Review of Income and Wealth
Series 41, Number 2, June 1995

ON EQUITY ASPECTS OF IMPERFECT INCOME REDISTRIBUTION

BY JEAN-YVES DUCLOS

Université Laval, Canada

We develop a theoretical analysis of the impact of imperfect targeting, participation costs and incomplete take-up upon the level of progressivity, vertical equity, horizontal inequity and redistribution exerted by state benefits. An illustration using the distribution of British Supplementary Benefits (now Income Support) indicates that progressivity is roughly untouched, vertical equity and horizontal inequity are increased, and total income redistribution is slightly increased by these redistributive imperfections in 1985 Britain.

INTRODUCTION

Imperfections in the conduct of redistributive policies can make the actual income redistribution quite different from the intended one. This is particularly true in the presence of incomplete take-up of state benefits, a phenomenon increasingly well documented in the literature on the efficacy of poverty relief.[1] There are, however, very few studies of the effect of such imperfections on the progressivity, vertical equity, horizontal equity and actual redistribution exerted by the state's support. This is despite the feature that the literature on the measurement of progressivity and equity is well developed.[2] As importantly, there exists no theoretical or empirical analysis of the related impact of the imperfect targeting of state benefits. By this, we will mean here the imperfect correspondence between the real distribution of benefit entitlement and the statutory one; this is because, in applying official benefit rules, welfare offices can only obtain and use imperfect information on the true characteristics and income of households. Thus, the impossibility of matching administrative benefit eligibility to the statutory one adds an additional redistributive imperfection to the existence of incomplete benefit take-up. Both of these features can affect the magnitude of the redistribution and equity exerted by state benefits.

Our main objective is thus to illustrate the extent to which redistribution, vertical equity and horizontal inequity can be affected by the following factors:

(F1) welfare office uncertainty over the true benefit entitlement of individuals, leading to administrative errors;

(F2) the presence of costs to welfare participation, generating incomplete benefit take-up.

Note: I am indebted to Tony Atkinson, Holly Sutherland, and to two anonymous referees for their valuable advice. I am also grateful to the Central Statistical Office and the ESRC Data Archive for permission to use FES data. The study would not have been possible without the financial support of the Commonwealth Scholarships Commission and the technical support of STICERD at the LSE.

[1]For a review of the literature, see Craig (1991) and Oorschot (1991). An official and thorough account of non-take-up in Britain is provided in Department of Social Security (1993).

[2]See, for instance, Kakwani (1986) and Lambert (1993).

The latter is well illustrated by the following account of the Supplementary Benefits Commission (1978) taken from Craig (1991, p. 543):

> "All we can say is that [the] reluctance to claim appears to come from some mixture of pride, ignorance, a sense of stigma, reluctance to make the efforts a claim calls for, a desire for self-sufficiency on the part of an individual or family, an unwillingness to become involved with a government agency and a feeling that the whole business is not worthwhile."

If we adopted an optimal taxation perspective, where the state's redistributive objective is a function of the net utility gains—and not of the increase in income *per se*—secured by the poorer, we could also add the following third factor limiting the net impact of redistributive transfers:

(F3) gross benefits exceeding net benefits because of the presence of take-up costs.

We apply our analysis to the impact of Supplementary Benefits (SB) in 1985 Britain.[3] The SB programme is intended to fill the gap between a specified minimum level of resources (determined by the household's characteristics) and a household's level of income. We call this the level of SB entitlement. A household reporting too high a level of income to the welfare office will be assigned a negative level of SB entitlement, which simply means that no benefit is then payable (and not that the household is liable to a SB tax). SB reaches mostly the non-working poor (the unemployed, the disabled and the pensioners). The benefit is payable for as long as the eligibility conditions are fulfilled.

We now proceed as follows. Firstly, we review briefly the features of our sample of Family Expenditure Survey (FES) data and present some estimates of entitlement uncertainty and participation costs derived from a study of SB welfare participation. Secondly, we recall the definition of widely used indices of progressivity, vertical equity and horizontal inequity, showing how these three indices can be linked to yield a final index of net redistribution. Thirdly, we discuss how such indices will be theoretically affected by the presence of redistributive imperfections. Finally, we indicate the empirical impact of imperfect targeting, incomplete take-up and take-up costs in the grant of SB.

I. Evidence of Imperfect Poverty Relief in Britain

The incompleteness of the take-up of SB in Britain has been the subject of many studies, including Altmann (1981), Fry and Stark (1987, 1992), and the periodic publications of the Department of Social Security (DSS), a department which is responsible for the administration of most state benefits (including SB). Table 1 and Figure 1 use the computation of SB entitlement provided by the tax and benefit model of Duclos (1992) to indicate the apparent extent of the problem.[4] Table 1 displays the number of calculated eligible and of "eligible"

[3]Now called Income Support since the 1988 social security reform.

