Review of Income and Wealth
Series 38, Number 4, December 1992

THE NON-CONSTANCY OF EQUIVALENCE SCALES

BY DENIS CONNIFFE

The Economic and Social Research Institute

Comparisons of households of differing composition are usually achieved through the use of equivalence scales. It is well known that the choice of scales can have considerable impact on the conclusions drawn from studies of welfare and poverty. There is a considerable literature on the theoretical issues relating to equivalence scales, but applied work on income distribution and related areas almost invariably takes scales to be constant irrespective of income. This paper focuses on the relation of scale to income, by applying theoretical analysis to some simple household types. The conclusion is that scales are not constant and that current practice should be changed.

1. INTRODUCTION

Equivalence scales are extremely important in studies of welfare and poverty. If the objective is to compare the living standards of groups of households and if family compositions differ between groups, straightforward comparisons of household incomes or expenditures are hardly appropriate. If two households of differing compositions have the same living standard, the ratio of their incomes is called an equivalence scale and some particular composition of household (usually either a single adult or married couple) is taken as the reference household. This paper is concerned with the issue of whether the equivalence scale should or should not be taken to vary with the income of the reference household.

Most studies assume the scale is constant and indeed some have just taken it to be inversely proportional to household size, so that comparisons were made on a per capita basis. Buhmann, Rainwater, Schmaus and Smeeding, 1988, surveyed equivalence scales in use for policy purposes and described them as extremely varied in how they allowed for family composition. However, in reality all the scales were constant as regards income and the variation between them was largely due to different degrees of adjustment for consumption economies due to larger household size.

The issue is important both for policy analysis in particular nations and for cross-national or intertemporal comparisons. For example, if a child is taken as equivalent to 0.5 of an adult and living standards of a "two adult-four children" household are compared with a "two adult" household the income of the former would be first divided by two. This implies that the "cost" of a child is £1,250 per annum if that household has an income of £10,000 per annum and £125,000 per annum if the household has an income of £1m. In the formulation of policy in the areas of welfare benefits and tax allowances, the assumption of constant scales could have serious, and perhaps unpalatable, implications. Even if one believes that, in reality, policy-makers ignore any estimates of scales, the issue still concerns economists and social scientists. Any comparative measure of poverty, for example, will need to adjust for family composition. How important is the constant scale in an assertion that (say) larger families are much more likely to be in poverty?

There is a considerable literature on equivalence scales and the methodological problems in estimating them. This literature is by no means unanimous in concluding that scales "ought" to be constant, that is independent of income or base utility level, although some authors (for example, Lewbel, 1989) have tried to make a constancy property central to both estimability and interpretation. In much of the literature the constancy issue is very difficult to disentangle from all the other contentious issues and complexities that surround the subject of scales. These include alleged incompleteness or conditionality, confounding with life-cycle effects, interpersonal utility comparisons and a host of issues pertaining to estimability. In the author's opinion, progress can best be made by initially considering households of particularly simple composition. This approach may mean that the arguments that follow in the next two sections are being applied to artifically simplified situations, but it will permit the focus to remain on the constancy of scales question.

2. A Two Identical Adult Household

First consider a household consisting of a single adult and suppose his spending on *broad* commodities can be described by the well-known linear expenditure system due to Stone (1954). For commodity i the quantity purchased (and consumed), q_i, is taken to equal some minimum or "subsistence" quantity, γ_i, plus an amount that is inversely related to price p_i and directly related to "discretionary" income, which is income, y, minus the expenditure required to purchase the subsistence quantities of all commodities. The equations are:

$$q_i = \gamma_i + \frac{b_i}{p_i}\left(y - \sum_j \gamma_j p_j\right) \tag{1}$$

where the b_i are some set of parameters indicating preferences for the commodities. If income is understood to be total expenditure, so that

$$y = \sum_j q_j p_j$$

these b's are constrained to sum to unity. The term "subsistence" may suggest misery, but all that is implied is that certain quantities of commodities must be consumed before preferences are operational. There are, of course, other possible formulations for demand equations besides (1), but it has been chosen because it is particularly plausible for broad commodities and because it is the most popular system in actual applications. Indeed, its only close rival as regards frequency of application will be employed in a later section to test the sensitivity of the findings to the choice of model.

Obviously the quantities of commodities purchased depend on the time interval over which (1) is presumed to hold. For short-time intervals, the assumed identity of purchasing and consumption may be invalid, especially for commodities of a durable nature. In practice, household expenditure surveys are often conducted over relatively short time periods and analyses of actual data often suppose that (1) holds for just a sub-set of commodities with a corresponding redefinition of total expenditure, or else the durable equations are modified to allow for past period purchases. For simplicity, this difficulty will be ignored and (1) will be taken as applying to all commodities.

