MORE ON THE MEASUREMENT OF TOTAL FACTOR PRODUCTIVITY

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"As the multiplicity of current and ancient debates has revealed, capital in the abstract sense is not an identifiable commodity like labour, land or specific capital goods, it is rather the willingness to wait."†

As compared with Hicksian, Harrodian measures of the concept of total factor productivity which rigorously take into account the reproducibility of commodity capital inputs and the technological interdependence of modern production economies are advocated. A number of recent measures of total factor productivity are shown to be variants of the Harrodian approach, and certain problems of aggregation associated with the Hicksian measures are shown to be resolved by the Harrodian measures. An examination of the concepts of technical progress and vertically integrated sectors advanced by Professor Luigi L. Pasinetti and their relation to the Harrodian measures of total factor productivity is made.

I. INTRODUCTION

A decade ago I argued that

"When the fact that capital inputs are produced means of production is rigorously and logically incorporated in the measurement of capital and technical change, support is provided for Harrod–Robinson concepts of technical change. The Hicks–Meade–Solow concepts of technical change are shown to be theoretically faulty."[1]

The measurement of the Hicks–Meade–Solow concept of technical change was a commonplace at the time, appearing as the estimates by Denison, Kendrick

*I am grateful to my colleagues, Keith Acheson, Jeff Bernstein, Ehsan Choudhri and Steve Ferris for their criticisms of an earlier version of this paper presented to the Workshop in Macroeconomics, Department of Economics, Carleton University, April 19, 1982 and to the meetings of the Canadian Economics Association, June 5, 1982. I am also indebted to Professor Don Daly, York University, Ms. Alexandra Cas, Statistics Canada, and to a referee of this journal for their valuable comments.

†Arrow, K. J. and Starrett, D. A., Cost- and Demand-Theoretical Approaches to the Theory of Price Determination, eds. J. R. Hicks and W. Weber, *Carl Menger and the Austrian School of Economics* (Oxford: Clarendon Press, 1973), 140.

[1]Rymes, T. K., The Measurement of Capital and Total Factor Productivity in the Context of the Cambridge Theory of Capital, *Review of Income and Wealth*, 18, March 1972, 80. See also my *On Concepts of Capital and Technical Change* (Cambridge: Cambridge University Press, 1971); Read, L. M., The Measurement of Total Factor Productivity Appropriate to Wage-Price Guidelines, *Canadian Journal of Economics*, I, May 1968, 349–358 and Rymes, T. K., Professor Read and the Measurement of Total Factor Productivity, *Canadian Journal of Economics*, I, May, 1968, 359–367.

and Griliches and Jorgenson to name only a few.[2] Though measures of such a concept of technical change continue to appear[3] it is clear that they are flawed by their failure to take account of the reproducibility of capital inputs[4] and that superior estimates—those related to the concepts of technical change advanced by Harrod and Robinson and made operational by Read—are now beginning to appear.[5] In this paper, I reaffirm the theoretical superiority of the Harrod–Robinson measures, briefly review some of the estimates now being made and show how the developing data system at Statistics Canada should permit, as my colleague Professor L. M. Read long ago suggested, the estimation of Harrod–Robinson–Read (hereinafter HRR) measures of technical change. In addition, I shall show that the HRR measures resolve certain aggregation problems encumbering the standard measures.[6] I shall finally attempt to show the relationships between the HRR measures and the concepts of technical change and vertical integration advanced by Professor L. L. Pasinetti.

I offer no theory of technical change—even of the type suggested by Nelson.[7] I argue merely that if economists are interested in knowing (say) how research and development expenditures may (or may not) affect total factor productivity

[2]Denison, E. F., *The Sources of Economic Growth* (New York: Committee for Economic Development, 1962); Kendrick, J. W., *Productivity Trends in the United States* (Princeton: Princeton University Press for the NBER, 1961) and Griliches, Z. and Jorgenson, D. W., The Explanation of Productivity Change, *Review of Economic Studies*, XXXIV, July 1967, 249–285, reprinted in *Survey of Current Business*, XLIX, May 1969, 50–61.

[3]See for example, Denison, E. F., *Accounting for Slower Economic Growth* (Washington: The Brookings Institution, 1979); Gollop, F. M. and Jorgenson, Dale W., U.S. Productivity Growth by Industry, 1947–73, eds. John W. Kendrick and Beatrice N. Vaccara, *New Developments in Productivity Measurement and Analysis* (Chicago: University of Chicago Press for the NBER, 1980) and Ostry, S. and Rao, P. S., Productivity Trends in Canada, eds. S. Maital and N. M. Meltz, *Lagging Productivity Growth: Causes and Remedies* (Cambridge, Mass.: Ballinger Publishing Company, 1980).

[4]In his recent study, *The Measurement of Economic Growth* (New York: Columbia University Press, 1980), Professor Dan Usher notes that, excluding increasing returns to scale, economic growth—the proportionate rate of increase in "real consumption" per head—is entirely technological in origin and that, as an accounting identity, the rate of economic growth is equal "in the long run" [*sic*] to the rate of (labour-embodied) technical change or the Harrod–Robinson concept. His empirical procedure results—as it must—in much higher estimates of the proportionate rate of change in total factor productivity as produced (say) by Griliches and Jorgenson and the Hicks-neutral estimates of technical change by Solow. Cf. Solow, R. M., Technical Change and the Aggregate Production Function, *Review of Economics and Statistics*, XXXIX, August 1957, 312–320. See also Zarembka, P., 'Real' Capital and the Neoclassical Production Function, eds. F. L. Altmann, O. Kýn and H.-J. Wagener, *On the Measurement of Factor Productivities* (Göttingen: Vandenhoeck and Ruprecht, 1976).

[5]See, for example, Hulten, Charles K., Technical Change and the Reproducibility of Capital, *American Economic Review*, XLV, December 1975, 956–965 and also his On the 'importance' of productivity change, *American Economic Review*, LXIX, March 1979, 126–136, and Hulten, Charles R., and Nishimizu, M., The Importance of Productivity Change in the Economic Growth of Nine Industrialized Countries, eds. S. Maital and N. M. Meltz, *Lagging Productivity Growth*, and Peterson, William, op. cit., Total Factor Productivity in the U.K.: A Disaggregated Analysis, eds. K. D. Patterson and Kerry Schott, *The Measurement of Capital: Theory and Practice* (London: Macmillan, 1979). Unpublished (to my knowledge) work relating productivity advance solely to direct and indirect labour has been done in Canada by Andrew Sharpe, cf. his paper, A disaggregated analysis of price changes and productivity in the Canadian economy, 1961–1976: a labour value approach, December 1980, presented at the 1981 CEA meetings.

[6]Professor E. R. Berndt noted that, with Hicksian measures of technical change in total factor productivity, "... comparison among sectors is very difficult." See his comments (p. 133) on the Gollop–Jorgenson paper cited above.

[7]Nelson, R. R., Research on Productivity Growth and Productivity Differences: Dead Ends and New Departures, *Journal of Economic Literature*, XIX, September 1981, 1029–1064.

or (say) why total factor productivity has supposedly been lower since the mid-70s, they had better in their measurement procedures ensure that the measured rate of technical progress or total factor productivity stands tests of economic theory and logical consistency.

II. A Simple Steady State Review

Total factor productivity measurement is the attempt to measure during some particular flow period of time the proportionate rate of change in the overall efficiency of an economic system arising from the supposed existence of technical progress. The measurement problem is concerned with the most appropriate way of expressing the aggregated and disaggregated prices and quantities of outputs and inputs in proportionate rate of change form such that the resulting measures can be reassembled to provide meaningful estimates of aggregated and disaggregated total factor productivity or technical progress. By meaningful I shall mean those estimates which correspond most closely with economic theory in general and that economic theory centrally concerned with the production of commodities by means of commodities.