[4]The model uses a subsample of the 1985 Family Expenditure Survey (FES) data. The FES has also been used for most of the tax and benefit and take-up analysis conducted in Britain. These computations of eligibility and entitlement can differ from those made by the DSS welfare offices, which are based on information revealed by claimants through interviews and written correspondence.

receipts according to some observable family characteristics. The take-up rate (Column C) for each group then equals the ratio of Column B over Column A. We see from Column C that the rate of group take-up appears not to vary too much from the global one, except for single parents—it exceeds 90 percent—and for the pensioners and self-employed, for whom survey deficiencies are however suspected.[5]

TABLE 1

THE RAW TAKE-UP OF SUPPLEMENTARY BENEFITS IN 1985 BRITAIN

	A Number of Calculated Eligible	B Number of "Eligible" Receipts	C % Take-up Rate (B/A*100)
Total	706	453	64.2
Pensioners	288	128	44.4
Single parents	73	67	91.8
Self-employed	16	4	25.0
Public housing	426	296	69.5
Out of labour market	617	405	65.6
Private tenants	61	39	63.9

Source: The data come from the 1985 British Family Expenditure Survey with computations by the author using the tax and benefit model of Duclos (1992a).

Consider now Figure 1. On the horizontal axis, we rank income units in increasing order (*viz*, in centiles) of their calculated SB entitlement. The dotted line displays the cumulative number of units calculated to be eligible (i.e., those appearing after centile 52, approximately). The filled line displays the cumulative number of *observed* receipts. Going from low centiles to higher ones, we note that there are some households with apparent negative entitlement who still manage to receive SB, while we suspect some non-take-up among the eligible since the total number of SB recipients, entitled or not, falls below the total size of the eligible population. Accounting for these phenomena therefore seems to require the joint presence of errors in measuring benefit eligibility and of take-up costs.[6]

We thus estimated the distribution of SB entitlement apparently assessed by the DSS welfare offices and the distribution of the perceived weekly costs to taking up SB.[7] To do this, we made use of: SB receipts observed in the FES, computations of entitlement by our tax and benefit model, and the various socio-economic characteristics of the income units of our FES sample. Households are assumed to choose whether to claim by comparing the benefit and the cost of taking up the benefit. A probit model based on a bivariate distribution of unobservables

[5]See Duclos (1993a) and Department of Social Security (1989).

[6]Such phenomena could be even more important for other types of transfers for which eligibility is subject to greater uncertainty or for which take-up costs are larger. In Britain, for instance [see, e.g., Fry and Stark (1992)], no fewer than 30 percent to 40 percent of the recipients of Family Credit and Housing Benefits are typically calculated as non-eligible using the FES—the main survey data on which take-up studies in Britain are based.

[7]The estimation is explained in detail in Duclos (1993a).

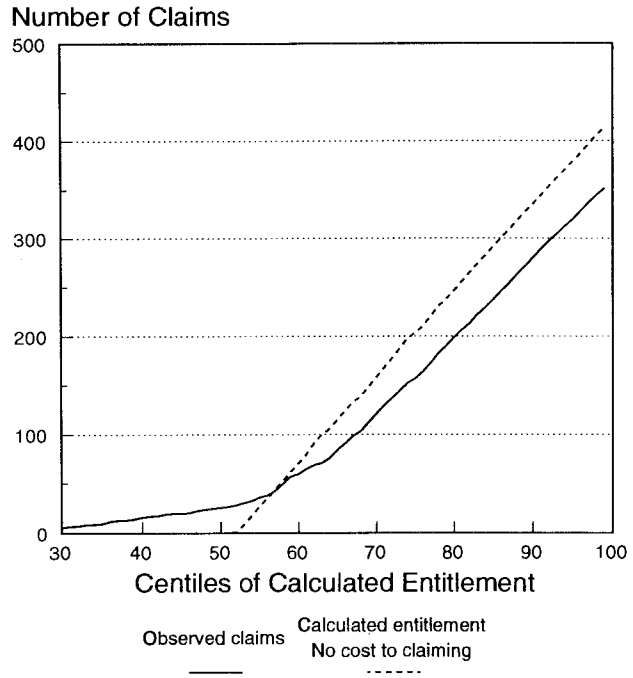

Figure 1. Cumulative Distribution of Receipts Excluding Pensioners

was then estimated using the method of maximum likelihood.[8] The estimation worked essentially as follows: (1) it identified the presence of entitlement discrepancies by the feature that without such discrepancies, some observed receipts could not be explained; (2) it identified the distribution of the weekly perceived costs to receiving SB by modelling the participation behaviour of households in the light of their imperfectly observable distribution of SB entitlement.

