ at the two reference price vectors (not shown here) are not as close together as in the relative case. However, since the changes in the mean presented in Table 2 dominate the changes in absolute inequality, our estimates of welfare change for all household sizes are again rather robust to the choice of the reference price vector.

\textsuperscript{12}This is the same pattern reported by Coulter et al. (1992a, b) for the U.K. and by Rodrigues (1993) for Portugal.
where $\Delta \mu(p_2) = \mu(z_2)/\mu(z_{12}), \Delta E(p_2) = [1 - I_1(z_2)]/[1 - I_1(z_{12})]$, and $\Delta W(p_2) = W(z_2)/W(z_{12})$. The results are in Table 5. The interpretation is as follows. In the left-hand side of the first row, for instance, we observe that at prices $p_1$, when economies of scale are assumed to be infinitely large (i.e. $\Theta = 0$), the mean has increased by 4.2 percent, while the expression $E(\cdot)$ has increased by 5.4 percent revealing an improvement in real inequality during the period. The product of these two factors, lead to a 9.9 percent welfare increase.

### Table 5

<table>
<thead>
<tr>
<th>Eq. Scales Adjustment Factor</th>
<th>At Prices $p_1$</th>
<th>At Prices $p_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Change in the Mean</td>
<td>Change in $E(\cdot)$</td>
</tr>
<tr>
<td>0.0</td>
<td>1.042</td>
<td>1.047</td>
</tr>
<tr>
<td>0.3</td>
<td>1.051</td>
<td>1.047</td>
</tr>
<tr>
<td>0.5</td>
<td>1.057</td>
<td>1.043</td>
</tr>
<tr>
<td>0.7</td>
<td>1.063</td>
<td>1.040</td>
</tr>
<tr>
<td>1.0</td>
<td>1.071</td>
<td>1.037</td>
</tr>
</tbody>
</table>

** $E(\cdot)$ in 1980–81/$E(\cdot)$ in 1973–74, where $E(\cdot) = 1 - I_1(\cdot)$ and $I_1(\cdot)$ = Theil inequality index.

We are interested in the robustness of our results to the choice of reference prices and to changes in the assumptions about the importance of economies of scale. In the first place, for every $\Theta$, our estimates for the lower and upper bounds for the change in the mean at $p_2$ and $p_1$, respectively, are very close together. The same is the case for the expression $E(\cdot)$. Hence, it is not surprising that our estimates for the welfare change practically coincide at both price situations. The conclusion is that, as we saw for each household size, our estimates for the total population are robust to the choice of the reference price vector. In the second place, the improvement in the mean grows monotonically as $\Theta$ goes from 0 to 1. The term $E(\cdot)$ slightly decreases as $\Theta$ increases. As a consequence, we observe that the welfare change is very robust to changes in $\Theta$, ranging from 10 to 11 percent.

**b. The Absolute Case**

In the absolute case, adjusted household expenditure for households of size $m$ is defined as

$$y^h(\lambda^m(\Theta)) = x^h - \lambda^m(\Theta)(m - 1), \quad \Theta \in [0, 1].$$

For each $m$, the parameter $\lambda^m(\Theta)$ is interpreted as the cost of a reference adult when the importance of the economies of scale is given by $\Theta$. Therefore, $\lambda^m(\Theta)$ varies inversely with $\Theta$. On the other hand, as can be seen in equation (1) in
Section I, $\lambda'(\cdot)$ is a decreasing function of $m$, reflecting the fact that the cost of an adult decreases with household size.\textsuperscript{13}

As can be seen in Table B in the Appendix, in both years absolute inequality shows a mild version of the $U$ pattern as a function of $\Theta$ which has been already discussed in the relative case. Due to the dominant role of the mean, which is a decreasing function $\Theta$, absolute welfare turns out to be decreasing with $\Theta$ in both surveys. The information about changes in absolute inequality and welfare is in Table 6.

