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CONSISTENCY-IN-AGGREGATION AND STUVEL INDICES

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In the National Accounts framework a frequent use is made of value, price, and quantity indices. Three requirements appear to be of vital importance. (i) For each aggregate the price index multiplied by the quantity index must be equal to the value index. (ii) The indices must be consistent-in-aggregation (which means something more than that a single-step calculation yields the same outcome as a two-or-more-step calculation). (iii) The indices must satisfy the equality test (defined in this paper). In this paper it is shown that the only indices satisfying these three requirements are the generalized Stuvel (1957) indices. These indices satisfy the Eichhorn and Voeller (1983) axioms for price and quantity indices. However, if one also requires that the indices be linearly homogeneous in current period prices and quantities then the only admissible indices are those of Laspeyres and Paasche.

1. INTRODUCTION

In 1957 G. Stuvel reported the discovery of a new pair of price and quantity indices. They attracted relatively little attention, except by Banerjee who incorporated them as a special case in his factorial approach (see for instance, Banerjee, 1980). As far as I know they have never been used, neither in econometric work nor in official statistics. Perhaps in an attempt to remedy this situation Stuvel wrote a slim monograph, published in 1989, with the rather pretentious title "The Index-number Problem and Its Solution." The index-number problem is described as the problem of finding "measures of price and volume development which take due account of the changes in the volume and price structures of commodity aggregates from base year to current year, and which by doing so might eliminate the duality or even plurality of index-number measures" (p. 9), and Stuvel thinks that "the new index numbers come as close to solving the index-number problem as we can ever hope to get" (p. 52).

Nevertheless, I think that the situation will not change drastically. This is not because computation of the Stuvel indices present unsurmountable difficulties. They are rather simple functions of Laspeyres and Paasche price and quantity indices and thus, given the latter indices, they can be computed easily. The reason is that although the Stuvel indices satisfy a remarkable number of so-called index tests, they are not linearly homogeneous in current prices or quantities. Thus in the sense of Eichhorn and Voeller's (1976) definition, the Stuvel indices are not genuine indices. However, they do satisfy the weaker axioms of Eichhorn and Voeller (1983).

I try to show in the present paper that the Stuvel indices remain important from a conceptual point of view. In most practical situations, the National

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Accounts being a good example, we are confronted with aggregates that consist of subaggregates, that in their turn consist of sub-subaggregates, etcetera until we reach the level of individual commodities. Three requirements are of vital importance in this context (see for the first two Al, Balk, De Boer and Den Bakker, 1986).

(i) For each (sub)aggregate the price index times the quantity index must be equal to the value index.

(ii) The indices must be consistent-in-aggregation, which roughly means that a single-step calculation yields the same result as two-or-more-step calculation.

(iii) If the indices for all sub-aggregates (on the same level) of an aggregate happen to be equal to each other numerically then the index for the aggregate must also have this common numerical value.

In index number theory the first requirement is known as the product test, due to Fisher (1922). The third requirement was formulated by Stuvel (1989). He called (iii) the equality test. It can be considered as a novel, but very natural test. The second requirement was formalized by Blackorby and Primont (1980). We show, however, that their formalization is not completely satisfactory from a practical point of view. Therefore a novel formal definition of consistency-in-aggregation has been developed. From this it can be shown that the generalized Stuvel indices (of which the original Stuvel indices are a particular instance) are the only indices satisfying these three requirements. If we also require linear homogeneity in current prices and quantities, we are left with the Laspeyres and Paasche indices. This is a rather remarkable result.

The plan of the paper is as follows. In section 2 the (generalized) Stuvel indices are introduced. They are compared to Fisher's "ideal" indices on the one hand and to Montgomery's pseudo indices on the other. In section 3 the requirement of consistency-in-aggregation and the equality test are discussed together with the main proposition of this paper. Section 4 closes with a discussion of the linear homogeneity issue.