The results indicate that weekly inconveniences to taking up are widely dispersed across the population, and that they are greater for younger people, tenants and the self-employed and lower for those with children, single parents and one-adult units. We also find that the unobservable variation in take-up costs is only half as dispersed as the unobservable divergence between the DSS welfare offices and our model in the modelling of entitlement. The general level of entitlement

[8]The two error terms of the bivariate distribution represent random discrepancies in the measurement of entitlement and unobservable errors in the specification of the costs to taking up. We also simultaneously estimate the size of the suspected underreporting of some SB receipts among the pensioner population as well as the underreporting of self-employment income. The estimation of SB underreporting was done by modelling in the likelihood function the probability that a pensioner mistook his receipt of SB for a receipt of a state pension. Underreporting of self-employment income was estimated by including, in the entitlement divergence equation, parameters attached to the varying levels of self-employment incomes declared by households.

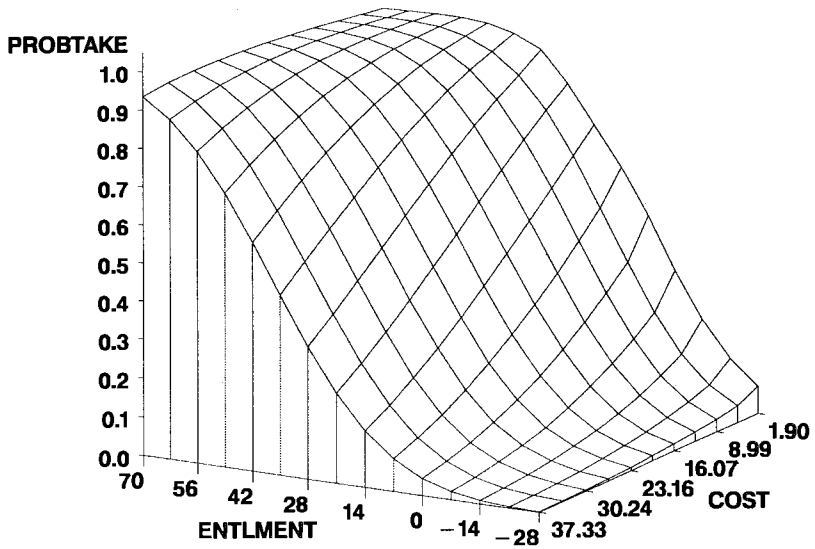

Figure 2. Cost, Entitlement and Probability of Receiving Supplementary Benefits

is underestimated by our model (relative to the DSS assessment) by about £4 a week.

Figure 2 displays on the vertical axis the estimated probability of a family receiving SB (PROBTAKE) against expected take-up costs (COST) and the expected DSS assessment of entitlement (ENTLMENT). The probability of receiving SB naturally decreases as the expected costs to welfare participation increase from as low as £1.90 to above £30 a week. The reverse trend occurs when expected entitlement rises from a negative value of −£28 to a maximum on Figure 2 or £70 a week. Due to random entitlement discrepancies, a positive probability of a receipt exists even when *expected* welfare entitlement is negative, particularly if the perceived participation costs are few.

Table 2 also illustrates our results by subtracting from gross SB received the size of the take-up costs incurred to indicate the net gain to receiving SB. Column

TABLE 2

BENEFITS, COSTS AND NET BENEFITS (POUNDS PER WEEK)
(Total average over the relevant sample of expected recipients)

	A Benefits	B Costs incurred	C Net benefits
Total	22,889	3,920	18,969
	(35.49)	(6.08)	(29.41)
Pensioners	8,598	1,560	7,038
	(29.96)	(5.44)	(24.52)
Self-employed	179	64	115
	(34.29)	(12.26)	(22.03)
Single parents	2,442	201	2,241
	(35.39)	(2.91)	(32.48)

Source: The data come from estimations in Duclos (1992b).

A displays the expected weekly grant of SB for our sample. Column B shows the expected weekly costs to be incurred in receiving the benefits of Column A. Subtracting B from A, we find the net benefits of Column C. The net money-metric utility C is about 80 percent of the gross benefit value, that figure being higher for the single parents and lower for the self-employed (respectively reflecting lower and higher costs to receiving SB).