I review the simplest cases. I start by comparing economies exhibiting the properties of the standard steady state growth model. That is, I have, taking the one commodity case first,

II-1[8] $$WL + RPK + \Gamma PK \equiv PQ$$

for the national accounts of such countries. When expressed in proportionate rate of change form during any period of time,[9] I have

II-2[10] $$\alpha(W^* - P^*) + \beta(R^* + P^* - P^*) + \gamma(\Gamma^* + P^* - P^*)$$
$$\equiv \alpha(Q^* - L^*) + \beta(Q^* - K^*) + \gamma(Q^* - K^*) \equiv T^*$$

where T^* is the standard or Hicksian measure of the proportionate rate of change of total factor productivity or technical progress. In steady state equilibrium, I would have

II-3 $$\alpha(W^* - P^*) = \alpha(Q^* - L^*) = T^*$$

[8]The notation is mnemonic and standard:
W is the money wage rate or money rental paid for labour services
L is the flow of labour services
R is the net rate of return
P is the money price of the commodity
K is the stock of the commodity
Γ is the rate of depreciation by evaporation
Q is the gross flow output of the commodity. Alternatively K is the flow of capital services and RP is the money *net* rental paid for capital services.

[9] Without commitment to any particular index number form (e.g. Divisia, Malmquist) the proportionate rate of change form matches Harrod's view that dynamics deal with rates of change at a moment in time.

[10]Again the notation is mnemonic and standard, i.e. X^* = the proportionate rate of change in any variable, i.e. $X^* = (1/x)(dx/dt)$ such that, for example, $W^* = P^*$ is the proportionate rate of change in the real wage rate and $Q^* - K^*$ is the proportionate rate of change in the output-capital ratio and Γ^* is the proportionate rate of change in the rate of depreciation by evaporation and α is, for example, the share of labour in gross national product, WL/PQ.

since R and Γ would be constant and the stock of capital would be, by production augmented by technical progress, growing at the same rate as output with no change being postulated in the preferences dictating the steady-state supply of labour and savings.

The Harrod–Robinson–Read conception of total factor productivity or HRR technical progress can be immediately derived by solving for H^* from the following

$$\text{II-4}\quad \alpha(W^*-P^*)+\beta([R^*+P^*+H^*]-P^*)+\gamma([\Gamma^*+P^*+H^*]-P^*)$$
$$\equiv\alpha(Q^*-L^*)+\beta(Q^*-[K^*-H^*])+\gamma(Q^*-[K^*-H^*])\equiv H^*$$

and for the steady state economy described above I would have

$$\text{II-5}\qquad \alpha(W^*-P^*)+(\beta+\gamma)H^*=\alpha(Q^*-L^*)+(\beta+\gamma)H^*=H^*$$

which yields, of course

$$\text{II-6}\qquad W^*-P^*=Q^*-L^*=H^*.$$

At this stage, a diagrammatic representation of the argument might be helpful.[11] Figure 1, which represents a once-over Harrod neutral technical change, has two panels: the left hand side is the Harrodian representation of the neutral change while the right hand side represents the Hicksian concept. In the initial position with a given technology, the standard story shows that a certain capital-labour ratio $(K/L)_0$ entails *via* the production function a certain gross output-

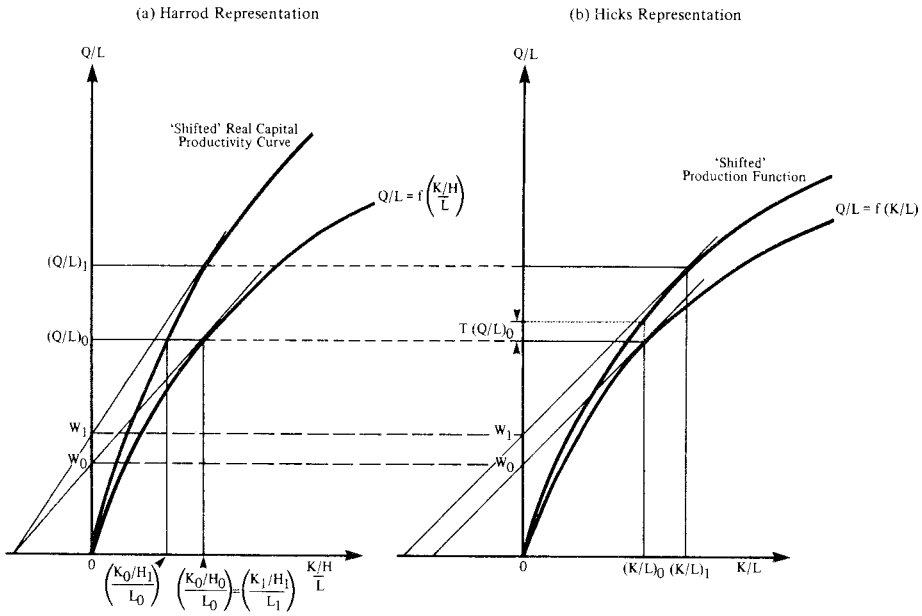

Figure 1. Once-Over Neutral Technical Change

[11]The diagram is drawn from my *On Concepts of Capital and Technical Change*, Diagram 5-1, 129 and is also related to that used by Frank Reid in his "Comment" in eds. Maital and Meltz, op. cit.

labour ratio $(Q/L)_0$ and under competitive assumptions a certain real wage rate W_0 [the vertical intercept of a line tangent to the production function at K/L equal to $(K/L)_0$] and a certain gross rate of return, $R+\delta$ [the slope of the tangent line]. The Harrod story where the horizontal axis is expressed in terms of waiting-labour ratio $[(K_0/H_0/L_0)]$ which, with a given technology $H=1$, is the same as $(K/L)_0$, tells the same story. The once-over technical change is represented in the Hicksian case by an increase in the output-labour ratio to (Q/L), which is said to be decomposed into two parts: that associated with the once-over improvement in technology or the shift in the production function, indicated in the figure as $T^* \cdot (Q/L)_0$ and that associated with the movement along the production function associated with the increase in the capital-labour ratio from $(K/L)_0$ to $(K/L)_1$ said to be induced by the technical change, though there is revealed no change in preferences to work or to save.

In the Hicksian case, to match the assumptions of Harrod neutrality, the diagram can be interpreted as representing a Hicksian neutral technical change and a Cobb–Douglas production function. In the Harrod representation, since $H_1>1$ is the expression of the once-over technical change, the effect can be, for comparison purposes, decomposed into the initial reduction of the primary input content of the initial $(K/L)_0$ from $[(K_0/H_0)/L_0]$ to $[(K_0/H_1)/L_0]$ followed by the greater production of commodity capital to $(K_1/L)_1$ which deflated by the technical advance yields $[(K_1/H_1)/L_1]$ equal to (K_0/L_0), the same higher real wage rate as is shown in the Hicksian case, W_1 [the vertical intercept of a line tangent to the "real capital productivity curve" at $[(K_1/H_1)/L_1\}=[(K_0/H_0)/L_0]$ and a proportionate increase in the real price of waiting $(R+\delta)H_1/H_0$ equal to the proportionate increase in the slope of the line tangent to the "real capital productivity curve" equal to the proportionate increase in the real wage rate $W_1/W_0=W_0 \cdot H_1/H_0$. It is only in this last sense that the Hicksian and Harrodian concepts of neutrality are similar. As Figure 1 shows, however, the reduction in the primary input content of reproducible capital which is inherent in the very idea of technical progress is captured in the representation of the Harrodian case and is missed in the Hicksian representation.

