# MODELING AGGREGATE PRODUCTIVITY GROWTH: THE IMPORTANCE OF INTERSECTORAL TRANSFER PRICES AND INTERNATIONAL TRADE

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The value-added model underlies current measures of aggregate productivity growth. Unbiased estimates result only if the economy is closed to trade in foreign-produced material inputs and all domestic intersectoral transactions are characterized by marginal cost pricing. Neither condition typically holds.

This paper identifies these biases and proposes a delivery-to-final-demand framework, a modified form of that first introduced by Domar. The rate of aggregate productivity growth is decomposed into terms identifying the contributions of total factor productivity growth within individual sectors, the reallocation of the economy's primary inputs among sectors, and changes in the allocative efficiency of markets for intermediate goods. The adjustments necessary to remove biases from existing value-added estimates are derived.

Economists long have reasoned that goods destined for final demand are the ultimate objective of economic production. The sum of sectoral deliveries to final demand, therefore, is the appropriate output criterion for evaluating aggregate productivity growth.

This is not a novel insight in the productivity literature. Domar (1961) first introduced it. Watanabe (1971), Star (1974), and Hulten (1978) followed. The papers establish the link between an aggregate delivery-to-final-demand framework and a sectoral model with intermediate input transfers among sectors.

Sectoral deliveries to intermediate demand, however, typically are ignored in studies of aggregate productivity. Intersectoral deliveries, even in the Domar (1961) *et. al.* papers, are viewed as wholly internal transactions, self-canceling in both real and money terms. Under these conditions, deliveries to final demand can be shown to equal national net output or, as conventionally described, aggregate value added. Furthermore, intermediate deliveries can be shown not to affect the measure of aggregate productivity growth. The result is that empirical studies of aggregate economic performance typically define productivity at the economy-wide level as the efficiency with which labor and capital inputs are converted into aggregate value added.[1]

The self-canceling properties of intersectoral transactions, however, are not economic truisms. They follow, instead, from two assumptions: (1) the economy is closed to trade in foreign-produced inputs and (2) intersectoral transfer prices equal marginal production costs.

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[1]For all their important differences, Solow (1957), Kendrick (1961, 1973), Denison (1962, 1974, 1979), Jorgenson and Griliches (1967), Christensen and Jorgenson (1973), and Kendrick and Grossman (1980) adopt this conventional approach to aggregate growth accounting.

Neither assumption need hold even in a competitive economy. In the presence of imported inputs, material inputs consumed in the domestic economy exceed the quantity supplied by domestic producers. Furthermore, given taxes and/or subsidies on intersectoral transactions, producer and consumer prices for intermediate goods are not equal. Transfer prices no longer reflect marginal production costs.

The implications for modeling and measuring aggregate productivity growth are direct. For a competitive economy both closed to trade in imported inputs and having marginal cost pricing in all intermediate input markets, value-added and delivery-to-final-demand models can be shown to produce equivalent measures of aggregate productivity growth. It follows, for those interested simply in measuring aggregate productivity trends, that the Domar (1961) model may be more elegant than but is not computationally superior to the value-added model. Once international trade, taxes, and subsidies are introduced, however, modeling aggregate productivity in terms of value added or deliveries to final demand does make a difference. The final measure of aggregate productivity growth can be shown to depend importantly on the initial description of the economy's macroeconomic objective for production, the process of intersectoral transfers, and the technical properties of microeconomic production. In short, one can characterize the principal objective of this paper as an attempt to demonstrate that a properly modified form of the delivery-to-final-demand framework first introduced by Domar (1961) is much more than a moot exercise.

Sections 1 and 2 of this paper contrast measures of aggregate productivity growth derived, respectively, from value-added and delivery-to-final-demand models of economic activity. Though the models produce substantively different measures of aggregate productivity growth, they can be related through terms adjusting for the productivity consequences of intersectoral and international trade.

Sections 3 and 4 investigate the properties of defining aggregate productivity growth for an open economy without marginal cost transfer prices. The value-added and delivery-to-final-demand rates of aggregate productivity growth are decomposed into their microeconomic sources. The decompositions not only demonstrate the unique productivity contributions of sectoral productivity growth, its intersectoral transmission, transfer prices, imported inputs, and the changing distribution of intermediate inputs but also make explicit the microeconomic assumptions underlying the alternative models of economic growth. Section 4 completes the analysis with a discussion of the practical consequences of these findings for intertempral and international productivity comparisons.

## 1. Value Added

Studies defining aggregate productivity growth in terms of value added implicitly view the macro economy as a set of horizontally independent sectors, each producing value added from labor and capital inputs. The derivation of the value-added model is familiar to most but the process by which the self-canceling nature of intermediate goods is embedded in the model requires emphasis.

The maximum value of aggregate value added ($\lambda$) is expressed as a function of all quantities of sectoral value added ($V_j$), aggregate labor ($L$) and capital ($K$) inputs, and time ($t$):[2]

$$\lambda = F(V_1, V_2, \ldots, V_n, L, K, t). \tag{1}$$

Society's economic problem is to maximize $\lambda$ given linearly homogeneous value-added functions

$$V_j = V^j(L_j, K_j, t), \tag{2}$$

market equilibrium conditions, and aggregate supplies of labor and capital, where $L = \sum L_j$ and $K = \sum K_j$.[3]

This particular characterization of macroeconomic activity has important technical implications for sectoral production. The existence of sectoral value-added functions $V^j$ implies that sectoral production of gross output is characterized by value-added separability:

$$Z_j = f^j[V^j(L_j, K_j, t), X_{1j}, X_{2j}, \ldots, X_{nj}], \tag{3}$$

where $Z_j$ is the gross output of the $j$th sector and $X_{ij}$ is the $i$th intermediate input used in the $j$th sector. In short, productivity improvements can affect output only through $V^j$. The important point is that while the decision to exclude intermediate inputs in the value-added model (1) may be based on the disarmingly straightforward assumption that transactions in intermediate goods are self-canceling, that decision effectively implies that all properties of sectoral productivity growth can be analyzed in isolation from intermediate inputs.