Suppose the single adult decides to join another and form a two adult household. Perhaps unrealistically, suppose the second adult has identical preferences and income to the first and that the only motivation is to improve living standards. The first adult, to be denoted by A, now faces a different set of prices than when constituting a single household. Since the second adult will pay his "share" the price of commodities that are jointly consumed, or that exhibit economies of scale, will fall so far as A is concerned. So he now perceives prices of $\theta_i p_i$, where $\frac{1}{2} \leq \theta_i \leq 1$. For example, if the availability of a commodity like accommodation, or heating, to A is unaffected by its simultaneous consumption by the other adult, rent or fuel price will have halved. Any problems of obtaining agreement about price sharing between the adults are being assumed away through the identical tastes and incomes. So A will now consume quantities

$$q_i = \gamma_i + \frac{b_j}{\theta_i p_i}\left(y - \sum_j \gamma_j \theta_j p_j\right), \tag{2}$$

as indeed will the other adult. A comparison of A's welfare, or living standard, in a single adult household with his standard in a two identical adult household will depend on how the quantities given by (1) compare with those given by (2). Put another way, the income, y_h, that A would require when in the two adult households to attain the same living standard as he possessed in a single household with income y_r, will depend on how the pattern of consumption represented by (2) compares with (1). This cannot be deduced by looking at just one commodity, because the relative prices of all commodities have changed (unless all θ_i are equal, which would be absurd) and so A will purchase in a different pattern when in the two adult household.

So some way of combining quantities to make an overall comparison is necessary. The natural choice would seem to be the utility function associated with (1)

$$\prod (q_j - \gamma_j)^{b_j}. \tag{3}$$

Substituting (1) into (3) with $y = y_r$ and (2) into (3) with $y = y_h$ and equating them, using $\sum b_j = 1$, gives

$$\frac{y_h}{y_r} = \delta + \frac{\sum_j \gamma_j p_j (\theta_j - \delta)}{y_r} \tag{4}$$

where

$$\delta = \prod \theta_j^{b_j}.$$

Clearly δ is a geometric type mean of the θ's and (4) is an equivalence scale that depends on the reference income y_r. If at one unlikely extreme, all $\theta_i = \delta = \frac{1}{2}$ then the scale would be a constant and equal to $\frac{1}{2}$; the "two living as cheaply as one" situation. At another extreme when all $\theta_i = \delta = 1$, so that there are no price reductions, then the scale is again constant and equal to unity. If the θ_j vary, which is to be expected, the scale will vary with income unless all $\gamma_i = \emptyset$. But then the linear expenditure system reduces to the Bergson (1936) system, which implies the same proportionate distribution of budget over commodities whatever the income.

The argument leading to (4) did not involve any interpersonal utility comparison, or any specification of a household utility, since the contrast was between A's welfare in a single adult household and his welfare in a two adult household. Although the present case may seem artificially simple, the implications of (4) for the question of the behaviour of equivalence scale with income are not trivial. The scale is not constant; instead it tends towards a constant with increasing income. Indeed, a little more can perhaps be deduced. In many societies, it seems plausible that luxury goods like consumer durables will show the greatest price reduction, that is the smallest values of θ_i, while necessities like food, or clothing, will show values of θ_i that are little below unity. So $\theta_i - \delta$ is likely to be positive for a necessity. However, it is also true that necessities will tend to have the largest values of committed, or subsistence, expenditures $\gamma_i p_i$. So the second term on the righthand side of (4) will be positive and the equivalence scale then decreases with income.

If the second adult has a lower income than A, although still with identical tastes, one modification of the previous approach would be to assume the two adults split payments for purchased commodities equally up to the exhaustion of the lower income.

Then, instead of (4) the scale that results is

$$\frac{y_h}{y_r} = \delta^* + \frac{\sum \gamma_j p_j (\theta_j - \delta^*)}{y_r} \tag{5}$$

where

$$\delta^* = \prod \left[\frac{\theta_j}{k + (1-k)\theta_j} \right]^{b_i},$$

with $k(<1)$ the ratio of the second adult's discretionary income to A's discretionary income.

However (5) may not really correspond to optimising behaviour on A's part. The portion of commodities that A alone has paid for will presumably still be available to the other adult, to at least some degree, in the case of goods like housing, heating etc. So A might conceivably charge the other adult for this benefit and, if he had the bargaining power, might compel the other adult to hand over such a portion of income as would leave him only marginally better off than if living alone. However, that bargaining power would be curtailed if there was actually a population of individuals seeking to form two adult households. However, if a distribution of income exists, A would only choose to set up a two adult household with someone with a lower income than himself, rather than with someone with an equal income, if he could do just as well from it. This implies a sufficient transfer of income to A to restore the situation and equivalence scale given by (4).