It is commonly asserted[12] that, in the one commodity case, if the underlying technology is Cobb-Douglas then Hicks neutral and Harrod neutral technical progress are equivalent. From the analysis, however, they clearly are not. In the Hicks neutral case, the measured rate of technical progress is said to indicate the equiproportionate rates at which the marginal physical product of labour and reproducible capital (for a *given* ratio of labour and *reproducible* capital) will rise with subsequent accumulation bringing about a decrease in the ratio of labour to commodity capital) without any change in the underlying propensities to work and to save. In the Harrod neutral case and using language only to sharpen the comparison between the two, the measured rate of technical progress is said to indicate the equiproportionate rate at which the "marginal physical products" of labour and waiting are rising. The accumulation of commodity or reproducible capital is correctly attributed to the technical progress taking place

[12]Layard, P. R. A. and Walters, A. A., *Microeconomic Theory* (New York: McGraw-Hill, 1978), 290.

because there has been no change in the underlying propensities to work and to save. The usual misinterpretation of Harrod-neutral progress as being solely labour-augmenting arises from consideration of expressions such as II-6 which shows the proportionate rate of Harrod-neutral technical progress as being equal to the proportionate rate of increase in the real wage rate and output per unit of labour.[13] As expressions II-4 and II-5 reveal, however, Harrod neutrality implies the real prices of both labour and waiting are increasing at the same rate and the output per unit of direct and indirect labour and waiting are rising at the same rate. The Harrod conception rigorously takes into account the reproducibility of capital goods and/or the services of such capital goods when, in the presence of technological advance, the efficiency with which the economic system is producing such inputs is being enhanced by the technical progress being measured.[14] It will be remembered that Harrod defined neutral technological progress as a state when, for a given representative individual exhibiting propensities to work and to save, the output price of inputs (i.e. the *real* wage rate and the *real* return to waiting) would be rising equiproportionately along with output per unit of labour and waiting.[15] In steady state one commodity equilibrium models this results in an unchanging fraction of time spent on work and an unchanging fraction of income saved,[16] and constancy in the rate of transformation between permanent streams of consumption (constancy in R) and in K relative to Q. The crucial distinction, then, between Hicksian and HRR concepts and measures of technical progress is that the latter logically takes into account the increasing efficiency with which the economic system is producing produced means of production simultaneously with the measurement of the increasing efficiency of the economic system. The Hicksian concepts and measures do not and are therefore internally inconsistent. Not only that but they automatically result, when technical progress does exist, in an understatement of the rate

[13]Such usual misinterpretations also arise from the representation of the Harrod case by $Q = Q(K, AL)$ as contrasted with the Hicks case by $Q = AQ(K, L)$ where A is often represented as (say) $A(t) = A_0 e^{at}$.

[14]It is not the durability of capital which matters but its reproducibility. For a survey of problems with respect to capital's durability but a complete neglect of the fact that commodity capital—in all its guises—is a reproducible input, see Diewert, W. E., Capital and the Theory of Productivity Measurement, *American Economic Review*, LXX, May 1980, 260–267.

[15]Harrod, R. F., The Neutrality of Improvements, *Economic Journal*, LXXI, June 1961, 300–304. Later, Solow reiterated the importance of the concept of waiting (cf. Solow, R. M., *Capital Theory and the Rate of Return* (Amsterdam: North Holland, 1963), 10–11, and most recently Yeager has again stressed the importance of the concept, cf., Yeager, Leland B., Capital Paradoxes and the Concept of Waiting, ed. M. J. Rizzo, *Time, Uncertainty and Disequilibrium: Exploration of Austrian Themes* (Lexington, Mass.: D. C. Heath, 1979).

[16]If one divides the time spent on work among time spent on the acquisition of consumption goods, time spent on the maintenance of the stock of capital (on the maintenance, that is, of a permanent stream of consumption) and time spent on the acquisition of net new capital goods for the growth of the stock of capital (the augmentation of the permanent stream of consumption), one has Pasinetti's concepts of direct vertically integrated labour, indirect vertically integrated labour and hyper-indirect vertically integrated labour, cf. Pasinetti, L. L., *Structural Change and Economic Growth* (Cambridge: Cambridge University Press, 1981), esp. Chap. VII. The two types of indirect vertically integrated "labour" are analytically equivalent, I would argue, to the concept of "waiting" employed by Harrod.

of technical progress or "the importance" of productivity change.[17] In addition, the Hicksian measures fail to "predict" or track the observed changes in relative prices—the observed rates of increase in the *real* wage rate and *real* return to waiting.

In the arithmetic formulation in expression II-4, one sees that, with respect to quantities, $Q^*-[K^*-H^*]$ measures the growth rate of output less the growth rate of capital adjusted for ("deflated" by) the fact that the commodity capital input is being produced and reproduced with the ever-increasing efficiency of the primary inputs, labour and waiting; that, with respect to prices, $[R^*+P^*+H^*]-P^*$ measures the growth rate in the real price of the primary input, waiting, just as W^*-P^* measures the growth rate in the real price of the primary input labour.

The theoretical and empirical error involved in the Hicksian measures and the validity of the HRR concepts and measures can be further seen by additional steady state comparisons. The Hicksian measures are notoriously brittle with respect to the static so-called partial elasticities of production which, under competitive assumptions, equal shares. Suppose the national accounts for an economy were rewritten in "net" terms,[18] i.e.

$$\text{II-7} \qquad PQ-\Gamma PK=PY=WL+RPK$$

and after the customary manipulation I would have

$$\text{II-8} \qquad \hat{\alpha}(W^*-P^*)+\hat{\beta}(R^*+P^*-P^*)\equiv\hat{\alpha}(Y^*-L^*)+\hat{\beta}(Y^*-K^*)\equiv\hat{T}^*$$

where, for example, $\hat{\beta}$, is now defined as the *net* partial elasticity of production

[17]The substantial understatement of the rate of technical progress implied in the use of the Hicksian measures as indicated by Hulten and Usher (*op. cit.*) indicates the erroneousness of Griliches' view that the differences in the concepts and measures of total factor productivity are "semantic." cf. Griliches, Z., Issues in Assessing the Contribution of Research and Development to Productivity Growth, *The Bell Journal of Economics*, X, Spring 1979, 92–116. Of course, Griliches deals with research and development as an input producing more output immediately and inducing more output as capital accumulation responds to the direct impact. The point is, however, the fact that the capital accumulation is not induced as an additional response to technical progress. As produced means of production, the capital accumulation is a logical part of that process. There are, in the literature, innumerable references in which this fundamental distinction between production and induction is and must be made. From Pigou, A. C., *The Economics of Stationary States* (London: Macmillan, 1935), one reads

> "For capital instruments are themselves produced by labour, existing capital instruments, *waiting* and, it may be, since our stationary state may have arisen out of one which was not stationary, uncertainty-bearing. If the analysis were pushed back far enough in time, capital instruments would all be found to originate ultimately in labour, land, *waiting*, and again, it may be, uncertainty-bearing, and thus to constitute a derivative, not an ultimate, factor of production. Moreover, had we been concerned to follow this line of thought, we should have needed to analyse further the stock of labour as it stands at any moment: for this is itself partly the product of *waiting* in the past. It is more convenient, however, to regard such elements as "waiting" and "uncertainty-bearing" as sources of factors of production than as factors themselves: the factors themselves being defined in the manner adopted here." (p. 26)

My emphasis. The last sentence refers, of course, to the fact that Pigou abstracts in his book on stationary states from technical progress.