2. Generalized Stuvel Indices

We consider an aggregate A consisting of a finite number of commodities. For each commodity $i \in A$ $x_i^t \geq 0$ denotes the quantity at period t and $p_i^t > 0$ denotes the price (unit value) at period t. We consider the time periods $t=0$ (base period) and $t=1$ (comparison period). The value of A at period t is given by $V^t \equiv \sum_{i \in A} p_i^t x_i^t$. It is well known that the product of the Laspeyres price index

$$P_L \equiv \sum_{i \in A} p_i^1 x_i^0 / \sum_{i \in A} p_i^0 x_i^0 \tag{1}$$

and the Laspeyres quantity index

$$Q_L \equiv \sum_{i \in A} p_i^0 x_i^1 / \sum_{i \in A} p_i^0 x_i^0 \tag{2}$$

is not equal to the value index V^1/V^0. Thus let us consider a factor t such that

$$(P_L t)(Q_L t) = V^1/V^0. \tag{3}$$

The solution of this equation is easy, and we obtain, using the positive root,

$$P_L t = (P_L P_P)^{1/2} \equiv P_F \tag{4}$$

where

$$P_P \equiv \sum_{i \in A} p_i^1 x_i^1 / \sum_{i \in A} p_i^0 x_i^1 \tag{5}$$

is the Paasche price index and P_F is Fisher's "ideal" price index. Likewise

$$Q_L t = (Q_L Q_P)^{1/2} \equiv Q_F \tag{6}$$

where

$$Q_P \equiv \sum_{i \in A} p_i^1 x_i^1 / \sum_{i \in A} p_i^1 x_i^0 \tag{7}$$

is the Paasche quantity index and Q_F is Fisher's "ideal" quantity index. The foregoing is an instance of one of Fisher's (1922) famous "rectifying principles": a pair of price and quantity indices that does not satisfy the product test equation $PQ = V^1/V^0$ is rectified so that it does. Notice that the pair (P_F, Q_F) is the positive solution of the pair of equations

$$PQ = V^1/V^0 \tag{8a}$$

$$P/P_L = Q/Q_L. \tag{8b}$$

These equations characterize the Fisher indices, as noticed by Van IJzeren (1958). See also Eichhorn and Voeller (1976: 42).

We now consider a variant of Fisher's "rectifying principle." Instead of relation (3) we solve t from

$$(P_L + t)(Q_L + t) = V^1/V^0. \tag{9}$$

The solution of this quadratic equation is

$$t = -(P_L + Q_L)/2 \pm [(P_L - Q_L)^2/4 + V^1/V^0]^{1/2} \tag{10}$$

and the desired indices become

$$P_L + t = (P_L - Q_L)/2 + [(P_L - Q_L)^2/4 + V^1/V^0]^{1/2} \equiv P_S(1/2, 1/2) \tag{11}$$

$$Q_L + t = (Q_L - P_L)/2 + [(P_L - Q_L)^2/4 + V^1/V^0]^{1/2} \equiv Q_S(1/2, 1/2). \tag{12}$$

Obviously we must choose the positive root in (10). The indices $P_S(1/2, 1/2)$ and $Q_S(1/2, 1/2)$ are Stuvel's (1957) price index and quantity index respectively. The parameters 1/2 will be explained in the sequel. The foregoing represents the way in which Siegel (1965) derived Stuvel's indices. Notice that this index pair is the positive solution of the pair of equations.

$$PQ = V^1/V^0 \tag{13a}$$

$$P - P_L = Q - Q_L. \tag{13b}$$

This fact was also discovered by Van IJzeren (1958). Notice the analogy between (8a, b) and (13a, b).