This model of totally selfish individual motivation is probably far removed from the reality of behaviour in two income households and many other assumptions are possible. For example, it could be supposed that both individuals pool their incomes and share them equally, and indeed Pahl (1983) has claimed that this system describes the management of finances in half of the households composed of British married couples. However, the married couple is really a more complex situation and will be left until later.

3. Single Adult with a Dependent

A dependent will be defined as someone with an income too low to purchase subsistence quantities of commodities even when forming part of a two adult household. Some further qualification of the idea will be made later. Suppose A makes up the difference in expenditure on the dependent's subsistence quantities, before devoting the rest of his income to himself. Obviously, there must be some familial, or legal obligation involved, or some emotional or altruistic motivation. Although A only directly provides for the dependant's subsistence quantities, the dependant will also benefit from A's own purchases of jointly consumable commodities. Direct support of the dependant costs A

$$\sum \gamma_j \theta_j p_j - y_0$$

where y_0 is the income of the dependent. Due to the economies associated with joint purchasing and consumption, his own expenditure on subsistence commodity i is now

$$(2\theta_i - 1)\gamma_i p_i.$$

So his remaining discretionary income is

$$y + y_0 - 2\sum \gamma_j \theta_j p_j$$

and he then consumes quantities

$$q_i = \gamma_i + \frac{b_i}{p_i}(y + y_0 - 2\sum \gamma_j \theta_j p_j). \tag{6}$$

Substituting into (3) and comparing, as before, with his utility as a single adult household gives

$$\frac{y_h}{y_r} = 1 - \frac{y_0}{y_r} + \frac{\sum \gamma_j p_j (2\theta_j - 1)}{y_r}. \tag{7}$$

Clearly, if $\theta_i = \frac{1}{2}$ for all i, A would be better off with a dependant as long as $y_0 > \emptyset$. However, it can be presumed that the third term will normally exceed the second, so that the equivalence scale (7) declines with income towards a value of unity.

In some cases of dependants, for example an aged parent or perhaps a child, it may be implausible to assume the same values of γ_i as for A. Letting γ_{i0} denote the quantities for the dependant, the same arguments give the scale

$$\frac{y_h}{y_r} = 1 - \frac{y_0}{y_r} + \frac{\sum \gamma_{j0} p_j (2\theta_j - 1)}{y_r}. \tag{8}$$

Again the scale decreases to unity with increasing income.

Several points are worth noting. First, there is still no household utility or interpersonal comparison involved in these scales, even though parameters associated with the dependant occur in (8). Second, the dependant's preferences, as measured by his b_i's, do not come into the reckoning since his income is below that required for his subsistence expenditure. Third, the average of the θ's, δ in equation (4), does not occur in (6) or (7). This is important, because as will be seen in section 5, δ cannot be estimated from a single household budget survey at a point in time. Fourth, although the scale given by (8) declines with income, it is clear that the income difference required to give A the same living standard with, or without, a dependant is constant for a fixed set of prices and is

$$y_h - y_r = 2 \sum \gamma_{j0} \theta_j p_j - y_0. \tag{9}$$

Indeed, it is clear that the utility function (3) is not really required at all as there is no need to consider weighting commodity quantities into some single criterion, because (6) with y_h becomes identical to (2) with y_r, given (9). Since the dependant does not have enough income to cooperate with A to reduce the prices of his discretionary purchases, there are no substitution effects, unlike the situation considered in section 2.

It is worth looking carefully at what is being assumed in this idea of dependence and normative aspects may be unavoidable here. It might be argued that A might feel compelled to treat his (adult) dependant much as he treats himself. In that case the cost he faces for every commodity increases, though not in proportion, and substitution effects will occur. For identical tastes, the formulae of section 2 would recur, except that now $1 \leq \theta_i \leq 2$ for all i. Scales would again not be constant, but would tend towards δ, now greater than unity, with increasing income.

This model may well be more plausible than the dependency model already discussed for many two adult households, where one adult has little, if any, independent income; a situation which may apply to many married couples. However, in such households the concept of dependant is rather different than that which implicitly underlay the arguments leading to (7) and (8). The incomeless or lower income adult may be providing significant labour and other inputs to a household production process that modifies the purchased commodities and this could affect the θ_i values. In addition, tastes or preferences may have been modified by cohabitation so that the assumed constancy of the b's irrespective of household type may be implausible. It seems better to treat the married couple as a single decision making unit, where the γ's and b's of equation (1) now refer to this unit.