[18]The assumption that Γ, the rate of "depreciation by evaporation," remains constant is consistent, of course, with the requirements of steady state analysis and, as emphasized by Feldstein and Rothchild, is only consistent with that analysis. See Feldstein, Martin S. and Rothchild, Michael, Towards an Economic Theory of Replacement Investment, *Econometrica*, XLII, May 1974, 393–423.

of output with respect to capital (services) as metered by

$$\text{II-9} \qquad \hat{\beta} \equiv \frac{RPK}{P(Q-\Gamma K)} = \frac{RPK}{PY} = \frac{RK}{Y}.$$

That is, $\hat{\beta}$ is now defined as the net marginal product of capital or the services of capital multiplied by the stock of capital divided by the flow of net output as metered by the *net* rate of return (equivalent to the *net* rental in the one commodity case) multiplied by the stock of capital (or the services of capital) divided by the net commodity output. By definition, $\hat{\beta} > \beta$ and $\hat{\alpha} > \alpha$ so that for the same economy the Hicksian *net* rate of technical progress will be greater than the Hicksian gross rate of technical progress, i.e.

$$\text{II-10} \quad \hat{\alpha}(W^*-P^*) \equiv \hat{\alpha}(Y^*-L^*) \equiv \hat{T}^* > \alpha(W^*-P^*) \equiv \alpha(Q^*-L^*) \equiv T^*.$$

Thus, the measured rate of Hicksian technical progress or total factor productivity is subject arbitrarily to whatever gross or net concept of output and partial elasticities of production are employed in measurement procedures. The HRR measures, of course, remain invariant, which is exactly what economic theory would suggest. At the theoretical level, it should not be the case that differences in *static* parameters of technology such as the partial elasticities of production should have any effects on the concept or measured rates of neutral technical *progress*. The HRR measures have this robustness whereas the Hicksian measures are brittle. Additional observations along this line may be made. In the one commodity context, the assumption of "depreciation by evaporation" is particular. Other assumptions and measures of depreciation may be entertained.[19] The different measures would be associated, of course, with different concepts and measures of the gross and net partial elasticities of production. For example, a one commodity steady state economy with "depreciation by sudden death" will have a vintage structure of capital goods such that the accounts would be

$$\text{II-11} \qquad PQ \equiv WL + R\sum_i P_i K_i + \sum_i -p_i P_i K_i$$

where P_i would be a vector of prices of new commodities, one year old commodities, etc., up to T year old commodities, K_i would be a vector of the number of commodities in such vintages and p_i would be the proportionate rate of decline in the price of any vintage (relative to the price of the latest vintage or newest commodity) as it aged. That proportionate rate of change would be, for instance,[20]

$$\frac{R\,e^{-RT}}{1-e^{-RT}}$$

[19]See my *On Concepts of Capital and Technical Change* (Cambridge: Cambridge University Press, 1971), Chapter 4. The measures of depreciation suggested there are all measures that a competitive price system would generate and are consistent with user cost measures, gross or net, of the services of capital goods of all kinds. See my comments on Young, A. H. and Musgrave, J. C., Estimation of Capital Stock in the United States, in ed. Dan Usher, *The Measurement of Capital* (Chicago: University of Chicago Press for the NBER, 1980).

[20]From Rymes, T. K., *op. cit.*, the price of any vintage with t "years" to live would be $P_iT(t) = Vi(t)/R[1-e^{-RT}]$ where $Vi(t)$ is the gross rent accruing to vintage T at time t, R is the equilibrium net rate of return and T is the "life" of commodity capital. Hence,

$$-pi = \frac{1}{PiT(t)}\frac{dPiT(t)}{dt} = \frac{R\,e^{-RT}}{1-e^{-RT}}.$$

so that the measured capital consumption allowances as a fraction of gross output would be a function of R and T. Consequently, the conception and measurement of the net partial elasticity of output with respect to any and all vintages of commodity capital services will be a function of R and T. Two economies with exactly the same HRR rate of technical progress would, if R (and T) differed between them, would exhibit different $\hat{\beta}$'s, $\hat{\alpha}$'s and different Hicksian rates of technical progress or total factor productivity. All Harrodian measures would, however, be invariant.

To probe even deeper into the one commodity steady state case and into the meaning of the postulated technological progress, suppose it was argued that no advances in the stock of knowledge were actually occurring, that the stock of knowledge was completed and because it was costly to implant that knowledge into technology in use only a fraction of that knowledge was in fact implanted each year. Since the implementation of the knowledge is costly, correct accounting for the resources used up in such implanting would wholly exhaust the "residual" and there would be recorded no technical progress or total factor productivity. Now the assumption that advances in knowledge are costly to implant sometimes appears as the distinction between disembodied and embodied (in output, labour and/or capital) technical advance. If implanted at cost in capital, the capital input would in fact be deemed to be growing sufficiently rapidly to account for observed technical progress and the therefore improperly-measured total factor productivity would, upon correction, be zero. As Jorgenson has shown, however, there is no distinction to be observed between disembodied and capital-embodied technical advances and therefore measured rates of technical advance on total factor productivity will not be affected.[21] Yet there is a way in which the Hicksian measures are *automatically* affected by such considerations. If it is argued that (say) research and development expenditures are really capital-type expenditures so that the partial elasticity of output with respect to "capital" is really greater than is conventionally measured, or equivalently, part of what is considered as part of the wages bill is really return to human capital (so that the steady and costly upgrading of human skills is treated as capital expenditures) and the stock of such reproducible skills and their services is capital and capital services so that the partial elasticity of output with respect to non-reproducible inputs is *definitionally* smaller, the Hicksian conceptions of technical progress and measures of total factor productivity would be lower. If all the advances were continuing to be neutral in Harrod's sense (it will be remembered we remain in a one-commodity steady state world), then the Harrod conception and measured rate of technical progress would be invariant to such static re-definitions of what is and what is not reproducible. If the argument is that all implantation of knowledge is costly and is capitalized accordingly then, in the context of the simple steady state story, *all* inputs would be considered

[21] Jorgenson, D., The Embodiment Hypothesis, *Journal of Political Economy*, LXXIV, February 1966, 1–17, and *On Concepts of Capital and Technical Change*, Chap. 4–5, Depreciation by Obsolescence. See also Nadiri, M. I., Producers Theory, eds. K. J. Arrow and M. D. Intriligator, *Handbook of Mathematical Economics*, II (Amsterdam: North Holland, 1982), 445.

reproducible and

(i) in the Hicksian case there would be no measured rate of technical progress or total factor productivity (the residual would be reduced to zero by definition) and the economy would be von Neumann;[22]

(ii) in the Harrodian case the measured rate of technical progress or total factor productivity would be invariant to such redefinition and would appear to be the same as the von Neumann rate of growth and net rate of return. That is, the accounts would simply become

$$RP_kK_k \equiv \sum P_k \Delta K_k$$

II-12

$$RP_kK_k \equiv g \sum P_kK_k$$

$$R \equiv g$$

Such a representation of an economic system is unexceptionable and would eliminate measures of technical progress or total factor productivity *in the Hicksian sense*. Yet in terms of the proportionate rate of increase in the efficiency of non-reproducible inputs the conception and measured rate of Harrodian technological progress and total factor productivity would be invariant and, in the limit (as α approaches zero) would equal the von Neumann R's and g's.