The expression for the rate of aggregate productivity growth $E_v$ can be derived by substituting the now familiar equilibrium conditions into the total logarithmic derivative of $F$ with respect to time:

$$E_v \equiv \frac{\partial \ln F}{\partial t} = \sum_j \frac{q_j^v V_j}{\sum_j q_j^v V_j} \frac{d \ln V_j}{dt} - \frac{p_L L}{\sum_j q_j^v V_j} \frac{d \ln L}{dt} - \frac{p_K K}{\sum_j q_j^v V_j} \frac{d \ln K}{dt}, \tag{4}$$

where $q_j^v$ is the producer price of value added produced in the $j$th sector and $p_L$ and $p_K$ are the economy-wide average prices paid by producers for labor and capital inputs, respectively; that is, $p_L = \sum p_{Lj} L_j / L$ and $p_K = \sum p_{Kj} K_j / K$.

Note that the expression for $E_v$ applies to all economies, whether open or closed to trade in foreign-produced inputs and regardless of the structure of taxes on intersectoral transactions. Intermediate inputs, whether purchased from domestic or foreign producers, do not enter either the production-possibilities frontier (1) or the sectoral value-added functions (2). That intersectoral transfer prices might not reflect true marginal production costs is of no consequence to the equilibrium and homogeneity conditions leading to the definition of $E_v$ in (4). The macro economy is viewed as if it were a set of horizontally independent sectors.

[2] Important differences in the demographic and occupational composition of labor input and the asset mix of capital input are suppressed to reduce notation.

[3] The function $F$ is homogeneous of degree minus one in quantities of sectoral value added, degree one in factor supplies, and degree zero in quantities of value added and inputs.

The important observation is that the model of aggregate productivity stated in terms of value added does not depend on the assumptions that the economy imports no material inputs and that internal transfers are allocatively efficient. Though these assumptions are used to rationalize the value-added model, nowhere are they introduced in the derivation of $E_v$. That derivation, instead, depends solely on the assumption of value-added separability. Not surprisingly, the resulting measure of aggregate productivity growth (4) is insensitive to both the economy's external trade in foreign-produced goods as well as the efficiency of intermediate input markets.

## 2. Deliveries to Final Demand

The delivery-to-final-demand model of aggregate productivity growth is constrained by sectoral production functions, not sectoral value-added functions. Value-added separability is not a maintained hypothesis. Intersectoral and international transactions in material inputs are not suppressed. The vertical structure of the aggregate economy is recognized explicitly. Consequently, the model must be sensitive both to the absence of marginal cost transfer pricing and to the degree of the economy's openness to trade in foreign-produced inputs.

The formal specifications of the delivery-to-final demand models appropriate to a "closed and untaxed" economy and to an "open and taxed" economy are developed separately below. The results are contrasted with the value-added model described in section 1.

*The closed economy with marginal cost transfer prices.* Society's objective is to maximize aggregate output defined as deliveries to final demand. The maximum value of aggregate deliveries to final demand ($\mu$) can be expressed as a function of all quantities of sectoral deliveries to final demand ($Y_j$), all primary inputs, and time:

$$\mu = G(Y_1, Y_2, \ldots, Y_n, L, K, t). \tag{5}$$

The economy-wide maximization problem is constrained by the supplies of primary inputs and by linearly homogeneous sectoral production functions:

$$Z_j = g^j(L_j, K_j, X_{1j}, X_{2j}, \ldots, X_{nj}, t). \tag{6}$$

It necessarily follows that the aggregate function $G$ is homogeneous of degree minus one in sectoral deliveries to final demand, degree one in labor and capital inputs, and degree zero in final demand deliveries and all inputs. Though intermediate inputs do not appear explicitly in the macro model (5), they enter the problem through the constraints (6).

Setting $\mu = 1$, necessary conditions for producer equilibrium in a competitive economy require the following relations:

$$\frac{\partial \ln G}{\partial \ln Y_j} = \frac{-q_j^y Y_j}{\sum_j q_j^y Y_j} \quad \frac{\partial \ln G}{\partial \ln L} = \frac{p_L L}{\sum_j q_j^y Y_j} \quad \frac{\partial \ln G}{\partial \ln K} = \frac{p_K K}{\sum_j q_j^y Y_j} \tag{7}$$

$$\sum_j \frac{q_j^y Y_j}{\sum_j q_j^y Y_j} = 1 \tag{8a}$$

and

$$(8b) \qquad \frac{p_L L}{\sum_j q_j^y Y_j} + \frac{p_K K}{\sum_j q_q^y Y_j} = 1,$$

where $q_j^y$ is the price of goods delivered to final demand in the $j$th sector.