II. Measuring Progressivity and Equity

Before illustrating the impact of redistributive imperfections (F1), (F2) and (F3) on the progressivity, equity and redistribution of SB, we must outline how we will measure these concepts. For this, we show how total redistribution—measured by the difference between original (X) and net income (N) Gini coefficients, $G_X - G_N$—can be decomposed into progressivity, average benefit, vertical equity, and horizontal inequity effects.[9]

Concentration curves can be defined as (e.g., Atkinson, 1979, p. 9)

$$C_{Y,Z}(p,\ Y,\ r(Z)) \tag{1}$$

with $0 \leq p \leq 1$. $C_{Y,Z}$ shows the cumulative total of the first $100*p\%$ of the observations of Y when such observations are ranked in increasing order $r(Z)$ of the variable $Z = Z(Y)$. This cumulative total is expressed as a proportion of the overall sum of Y. The ordinary Lorenz curve, $L_Y(p)$, is a special case of $C(p,\ Y, r(Z))$ occurring when the ranking $r[Z(Y)]$ according to Z is identical to the ranking $r(Y)$ according to Y; that is, $L_Y(p) = C(p,\ Y, r(Y))$.

To each concentration curve, we can assign a concentration index $I_{Y,Z}$, defined as

$$I_{Y,Z} = 2\int_0^1 [(p - C(p,\ Y, r(Z)))]\, dp = 1 - 2\int_0^1 C_{Y,Z}\, dp. \tag{2}$$

When $r(Y) \equiv r(Z)$, $I_{Y,Z}$ simply becomes the Gini coefficient, G_Y.

We define a progressive benefit as one whose average rate falls as gross income increases (see Jakobsson, 1976). The Kakwani (1977) index of benefit progressivity is:

$$\Pi^K = G_X - I_{B,X} \tag{3}$$

and will necessarily be positive for a uniformly progressive benefit allocation. G_X is the Gini coefficient of the distribution of original income X, and $I_{B,X}$ is the concentration index of the distribution of B when units are ranked in order of X.[10] We can also show (see, e.g., Lambert, 1993, p. 43) that

$$I_{B,X} = 2 \cdot \mathrm{cov}\left(\frac{B(X)}{\mu_B},\ F(X)\right) \tag{4}$$

with μ_B being the mean benefit over the whole distribution of income and $F(X)$

[9]See Kakwani (1986), Jenkins (1988) or Duclos (1993b).

[10]Being scale invariant, this index of progressivity is probably better described as an index of tax or benefit departure from proportionality. On this, see Lambert (1993, ch. 7) and Blackorby and Donaldson (1984).

being the cumulative distribution of X. Thus, the more negatively correlated are $B(X)$ and $F(X)$, the smaller is $I_{B,X}$ and the greater is the progressivity index Π^K.

We measure vertical equity by the pressure exerted by benefit B in compressing the distribution of income. For such a measure, we simply multiply Π^K by mean benefit as a proportion of mean net income, μ_N

$$\text{(5)} \qquad \text{vertical equity of benefit } B = \frac{\mu_B}{\mu_N} \cdot \Pi^K.$$

With this measure, the vertical equity of one benefit or tax is computationally independent of the vertical equity of others. We can also show that $I_{N,X}$ equals the initial Gini coefficient minus the sum of the vertical equity exerted by all taxes and benefits:

$$\text{(6)} \qquad I_{N,X} = G_X - \sum_{\substack{\text{all taxes} \\ \text{and benefits}}} (\text{vertical equity of individual tax or benefit}).$$

The change in income inequality is given by $G_X - G_N$. We can show (see Atkinson, 1979) that $G_X - G_N$ is always larger than $G_X - I_{N,X}$ whenever the tax and benefit system induces a reranking of units. Adopting the no-reranking criterion for horizontal inequity and using (as we do in the illustration below) *equivalent* and not money incomes, $G_N - I_{N,X}$ can then provide an index of the horizontal inequity operated by the combination of all taxes and benefits.[11,12] We can then summarise our treatment of progressivity, equity and redistribution as follows:

$$\text{(7) net redistribution} \begin{cases} = G_X - G_N \\ = (G_X - I_{N,X}) - (G_N - I_{N,X}) \\ = (\text{sum of vertical equity}) - (\text{total horizontal inequity}). \end{cases}$$

III. Equity and Redistributive Imperfections

A. The Presence of Administrative Errors

Administrative entitlement errors may come in at least two shapes. DSS welfare offices may systematically over- or underestimate the entitlement of all or of a group of identifiable units. Alternatively, entitlement errors can be distributed randomly and unobservably across the population of units.

A systematic reduction in everyone's benefit entitlement will increase the Kakwani index of a benefit whose absolute size decreases with income since the benefit share of those at the bottom of the income distribution will increase. That systematic fall in benefit entitlement will, however, cause a drop in the average benefit rate that will decrease the level of vertical equity exerted by the benefit. The redistributive impact of the benefit will also fall, and horizontal inequity

[11]See, for instance, Feldstein (1976), Plotnick (1982), or Jenkins (1988).