The von Neumann representation cannot handle natural agents of production (either fixed or exhausting). Relative prices and quantities are unchanging in the von Neumann steady state whereas between reproducible and non-reproducible inputs they are changing at the rate given by the Harrod neutral rate of technical advance. Even if all inputs were treated as capital so that in the Hicksian sense the concept and measure of technical progress or total factor productivity shrank to zero (and we would have "explained away" the residual or measure of our ignorance) the Harrod–Robinson–Read conception and measures remain logically and rigorously intact.[23]

Turning now briefly to the two sector steady state case, I can provide a further review illustrating the superiority of the HRR case. The accounts are

II-13

$$P_cC = WL_c + RP_kK_c + \Gamma P_kK_c$$

$$P_k\Delta K = WL_k + RP_kK_k + \Gamma P_kK_k$$

[22]I should argue as well that the economy would exhibit the characteristics apparently advocated by Frank Knight in early writings in which he argued against any meaningful distinction between reproducible and non-reproducible inputs and diminishing returns to investment in the presence of given knowledge. See Knight's summary position where he seemingly abandoned this argument in his Diminishing Returns from Investment, *Journal of Political Economy*, LII, March 1944, 26–47. See also Buechner, M. Northrup, Frank Knight on capital as the only factor of production, *Journal of Economic Issues*, X, September 1976, 598–617.

[23]I am indebted to Professor Keith Acheson and Professor L. Pasinetti (cf., Rymes, *op. cit.*, 104) and Professor Ian Steedman for discussions and correspondence about the links between von Neumann and Harrod. Professor Pasinetti objects, of course, to the inadequacy (non-existence) of the treatment of technical progress in the von Neumann model—particularly on the demand side. See Pasinetti, L. L., *A New Theoretical Approach to the Problems of Economic Growth* (Vatican: Pontificia Academia Scientiarum, 1965), appendix to Chap. VI [reprinted in *The Econometric Approach to Development Planning* (Amsterdam: North Holland, 1965) and *Structural Change and Economic Growth* (Cambridge: Cambridge University Press, 1981), appendix to Chap. VI].

so that the Hicksian measures of total factor productivity are

$$\text{II-14}\qquad \begin{aligned} &\alpha_c(W^*-P_c^*)+\beta_c(R^*+P_k^*-P_c^*)+\gamma_c(\Gamma^*+P_k^*-P_c^*)\\ &\quad\equiv\alpha_c(C^*-L_c^*)+\beta_c(C^*-K_c^*)+\gamma_c(C^*-K_c^*)\\ &\quad\equiv T_c^*\\ &\alpha_k(W^*-P_k^*)+\beta_k(R^*+P_k^*-P_k^*)+\gamma_k(\Gamma^*+P_k^*-P_k^*)\\ &\quad\equiv\alpha_k(\Delta K^*-L_k^*)+\beta_k(\Delta K^*-K_k^*)+\gamma_k(\Delta K^*-K_k^*)\\ &\quad\equiv T_k^* \end{aligned}$$

where again the notation is mnemonic. No account is taken in such representations of the increasing efficiency with which the capital goods (or their services) *when considered as inputs* are being reproduced even though it is clear that a Hicksian representation of technical progress or total factor productivity in the capital good sector is being presented. The HRR conception and empirical representations rigorously take into account the ever-increasing efficiency with which capital goods as inputs are being produced. The Hicksian measures show technical progress in the capital goods sector and then ignore that for the consumption good sector. The sectoring is static and the internal logical inconsistency in the neo-classical or Hicksian conceptions and measures is clear.

The HRR measures would be

$$\text{II-15}\qquad \begin{aligned} &\alpha_c(W^*-P_c^*)+\beta_c([R^*+P_k^*+H_k^*]-P_c^*)+\gamma_c([\Gamma^*+P_k^*+H_k^*]-P_c^*)\\ &\quad\equiv\alpha_c(C^*-L_c^*)+\beta_c(C^*-[K_c^*-H_k^*])+\gamma_c(C^*-[K_c^*-H_k^*])\\ &\quad\equiv H_c^*\\ &\alpha_k(W^*-P_k^*)+\beta_k([R^*+P_k^*+H_k^*]-P_k^*)+\gamma_k([\Gamma^*+P_k^*+H_k^*]-P_k^*)\\ &\quad\equiv\alpha_k(\Delta K^*-L_k^*)+\beta_k(\Delta K^*-[K_k^*-H_k^*])+\gamma_k(\Delta K^*-[K_k^*-H_k^*])\\ &\quad\equiv H_k^*. \end{aligned}$$

The difference between the Hicksian and HRR concepts and measures now becomes striking. The specialization of the one commodity economy into two activities results immediately, in the Hicksian case, in different measures of the rate of technical progress and total factor productivity in the two sectors solely again on the basis of differences in the partial elasticities of output with respect to labour in the two sectors. That is, different rates of technical progress would be deemed to be taking place solely because of differences in the static parameters of technology in the two sectors—the differences being one of the reasons why the specialization of the economic system has presumably taken place. That is, the Hicksian measures would be

$$\text{II-16}\qquad \begin{aligned} &\alpha_c(W^*-P_c^*)\equiv\alpha_c(C^*-L_c^*)\equiv T_c^*\\ &\alpha_k(W^*-P_k^*)\equiv\alpha_k(\Delta K^*-L_k^*)\equiv T_k^*. \end{aligned}$$

Since by virtue of the steady state assumption

$$P_c^*=P_k^*,\ C^*=\Delta K^*\text{ and }L_c^*=L_k^*,$$

it would follow immediately that

$$\alpha_c(W^*-P_c^*)+(\beta_c+\gamma_c)H_k^* \equiv \alpha_c(C^*-L_c^*)+(\beta_c+\gamma_c)H_k^* \equiv H_c^*$$

$$\alpha_k(W^*-P_k^*)+(\beta_k+\gamma_k)H_k^* \equiv \alpha_k(\Delta K^*-L_k^*)+(\beta_k+\gamma_k)H_k^* \equiv H_k^*$$

or

II-17 $$H_c^* \equiv H_k^* \equiv W^*-P_k^* \equiv W^*-P_c^* \equiv C^*-L_c^* \equiv \Delta K^*-L_k^*.$$

Thus, we have again

II-18 $$\alpha_c H_c^* \equiv T_c^*$$
$$\alpha_k H_k^* \equiv T_k^*.$$

It is seen dramatically that, under the usual steady state conditions, the HRR measures in the two sectors are (i) invariant to static parameters of technology and (ii) the same—as by the assumption of steady state they must be. The Hicksian concept and measures of technical progress or total factor productivity are, it would appear, arbitrary. Given that there is no reason, save a fluke, for $\alpha_c = \alpha_k$, we have

$$T_c^* \lesseqgtr T_k^* \text{ as } \alpha c \lesseqgtr \alpha k.$$

The Hicksian measures fail, in the dynamic sense, to take into account the technological interdependence of the economic system. This the HRR measures do rigorously, which means that with technical advance in the capital good sector the flow of consumption goods is being produced with an ever increasingly efficient direct and indirect application of labour and waiting. The HRR measures take into account reproducibility of commodity capital inputs in the form of the using up of capital and the services of capital goods in both the consumption and capital goods sectors. The Hicksian measures do not—and generate correspondingly meaningless results. It is to be noted that the HRR measures (though H_k^* is independent) are simultaneously determined. The partial nature of the Hicksian measures which deny the technological interdependence of the economic system is manifest. The HRR measures are consistent with the movement of relative prices and quantities—i.e., consistent with the demand assumptions of the steady state, one would argue that, in the two sector case, if rates of technical advance were the same the relative commodity prices would be unchanged. This is what the HRR measures would show. Again the Hicksian concepts and measures would fail to be associated with the observed movement in relative commodity prices. It is finally of considerable instruction to note that only in the special case where $H_k^* =$ zero would the Hicksian and HRR concepts and measures be the same.