Stuvel himself arrived at his index pair in a rather different way. A slight generalization of the derivations provided by Stuvel (1957), (1989) and Banerjee (1959) runs as follows. Consider the following decomposition of the value change of a single commodity into a quantity effect and a price effect. For each $i \in A$

$$p_i^1 x_i^1 - p_i^0 x_i^0 = (ap_i^1 + bp_i^0)(x_i^1 - x_i^0) + (bx_i^1 + ax_i^0)(p_i^1 - p_i^0) \tag{14}$$

where a and b are arbitrary positive constants such that $a+b=1$. Rewriting (14) as an equation in elementary value relatives $p_i^1 x_i^1 / p_i^0 x_i^0$, price relatives p_i^1/p_i^0 and quantity relatives x_i^1/x_i^0 we obtain

$$p_i^1 x_i^1 / p_i^0 x_i^0 - 1 = (ap_i^1/p_i^0 + b)(x_i^1/x_i^0 - 1) + (bx_i^1/x_i^0 + a)(p_i^1/p_i^0 - 1). \tag{15}$$

Summing equation (14) over all commodities $i \in A$, we obtain for the aggregate

$$\sum_i p_i^1 x_i^1 - \sum_i p_i^0 x_i^0 = \sum_i (ap_i^1 + bp_i^0)(x_i^1 - x_i^0) + \sum_i (bx_i^1 + ax_i^0)(p_i^1 - p_i^0) \tag{16}$$

(where no danger of confusion exists we will abbreviate $\sum_{i \in A}$ as $\sum_i$), or

$$\begin{aligned} \sum_i p_i^1 x_i^1 / \sum_i p_i^0 x_i^0 - 1 &= \sum_i (ap_i^1 + bp_i^0)(x_i^1 - x_i^0) / \sum_i p_i^0 x_i^0 \\ &\quad + \sum_i (bx_i^1 + ax_i^0)(p_i^1 - p_i^0) / \sum_i p_i^0 x_i^0. \end{aligned} \tag{17}$$

Now we would like to rewrite the righthand side of (17) in a form analogous to the righthand side of (15), replacing elementary relatives by indices. Thus a pair (P, Q) is defined such that

$$(aP+b)(Q-1) = \sum_i (ap_i^1 + bp_i^0)(x_i^1 - x_i^0) / \sum_i p_i^0 x_i^0 \tag{18a}$$

$$(bQ+a)(P-1) = \sum_i (bx_i^1 + ax_i^0)(p_i^1 - p_i^0) / \sum_i p_i^0 x_i^0. \tag{18b}$$

Adding (18a) and (18b) yields

$$PQ = V^1/V^0. \tag{19a}$$

Subtracting (18b) from (18a) and using (19a) yields

$$a(P - P_L) = b(Q - Q_L). \tag{19b}$$

This is clearly a generalization of (13b). The solution of (19a)–(19b), using the positive root, is

$$P_S(a,b) \equiv (P_L - (b/a)Q_L)/2 + [(P_L - (b/a)Q_L)^2/4 + (b/a)V^1/V^0]^{1/2} \tag{20}$$

$$Q_S(a,b) \equiv (Q_L - (a/b)P_L)/2 + [(Q_L - (a/b)P_L)^2/4 + (a/b)V^1/V^0]^{1/2}. \tag{21}$$

Choosing $a=0$, $b=1$ we obtain $P_S(0,1) = P_P$ and $Q_S(0,1) = Q_L$. Similarly $P_S(1,0) = P_L$ and $Q_S(1,0) = Q_P$. Finally $P_S(1/2,1/2)$ and $Q_S(1/2,1/2)$ are Stuvel's indices (11) and (12) respectively. We call $P_S(a,b)$ and $Q_S(a,b)$ the *generalized Stuvel indices*.

A distinctive feature of the generalized Stuvel indices is that they permit us to decompose the value change of the aggregate, $\sum_i p_i^1 x_i^1 / \sum_i p_i^0 x_i^0 - 1$, additively into a quantity effect and a price effect in a way corresponding to the decomposition of the value change of a single commodity, $p_i^1 x_i^1 / p_i^0 x_i^0 - 1$. Choosing P and Q such that (18a, b) is satisfied, we obtain a structural similarity between expression (17) for the aggregate and expression (15) for each single commodity.