For cases of "real" dependency—a child, a disabled adult, an aged relative etc.—the method leading to (8) seems appropriate. The dependant's income is supplemented by A to subsistence level and he also gains somewhat from A's discretionary expenditures, even if these were selfishly decided upon. The subsistence quantities γ_{i0} have been treated as constant for a given type of dependant in a population of interest at a given time. They are assumed independent of A's income and to be norm values for the society.

Lewbel (1985) has argued that γ's are functions of income: "Paying the maid may be just as much a part of a rich man's committed expenditure as a loaf of bread is a poor man's." Even if one found that view acceptable for a rich man's own γ_i, it would not follow that society exerts any pressure on him to support his dependants beyond the norms. If he chooses to do so, it is because he derives greater satisfaction than by meeting the norms and spending his discretionary income on himself. It is also rather odd taking parameters as implicit functions of income, when income occurs explicitly in equations.

4. Adult Couple and Child

The commodity purchases of a couple can be assumed describable by (1), with the γ's now representing necessary minimum quantities for the adult couple and the preference indicating b's referring to the joint decision making. It will be assumed that either the child is incomeless, or that the income accrues directly to the parents. The parents provide the subsistence quantities for the child, providing a portion of their own necessary quantities in the process and have available for their consumption

$$q_i = \gamma_i + \frac{b_i}{p_i}[y - \sum \gamma_j p_j - \sum{}_{j1} p_j (2\phi_{j-1})] \tag{10}$$

where γ_{i1} is the subsistence quantity of commodity i for a child. The ϕ's replace θ's to emphasise these may be different. Quantities of all commodities given by (10) are identical to (1) if income is incremented by

$$\sum \gamma_{j1} p_j (2\phi_j - 1). \tag{11}$$

The equivalence scale is

$$\frac{y_h}{y_r} = 1 + \frac{\sum \gamma_{j1} p_j (2\phi_j - 1)}{y_r} \tag{12}$$

which declines to unity with increasing income. These formulae do not assume that the child does not benefit from the parents discretionary expenditure; the child does to the extent that commodities are jointly consumable etc. However, they do assume that, once parents have provided the child with commodities to the levels society considers the norm, their own preferences for commodities, and hence the b_i values, are unchanged.

If preferences do change, that is, if parents reallocate discretionary spending away from that preferred by a childless married couple, presumably because they themselves enjoy observing the child's consumption, it is obviously still true that the income increment (11) would have permitted them to purchase the same set of quantities for their own consumption should they have wanted to. If they preferred some other set, this must be because they considered it offered greater utility. There are then two possible lines of argument. The first is to define the income increment and scale to be (11) and (12), which is equivalent to saying that comparisons ought to be made as if preferences are the same. This may seem odd in suggesting that welfare may differ from preferences, but Fisher (1987) has given some examples that suggest changes in preferences should not always be allowed for. The second line or argument is to say that household H is now actually better off than household R, so that (11) and (12) are upper bounds. As regards the relationship between scale and income, both arguments lead to the same conclusion, since if an upper bound decreases towards unity so must the true value, given that it can never be below unity if child costs are positive at all. It may be worth saying that constancy of the b's may often be empirically testable.

A child can affect a parent's welfare, other than by its effect on consumption patterns and Pollak and Wales (1979) argued that welfare comparisons should be based on "unconditional" utility, which takes account of the impact of the child in every way. Measuring this utility is an obvious problem and they suggested "psychometric," or subjective measures of the type developed by Kapteyn and Van Praag (1976). Most researchers feel that observations of expenditure patterns are still meaningful and useful even accepting the existence of other dimensions, a view expressed by Deaton and Muellbauer (1986), for example.

However, even if compensatory income increments and scales could be calculated from evaluative survey indicators, the final result in terms of relationship of scale to income need not be any different than already described. Bradbury (1989) has given a good account of these subjective measures. One approach is based on trying to measure utility directly, if subjectively, by weighting answers to questions like—"How do you feel? (Delighted... Terrible.) Indicate as appropriate"—into some kind of index. Another approach asks opinions about the income level a household thinks it would require to attain some satisfaction level. Depending on the functional form chosen to relate utility and income, scales can be constant, or change with income, and arguments analogous to previous ones are possible. It is probably not worth going into in detail, because Bradbury was not encouraging about the value of the approach, believing reference group effects and "preference drift" invalidated much empirical estimation.

So far, a single child of unspecified age has been mentioned, but clearly (10), (11) and (12) could apply to several children in a range of ages by replacing γ_{i1} by, say γ_{i2}, which could be the subsistence quantity for two young children. It is the author's opinion that it is unwise to try to parameterise the quantities in terms of functions of number, or age, in advance of estimation because then unsuitable functional forms, or assumptions (absence of economies of scale, say, if a linear function was taken), may be embedded without testing their plausibility.