III. Empirical Implementation of Harrod–Robinson–Read Measures of Total Factor Productivity

Though steady state exercises are powerful tools which illuminate the theoretical and logical inconsistencies in Hicksian concepts and measures of

technical progress and the theoretical and logical precision of the superior HRR counterparts, it is clear that the importance of the HRR measures awaits generalization and empirical implementation. I now extend the discussion to a disaggregated Sraffa–Leontief world and point out pitfalls associated with Hicksian concepts and measurements such as net output by "industry." I also consider in an exploratory way the implementation of the HRR measures in the Canadian context.

Consider what is called an "industry" in a "standard" Leontief framework. For the accounts of that "industry" I will have

$$\text{II-19} \qquad WL + RP_k K + \Gamma P_k K + P_m M \equiv PQ$$

where again the notation is mnemonic and $P_m M$ stands for the value of intermediate inputs. It is now understood that, with the exception of output, the symbols stand for vectors. Thus, for example WL are row and column vectors of the many different prices and kinds of labour employed in the "industry." The disaggregation of labour, and the other inputs, can be as detailed as theory dictates and the data permit. The standard transformations yield

$$\text{II-20} \qquad \alpha(W^* - P^*) + \beta(R^* + P_k^* - P^*) + \gamma(\Gamma^* + P_k^* - P^*) + \varepsilon(P_m^* - P^*)$$
$$\equiv \alpha(Q^* - L^*) + \beta(Q^* - K^*) + \gamma(Q^* - K^*) + \varepsilon(Q^* - M^*) \equiv T^*.$$

In standard terminology, for example, one would argue that M^* is the proportionate rate of change of the constant dollar value of the intermediate inputs used in the "industry," K^* would be the proportionate rate of change respectively of the constant dollar value of the net stock of capital (or the net service flows of the stock of capital) and the constant dollar value of the capital consumption allowances or depreciation of (or the constant dollar value of the capital used up in) the "industry." (Of course, one could, in principle, disaggregate to the level of the individual prices and quantities.)

It follows immediately from the foregoing discussion that these "industry" concepts and measures of Hicksian technical progress or total factor productivity will be smaller the "grosser" the concept of output employed—even when for all inputs (primary and intermediate) no relative changes in quantities or prices might be taking place. That is, even if all "industries" exhibit all the properties associated with a steady state economy it will be the case (similarly to the argument advanced above about net output) that the Hicksian measures of technical progress will be lowest when the grossest output concept of an "industry" is employed (when (say) the output concept is gross output including all intra-industry intermediate output-intermediate input flows), will be higher when such intra-industry flows are "netted" out, will be even higher when the output concept is real value added (which, in Canada, is called constant dollar gross domestic product at factor cost) obtained by the route of double deflation and again higher still when the output concept is net real value added (in Canada, this would be called constant dollar net domestic product at factor cost) obtained

by the route of double deflation and subtraction of the constant dollar capital consumption allowances.[24]

It is sometimes argued that real value added will be a "correct" measure of output in some sense if certain conditions, such as those holding in a steady state (wherein the ratios of intermediate inputs to gross output and their prices remain constant) prevail or if one argues that the gross output concept is drawn from a gross output production function separable into intermediate and "primary" inputs.[25] Yet, as was always well understood, the construction and use of industry output concepts such as real value added as distinct from the duplicated gross output concepts was advocated and the corresponding measures developed precisely because steady state conditions (such as those outlined above) did not prevail. The so-called aggregation and separability problems discussed in the extensive production literature are, however, beside the point! Even when conditions necessary for aggregation theorems to go through prevail, it is still the case that Hicksian concepts and measures of technical progress or total factor productivity remain invalid and arbitrarily susceptible to the particular grossness of the output concept employed.

The HRR measures, with specific reference to vector notation being borne in mind, are for the industry in question (and there will be one such equation for each industry)

$$\text{II-21} \quad \alpha(W^*-P^*)+\beta([R^*+P_k^*+H_i^*]-P^*)+\gamma([\Gamma^*+P_k^*+H_i^*]-P^*)$$
$$+\varepsilon([P_m^*+H_i^*]-P^*)$$
$$\equiv\alpha(Q^*-L^*)+\beta(Q^*-[K^*-H_i^*])+\gamma(Q^*-[K^*-H_i^*])$$
$$+\varepsilon(Q^*-[M^*-H_i^*])\equiv H^*$$

and their immediate generality and power is easily seen. (a) Consider the entry $Q^*-[K^*-H_i^*]$. That vector is a measure of the proportionate rate of change of output, Q^*, minus the proportionate rate of change of the net capital stock, K_i^* (or services of that commodity capital) *from* (say) *the i-th industry* and H_i^* is the HRR rate of technical progress or total factor productivity of the i-th industry. That is, in the "industry" in question, one rigorously takes into account that, when it uses reproducible inputs produced in other "industries" and those industries are experiencing technical progress, the sector in question is simultaneously experiencing technical progress because it is using labour and waiting, directly and *indirectly* more efficiently than before. (b) The use of a scientific

[24]The phenomenon, that referred to by Professor Berndt (cf. footnote 6), is well-known (cf. Domar, E., On the Measurement of Technological Change, *Economic Journal*, LXXI, December 1961, 709–729). Some years ago, I constructed total factor productivity estimates for Canadian manufacturing. On a gross domestic product basis, total factor productivity was for nine major groups in manufacturing for the years 1946–53 an average 143 percent higher than the total factor productivity estimates on a gross output basis; cf. Lithwick, N. H., Post, G. and Rymes, T. K., Postwar Production Relationships in Canada, ed. M. Brown, *The Theory and Empirical Analysis of Production* (New York: Columbia University Press for the NBER, 1967), Table II.

[25]See, for example, Bruno, Michael, Duality, Intermediate Inputs and Value-Added, and Diewert, W. Erwin, Hicks' Aggregation Theorem and the Existence of a Real Value-Added Function, in eds. M. Fuss and D. McFadden, *Production Economics: A dual approach to theory and applications, II* (Amsterdam: North Holland, 1978).

tool of great importance—the inter-industry Sraffa–Leontief framework—is immediately seen. It is precisely that framework that permits construction of disaggregated "industry" HRR technical progress because it provides the knowledge of the technological interdependence which characterizes modern economies producing commodities by means of commodities. If one has n such "industries," each sector using reproducible inputs produced by all n "industries," one cannot solve for the HRR rate of technical progress or total factor productivity in any one "industry" without solving simultaneously for the rates in all "industries." With modern analytical Sraffa–Leontief tools, this is easy to do. (c) The HRR measures provide one answer to Professor Berndt's suggestion that measures of technical change and total productivity invariant to the "grossness" of output be "... high on the list of future research priorities." For it is also immediately clear that, at least in the context of the discussion so far, the HRR measures will provide precisely this invariance. Indeed, the non-invariance of the Hicksian measures, which leads to their rejection in the measurement of productivity, arises precisely because of the failure to take into account the reproducibility of intermediate capital inputs. (d) The use of real value added output measures, where technical progress is occurring at different rates in different sectors, remains, however, incorrect in a world of technical progress because such *output* measures do not take into account the reproducibility of the intermediate inputs.[26]

Thus, the use of modern Sraffa–Leontief systems permit the derivation of disaggregated "industry" measures of HRR technical progress or total factor productivity which, if the foregoing argument is accepted, would provide measures consistent with the observed behaviour of relative prices and quantities in the economic system. They are measures, then, of empirical content and conformable to the tenets of economic theory. Such measures can be now prepared with data available from Statistics Canada.[27]

It will be noted that in both the two sector and n-"industry" case I have not suggested the construction of an aggregate measure of technical progress or total factor productivity. In my view, a relative "industry" ranking is all that is required for the examination of the movements of relative prices and quantities (indeed, in the context of steady state analysis, all industries would show the same rate of HRR progress so no aggregation would be required).[28]

One final point of empirical importance must be made. The HRR measures are not restricted to steady states for their validity nor will they generate measures of technical advance or total factor productivity where none exist. If I return to

[26]See my *On Concepts of Capital and Technical Change*, Chap. 7, On the Concept of Net Output.