There also exists a correspondence between the generalized Stuvel price index (quantity index) of the aggregate and the price relatives (quantity relatives) of the single commodities. This correspondence can be demonstrated most clearly by a simple manipulation of the equations (19a, b). Substituting (19a) into (19b), we obtain

$$a(P - P_L) = b(V^1/V^0 P - V^1/V^0 P_P), \tag{22}$$

or

$$aP - bV^1/V^0 P = aP_L - bV^1/V^0 P_P. \tag{23}$$

Multiplying both sides by V^0 and using (1) and (5), we obtain

$$aV^0 P - bV^1/P = \sum_i (av_i^0 r_i - bv_i^1/r_i) \tag{24}$$

where $v_i^t \equiv p_i^t x_i^t$ ($i \in A$; $t = 0, 1$) and $r_i \equiv p_i^1/p_i^0$ ($i \in A$). Recall that $V^t = \sum_i v_i^t$. Expression (24) implicitly defines the generalized Stuvel price index $P_S(a, b)$ and is of the form

$$\psi(P, V^0, V^1) = \sum_i \psi(r_i, v_i^0, v_i^1) \tag{25}$$

with $\psi(\alpha, \beta, \gamma) \equiv a\beta\alpha - b\gamma/\alpha$. Of course, there can be derived a similar expression relating the generalized Stuvel quantity index $Q_S(a, b)$ to the quantity relatives $s_i \equiv x_i^1/x_i^0$.

The generalized Stuvel indices are not the only indices which permit a structurally similar decomposition of the value change of the aggregate and the value change of each single commodity. Consider for each $i \in A$ the following decomposition of the value change into a price effect and a quantity effect,

$$p_i^1 x_i^1 - p_i^0 x_i^0 = L(v_i^0, v_i^1) \ln p_i^1/p_i^0 + L(v_i^0, v_i^1) \ln x_i^1/x_i^0, \tag{26}$$

where the logarithmic mean is defined by

$$\begin{aligned} L(\alpha, \beta) &\equiv (\alpha - \beta)/\ln(\alpha/\beta) && \text{if } \alpha \neq \beta \\ &\equiv \alpha && \text{if } \alpha = \beta. \end{aligned} \tag{27}$$

On the properties of $L(\alpha, \beta)$ see Lorenzen (1990). Summing equation (26) over all commodities $i \in A$ we obtain for the aggregate

$$V^1 - V^0 = \sum_i L(v_i^0, v_i^1) \ln r_i + \sum_i L(v_i^0, v_i^1) \ln s_i. \tag{28}$$

If we require that the pair (P, Q) satisfies the product test equation $PQ = V^1/V^0$ we also obtain, using (27) again,

$$V^1 - V^0 = L(V^0, V^1) \ln P + L(V^0, V^1) \ln Q. \tag{29}$$

Defining P and Q such that

$$L(V^0, V^1) \ln P = \sum_i L(v_i^0, v_i^1) \ln r_i \tag{30}$$

$$L(V^0, V^1) \ln Q = \sum_i L(v_i^0, v_i^1) \ln s_i \tag{31}$$

we again obtain a structural correspondence between the decomposition of the value change of each single commodity (26) and that of the aggregate (29). The expressions (30) and (31) define the Montgomery (1937) pseudo price index P_M and quantity index Q_M respectively. These indices were independently rediscovered by Vartia (1976). Since then they are also known as Vartia-I indices. They do not satisfy the Eichhorn and Voeller (1983) axioms. In particular they fail the proportionality property. It is important to observe that (30) is also of the form (25), but now with $\psi(\alpha, \beta, \gamma) \equiv L(\beta, \gamma) \ln \alpha$.