**TABLE 6**


<table>
<thead>
<tr>
<th>Equiv. Scale Adjustment Factor $\Theta$</th>
<th>Mean Gain in Pesetas</th>
<th>Inequality Gain in Pesetas</th>
<th>Welfare Gain in Pesetas</th>
<th>Inequality Gain in Pesetas</th>
<th>Welfare Gain in Pesetas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma = 1.75 \times 10^{-6}$</td>
<td>$\gamma = 5 \times 10^{-7}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>33,419</td>
<td>21,673</td>
<td>55,092</td>
<td>14,851</td>
<td>48,270</td>
</tr>
<tr>
<td>0.3</td>
<td>26,627</td>
<td>20,503</td>
<td>47,130</td>
<td>13,425</td>
<td>40,052</td>
</tr>
<tr>
<td>0.5</td>
<td>22,350</td>
<td>21,903</td>
<td>44,253</td>
<td>13,370</td>
<td>37,270</td>
</tr>
<tr>
<td>0.7</td>
<td>18,566</td>
<td>23,391</td>
<td>31,957</td>
<td>13,446</td>
<td>32,012</td>
</tr>
<tr>
<td>1.0</td>
<td>13,924</td>
<td>24,563</td>
<td>38,487</td>
<td>13,550</td>
<td>27,474</td>
</tr>
</tbody>
</table>

The main conclusion is that the improvement in inequality in Spain during this period is so large that, in the presence of a moderate increase in the mean household expenditures of about 4–7 percent, we find an unprecedented decrease in absolute inequality. In absolute terms, the gain in absolute inequality is very robust to any of the equivalence scales adjustment factors we care to use. However, given the larger variation of mean household expenditures as a function of $\Theta$, the change in welfare ranges from 38,000 to 55,000 pesetas, or from 27,000 to 48,000 pesetas, depending on the choice of the aversion to inequality parameter $\gamma$. For an intermediate value of $\Theta = 0.5$, such welfare gains represent 10 or 8.5 percent of the corresponding mean household expenditures, depending on whether we choose the larger or the smaller aversion to inequality.

### III.3. Using a Single Price Index for all Households

That intertemporal comparisons of welfare require an adjustment for price change is, of course, widely recognized. However, researchers often correct the original distributions with a single measure of price change for all households [see, for instance, Jenkins (1991)]. As a final exercise, we have done that for the population as a whole in the relative case. We take 318.1 percent as the common inflation rate from 1973–74 to the winter of 1981, which is the average of

\textsuperscript{13}We find that the greater the household size, the greater the percentage of $\lambda''$ relative to the mean household expenditures per capita. In 1980–81, when $\Theta = 0.3$, $\lambda''$ amounts to from 37 to 60 percent of mean household expenditures per capita as household size increases from 2 to 17 people. When $\Theta = 0.5$ or 0.7, the range of variation goes from 58 to 80 or from 77 to 92 percent, respectively. Such orders of magnitude are practically the same for the 1973–74 household expenditures distribution.
our household specific inflation rates. A single inflation rate precludes the distributional impact of changes in relative prices over the period. Therefore, the ratio

\[ \Delta E(\Theta) = \frac{E(z_2(\Theta))}{E(z_{12}(\Theta))} = \frac{[1 - I_1(z_2(\Theta))]}{[1 - I_1(z_1(\Theta))318.1]} \]

\[ = \frac{[1 - I_1(z_2(\Theta))]}{[1 - I_1(z_1(\Theta))]} \]

captures only the change in \textit{money} inequality. The estimates for the change in the mean, relative inequality and welfare appear in Table 7.

We see that, for all \( \Theta \), the change in welfare in real terms is only slightly lower than what we obtained using household specific inflation rates (compare column 3 in Table 7 to column 6 in Table 5). However, because the change of relative prices has damaged the rich more than the poor, the estimates of Table 7 tell the wrong story. On the one hand, the change in money inequality underestimates the change in real inequality. On the other hand, since the rich had a greater inflation rate than the poor, using the average inflation rate underestimates the mean household expenditures from 1973-74 at winter of 1981 prices, so that the improvement of the mean in real terms is now overvalued.