[27]I am grateful to various officers of Statistics Canada who have answered queries in this regard. I do not wish to leave the impression that there are no problems (the treatment of *imported* intermediate inputs requires inter-country knowledge of HRR measures and current rectangular IO systems in Canada imply joint production problems). Nonetheless, research work in which I am currently engaged at Statistics Canada has so far indicated that preliminary industry HRR measures of total factor productivity are feasible. As can be understood from a comparison of II-20 and II-21, industry Hicksian measures will be also derived as a by-product from this research.

[28]Tentative suggestions for aggregation to the economy wide level are found in my The Measurement of Capital and Total Factor Productivity in the Context of the Cambridge Theory of Capital, *Review of Income and Wealth*, XVIII, March 1972, 79–108.

the one commodity case, I can easily show these points. Dealing only quantities, I have

$$\text{II-22} \qquad \alpha[Q^*-L^*]+\beta[Q^*-(K^*-H^*)]+\gamma[Q^*-(K^*-H^*)]=H^* \quad \text{or}$$
$$\alpha[Q^*-L^*]+\beta[Q^*-K^*]+\gamma[Q^*-K^*]=\gamma H^*$$

so that if there is no technical progress to be measured, H^* will equal zero or, following neoclassical lines of thought, if extra saving is occurring, then Q^*-K^* will be negative, Q^*-L^*will be positive, the two effects will net out—and now of course the static parameters of technology play their role—so that $\alpha[Q^*-L^*]+\beta[Q^*-K^*]+\gamma[Q^*-K^*]=$ zero. Such neoclassical cases may be extended to the two- and n-sector cases. Thus, the HRR measures have all the generality and more, of course, of their Hicksian counterparts.

As a further illustration of the relaxation of the steady state assumption (which is and has always been a mythical state of affairs designed, by keeping the index number birds off the fields, to shed the maximum penetrating light on the theoretical concepts employed), let the underlying rate of Harrod technical progress be non-neutral in the sense that the efficiency of waiting changes relatively to the efficiency of labour, such that $Q^*=K^*+k$, $k \gtrless 0$, with R unchanged. The expression II-22 would be then rewritten as

$$\alpha[Q^*-L^*]+\beta[Q^*-(K^*-H^*)]+\gamma[Q^*-(K^*-H^*)]=H^*$$

which reduces to

$$H^*=Q^*-L^*+\frac{\beta+\gamma}{\alpha}k.$$

Again, the static parameters of technology play a role because of the non-neutrality assumption (and in discrete representations index number problems arise). It is clear, however, that if $k \gtrless 0$, then the rate of non-neutral Harrod technical progress exceeds (is less than) the growth rate of labour productivity and the rate of growth of waiting, K^*-H^*, will be L^*-k/α' or less (greater) than the rate of growth of labour so that the rate of increase in the efficiency of waiting exceeds (is less than) the rate of increase in the efficiency of labour.

IV. Recent Estimates and the Concept of "Sectors" Employed by Professor Pasinetti

Peterson[29] presents estimates which are a first step towards what I call the HRR measures. The first steps in his accounting are very similar to mine but he argues (213)

> "The approach adopted in this paper leads naturally to the alternative treatment suggested by Rymes, in which capital goods are regarded as a form of intermediate product and labour is the sole primary input."

[29]Peterson, William, Total Factor Productivity in the U.K.: A Disaggregated Analysis, eds. K. D. Patterson and K. Schott, *The Measurement of Capital: Theory and Practice* (London: Macmillan, 1979).

It seems to me that the essence of HRR measures is that they take account of the growing efficiency of *both* labour and waiting (and, in a fuller statement, inputs like land), however many kinds of labour and waiting are directly and indirectly involved in economic production. The HRR measures do not reduce to labour productivity, directly and indirectly, alone. Labour is not the sole primary input.[30] With this understanding, I believe the Peterson estimates are the best estimates of Harrod technical progress and total factor productivity currently available.

Usher as well gets results consistent with the interpretation of aggregate Canadian data as if they had been generated by Harrod neutral advance. His data indicate that, from 1926 to 1974, "... the rate of technical change is approximately equal to the rate of growth of output per head".[31] The Usher estimates are, therefore, consistent with the one sector observation noted above, namely

$$H^* = Q^* - L^*.$$

Usher derives his result by concentrating on the sustainable rate of growth of consumption (per head) in Canada *and* the re-expression of different kinds of capital (physical and educational) in terms of foregone consumption. He is, of course, perfectly aware that capital is endogenous, draws out the conclusions, but does not focus upon the obvious fact that the endogeneity of capital comes not so much from the willingness to save but from the fact that it is a reproducible input (see the distinction made earlier between induction and production).

Hulten's work also incorporates the reproducibility of capital as an important fact in deriving his results that the conventional (i.e. Hicksian) measure of total factor productivity "... understates the impact of factor efficiency on economic growth."[32]

Hulten has argued that his various concepts such as the long-run Fisherian rate of technical progress[33] (which is the same as Harrod-neutral progress) and the dynamic residual ("... when the extra capital forthcoming as a result of productivity change is taken into account, the importance of productivity change nearly doubles."[34]), which are very similar to the Harrodian concepts, all stand higher than the Hicksian measures. They must. Yet, while I am in full sympathy with Hulten's basic approach, I would claim that his variants are contained within the HRR concepts. Hulten's latest estimates, I would assert, amount to no more than the estimate, on the price side of the HRR measures, of the proportionate rate of change of the real price of *waiting* and labour—or the steady rise in permanent consumption streams and hence the emphasis on dynamic efficiency and long-run Fisherian total factor productivity estimates.

[30]I would have the same questions about the Sharpe measures earlier cited (see footnote 5).

[31]See Usher, Dan, *The Measurement of Economic Growth* (New York: Columbia University Press, 1980), 289.

[32]See Hulten, Charles R. and Nishimizu, M., The Importance of Productivity Change in the Economic Growth of Nine Industrialized Countries, eds. S. Maital and N. M. Meltz, *Lagging Productivity Growth*, 85.

[33]Hulten, Charles R., Technical Change and the Reproducibility of Capital, *American Economic Review*, XLV, December 1975, 960.

[34]Hulten, Charles R., On the 'Importance' of Productivity Change, *American Economic Review*, LXIX, March 1979, 134.

Some time ago Edward Denison, using the movements of relative prices as predicted by the Harrodian measures, adjusted his estimates of total factor productivity to approximate the Harrodian variant.[35] I have already demonstrated that the "netter" is the measure of output being employed—i.e. the smaller is the weight given to the reproducible commodity capital input—the higher will be the Hicksian measure of total factor productivity. They will be equal to the steady state Harrod measures when, in the Hicksian measures, the weight given to the reproducible commodity input equals zero. Because Denison uses the output concept net national income (and ignores capital consumption allowances) he obtains a Hicksian measure of total factor productivity which is the closest possible approximation to the Harrodian measures. Even in his case, the growth rate of total factor productivity (what Denison calls the growth rate of advances in knowledge and economies of scale) is increased by some 15 percent. Denison considers this a negligible increase.[36] The "negligible" difference arises simply because an output concept is chosen such that the weight given to reproducible capital is as small as possible. Had Denison dealt with gross national income and included capital consumption allowances as part of the flow of reproducible inputs, the differences would no longer be "negligible"—as has been conjectured by this paper. Denison's treatment of the Harrodian capital concept as consumption foregone is similar to Usher's and is insightful in that it sheds further light on the concept of waiting as a primary input on all fours with labour.