3. Consistency-in-Aggregation and the Equality Test

In the previous section we considered an aggregate consisting of a finite number of commodities. Usually, however, aggregates have more structure. In official statistics, e.g. the aggregate "household consumption" consists of the subaggregates "food, beverages and tobacco," "clothing and footwear," "gross rent, fuel and power," etcetera. However, each of these subaggregates is built up from subsubaggregates, e.g. "food, beverages and tobacco" from "food," "non-alcoholic beverages," "alcoholic beverages" and "tobacco." The entire structure usually contains four or five levels. At the lowest level we have the subaggregates directly consisting of commodities. Besides the structure given in official publications one can consider other decompositions of an aggregate. One can partition, e.g. "household consumption" into the subaggregates "food" and "other commodities." An important requirement for indices is that they are consistent-in-aggregation. What does this mean?

Let the aggregate A be partitioned arbitrarily into K subaggregates A_k, symbolically

$$A = \bigcup_{k=1}^{K} A_k, \; A_k \cap A_l = \varnothing (k \neq l), \tag{32}$$

where each subaggregate consists of a number of commodities. Following Vartia (1974), (1976) we say that an index is consistent-in-aggregation if

(i) the index for the aggregate, which is defined as a single stage index, can also be computed in two stages, namely by first computing the indices for the subaggregates and from these the index for the aggregate;

(ii) the indices used in the single stage computation and those used in the first stage computation have the same functional form (only the numbers of variables can be different);

(iii) the formula used in the second stage computation has the same functional form (except possibly for the number of variables) as the indices used in the single and in the first stage after the following transformation has been applied: commodity indices are replaced by subaggregate indices and commodity values are replaced by subaggregate values.

All price indices which are implicitly defined by an equation of the form (25), where ψ is a continuous function which is strictly increasing in its first argument, are consistent-in-aggregation in the sense described above. This can be demonstrated easily as follows. The single stage price index for the aggregate is defined by (25). The first stage price indices for the subaggregates are similarly defined by

$$\psi(P_k, V_k^0, V_k^1) = \sum_{i \in A_k} \psi(r_i, v_i^0, v_i^1) \tag{33}$$

where $V_k^t = \sum_{i \in A_k} v_i^t$ for $t=0, 1$. The link between P and $P_1, \ldots, P_K$ is given by

$$\begin{aligned} \psi(P, V^0, V^1) &= \sum_{k=1}^{K} \sum_{i \in A_k} \psi(r_i, v_i^0, v_i^1) \\ &= \sum_{k=1}^{K} \psi(P_k, V_k^0, V_k^1) \end{aligned} \tag{34}$$

(notice that $V^t = \sum_{k=1}^{K} V_k^t$ for $t=0, 1$). It is clear that the requirements (i)–(iii) are satisfied. In particular we can conclude that the generalized Stuvel price index $P_S(a, b)$ and the Montgomery pseudo price index P_M are consistent-in-aggregation. Of course, analogous results can be established for quantity indices.

An example of a price index which is not consistent-in-aggregation is the Walsh index. It is defined by

$$\sum_{i \in A} (v_i^0 v_i^1)^{1/2} \ln P_W = \sum_{i \in A} (v_i^0 v_i^1)^{1/2} \ln r_i. \tag{35}$$

The single stage Walsh index for the aggregate can be calculated in two stages as follows

$$\begin{aligned} \sum_{i \in A} (v_i^0 v_i^1)^{1/2} \ln P_W &= \sum_{k=1}^{K} \sum_{i \in A_k} (v_i^0 v_i^1)^{1/2} \ln r_i \\ &= \sum_{k=1}^{K} \left(\sum_{i \in A_k} (v_i^0 v_i^1)^{1/2} \right) ln\, P_{W,k}. \end{aligned} \tag{36}$$

It is clear that requirements (i) and (ii) are satisfied. However, requirement (iii) is not satisfied since in general

$$\sum_{i \in A_k} (v_i^0 v_i^1)^{1/2} \neq (V_k^0 V_k^1)^{1/2}. \tag{37}$$

The aggregate index P_W cannot be calculated from the subaggregate indices $P_{W,k}$ and the subaggregate values V_k^0 and V_k^1. This example demonstrates that Blackorby and Primont's (1980) definition of consistency-in-aggregation is not entirely appropriate. They apparently overlooked the important requirement (iii). This requirement permits us to derive the index for the aggregate from the indices for the subaggregates using *only* the base period and comparison period values of these subaggregates (cf. Stuvel 1989: 36).