Condition (8b) follows from the equality of the value of a sector's total production both to the sum of the values of that sector's deliveries to intermediate and final demands and to the sum of that sector's payments to all factors of production:

$$(9) \qquad \begin{aligned} q_j Z_j &= \sum_i q_j^x X_{ji} + q_j^y Y_j \\ &= p_{Lj} L_j + p_{Kj} K_j + \sum_i p_i X_{ij}, \end{aligned}$$

where $q_j$ is the producer price of gross output in the $j$th sector, $q_j^x$ is the price of goods delivered to intermediate demand by the $j$th sector, and $p_i$ is the price paid for intermediate input produced in the $i$th sector.[4] Summing (9) over all sectors and equating the results produces the condition required by (8b) since, given the equivalence of producer and consumer prices in markets for intermediate goods ($q_j^x = p_j, \forall_j$),

$$\sum_j \sum_i q_j^x X_{ji} = \sum_j \sum_i p_i X_{ij}.$$

The rate of aggregate productivity growth $E_y$ can be derived by substituting the equilibrium conditions (7) into the total logarithmic derivative of $G$ taken with respect to time:

$$(10) \qquad E_y = \sum_j \frac{q_j^y Y_j}{\sum_j q_j^y Y_j} \frac{d \ln Y_j}{dt} - \frac{p_L L}{\sum_j q_j^y Y_j} \frac{d \ln L}{dt} - \frac{p_K K}{\sum_j q_j^y Y_j} \frac{d \ln K}{dt}.$$

Distinctly different characterizations of macroeconomic objectives and microeconomic activity are modeled by the value-added and delivery-to-final demand production-possibilities frontiers $F$ and $G$ defined, respectively, in (1) and (5). The frontier $F$ is concerned with maximizing aggregate net output, is defined in terms of sectoral value added, and is based on value-added functions for each sector. The macro economy is viewed as if it were a set of horizontally independent sectors. Value-added separability, with all its implications for sectoral and aggregate productivity growth, is a maintained assumption. In contrast, the frontier $G$ is concerned with maximizing deliveries to economy-wide final demand, is stated in terms of sectoral deliveries to final demand, and is based on gross output production functions for each producing sector. The macro economy is modeled as a collection of vertically interdependent sectors. An active market in intersectoral transactions is recognized explicitly.

[4]Note that there is no requirement that the price of the $j$th sector's deliveries to final demand equal the price of that sector's deliveries to intermediate demand. Do note, however, that intermediate input prices $p_i$ do not have $j$ subscripts. All sectors purchasing the $i$th intermediate input ($X_{ij}, \forall_j$) pay the identical f.o.b. price $p_i$.

In spite of these fundamental differences, the following proposition holds:

*Proposition* 1: For an economy both closed to trade in foreign-produced inputs and with marginal cost pricing on intersectoral transfers, value-added and delivery-to-final demand models of macroeconomic production lead to identical measures of aggregate productivity growth.

Verifying this proposition is a straightforward exercise. Consider first the value-added separable production function (3) and the functional expression for gross output implicit in (9):

$$Z_j = Z^j(Y_j, X_{j1}, X_{j2}, \ldots, X_{jn}). \tag{11}$$

Totally differentiating (3) and (11) with respect to time, substituting the latter into the former, and applying equilibrium conditions derived above allows the expression for $E_v$ in (5) to be rewritten in the form:

$$E_v = \sum_j \frac{q_j^y Y_j}{\sum_j q_j^v V_j} \frac{d \ln Y_j}{dt} - \frac{p_L L}{\sum_j q_j^v V_j} \frac{d \ln L}{dt} - \frac{p_K K}{\sum_j q_j^v V_j} \frac{d \ln K}{dt}. \tag{12}$$

Multiplying this expression by the ratio of the value of aggregate value added to the value of aggregate deliveries to final demand produces an expression identical to the definition of $E_y$ in (10) so that

$$E_v = \frac{\sum_j q_j^y Y_j}{\sum_j q_j^v V_j} E_y. \tag{13}$$

Finally, the ratio premultiplying $E_y$ in (13) equals unity. As defined above, the value of sectoral value added equals the value of sectoral output less that sector's purchases of intermediate input, where the value sum of deliveries to intermediate and final demands, equation (9), can be substituted for the value of total output:

$$q_j^v V_j = \sum_i q_j^x X_{ji} + q_j^y Y_j - \sum_i p_i X_{ij}. \tag{14}$$

Summing (14) over all sectors produces the expected result $\sum q_j^v V_j = \sum q_j^y Y_j$, since $\sum \sum q_j^x X_{ji} = \sum \sum p_i X_{ij}$.

Proposition 1 follows directly: $E_v = E_y$. The different initial descriptions of macroeconomic objectives and microeconomic production offered by value-added and delivery-to-final-demand models are of no consequence to productivity accounting in a closed economy with marginal cost transfer prices. The models produce identical measures of aggregate productivity growth.

*The open economy with tax distorted transfer prices.* The development of this model differs from its untaxed closed economy counterpart in two important ways. First, imported inputs enter both the aggregate production-possibilities frontier and the sectoral production functions. Second, taxes and/or subsidies on intersectoral transactions enter the analysis through market equilibrium conditions.

The model, though focusing on imported inputs and taxes on intersectoral transfers, places no restrictions on wider aspects of the economy's international trade or tax structure. In particular, all exports enter the model. Since exports are goods and services produced domestically but no longer consumed internally, exports simply are part of the producing country's deliveries to final demand. In contrast, only imported inputs need enter the model. Imported final products may exist but need not be considered. Imports that are delivered directly to domestic final demand are not included in the model since they bear no influence on domestic productivity measurement. Similarly, final demand deliveries may be subject to transaction taxes yet, since productivity is modeled in terms of prices faced by producers, prices paid by consumers of products delivered to final demand never enter the model. In contrast, prices paid by consumers of intermediate inputs do enter the model. Taxes or subsidies differentiating consumer and producer prices for goods delivered to intermediate demand must be considered explicitly.[5]

One assumption, however, is introduced. All taxes collected on intermediate demand deliveries are assumed to be disbursed as subsidies to other intersectoral transactions $\sum\sum p_i X_{ij} = \sum\sum q_j^x X_{ji}$, where $p_i \gtreqless q_i^x$ for all $i$. This simplifies the comparison between delivery-to-final-demand and value-added models without compromising the importance of accounting for intersectoral transfer prices.