I conclude this paper by referring to recent work on concepts of sectors and technical progress by Professor Luigi L. Pasinetti.[37] It is clear that it is the Harrod (indeed the Harrod-neutral) conception of technical progress which is important.[38] Yet I believe Pasinetti overstates the case. Harrod conceived of neutral progress as a state where the efficiency of labour and waiting was increasing at the same rate. In Pasinetti's terms, this would be the same as equal rate efficiency increases in direct labour, indirect vertically integrated labour and hyper-indirect vertically integrated labour. From this, Pasinetti argues "Labour alone can make all the capital goods .... Capital goods alone can make nothing."[39] This fails to take into account the waiting time which is as essential as labour in intertemporal capitalistic production. Pasinetti, I believe erroneously, conceives HRR concepts as labour-augmenting when, as I have pointed out, they should be considered as labour- *and* waiting-augmenting.

More important, however, is Pasinetti's *related* work on vertically integrated "sectors" as distinct from "industries." From the "standard" Sraffa–Leontief

[35]Denison, E. F., *Accounting for United States Economic Growth, 1929–1969,* (Washington, D.C.: The Brookings Institution, 1974), 133–135.

[36]Denison, E. F., The Contribution of Capital to Economic Growth, *American Economic Review*, LXX, May 1980, 220–224.

[37]Though I was able to discuss some of these ideas with him when Professor Pasinetti was a Visiting Scholar at the University of Ottawa and Carleton University in the Fall Term 1981, he is not in any way responsible for my interpretation of his ideas.

[38]See Pasinetti, L. L., *Structural Change and Economic Growth*, (Cambridge: Cambridge University Press, 1981), 208–214. See also his appendix to On 'Non-Substitution' in Production Models, *Cambridge Journal of Economics*, I, 1977, 394.

[39]*Ibid.*, 200.

framework, where

$$A=[a_{ij}], \qquad a_{ij}\equiv\frac{X_{ij}}{X_j} \qquad i,j=1,\ldots,n$$

$$b_{oj}\equiv\frac{L_j}{X_j}, \qquad j=1,\ldots,n$$

$$B=[b_{ij}], \qquad b_{ij}=\frac{K_{ij}}{X_j} \qquad i,j=1,\ldots,n.$$

I can write

$$X=[I-A]^{-1}y$$
$$bX=b[I-A]^{-1}y=\bar{L}$$

and

$$BX=B[I-A]^{-1}y=K$$

where y is the $n\times 1$ vector of final demands, X is the $n\times 1$ vector of gross outputs by industry, K is the $n\times 1$ matrix of capital inputs, and $b[I-A]$ can be interpreted as the direct and indirect labour requirements for the n sectors making up n components of final demand.

I can decompose the B matrix into capital consumption allowances and the services of the net capital stock. I can also decompose the final demand vector into consumption and net capital formation so that I can write

$$AX+\Gamma BX+C+gK=X$$
$$bX=\bar{L}$$
$$BX=K$$

where Γ and g would be the diagonal vectors of rates of capital consumption allowances and growth rates of net capital stock (taken here for simplicity as scalars) such that

$$[I-A-\Gamma B]^{-1}C+g[I-A-\Gamma B]^{-1}K=X$$
$$b[I-A-\Gamma B]^{-1}C+gb[I-A-\Gamma B]^{-1}K=\bar{L}$$
$$B[I-A-\Gamma B]^{-1}C+gB[I-A-\Gamma B]^{-1}K=K.$$

The last part of the second equation illustrates Pasinetti's concepts of direct and indirect vertically integrated labour requirements for the n sectors making up final *consumption* demand. The third equation can immediately be written as

$$B[I-A-\Gamma B]^{-1}C=[I-gB(I-A-\Gamma B)^{-1}]K$$

such that I have

$$[I-gB(I-A-\Gamma B)^{-1}]^{-1}B[I-A-\Gamma B]^{-1}C=K$$

such that equation 2 is then rewritten as

$$[I-A-\Gamma B]^{-1}C+gb[I-A-\Gamma B]^{-1}(I-gB[I-A-\Gamma B]^{-1})^{-1}$$
$$\times B[I-A-\Gamma B]^{-1}C=\bar{L}$$

which now illustrates Pasinetti's concepts of direct, indirect and hyper-indirect vertically integrated labour requirements[40] (see footnote 16). Thus labour has to be set aside not only to maintain the stock of capital but also to maintain its rate of growth consistent with full employment. The important point to note in Pasinetti's concepts of vertically integrated sectors and labour is that labour is the sole primary input and that capital inputs in *all* forms are taken as reproducible inputs. The role of waiting as a primary input on all fours with labour is not so treated by either Pasinetti or Peterson—yet it is, I suggest, implicit in the concept of indirect and hyper-indirect vertically integrated labour requirements.[41]

Pasinetti's notion of vertically integrated "sectors" is an important addition to economic analysis.[42] It is more important to see it in relation to the HRR measures of technical progress. The discussion surrounding expression II-21 indicates clearly that, in preparing measures of the HRR rate of technical progress or total factor productivity in all "industries," the reproducible commodity inputs are being simultaneously re-expressed in terms of their direct and indirect labour and waiting primary input content automatically by the set of expressions II-21.

The difference between Pasinetti's sectoral concepts and the HRR measures of technical progress is simply that Pasinetti can generate, with the Sraffa–Leontief data, his vertically integrated "sectors" in any period of time as a static concept so that his primary inputs are, as a consequence, reduced to the levels of labour alone. The Harrod concepts do *not* have to perform the static vertical integration; they deal with rates of technical change in any period of time and reduce primary inputs to the rates of change of labour and waiting (and land when Ricardian inputs are taken into account). Both concepts are now empirically attainable because of the development of the same tools of national accounting. Both shed light on the changing economic efficiency of economies where production of commodities takes place by means of reproducible commodities.

[40]Pasinetti's concepts of vertically integrated sectors is set out in his The Notion of Vertical Integration in Economic Analysis, ed. Pasinetti, L. L., *Essays on the Theory of Joint Production* (New York: Columbia University Press, 1980).

[41]The Sraffa–Leontief price system yields the matching prices where

$$PA+Wb+(R+\Gamma)PB=P$$

where P is the $1\times n$ row vector of n relative prices such that I have

$$wb[I-A-(R+\Gamma)B]^{-1}=P$$

such that for the set of n relative prices, "the" wage rate 'drops out', leaving the rate of return (and technology) for the determination of relative prices. See Pasinetti's *Structural Change*, 191-192. The point is, however, that in Pasinetti's analysis R and g are related but that only labour is considered a primary input.

[42]Pasinetti stresses as well the empirical problems in constructing "industry" statistics. The real problem, however, as he mentions in his *Structural Change and Economic Growth*, is that we utterly lack a theory of institutional form which would lead us to a satisfactory set of input-output statistics based on the concepts of the firm, the corporation, the enterprise and the industry. All the current input-output statistics are and will remain *ad hoc* until this conceptual problem is resolved.