The other example discussed by Blackorby and Primont (1980) is the Walsh–Vartia pseudo price index. This index is defined by

$$(38) \qquad (V^0V^1)^{1/2} \ln P_{WV} = \sum_{i \in A} (v_i^0 v_i^1)^{1/2} \ln r_i.$$

This equation is of the form (25) with $\psi(\alpha, \beta, \gamma) = (\beta\gamma)^{1/2} \ln \alpha$. Thus the Walsh–Vartia pseudo index P_{WV} is consistent-in-aggregation.

Summarizing the foregoing discussion I propose the following definition. A (price or quantity) index I is *consistent-in-aggregation* if the following relation holds between the index I for an aggregate and the indices I_k for the subaggregates $k = 1, \ldots, K$,

$$(39) \qquad \psi(I, V^0, V^1) = \sum_{k=1}^{K} \psi(I_k, V_k^0, V_k^1),$$

where ψ is a continuous function which is strictly increasing in its first argument. The latter condition implies that an explicit solution for I exists. A little reflection shows that this definition encompasses all three requirements (i)–(iii). Consider the case where the lowest level subaggregates consist of single commodities. It is assumed that for each single commodity the index is given by the ratio r_i or s_i. Then (39) defines the indices for all aggregates at all higher levels. Thus condition (ii) is satisfied. The additive structure of (39) together with the additive nature of the values implies that (i) and (iii) are also satisfied.

When an aggregate consists of subaggregates a second requirement for index formulas is of great importance. If the price (quantity) indices for the subaggregates are all equal to each other then the price (quantity) index for the aggregate must be equal to the price (quantity) indices for the subaggregates. Stuvel (1989) called this the *equality test*. As far as I know this test was not mentioned by other authors. Van IJzeren (1958) preluded on it. Notice that if the subaggregates consist of single commodities, the equality test becomes the well-known proportionality test (cf. Fisher 1922: 420 and Eichhorn and Voeller 1976: 27). Thus the proportionality test is a specific case of the equality test. For instance, Fisher's indices satisfy the proportionality test but they do not satisfy the equality test, as was demonstrated by Stuvel (1989: 39).

That the generalized Stuvel indices satisfy the equality test can be demonstrated as follows. From the defining equation (24) we obtain

$$(40) \qquad aV^0P - bV^1/P = \sum_{k=1}^{K} (aV_k^0P_k - bV_k^1/P_k)$$

where P_k is the generalized Stuvel price index for subaggregate A_k ($k = 1, \ldots, K$). If $P_k = \lambda$ for $k = 1, \ldots, K$ we obtain

$$(41) \qquad aV^0P - bV^1/P = aV^0\lambda - bV^1/\lambda.$$

Since $f(P) \equiv aV^0P - bV^1/P$ is strictly increasing in P we obtain $P = \lambda$.

The Montgomery pseudo indices do not satisfy the equality test. From the defining equation for the pseudo price index (30) we obtain

$$L(V^0, V^1) \ln P = \sum_{k=1}^{K} L(V_k^0, V_k^1) \ln P_k \tag{42}$$

where P_k is the Montgomery pseudo price index for subaggregate A_k ($k = 1, \ldots, K$). Setting $P_k = \lambda$ ($k = 1, \ldots, K$) in (42) we do *not* obtain $P = \lambda$ since in general

$$L(V^0, V^1) \neq \sum_{k=1}^{K} L(V_k^0, V_k^1). \tag{43}$$

For the same reason the Montgomery pseudo indices do not satisfy the proportionality test. The same applies to the Walsh–Vartia pseudo indices [see equation (38)]: they satisfy neither the equality test nor the proportionality test. However, the Walsh indices [see equation (35)] satisfy the equality test. This is clear from (36).