[12]Aronson, Johnson and Lambert (1994) provide a more sophisticated treatment of horizontal inequity by distinguishing between the "classical" type (unequal treatment of equals) and the "Atkinson/Plotnick" type (reranking of individuals) and by assuming that individuals whose incomes fall within a certain range can be considered as equals. Here, we implicitly combine both types into one index of horizontal inequity.

should also drop slightly due to the lesser importance of income redistribution. The reverse argument can be made for an overestimation of everyone's entitlement.

To understand the impact of random administrative errors upon progressivity, equity and redistribution, denote by $B^*(X)$ the mean level of assessed entitlement for a unit with income X, and by $B(X)$ the actual level of benefit entitlement. We have

$$B(X) = B^*(X) + \varepsilon \tag{8}$$

with $\partial B^*(X)/\partial X \leq 0$ and ε a random error term with mean zero.[13] Benefits payable are:

$$B^b(X) = \max [0, B(X)]. \tag{9}$$

Clearly, random errors ε are on average beneficial to *all* units for, although they never force a negative benefit level, they may lead to a sizeable exaggeration of entitlement relative to $B^*(X)$. It is incidentally those at the limit of benefit eligibility who will gain the most on average from these errors.

Random errors have thus two effects on the Kakwani index. First, average benefit increases and, second, the largest gainers lie around the threshold of eligibility, not at the bottom of the income distribution. Hence, the global impact of random entitlement discrepancies on the Kakwani index will be indeterminate. Random errors can nevertheless be expected to increase the degree of vertical equity and redistribution exercised by a means-tested benefit such as SB since they raise the average benefit level and redistribute income in favour of a relatively poor (though not the poorest) subset of the population. Finally, because random errors may induce a substantial reranking of units, they should generate increased horizontal inequity.

B. Incomplete Take-up of State Benefits

Consider first the new benefit function:

$$B^b(X) = \max [0, p(X) \cdot B^*(X)] \tag{10}$$

where the take-up probability $p(X)$ lies between 0 and 1. This benefit specification can describe well *ex ante* expected benefits across the population in the presence of imperfect take-up. We can verify that a constant take-up rate of $p(X) \equiv \bar{p}$ does not change the value of the Kakwani index—B^b is simply a proportionately scaled down measure of $B^*(X)$. Vertical equity then drops proportionally by $1 - \bar{p}$, and horizontal inequity and redistribution should fall similarly.

We do anticipate, however, that $p(X) \not\equiv \bar{p}$ and that, in particular, $p'(X) \leq 0$: the probability of benefit take-up decreases as income increases (or as entitlement falls). The progressivity implications of this are described in Loomis and Revier (1988) in the context of excise taxes on goods which not everyone purchases in the same proportion. Translated to our framework, Loomis and Revier find that imperfect take-up increases benefit progressivity if the take-up rate falls with

[13]This is analogous to the disturbance term introduced in Aronson *et al.* (1994) to describe randomness in the taxation of incomes.

income.[14] The average benefit will, of course, be lower than when take-up is complete. Hence, notwithstanding the greater progressivity, we expect vertical equity and the level of final redistribution to drop, even in the case in which $p'(X) \leq 0$.[15] Horizontal inequity will similarly fall. This is because a fall in the average benefit will lower the "harm" done by the reranking of units.

A second approach to describing the effect of imperfect take-up is by specifying (0, 1) events of a benefit receipt. We use this stochastic approach in our illustration below. Relative to the first one, this second approach does not alter the assessment of progressivity and vertical equity (since units are allocated, on average, the same benefit) but it simulates much better the *ex post* stochastic distribution of receipts and non-receipts. It therefore allows for a better analysis of horizontal inequity: for two units with similar income and relevant characteristics, one unit may choose to request the state's support and the other may not, the difference in behaviour being explained by different take-up costs. This effect clearly raises the level of horizontal inequity; combined with its downward pressure on the size of the average benefit, imperfect take-up has, under this second, *ex post*, specification, an ambiguous horizontal equity impact.

C. *Gross and Net Benefits of State Support*

As noted above, net SB can lie well below gross SB if there are important take-up costs. *Fixed* costs to taking up decrease net redistribution since they hit absolutely and relatively more those with lower income and greater entitlement. Those with relatively low benefit entitlement and a relatively high income can always avoid the take-up costs by not receiving the benefit; well-off units with no benefit entitlement will clearly not suffer from the presence of take-up costs. Those, however, with a sufficiently high benefit entitlement will still prefer to claim: they will then bear fully the take-up costs.[16]

Take-up costs generally have an ambiguous effect on horizontal inequity. On the one hand, horizontal inequity tends to fall since take-up costs reduce the net advantage of a unit choosing to take up when a similar one does not. On the other hand, however, variability in take-up costs across the population increases the variability of the net SB benefit, and can thus generate reranking and further horizontal inequity.