5. Estimation and Related Issues

The mechanics of estimation are not of primary interest in this paper, but some discussion is essential because some authors (Lewbel, 1989, Blundell and Lewbel, 1991; Nicol, 1990) are quite pessimistic about what can be estimated from budget surveys (even when there is a time series of them) and have considered formally imposing constancy of scales as a constraint to permit estimation. Other authors, for example, Ray (1983) and Jorgenson and Slesnick (1987) have implicitly assumed constancy of scales in the estimation process. So it is important to show that non-constancy does not mean non-estimability. Much can be estimated for scales relating to dependants from even a single budget survey, although this depends crucially on the fact that the scales are of the form (7) and not (4).

Some of the arguments that follow are certainly not original, but are drawn from the extensive literature that has focused on the identification of scales issue.

The equations of the previous sections involved quantities that are not necessarily observable. Most household budget surveys will only record total household consumption on commodities and this, along with the intrinsic joint consumption properties of certain commodities, will impose constraints on what can actually be estimated. So in the two identical adult cases of section 1, total household consumption of commodity i will be

$$q_{ic} = 2q_i - 2(1-\theta_i)q_i,$$

where q_i is given by (2). So the equation relating observable household expenditure $p_i q_{ic}$ to observable household income y_c is

$$p_i q_{ic} = 2\gamma_i\theta_i p_i + b_i(y_c - 2\sum \gamma_j\theta_j p_j). \tag{13}$$

The equation for a single adult household, taken as the "reference" household in this case would be

$$p_i q_{ir} = \gamma_i p_i + b_i(y_r - \sum \gamma_j p_j). \tag{14}$$

In terms of relationships between expenditure and income these are parallel straight lines with slopes b_i and intercepts

$$2(\gamma_i\theta_i p_i - b_i \sum \gamma_j\theta_j p_j) \quad \text{and} \quad \gamma_i p_i - b_i \sum \gamma_j p_j \tag{15}$$

respectively. Assuming that only one household budget survey is available, that is that prices are fixed, the b's are obviously estimable from samples of households of different income. Indeed, the constancy of the b's between (13) and (14) is testable. The intercepts (15) are obviously estimable, but since both sets sum to zero over commodities there are just $2r-2$ algebraically independent quantities from which to estimate the $2r$ unknown θ's and γ's. If it were known that some $\gamma_i = 0$, which seems most unlikely, this commodity would estimate $\sum \gamma_j\theta_j p_j$ and $\sum \gamma_j p_j$, and the others would yield estimates of the γ's and θ's since the p's are known. Otherwise, variation in prices via a time series of budget surveys is necessary to estimate coefficients of prices as well as of incomes in (13) and (14). The root reason for this is that if both adults have discretionary income, varying "sharability" of commodities leads to effective relative price changes. With only a single household budget survey, effects are totally confounded.

In the case of a single adult with an adult dependant, where (6) and (7) apply, the equation relating observed expenditure on commodity i to total household income is exactly the same as (13). From observations of expenditures alone, one could not distinguish between two adults with equal discretionary income and one adult (with dependant) with twice the discretionary income. However, if it is known that one adult is a dependant with income (less the subsistence income) y_0 then $y_c = y + y_0$ and the intercept is

$$2\gamma_i\theta_i p_i + b_i y_0 - 2b_i \sum \gamma_j\theta_j p_j. \tag{16}$$

Subtracting the intercept for a single adult household gives

$$(2\theta_i - 1\gamma_i p_i + b_i y_0 - b_i \sum \gamma_j p_j(2\theta_j - 1) \tag{17}$$

which, apart from the first term, is b_i by the negative of the income increment required to give the independent adult the same quantities for his own consumption as his single adult household counterpart. Now the first term will be zero for a commodity for which $\theta_i = \frac{1}{2}$ and so even one highly sharable good permits assessment of the cost of an adult dependant since b_i is estimable as a slope coefficient. The actual scales as given by (7) are just a trivial further step.

Similarly, the scales (8) and (12) can be estimated from intercept differences given the same assumption. However, the child dependant case is worth more attention. The difference in intercepts is

$$(2\phi_i - 1)\gamma_{i1}p_i - b_i \sum \gamma_{j1}p_j(2\phi_j - 1) \tag{18}$$

so that, if the first term is zero, the appropriate income increment (11) is estimable as are the scales (12). However, besides $\phi_i = \frac{1}{2}$, another possible identifying assumption is $\gamma_{i1} = 0$. This is the "adult good" idea and an estimate based on (18) for a single adult good commodity will be identical to that obtained by measuring the difference in household incomes required to equalise consumption of the adult good, which is the well-known Rothbarth (1943) method of estimating scales.