Given this setting, the economy's production problem is to maximize aggregate deliveries to final demand ($\mu$)

$$\mu = H(Y_1, Y_2, \ldots, Y_n, L, K, M_1, M_2, \ldots, M_u, t), \tag{15}$$

where $M_m$ is the quantity of the $m$th imported material input, subject to fixed supplies of domestic labor and capital inputs, market equilibrium conditions, and linearly homogeneous sectoral production functions:

$$Z_j = h^j(L_j, K_j, X_{1j}, X_{2j}, \ldots, X_{nj}, M_{1j}, M_{2j}, \ldots, M_{uj}, t). \tag{16}$$

Setting the aggregate value of $\mu$ equal to unity, the constant-returns-to-scale function $H$ can be expressed as a production-possibilities frontier. Homogeneity and market equilibrium conditions are an obvious extension of equations (7)–(8b).

The rate of aggregate productivity growth ($E_y'$) appropriate for an open economy is derived by substituting the necessary conditions for producer equilibrium into the logarithmic time derivative of the aggregate frontier $H$:

$$\begin{aligned} E_y' \equiv \frac{\partial \ln H}{\partial t} = \sum_j \frac{q_j^y Y_j}{\sum_j q_j^y Y_j} \frac{d \ln Y_j}{dt} - \frac{p_L L}{\sum_j q_j^y Y_j} \frac{d \ln L}{dt} \\ - \frac{p_K K}{\sum_j q_j^y Y_j} \frac{d \ln K}{dt} - \sum_m \frac{p_{Mm} M_m}{\sum_j q_j^y Y_j} \frac{d \ln M_m}{dt}, \end{aligned} \tag{17}$$

where $p_{Mm}$ is the price of the $m$th imported material input.[6] The expressions $E_y$

[5]These arguments suggest only that imported final products and taxes on final deliveries do not enter the formula for productivity measurement. They do not argue that these variables have no effect on aggregate productivity growth. Clearly, they do by influencing the economy's primary input requirements and its level of final production.

[6]The fact that the input shares in aggregate output sum to unity can be verified from the sectoral accounting identity:

$$\sum_i q_j^x X_{ji} + q_j^y Y_j = p_{L_j} L_j + p_{K_j} K_j + \sum_i p_i X_{ij} + \sum_m p_{Mm} M_{mj}.$$

Summing this over all $n$ sectors produces the required result. Note also that import prices $p_{Mm}$ do not have $j$ subscripts. It is assumed that all domestic sectors pay the same f.o.b. price for identical imported materials.

in (10) and $E_y'$ differ only by the presence of the last term in (17), reflecting the fact that the list of primary inputs in an open economy includes domestic labor and capital inputs as well as imported materials.

The comparison of $E_y'$ with $E_v$ in (4) is of more interest and leads to the following result:

*Proposition* 2: For an economy open to trade in foreign-produced material inputs and having taxes and/or subsidies on deliveries to intermediate demand, the rate of aggregate productivity growth derived from a value-added model of aggregate production differs from the corresponding rate resulting from a delivery-to-final-demand model. The two rates differ by an amount that depends on (1) the relative importance of imported inputs in domestic production and (2) changes in the sectoral distribution of domestically produced intermediate inputs.

Proving this proposition begins with both the sectoral output function (11) defining each sector's output in terms of its delivery to final and intermediate demand components and the value-added separable production function for any sector in an open economy:

$$Z_j = f^j[V^j(L_j, K_j, t), X_{1j}, X_{2j}, \ldots, X_{nj}, M_{1j}, M_{2j}, \ldots, M_{uj}]. \tag{18}$$

The growth rate of value added can be derived from the logarithmic time derivative of (18):

$$\frac{d \ln V_j}{dt} = \frac{q_j Z_j}{q_j^v V_j} \frac{d \ln Z_j}{dt} - \sum_i \frac{p_i X_{ij}}{q_j^v V_j} \frac{d \ln X_{ij}}{dt} - \sum_m \frac{p_{Mn} M_{mj}}{q_j^v V_j} \frac{d \ln M_{mj}}{dt} \tag{19}$$

Replacing $d \ln Z_j/dt$ with its delivery to final and intermediate demand components and substituting the resulting expression for the growth of sectoral value added into equation (4) yields:

$$\begin{aligned} E_v = \sum_j \frac{q_j^y Y_j}{\sum_j q_j^v V_j} \frac{d \ln Y_j}{dt} - \frac{p_L L}{\sum_j q_j^v V_j} \frac{d \ln L}{dt} - \frac{p_K K}{\sum_j q_j^v V_j} \frac{d \ln K}{dt} \\ - \sum_j \sum_i \frac{\Gamma_i X_{ij}}{\sum_j q_j^v V_j} \frac{d \ln X_{ij}}{dt} - \sum_j \sum_m \frac{p_{Mm} M_{mj}}{\sum_j q_j^v V_j} \frac{d \ln M_{mj}}{dt}, \end{aligned} \tag{20}$$

where $\Gamma_i \equiv (p_i - q_i^x) \gtreqless 0$ is the tax imposed on the transfer price of the $i$th commodity delivered to intermediate demand. Multiplying (20) by the ratio $\sum q_j^v V_j / \sum q_j^y Y_j$ and substituting the definition of $E_y'$ in (17) into the resulting expression produces the following relation:

$$E_v = \frac{\sum_j q_j^y Y_j}{\sum_j q_j^v V_j} E_y' - \sum_j \sum_i \frac{\Gamma_i X_{ij}}{\sum_j q_j^v V_j} \frac{d \ln X_{ij}}{dt}. \tag{21}$$