\textbf{IV. CONCLUDING REMARKS}

In this paper we have presented a social welfare model to assess the evolution of the standard of living of a country with the help of two cross-sections of household budget data. The model combines two elements: the use of conceptual arguments to single out a social evaluation function for applied work; and the adoption of convenient methodological strategies to address the vexing problems of non-income needs and intertemporal comparisons at constant prices. Using current consumption expenditures as the best proxy for a household standard of living, the main lessons obtained in the Spanish case during the 1973-74 to 1980-81 period are the following four.
(i) Intertemporal comparisons require the expression of original money income distributions at constant prices. The use of household specific price indices for this task permits the study of distributional implications of the changes in relative prices. With this aim in mind, we have used statistical price indices which provide convenient bounds for the true cost-of-living indices. We have presented the theoretical bounds for the change in the mean and inequality at prices of the initial and the final situation under comparison. Our empirical work shows that we can narrowly bound the corresponding theoretical magnitudes. Consequently, our estimates of welfare change are very robust to the choice of the reference price vector. We conclude that, at least in this case, we have been justified in avoiding the cost of estimating a complete demand system in order to recover the true cost-of-living household indices.

(ii) The construction of household specific inflation rates has paid some empirical dividends in this case. The reason is that we have found that the evolution of relative prices has damaged the rich more than the poor [see also Ruiz-Castillo (1995a)]. In a situation of this type, if we use a common inflation rate for all households we can be sure that our estimates of the inequality improvement would be biased downwards, while our estimates of the change in the mean would be biased upwards.

(iii) The separate study of each subgroup by partition into household size has been worthwhile. We have found considerable differences among them, but the use of decomposable measurement instruments allow us to understand how results at the household size level get translated to the population as a whole. In particular, given that larger households do worse than smaller ones, had we estimated the welfare change for the unweighted household distribution, we would have found a greater welfare improvement than with the household size weighted distribution in which each individual is assigned the adjusted expenditure of the household to which she belongs.

(iv) We have provided additional evidence on the usefulness of parametrizing the equivalence scales in the relative case when household size is assumed to be the only characteristic determining non-income needs. The extension of that model to the absolute case presented in this paper, has made possible the application of this notion which, in spite of its conceptual and practical interest, has been very seldom used in the empirical literature.

**STATISTICAL APPENDIX**

**TABLE A**

| | MEAN HOUSEHOLD EXPENDITURES, RELATIVE INEQUALITY AND WELFARE FOR THE TOTAL POPULATION AT PRICES $p = \text{Winter 1981}$ | | |
|---|---|---|---|---|
| | 1973–74 | 1980–81 | | |
| | Mean $E(\cdot)^*$ Welfare | Mean $E(\cdot)^*$ Welfare | | |
| 0.0 | 931,092 0.7859 730,992 | 964,511 0.8320 802,505 | | |
| 0.3 | 590,187 0.8068 476,195 | 616,815 0.8490 523,667 | | |
| 0.5 | 439,452 0.8129 357,241 | 461,802 0.8525 393,676 | | |
| 0.7 | 329,717 0.8115 267,579 | 348,284 0.8489 295,671 | | |
| 1.0 | 217,553 0.7938 172,695 | 231,477 0.8286 191,800 | | |

* $E(\cdot) = 1 - I_1(\cdot)$, where $I_1(\cdot) =$ Theil inequality index.
### TABLE B
Mean Household Expenditures, Absolute Inequality and Welfare for the Total Population at Prices $p_2 = \text{Winter 1981}$: Household Expenditures Distributions Weighted by Household Size

<table>
<thead>
<tr>
<th>Aversion to inequality parameter $\gamma = 1.75 \times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta$</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.7</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aversion to inequality parameter $\gamma = 5 \times 10^{-7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta$</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.7</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

### REFERENCES

Sanz, B., La articulación micro-macro en el sector hogares: de la Encuesta de Presupuestos Familiares a la Contabilidad Nacional, in La desigualdad de recursos, II Simposio sobre la Desigualdad de la Renta y la Riqueza, Vol. 6, 45-86, Fundación Argentaria, Madrid, 1996.