In the foregoing we discussed two important requirements for price and quantity index formulas, namely that they be consistent-in-aggregation and that they satisfy the equality test. We showed that these requirements are independent: the Montgomery pseudo indices are consistent-in-aggregation but they do not satisfy the equality test. The Walsh indices satisfy the equality test but they are not consistent-in-aggregation. The set of indices satisfying both requirements is however not empty: the generalized Stuvel indices are consistent-in-aggregation and they satisfy the equality test. This raises the question whether there are other indices satisfying both requirements.

The answer to this question appears to be negative. This is stated in the following proposition. The rather tedious proof is provided by Balk (1995) and is based upon a derivation given by Gorman (1986).

PROPOSITION: *Assume that the product of the price index and the quantity index for a (sub)aggregate is equal to the corresponding value ratio and that both indices satisfy the equality test. If the price index is consistent-in-aggregation then it is a generalized Stuvel index.*

This proposition, together with the fact that the generalized Stuvel indices are consistent-in-aggregation and satisfy the equality test, thus provides a second characterization of the generalized Stuvel indices. Recall that the first characterization was given by the equations (19a)–(19b). We see that the rather simple looking relation (19b) appears to be equivalent to the requirement of consistency-in-aggregation and the satisfaction of the equality test.

4. Concluding Remarks

A defect of the generalized Stuvel indices (for $a, b \neq 0$) is that they do not satisfy the linear homogeneity axiom (see Eichhorn and Voeller 1976: 24). For a price index it reads

$$P(x^0, p^0, x^1, \lambda p^1) = \lambda P(x^0, p^0, x^1, p^1) \qquad (\lambda > 0) \tag{44}$$

where x^t denotes the vector of x_i^t and p^t denotes the vector of p_i^t $(i \in A;\ t = 0, 1)$. Stuvel (1989) called it the "comparative proportionality test." That $P_S(a, b)$ for $a, b \neq 0$ does not satisfy (44) is immediately clear from the explicit definition (20). Only the degenerate cases $P_S(0, 1) = P_P$ and $P_S(1, 0) = P_L$ satisfy (44). Thus we can formulate the following.

COROLLARY: *Assume that the product of the price index and the quantity index for a (sub)aggregate is equal to the corresponding value ratio and that both indices satisfy the equality test. The only price indices which are consistent-in-aggregation and satisfy the linear homogeneity axiom are the Laspeyres and the Paasche index.*

Stuvel (1989: 105–6) tried to argue that the failure of the Stuvel indices "to satisfy the comparative proportionality test is not as serious as one might imagine. The reason for this is the following. Unlike Fisher's proportionality test, which deals with the case in which all prices change by a constant factor λ from base year to current year, the comparative proportionality test deals with the case in which two different current-year situations are compared with the base-year situation. The difference between these two current-year situations is that in the one the prices of the single commodities in the aggregate are λ times what they are in the other. Such a difference can only arise in one of two ways and neither of these is really relevant in the binary context."

The problem however is that indices are mostly used in the context of multiple comparisons (a number of time periods) and are calculated as $P(x^0, p^0, x^t, p^t)$ for $t = 0, 1, \ldots, T$. When we consider more than two time periods the linear homogeneity axiom (44) seems to me at least as natural as the proportionality test

$$P(x^0, p^0, x^1, \lambda p^0) = \lambda. \tag{45}$$

It seems that maintaining the linear homogeneity axiom or denying its importance is largely a matter of taste. The debate can only be resolved if one is prepared to take into account considerations from a different angle. For instance, the failure of the non-degenerate Stuvel price index to satisfy the linear homogeneity axiom implies that, in the consumer context, it cannot be interpreted as a cost-of-living index. As is well known, a cost-of-living index is defined as a ratio of values of an expenditure function, and an expenditure function is linearly homogeneous in prices by construction. Thus the Stuvel price index is devoid of any welfare-theoretic meaning.

The Stuvel indices remain rather artificial constructs. Their importance lies in the fact that they throw light on the requirement of consistency-in-aggregation, which has been an important issue in recent discussions.

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