The importance of taxes on intersectoral transactions is explicit. The role of trade in foreign-produced inputs is implicit in the ratio $\sum q_j^y Y_j / \sum q_j^v V_j$. While that ratio equals unity for a closed economy, it is greater than unity for an open economy. Substituting (9) for the value of total output, $q_j Z_j$, into the accounting identity

$$q_j^v V_j = q_j Z_j - \sum_i p_i X_{ij} - \sum_m p_{Mm} M_{mj} \tag{22}$$

and summing over all $n$ sectors yields:

$$\sum_j q_j^v V_j = \sum_j q_j^y Y_j - \sum_j \sum_m p_{Mm} M_{mj}. \tag{23}$$

The value of aggregate deliveries to final demand exceeds the value of aggregate value added by an amount equal to the value of imported material inputs. Substituting (23) into (21) makes explicit the importance of imported inputs and Proposition 2 follows directly:

$$E_v = E'_y + \left[\frac{\sum_j \sum_m p_{Mm} M_{mj}}{\sum_j q_j^y Y_j - \sum_j \sum_m p_{Mm} M_{mj}}\right] E'_y - \sum_j \sum_i \frac{\Gamma_i X_{ij}}{\sum_j q_j^y Y_j - \sum_j \sum_m p_{Mm} M_{mj}} \frac{d \ln X_{ij}}{dt}. \tag{24}$$

The two rates of productivity growth $E_v$ and $E'_y$ differ by an amount that depends on (1) the relative importance of imported inputs in domestic production and (2) changes in the sectoral distribution of domestically produced intermediate inputs.

If either taxes and subsidies equal zero ($\Gamma_i = 0, \forall_i$) or the distribution of intermediate inputs is unchanging ($d \ln X_{ij}/dt = d \ln X_j/dt, \forall_i$), equation (21) reduces to

$$E_v = E'_y + \left[\frac{\sum_j \sum_m p_{Mm} M_{mj}}{\sum_j q_j^y Y_j - \sum_j \sum_m p_{Mm} M_{mj}}\right] E'_y. \tag{25}$$

It necessarily follows that $E_v > E'_y$.

Conversely, if the economy is closed to trade in foreign-produced inputs ($M_m = 0, \forall_m$), equation (21) reduces to

$$E_v = E'_y - \sum_j \sum_i \frac{\Gamma_i X_{ij}}{\sum_j q_j^y Y_j} \frac{d \ln X_{ij}}{dt}. \tag{26}$$

The relation between $E_v$ and $E'_y$ is no longer unambiguous but depends on the changing composition of intermediate inputs. For any period during which the distribution of intermediate inputs shifts to relatively higher taxed products, $E'_y > E_v$. Conversely, $E'_y < E_v$ during any period when the composition of intermediate inputs shifts to more heavily subsidized transactions.

The important summary conclusion is that value-added and delivery-to-final-demand models of aggregate production produce identical measures of aggregate productivity growth if and only if the economy is closed to trade in foreign-produced inputs and has either no taxes on deliveries to intermediate demand or an unchanging distribution of intermediate inputs. In the absence of these limiting conditions, $E_v \neq E'_y$.

## 3. Sources of Growth

The value-added and delivery-to-final-demand models of aggregate production are constrained by different models of microeconomic production and intersectoral transfers. One objective of this section is to make these differences explicit by decomposing the alternative measures of aggregate productivity growth, $E_v$ and $E'_y$, into their microeconomic source components.

*Sectoral productivity growth.* The economy's value-added maximization problem described in (1) is constrained by the value-added subfunctions $V^j$ defined in (2). The rate of sectoral productivity growth $e^j_v$ is derived by substituting conditions for producer equilibrium into the logarithmic time derivative of $V^j$:[7]

$$(27) \qquad e^j_v \equiv \frac{\partial \ln V^j}{\partial t} = \frac{d \ln V_j}{dt} - \frac{p_{Lj} L_j}{q^v_j V_j} \frac{d \ln L_j}{dt} - \frac{p_{Kj} K_j}{q^v_j V_j} \frac{d \ln K_j}{dt}.$$

Given value-added separability, neither foreign-produced inputs nor domestically-produced intermediate inputs enter the sectoral functions $V_j$ or the resulting expression for sectoral productivity growth.

Microeconomic production in the delivery-to-final-demand model, in contrast, is characterized by the sectoral production functions $h^j$ defined in (16). The rate of sectoral productivity growth $e^j_z$ is solved by imposing equilibrium conditions on the logarithmic time derivative of $h^j$:[8]

$$(28) \qquad e^j_z \equiv \frac{\partial \ln h^j}{\partial t} = \frac{d \ln Z_j}{dt} - \frac{p_{Lj} L_j}{q_j Z_j} \frac{d \ln L_j}{dt} - \frac{p_{Kj} K_j}{q_j Z_j} \frac{d \ln K_j}{dt} - \sum_i \frac{p_i X_{ij}}{q_j Z_j} \frac{d \ln X_{ij}}{dt} - \sum_m \frac{p_{Mm} M_{mj}}{q_j Z_j} \frac{d \ln M_{mj}}{dt}.$$

The variables $e^j_v$ and $e^j_z$ provide different descriptions of sectoral productivity growth, leading to the following proposition:

*Proposition* 3: For an economy open to trade in foreign-produced inputs and subject to taxes on deliveries to intermediate demand, the rate of sectoral productivity growth derived from a sectoral value added production function is *greater* than the corresponding rate resulting from a sectoral gross output production function. The two rates differ by an amount that is proportional to the share of the sector's purchases of domestic intermediate and foreign-produced inputs in the value of sectoral gross output.