The γ's and ϕ's in (11) cannot be disentangled at the estimation stage of course. Either the whole expression (11) can be estimated if only one commodity equation (which must have an identifying property) is employed, or all the $a_i = \gamma_{i1}(2\phi_i - 1)$ if all commodity equations are estimated with sufficient identifying constraints. The latter situation is by far the better because then scales can be constructed for years other than the survey year by simply calculating the index

$$\sum_j a_j p_{jk} \tag{19}$$

where p_{ik} is the price of commodity i in year k. Of course (19) is now a Lespeyres type index where weights are based on the survey year and can only be updated when another household budget survey is conducted. However, in many countries this is the situation even for the consumer price index. An example of updating scales in this way is given in Conniffe and Keogh (1988).

Household utility functions have not featured in the discussion so far. In cases of dependency, scales have been defined to enable the supporting individual, or couple, to attain the same consumption levels of all commodities as in the absence of a dependant. The case of identical adults in section 1 did make explicit use of a utility function, but the comparison was on an individual and not a household basis. However, the progression from (1) for an individual to (13) for observable consumption by a household could be seen as replacing the utility function (3) by

$$\prod [q_j/(2\theta_j) - \gamma_j]^{b_j}.$$

The idea that a household utility function could be generated from an individual one by replacing the q_i's by q_i/m_i is due to Barten (1964). The corresponding progression in the case of the dependency examples could be regarded as a case of the "translation" modification to a utility function as described by Pollak and Wales (1978). This author prefers to avoid household utility functions if at all possible.

Before concluding this section it should be said that there has been debate about the identifiability of scales ever since Prais and Houthakker (1955), with such contributors as Forsyth (1960), Singh and Nagar (1973) and Muellbauer (1974) and continuing to include some references already mentioned earlier. Some authors have worked in the context of a single available survey, while others have assumed the existence of a time series of surveys. In the latter case, the possibilities of estimating intertemporal substitutions have even led to advocacy of life-cycle equivalence scales (Banks, Blundell and Preston, 1991; Pashardes, 1991).

6. The Almost Ideal Demand System

So far the discussion has utilised the linear expenditure system (1) to obtain explicit formulae. It is interesting to look briefly at how arguments might be affected by a change to the "almost ideal demand system." This demand model was introduced by Deaton and Muellbauer (1980) and rivals the linear expenditure system in popularity. It arises from the cost function

$$\log y = S + TU \tag{20}$$

where

$$S = A_0 + \sum A_j \log p_j + \tfrac{1}{2} \sum C_{jk} \log p_j \log p_k \tag{21}$$

and

$$T = \sum b_j \log p_j \tag{22}$$

where

$$\sum_j A_j = 1, \qquad \sum_j C_{jk} = \sum_k C_{jk} = 0 \quad \text{and} \quad \sum b_j = 0.$$

It gives demand functions

$$w_i = A_i + \sum C_{ij} \log p_j + b_i \log (y/S). \tag{23}$$

Scales have been estimated from this system by many authors including Deaton and Muellbauer (1986), Pashardes (1988), Deaton, Riuz-Castillo and Thomas (1989), Blundell and Lewbel (1991) and Tsakloglou (1991). Constant scales have been derived by either explicit assumptions or implicit ones. The explicit argument starts from (20), where one assumes that T is unaffected by a change in household composition so that for constant utility U

$$\log y_h - \log y_r = S_h - S_r$$

giving a constant scale. This was the line taken by Blundell and Lewbel who also assumed that the C's in (21) were unchanged with household type so that

$$\log y_h - \log y_r = (A_{0h} - A_{0r}) + \sum (A_{jh} - A_{jr}) \log p_j.$$

The implicit argument starts from (23) in constant price form.

$$w_c = d_i + b_i \log y,$$

assumes that only the intercept changes with household type and takes the scale given by equating the food shares, so that

$$\log y_h - \log y_r = (d_{fr} - d_{fh})/b_f.$$

The equating of food shares is a device that goes back to Engel (1857), but can be shown (for example, Deaton and Muellbauer, 1986) to overestimate the correct scale.