IV. Equity and the Take-up of State Benefits in 1985 Britain

This last section illustrates how the equity and redistribution exerted by Supplementary Benefits in 1985 Britain might have been distorted by redistributive imperfections. For this, we make throughout the implicit assumption that our

[14]Intuitively, a fall in the propensity of units to claim as their benefit entitlement falls tends to concentrate benefit shares more on those with lower incomes.

[15]Decreasing take-up rates could nevertheless conceivably increase income redistribution if benefit eligibility was widespread enough to make those at the top of the distribution fail to claim sizeable benefit amounts.

[16]The only rare circumstances in which take-up costs could increase redistribution is when they mimic a progressive tax: they are then variable, borne across the whole population, and low for the poor and proportionately higher for the rich.

model measures SB entitlement accurately. It is, of course, possible that the discrepancies reported in Section I arise from our inability to model entitlement accurately. As discussed in Duclos (1993a), however, because our own errors in appraising entitlement may be positively correlated with DSS errors, this assumption does not necessarily overestimate the presence of DSS errors and could well underestimate it.

We use the implicit 1985 SB equivalence scale to build equivalent incomes.[17] We focus our analysis on individuals, not families, assuming that family income is equally divided across members. For each family, we create two observations, with separate weights accounting for the probability that the respective observation occur. In the first of these observations, the unit does not receive SB but, in the other, it is granted the level of benefit and net benefit conditional on the unit being in receipt of SB. As discussed above, this mildly stochastic procedure can approximate the wide *ex post* distribution of SB benefit and net benefit in the population.

Figure 3 indicates the movement of the Lorenz curve (A) of original income X to concentration curves of two income distributions (B and C, ordered by X),

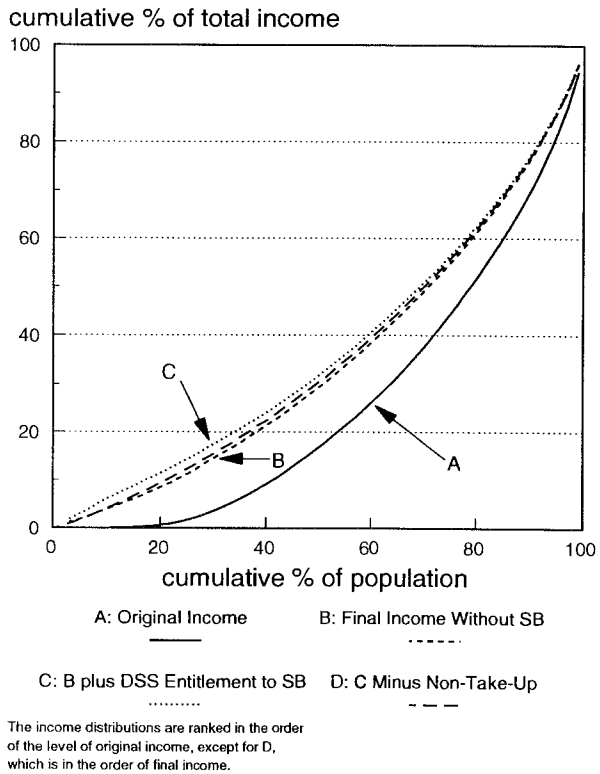

Figure 3. Income Redistribution With Administrative Errors and Incomplete Take-up

[17]We exclude from that scale the element of SB dealing with the payment of mortgage interest by owner-occupiers.

towards the Lorenz curve (D) of final income. The movement of A to B shows the sizeable vertical equity exerted by the combination of all benefits and taxes apart from SB. The wobbly increasing and decreasing slope at the bottom of B suggests a significant amount of horizontal inequity in the movement of A to B. This horizontal inequity is reduced as DSS entitlement is added, leading to the concentration curve C. Since C incorporates all expected gross benefits and taxes, it lies closest to an equal income distribution curve. To picture true income redistribution we must, however, subtract from C the amount of SB not taken up. We must also rerank the distribution of individuals according to the size of their final income. This is shown on curve D, which lies quite close to curve B.

Table 3 disaggregates more clearly the impact of SB upon progressivity, vertical and horizontal equity, and redistribution. Lines (a) to (e) of Table 3 show the variations in equity effected by SB when statutory entitlement is replaced by expected DSS entitlement (b), when we allow for random entitlement errors (c), when the take-up rate is incomplete and varies according to observable and unobservable characteristics (d), and when the net value of SB is diminished by the presence of take-up costs (e).