Formally demonstrating this proposition begins with substituting equation (19), the logarithmic time derivative of value added, into equation (27):

$$(29) \qquad e^j_v = \frac{q_j Z_j}{q^v_j V_j} \frac{d \ln Z_j}{dt} - \sum_i \frac{p_i X_{ij}}{q^v_j V_j} \frac{d \ln X_{ij}}{dt} - \sum_m \frac{p_{Mm} M_{mj}}{q^v_j V_j} \frac{d \ln M_{mj}}{dt} - \frac{p_{Lj} L_j}{q^v_j V_j} \frac{d \ln L_j}{dt} - \frac{p_{Kj} K_j}{q^v_j V_j} \frac{d \ln K_j}{dt}.$$

Substituting (28) into (29) and rearranging terms yields:

$$(30) \qquad e^j_z = 1 - \left[\frac{\sum_i p_i X_{ij} + \sum_m p_{Mm} M_{mj}}{q_j Z_j}\right] e^j_v.$$

[7]Competitive equilibrium conditions at the sectoral level require that output elasticities with respect to inputs equal the corresponding value shares of input payments in sectoral output. Linear homogeneity requires that both the output elasticities and the value shares each sum to unity.

[8]See note 7.

It necessarily follows that $e_v^j > e_z^j$. The two rates differ by an amount that is proportional to the share of sectoral purchases of domestically-produced and foreign-produced inputs in the value of sectoral gross output.[9]

Note that the relationship between $e_z^j$ and $e_v^j$ is unaffected by the presence or absence of taxes on intersectoral transfers. Both measures $e_z^j$ and $e_v^j$ are sector specific and depend on equilibrium conditions unique to the $j$th sector. Only prices *paid* by the $j$th sector for intermediate inputs matter. Prices *received* by producers of intermediate inputs delivered to the $j$th sector do not enter the formulas defining $e_z^j$ or $e_v^j$.

*Sources of productivity growth.* The formal decomposition of $E_v$ into its source components is simple and direct.

*Proposition* 4: The rate of aggregate productivity growth $E_v$ for a value-added model of aggregate production equals the sum of two components: (1) a weighted *average* of the sectoral rates of productivity growth $e_v^j$, with weights equal to sectoral value shares of value added in total value added; and (2) reallocation terms reflecting the aggregate productivity gains resulting from the economy's recomposition of labor and capital inputs among sectors.

This result is derived by substituting the expression for $e_v^j$, equation (27), into equation (4), the expression for $E_v$:

$$(31) \qquad E_v = \sum_j \frac{q_j^v V_j}{\sum_j q_j^v V_j} e_v^j + \sum_j \frac{(p_{Lj} - p_L) L_j}{\sum_j q_j^v V_j} \frac{d \ln L_j}{dt} + \sum_j \frac{(p_{Kj} - p_K) K_j}{\sum_j q_j^v V_j} \frac{d \ln K_j}{dt}.$$

The labor and capital reallocation terms identify the productivity consequences resulting from any sectoral recomposition of the economy's primary inputs. Each term has a straightforward interpretation. Consider the one corresponding to capital. The variable $p_K$, recall, refers to the average price of capital input in the aggregate economy; $p_{Kj}$ represents the return to capital input in the $j$th sector. Aggregate productivity growth is affected by the reallocation of capital among sectors with varying rates of return. If, for example, capital input moves from a sector with a relatively low rate of return ($p_{Kj} < p_K$) to a sector with a high rate of return ($p_{Kj} > p_K$), the quantity of capital input for the economy as a whole is unchanged, but the level of output is increased—that is, aggregate productivity is enhanced. In this example, the capital reallocation component would have a positive value. Similarly, sectoral shifts in labor resources are monitored by the labor reallocation term. If labor input moves from a sector where it has relatively low marginal productivity to a sector where it has relatively high marginal productivity, economy-wide labor input is unchanged but aggregate productivity is improved.

The important but not surprising observation that follows from (31) is that neither international nor intersectoral transactions, whether stated in nominal or real terms, enters the source decomposition of $E_v$. Imported inputs enter neither the aggregate nor sectoral production accounts. Since sectors are viewed as

[9]Note that even if the $j$th sector imports no foreign-produced inputs ($M_{mj} = 0, \forall_m$), the measure $e_v^j$ still will be greater than $e_z^j$.

horizontally independent producers, sectoral reallocations of deliveries to intermediate demand and their transfer prices are ignored.

The delivery-to-final-demand model of aggregate production and its microeconomic production constraints, in contrast, view the economy as consisting of sectors depending not only on each other but on international trade as well. Advances in productivity in an individual industry contribute to aggregate economic growth both directly through deliveries to final demand and indirectly through increased deliveries to sectors dependent on its output as intermediate input. The source decomposition of $E'_y$ must capture not only both direct and indirect transmissions of sectoral productivity growth but also changes in the efficiency of markets linking buyers and sellers of intermediate inputs.

*Proposition* 5: The rate of aggregate productivity growth $E'_y$ for a delivery-to-final-demand model of aggregate production equals the sum of three components: (1) a weighted *sum* of the sectoral rates of productivity growth $e^j_z$, with weights equal to sectoral ratios of the value of gross output to the value of total deliveries to final demand; (2) reallocation terms for labor and capital inputs; and (3) a term measuring the net productivity effect of changes in allocative efficiency across all intermediate demand markets.