Taking the case of two identical adults, as in section 2 and replacing p_i by $\theta_i p_i$, it is quite clear that T does change with household. S also changes more elaborately than Blundell and Lewbel assumed. Following through, the scale that actually results is

$$\frac{y_h}{y_r} = \left(\prod \theta_j\right)^{A_j} \operatorname{Exp}(Z) \tag{24}$$

where

$$Z = \sum\sum C_{jk}[\log \theta_j \log \theta_k + 2 \log p_j \log \theta_k] + \frac{\sum b_j \log \theta_j}{\sum b_j \log p_j}(\log y_r - S_r). \tag{25}$$

This scale is actually very similar to (4). The conditions on the b's and c's are such that Z becomes zero if the θ's are all equal and so for all $\theta_i = \frac{1}{2}$ the scale becomes $\frac{1}{2}$ and for $\theta_i = 1$ it becomes unity. The A's in an almost ideal demand system play the role of the b's in a linear expenditure system. Since Z is a function of y_r the scale is not constant with income. As in the case of (4) it seems the scale decreases with income, because logarithms of the θ's are negative or zero and the large negatives will tend to be associated with the positive b's, so making the second term of (25) negative.

As regards the case of section 3, an adult with a dependant, the choice, as before, is between assuming he must treat the dependant as he treats himself, or must meet some societal norm. With the former assumption (25) will appear again, with θ's greater than or equal to unity. With the latter assumption, which the author considers more plausible, the idea would again be that θ's will only appear with the components of consumption relating to this norm. The almost ideal system is not nearly as convenient as the linear expenditure system in implementing a separation between "subsistence" and "discretionary" expenditures. S in (20) could be interpreted as the logarithm of subsistence expenditure, so that S_h might be a function of the θ's and perhaps

$$y_h - \operatorname{Exp}(S_h)$$

is then discretionary income, with an allocation to commodities independent of the θ's. But the logarithm of this is not equal to TU. However, perhaps enough has been said to demonstrate that constant scales have little prior support with this model either.

7. Summary and Discussion

Based on the results in sections 2, 3 and 4, the following summarises the situation: Scales are not constant with income; they decline towards a constant with increasing income and that constant is unity in the case of dependants. An alternative statement of the situation as regards dependants is that the compensatory, or equivalent, income increment is a constant.

Yet scales used in comparative and policy-related welfare studies are almost invariably constant. It is hard to see what other considerations should overrule the sort of assessment conducted in previous sections. Nowadays, no researcher seriously suggests that within a particular society higher income groups should be deliberately granted greater monetary compensation for having children (through child allowances, or whatever) than lower income groups. It was seriously suggested in the past, for example, by Fisher (1930, 1931), but the greatest statistician of this century had ideas about the genetic superiority of the higher income classes that few social scientists could feel comfortable with. Becker (1981) has, of course, envisaged a "quality" measure of a child as explicitly appearing in the parents' utility function, but that whole approach would cancel out costs with benefits.

It can hardly be claimed that greater simplicity make constant scales preferable. Two types of household, differing in dependent status, could have the appropriate income increments subtracted from (average) incomes before making the comparison, rather than by first dividing by "adult equivalents". Of course, groups of households may be compared containing mixtures of household types. If the frequencies of types are known, the correct weighted average equivalent income increment can be obtained and if they are unknown, with perhaps only average family size known, simple interpolation formulae, can be employed. If the types of household differ in the number of income earning adults, scales of the type (4) will occur. As outlined in previous sections, estimation may demand more complex data and other difficulties may arise because of inputs to household production, but assuming (4) is applicable and estimable, employing it is not difficult.

An "income increment"

$$\sum \gamma_j p_j(\theta_j - \delta)$$

should be subtracted, before a scale adjustment by δ.

When comparing groups in different populations—such as countries at different levels of economic development, or the same country at different time points—subsistence quantities and consequent costs of dependants could well be different in the two populations. Then even households of the same composition in the two populations could have different income increments subtracted before comparison of their average incomes. However, "constant" scales would presumably also differ between such populations.

A final reason for the dominance of constant scales in the applied literature might be thought to be that they were all that were available. It is true that some scales were calculated with constraints to force constancy. However, many were calculated from methods that actually gave scales varying with income, but were then "converted" to a constant scale by fixing them at some income value.

In this paper, the simple household examples were chosen to permit explicit algebraic derivation of the relationship of scale to income. To some extent the main demand system considered was also chosen because it was easily manipulated. That is not to say it is implausible, or less sophisticated than most of the equation systems used to derive scales. Some of these have been very ad hoc, paying little attention to the theory of consumer demand. If analyses of simple household types show that scales change with income, it seems most unlikely that analyses of complex households will show they do not. Constant scales are not plausible and their use needs reconsidering.

References

Banks, J., Bundell, R., and Preston, I., Adult Equivalence Scales; A Life Cycle Perspective, *Fiscal Studies*, 12, 17-29, 1991.

Barten, A. P., Family Composition, Prices and Expenditure Patterns, *Colston Papers*, 16, 227-292, 1964.