TABLE 3

INCOME SUPPORT AND EQUITY

	Kakwani Index*	SB as % of Final Income	Vertical Equity of SB ($\times 10^{-2}$)	Total Horizontal Inequity of System ($\times 10^{-2}$)**	Final Gini Coefficient
(a) Statutory SB entitlement	1.31	2.7	3.5	1.8	0.289
(b) Average DSS entitlement	1.32	3.1	4.0	1.9	0.286
(c) Actual DSS entitlement (including random entitlement errors)	1.30	3.5	4.5	2.1	0.283
(d) Benefit level, imperfect take-up	1.31	3.2	4.1	2.1	0.287
(e) Net benefit level, imperfect take-up	1.31	2.6	3.3	1.9	0.292

*This indicates the difference $G_X - I_{B,X}$.

**Indicates the difference $G_X - I_{N,X}$. For all computations, revised figures for National Insurance basic pensions and related corrections for SB misreporting by pensioners are applied.

The Kakwani index of progressivity varies very little across these different lines. The biggest change occurs when random errors are introduced (line c), an imperfection which makes the SB safety net less reliable and less globally progressive. As discussed above, imperfect take-up rates tend to make SB more progressive since those with a smaller entitlement are also less likely to claim SB.

Since progressivity changes little between (a) and (e), it will be the variation in the average SB that will shift vertical equity. DSS entitlement appears systematically greater than our computation of entitlement, and this pushes average SB from 2.7 percent to 3.1 percent when we move from (a) to (b), and it then increases vertical equity. Random errors also cause a substantial increase in average SB: moving from (b) to (c), SB increases from 3.1 percent to 3.5 percent. This falls to 3.2 percent when we move to (d) with an imperfect take-up of SB. SB vertical

equity correspondingly falls by about 10 percent of its peak at (c). The fall is even greater when we consider in line (e) the additional imperfection that, for those who do receive SB, the net benefit is only about 80 percent of the gross benefit. SB net of take-up costs (line e) then contributes significantly less in vertical equity than either payable SB (c) or taken up SB (d), and even less than the statutory SB entitlement.

Since SB represents less than 5 percent of the sum of the absolute sizes of the taxes and benefits included in net income, the increase from 0.018 to 0.021 of the index of total horizontal inequity when we add SB imperfections—a move from (a) to (d)—suggests that a significant degree of reranking can be introduced by redistributive imperfections. We note, however, that imperfect take-up *per se* does not have a clear impact on horizontal inequity. Also observe that horizontal inequity is decreased to 0.019 when take-up costs reduce the net advantage of those who do receive SB.

Combining changes in SB vertical equity and in horizontal inequity can account for the movement in the Gini coefficient shown in the last column of Table 3. The coefficient falls to 0.286 when average DSS (b) rather than statutory (a) entitlement is used since average SB is then greater, raising vertical equity with a slight decrease in horizontal inequity. The increase in vertical equity when random errors appear is offset by 40 percent by a rise in horizontal inequity, and the Gini coefficient falls to its overall low, 0.283. Income redistribution is thus at its highest on line (c), where only one of the redistributive imperfections is taken into account. Imperfect take-up rates (line d) push the Gini coefficient above the level at (b) as vertical equity drops without an offsetting fall in horizontal inequity. Finally, deducting take-up costs on line (e) raises the Gini coefficient to an overall high, and thus decreases redistribution to an overall low. At (e), progressivity, total net income support, and vertical equity all lie below their statutory SB levels at (a), with horizontal inequity slightly higher than at (a) but much decreased relative to its peak at (c).

Conclusion

We discuss the redistribution effected by a state benefit in the presence of two major imperfections: the difficulty of enforcing true benefit entitlement, and the incompleteness of benefit take-up, plausibly generated by the presence of take-up costs. Our analysis could in principle be extended to the use of redistributive taxation instead of redistributive transfers. Tax evasion could then be a clear source of imperfection. In analogy to the three factors discussed in the introduction, we would consider government uncertainty over the true tax liability of households, the ability and willingness of some to dodge the payment of taxes, and the feature that the net gain to tax evasion would deduct from the taxes successfully evaded the costs incurred in the evasion process.

We find that random entitlement errors likely increase vertical equity, that imperfect take-up generally decreases it, and that both factors can easily raise the degree of horizontal inequity generated by a tax and benefit system. We then consider the impact of imperfections in allocating Supplementary Benefits in 1985 Britain. The application is illustrative since we can rarely be wholly certain of the

distribution of entitlement errors made by a welfare agency. Given our assumptions, we find that random and systematic errors can raise by a third the potential level of income support granted and can thus expand the degree of vertical equity and redistribution exerted. Around 30 percent of the additional redistributive impact would nevertheless be cancelled by the presence of greater horizontal inequity. Imperfect take-up decreases the average SB level granted and lowers the degree of SB vertical equity by close to 10 percent, but it does not affect much the extent of horizontal inequity. Discounting the level of SB received by the estimated take-up costs further decreases vertical equity and redistribution.