Demonstrating this proposition begins with the logarithmic time derivative expressing the growth in sectoral output as the sum of its final and intermediate demand components:

$$\frac{d \ln Z_j}{dt}=\sum_i \frac{q^x_j X_{ji}}{q_j Z_j}\frac{d \ln X_{ji}}{dt}+\frac{q^y_j Y_j}{q_j Z_j}\frac{d \ln Y_j}{dt}. \tag{32}$$

Replacing $d \ln Z_j/dt$ in (28) with this expression, multiplying the amended form of (28) by the ratio $q_j Z_j/\sum q^y_j Y_j$, summing over all $n$ sectors, and substituting the resulting expression into (17) identifies the unique source components contributing to aggregate productivity growth:

$$E'_y=\sum_j \frac{q_j Z_j}{\sum_j q^y_j Y_j} e^j_z+\sum_j \frac{(p_{Lj}-p_L)L_j}{\sum_j q^y_j Y_j}\frac{d \ln L_j}{dt}+\sum_j \frac{(p_{Kj}-p_K)K_j}{\sum_j q^y_j Y_j}\frac{d \ln K_j}{dt} \tag{33}$$

$$+\sum_j\sum_i \frac{(p_i-q^x_i)X_{ij}}{\sum_j q^y_j Y_j}\frac{d \ln X_{ij}}{dt}.$$

The first term reflects the sum contribution of productivity growth across individual producing sectors. This term is formed as a weighted sum, not weighted average. The sum of the weights exceeds unity by an amount equal to the ratio of the aggregate value of deliveries to intermediate demand to the aggregate value of deliveries to final demand:

$$\sum_j \frac{q_j Z_j}{\sum_j q^y_j Y_j}=\sum_j \frac{(q^y_j Y_j+\sum q^x_j X_{ji})}{\sum_j q^y_j Y_j}=1+\sum_j\sum_i \frac{q^x_j X_{ji}}{\sum_j q^y_j Y_j}. \tag{34}$$

The weights sum to a value larger than unity because each sector contributes to the rate of productivity growth for the aggregate economy through its deliveries to both final and intermediate demands.

The remaining three terms in (33) measure the aggregate productivity consequences of the changing composition of economic activity. The labor and capital reallocation terms have exactly the same interpretation as their respective counterparts in equation (31), the source decomposition of $E_v$. Shifts in either primary input to higher (lower) marginal productivity uses enhances (diminishes) aggregate productivity performance. The final term in (33) likewise depends on input shifts but has a uniquely different interpretation. Whereas the labor and capital reallocation effects focus on primary input price differences among sectors, the market term concentrates on the differentials between consumer and producer prices for each good delivered to intermediate demand markets.

The productivity effect represented in the market term depends on the tax-induced differentials $(p_i - q_i^x)$ and the growth rates of products delivered to intermediate demand $(d \ln X_{ij}/dt)$. Market equilibrium conditions in a competitive economy require that $p_i$ equal the marginal value product $(MVP_i)$ of the $i$th intermediate input to consuming sectors while $q_i^x$ equals the marginal cost $(MC_i)$ incurred by the sector producing that input. If there is a tax on the $i$th product $(p_i > q_i^x)$, then $MVP_i > MC_i$ and aggregate productivity growth will increase (decrease) if deliveries of $X_i$ to intermediate demand are increased (decreased). Subsidized transactions have exactly the opposite effect since $\Gamma_i < 0$ necessarily implies $MVP_i < MC_i$. Finally, should there be neither taxes nor subsidies on the economy's internal transactions, then $MVP_i = MC_i$ for all $i$ and the market terms equal zero. In this case, proper transfer prices guide transactions among vertically related sectors. When transfer prices reflect marginal costs, changes in the composition of deliveries to intermediate demand have no independent effect on aggregate productivity growth.

The decomposition presented in equation (33) makes clear that the measure $E'_y$ is sensitive to the economy's internal transfers among interdependent sectors and to its external relations with other economies. The role of imported inputs is captured in the sectoral productivity measures $e_z^j$. The transmission of each sector's productivity growth throughout the economy is reflected in the sectoral productivity weights $(q_j Z_j / \sum q_j^y Y_j)$. The market term models the efficiency of intermediate demand markets. The assumption of value-added separability assures that none of these effects appears in the source decomposition of $E_v$.

## 4. Conclusion

The source decompositions derived above demonstrate that the delivery-to-final demand model is sensitive to intertemporal and international variations in both imported material input requirements and the allocative efficiency of domestic markets. The value-added model is not. This distinction is not unimportant. A measure of aggregate productivity growth is useful only as a comparative measure of economic performance over time or across countries. The value-added model leads to biased estimates.

This is not a moot point. As an illustration, consider the $E_v$ and $E'_y$ rates of aggregate productivity growth reported in Table 1 for eight industrial nations. The growth rates for the value-added model are calculated from the annual growth rates reported in Christensen, Cummings, and Jorgenson (1980). The

TABLE 1

AVERAGE ANNUAL RATES OF AGGREGATE PRODUCTIVITY GROWTH[a]

| | 1957–1966 | | 1966–1973 | |
|---|---|---|---|---|
| | $E_v$ | $E'_y$ | $E_v$ | $E'_y$ |
| Canada | 0.0174 | 0.0147 | 0.0136 | 0.0114 |
| France | 0.0322 | 0.0286 | 0.0284 | 0.0248 |
| Germany | 0.0314 | 0.0274 | 0.0299 | 0.0259 |
| Italy | 0.0374 | 0.0327 | 0.0263 | 0.0227 |
| Japan | 0.0428 | 0.0393 | 0.0387 | 0.0361 |
| Netherlands | 0.0144 | 0.0103 | 0.0345 | 0.0253 |
| United Kingdom | 0.0151 | 0.0128 | 0.0260 | 0.0219 |
| United States | 0.0167 | 0.0161 | 0.0067 | 0.0064 |

[a]The average annual growth rates $E_v$ reported for the value-added model are calculated from annual growth rates reported in Christensen, Cummings, and Jorgenson (1980), Table 11.10.

corresponding rates for the delivery-to-final-demand model are derived from equation (25).[10]

Table 1, note, focuses only on the importance of considering imported inputs. Quantifying the productivity effect of tax-distorted transfer prices (see equation (24)) requires detailed information on both the changing sectoral composition of deliveries to intermediate demand and the sector-specific incidence of excise taxes, subsidies, and the increasingly important value-added tax—information which is not readily available. In contrast, the trade-induced spread between $E_v$ and $E'_y$ can be calculated quite easily. It depends only on the current price ratio of imported inputs to value added.