Becker, G. S., *A Treatise on the Family*, Harvard University Press, Cambridge, MA, 1981.

Bergson, B. A., Real Income, Expenditure Proportionality, and Frisch's "New Methods of Measuring Marginal Utility", *Review of Economic Studies*, 4, 33-52, 1936.

Blundell, R. and Lewbel, A., The Information Content of Equivalence Scales, Journal of Econometrics, 50, 49-68, 1991.

Bradbury, B., Family Size Equivalence Scales and Survey Evaluations of Income and Well Being, *Journal of Social Policy*, 18, 383-408, 1989.

Buhmann, B., Rainwater, L., Schmaus, G., and Smeeding, T. M., Equivalence Scales, Well-Being, Inequality and Poverty: Sensitivity Estimates Across Ten Countries Using the Luxembourg Income Study (LIS) Database, *Review of Income and Wealth*, 34, 115-143, 1988.

Conniffe, D. and Keogh, G., *Equivalence Scales and Costs of Children*, ESRI, Dublin, 1988.

Deaton, A. and Muellbauer, J., An Almost Ideal Demand System, *American Economic Review*, 70, 312-326, 1980.

———, On Measuring Child Costs: With Applications to Poor Countries, *Journal of Political Economy*, 94, 720-744, 1986.

Deaton, A., Ruiz-Castillo, J., and Thomas, D., The Influence of Household Composition on Household Expenditure Patterns: Theory and Spanish Evidence, Journal of Political Economy, 97, 179-200, 1989.

Engel, E., Die Produktions und Konsumpions ver Haltnisse des Kong Reichs Sachen, (Reprint in the *Bulletin of the International Statistical Institute*, 1895), 1857.

Fisher, F. M., Household Equivalence Scales and Interpersonal Comparisons, *Review of Economic Studies*, 54, 519-524, 1987.

Fisher, R. A., *The Genetical Theory of Natural Selection*, Oxford University Press, Oxford, 1930.

———, The Biological Effects of Family Allowances, *Family Endowment Chronicle*, 1, 21-25, 1931.

Forsyth, F. G., The Relationship between Family Size and Family Expenditure, *Journal of the Royal Statistical Society* A, 123, 367-393, 1960.

Jorgensen, D. W. and Slesnick, D. T. S., Aggregate Consumer Behaviour and Household Equivalence Scales, *Journal of Business and Economic Statistics*, 5, 219-232, 1987.

Kapteyn, A. and Van Praag, B., A New Approach to the Construction of Family Equivalence Scales, *European Economic Review*, 7, 313-335, 1976.

Lewbel, A., A Unified Approach to Incorporating Demographic or other Effects into Demand Systems, *The Review of Economic Studies*, 52, 1-18, 1985.

———, Household Equivalence Scales and Welfare Comparisons, *Journal of Public Economics*, 39, 377-391, 1989.

Muellbauer, J., Household Composition, Engel Curves and Welfare Comparisons between Households, *European Economic Review*, 5, 103-122, 1974.

Nicol, C. J., Household Equivalence Scales without Exact Aggregation, paper presented at *6th World Congress of the Econometric Society*, Barcelona, Spain, 1990.

Pahl, J., The Allocation of Money and the Structuring of Inequality within Marriage, *Sociological Review*, 31, 237-262, 1983.

Pashardes, P., Contemporaneous and Intertemporal Child Costs: Equivalent Expenditure vs Equivalent Income Scales, Journal of Public Economics, 45, 191-213, 1991.

Pollak, R. A., and Wales, T. J., Estimation of Complete Demand Systems for Household Budget Data: the Linear and Quadratic Expenditure Systems, *American Economic Review*, 68, 348-359, 1978.

———, Welfare Comparisons and Equivalence Scales, *American Economic Review*, 69, 216-221, 1979.

Prais, S. J. and Houthakker, H. S., *The Analysis of Family Budgets*, Cambridge University Press, London, 1955.
Ray, R., Measuring the Costs of Children: An Alternative Approach, *Journal of Public Economics*, 22, 89-102, 1983.
Rothbarth, E., Note on a Method of Determining Equivalent Income for Families of Different Composition, in Appendix to *War Time Patterns of Saving and Spending*, Cambridge, 1943.
Singh, B. and Nagar, A. L., Determinants of Consumer Unit Scales, *Econometrica*, 41, 347-356, 1973.
Stone, J. R. N., Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand, *Economic Journal*, 64, 511-527, 1954.
Tsakloglou, P., Estimation and Comparison of Two Simple Models of Equivalence Scales for the Cost of Children, *The Economic Journal*, 101, 343-357, 1991.