Comparing, then, the actual equity of income support with that predicted by statutory SB rules, we find that: (1) the progressivity of SB is essentially unchanged; (2) vertical equity is raised overall by about 18 percent, with the positive impact of entitlement errors and greater average generosity (+29 percent) dominating the negative effect of incomplete take-up (−11 percent); (3) total horizontal inequity is increased significantly, especially given the modest contribution of SB to the overall British tax and benefit system; (4) income redistribution is increased (as measured by the fall in the Gini coefficient) but, in generating greater redistribution, approximately half of the increase in vertical equity is offset by a rise in horizontal inequity. Not only, therefore, can the modelling of imperfections change the level of apparent income redistribution, but it can also alter significantly the cost of redistribution in terms of state resources and equity.

References

Altmann, R. M., Take-up of Supplementary Benefit by Male Pensioners, Discussion Paper No. 25, SSRC Programme on Taxation, Incentives and the Distribution of Income, London School of Economics, 1981.

Aronson, J. R., Johnson, P., and Lambert, P. J., Redistributive Effect and Unequal Income Tax Treatment, *Economic Journal*, 104(423), 262–270, March, 1994.

Atkinson, A. B., Horizontal Equity and the Distribution of the Tax Burden, Ch. 1 in Aaron, H. J. and Boskin, M. J. (eds.), *The Economics of Taxation*, Brookings Institution, Washington, DC, 3–18, 1979.

Blackorby, C. and Donaldson, D., Ethical Social Index Numbers and the Measurement of Effective Tax/Benefit Progressivity, *Canadian Journal of Economics*, 17(4), 683–694, November, 1984.

Craig, P., Costs and Benefits: A Review of Research on Take-up of Income-Related Benefits, *Journal of Social Policy*, 20(4), 537–565, October, 1991.

Department of Social Security, Analytical Services Division, Supplementary Benefit Take-up 1985, Technical Note, 1989.

Department of Social Security, *Income Related Benefits Estimates of Take Up in 1989*, 1993.

Duclos, J.-Y., A Computer Model of the 1985 British Personal Tax and Transfer System, Ch. 1 in *Progressivity, Equity and the Take-up of State Benefits, with Application to the 1985 British Tax and Benefit System*, Ph.D. Thesis, University of London, 1992.

Duclos, J.-Y., Estimating and Testing a Model of Welfare Participation, working Paper 93-31, Département d'économique de l'Université Laval, 1993a.

———, Progressivity, Redistribution and Equity, with Application to the British Tax and Benefit System, *Public Finance/Finances Publiques*, 48(3), 350–365, 1993b.

Feldstein, M., On the Theory of Tax Reform, *Journal of Public Economics*, 6, 77–104, 1976.

Fry, V. and Stark, G., The Take-up of Supplementary Benefit: Gaps in the 'Safety Net'? *Fiscal Studies*, 8(4), 1–14, November, 1987.

———, The Takeup of Means-Tested Benefits in the UK: The Transition to Income Support and Family Credit, mimeo, May, 1992.

Jakobsson, U., On the Measurement of the Degree of Progression, *Journal of Publics Economics*, 5(1–2), 161–168, January–February, 1976.

Jenkins, S. P., Reranking and the Analysis of Income Redistribution, *Scottish Journal of Political Economy*, 35(1), 65–76, February, 1988.
Kakwani, N. C., Measurement of Tax Progressivity: An International Comparison, *Economic Journal*, 87, 71–80, March, 1977.
———, *Analysing Redistribution Policies: A Study Using Australian Data*, Cambridge University Press, 1986.
Lambert, P., *The Distribution and Redistribution of Income: a Mathematical Analysis*, 2nd edition, Manchester University Press, Manchester, 1993.
Loomis, J. B. and Revier, C. F. Measuring Progressivity of Excise Taxes: A Buyers Index, *Public Finance Quarterly*, 16(3), 301–314, July, 1988.
Oorschot, W. van, Non-Take-up of Social Security Benefits in Europe, *Journal of European Social Policy*, 1(1), 15–30, 1991.
Plotnik, R., The Concept and Measurement of Horizontal Inequity, *Journal of Public Economics*, 17, 373–391, April, 1982.
Supplementary Benefits Commission, Take-up of Supplementary Benefits, Supplementary Benefits Administration Paper 7, HMSO, London, 1978.