It follows from only the most casual familiarity with trade patterns that $E_v - E'_y$ differentials will vary considerably across countries. This is confirmed in Table 1. Compare each country's $E_v$ and $E'_y$ productivity growth rates in the 1966–73 period. As expected, the differential is small for the United States while sizable for most other nations—most notably, the Netherlands. The rates $E_v$ and $E'_y$ differ by only three one-hundredths of a percentage point in the U.S. but by an average 0.42 percentage points for the other seven countries. The $E_v$ and $E'_y$ gap nearly equals a full percentage point (0.92) for the Netherlands. The important conclusion is that an evaluation of relative productivity growth rates across nations will be biased by a value-added model, especially comparisons involving nations with very different imported input requirements.

An intertemporal bias can surface as well. Again consider Table 1. The $E_v - E'_y$ differential is a constant 0.36 percentage points in both 1957–66 and 1966–73 subperiods for France. It is a constant 0.40 points for Germany. However, the gap broadens in two countries, the U.K. and the Netherlands, and narrows in the remaining four. The largest change occurs in the Netherlands where the

[10]Current price imported inputs required by equation (25) are defined as the product of gross imports reported in OECD (1984a, 1984b) and the ratio of imported inputs to total imports calculated from data reported in U.N. Department of International Economic and Trade Affairs (1982). Value added is defined as the sum of gross domestic product plus subsidies less indirect business taxes. Tornquist weights are applied.

differential more than doubles between the two subperiods, rising slightly more than one-half percentage point from 0.41 to 0.92 points.

It is important to note that this bias has increased in the post-1973 period. Though growth rates comparable to those reported in Table 1 are not available post 1973, equation (25) makes clear that the value-added model bias, stated as a percent of $E'_y$, simply equals the ratio of imported inputs to aggregate value added. During the 1966–73 period, that bias for the eight countries listed in Table 1 averages 16.5 percent, ranging from less than 5 percent for the U.S. to 36 percent for the Netherlands. Reflecting the ubiquitous increase in the importance of trade in recent years, the bias that would result from the application of the value-added model to 1983 data averages 21.5 percent, the result of an increased bias in every country. The U.S. and the Netherlands again are the outliers. The 1983 bias for the U.S. equals 7.5 percent; the Netherlands bias climbs a full 10 percentage points to 46 percent. If one accepts the premise that aggregate economic growth should be evaluated in a delivery-to-final-demand framework, then one concludes that the value-added model produces not only an upward biased but also an increasingly upward biased measure of postwar aggregate economic performance.

This conclusion is only strengthened when one considers the role of transfer prices in intermediate good transactions. In particular, transactions in intermediate inputs can be suppressed if and only if the composition of intermediate input purchases is unchanging or transfer prices equal marginal costs. Neither condition is true. Between 1967 and 1972, for example, the U.S. motor vehicle and equipment sector increased its deliveries to intermediate demand by 40 percent. During this same period, the construction and mining machinery sector decreased its intermediate demand deliveries by 19 percent.[11] The pattern in each country depends on each economy's cyclical behavior.

The allocative efficiency implications of these redistributions depend on the structure of intersectoral transfer prices. Excise taxes are just one example of how transfer prices may fail to reflect marginal costs. More importantly, the structure of these taxes differs significantly across countries. Between 1973 and 1979, for example, excise taxes increased by 13 percent in the United States while increasing by 50 percent in both Canada and the Netherlands.[12]

The value-added tax must be considered as well, especially since its application is so uneven both across countries and over time. Canada, Japan, and the United States simply have no value-added tax. Major European countries do, though rates and introduction dates differ considerably. Among the countries listed in Table 1, France and Germany introduced the tax in 1968. The Netherlands followed in 1969. No value-added tax appeared in Italy or the U.K. until 1973. By 1978 value-added tax receipts equalled 3.1 and 4.9 percents of GDP in the U.K. and Italy, respectively. It equalled 5.7 percent in Germany and was as high as 7.5 and 8.6 percents, respectively, in the Netherlands and France.[13] The growth paths differ as well. Relative to their first full year after introduction, general

[11]These percentages were calculated from the current price 1967 and 1972 input-output transaction matrices reported respectively in Bureau of Economic Analysis (1974) and Ritz, Roberts, and Young (1979).

[12]United Nations Department of International Economic and Social Affairs (1977–80).

[13]Aaron (1981), p. 14.

value-added tax rates in 1980 had increased by only 5 percent in France, 17 percent in Italy, 30 percent in Germany, and by as much as 50 percent in both the Netherlands and the U.K.[14] Accounting for intertemporal and international differences in both the structure of transfer prices and the changing distribution of intermediate inputs cannot be ignored.

The summary conclusion of this paper follows from equation (24): The rates $E_v$ and $E_y'$ maintain no constant relation either across countries or over time. In short, conclusions regarding relative rates of productivity growth can be biased by a value-added model. Stated positively, the flexibility of the delivery-to-final-demand model provides an opportunity to generalize productivity modeling significantly. It offers a framework for evaluating the productivity consequences of imperfect competition in domestic markets. It provides a formal structure for analyzing regional productivity growth among trading nations. The effects of tariffs and quotas on international transfer prices can be modeled explicitly. Unless affirmed by economic evidence, value-added separability is an inappropriate as well as unnecessary assumption.

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[14] *Ibid.*

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