CAPITAL AGGREGATION IN THE PRESENCE OF OBSOLESCENCE-INDUCING TECHNICAL CHANGE

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The major question addressed is the treatment of capital embodied technical progress. Should obsolescence be deducted to calculate a net stock, or should quality adjustments be made in each vintage of new capital, or both, or neither? In order to estimate the contribution of new investment to growth it is necessary to use a capital stock where different vintages are weighted in proportion to their marginal products. The commonly used gross capital measures do not do this, because they do not allow for the higher marginal product of more modern capital. Such an allowance for capital embodied technical progress can be made either by quality adjusting new capital or by incorporating obsolescence into the valuation of the old capital (but not both). However, even if new capital incorporates an allowance for improved quality, it will still be necessary to revalue the old capital. Frequently, a reasonable approximation to the net capital stock results from a linear decline in quasi-rents and can be approximated by published estimates of the stock of capital net of straight line depreciation. Steady technical progress will not lead to the commonly used exponential service decline functions. To avoid overestimating the return to investment when technology changes it will be necessary to use information on capital embodied technical change to revalue old capital, rather than to change the price indices for new capital.

This paper is about the treatment of obsolescence, or capital embodied technical change. Strictly speaking obsolescence is not allowable in the many models of the production process which assume that a capital aggregate exists. Such constructions implicitly assume that the contribution to output or marginal product of one item of capital is independent of what other items of capital also exist. Yet the marginal product of old capital such as calculators depends on whether the capital stock also includes new capital, i.e. computers. One cannot say whether calculators should be included in the stock or what their weight should be without knowing whether they have been made obsolete by the presence of computers in the capital stock. The inclusion of new improved capital lowers the marginal product of older pre-technical change capital.

This paper seeks useful insights from directly confronting the question of how obsolescence is to be handled. Unfortunately, discussions of productivity estimation that do not state how obsolescence is handled (which is true of much of the literature) risk either excluding it or double counting it. Indeed, the well known differences between the Jorgenson–Griliches (1967, 1972) and the Denison (1969, 1972) estimates are fundamentally about obsolescence. Denison excludes it and Jorgenson and Griliches first double count it and then put it into only service decline.

EQUATING THE MARGINAL PRODUCT OF INVESTMENT AND THE MARGINAL PRODUCT OF CAPITAL

For many policy analyses, the marginal product of investment is needed. This is the increase in quantity of output measured at constant prices from a one dollar increase in investment in a particular year. A common use for this figure is to estimate the contribution of additional investment to national income. It is also commonly used in growth accounting to determine how much of recent growth can be explained by increases in the quantity of capital.[1]

In both uses attention is focused on the current vintage technology. The policy maker who is thinking of expanding investment is not going to make an equiproportional expansion in all vintages of capital. The capital added will be current vintage (computers rather than calculators).

The key question for policy is the magnitude of return from new investment, or what is dQ/dI, where Q is quantity of output and I is services from current period investment. If this is to be measured from the marginal product of capital (or dQ/dK), where K represents capital services, it is necessary that $dQ/dI = dQ/dK$. Since current investment is in the current vintage, $dQ/dI = dQ/dV_n$ where V_n is the current vintage of capital.

The necessary conditions across vintages are:

$$\frac{dQ}{dI} = dQ/dK = dQ/dV_1 = dQ/V_2 = \ldots dQ/dV_n.$$

Let us use computers as an example. Any new computer built will be of the current vintage. Thus the increment in output per dollar invested in new computers should equal the increment in output per dollar of computer capital. Both should equal the output increment from adding non-computer capital.

The above can be estimated by aggregating capital using weights based on current prices for capital services, or the current quasi-rents. Thus it is proposed that capital be measured by the value of services, and that the weights given to different vintages in aggregation be proportional to each vintage's price of services, which in a competitive market will be the marginal product of each vintage, or dQ/dV. In addition, for current vintage items, use of current prices for services gives weights proportional to current production costs.

Conceptually, the idea of weighting by current prices of services is well accepted and is used by most authors in weighting different factors of production and types of capital and labor. However, authors supporting the concept in principle still use a gross capital concept inconsistent with it.

The Gross Capital Concept

The gross capital concept has been used by numerous authors.[2] While details vary, the concept has been that capital is measured by *capacity* to produce output. The conceptual idea underlying these authors' work is that "The appearance of better goods does not reduce the ability of existing goods to produce and therefore should not be allowed to affect capital input" (Denison [1972, p. 101]). This concept is implemented by requiring that any capital item be entered

[1]Denison [1974, 1979], Jorgenson [1980*b*], and Kendrick and Grossman [1980].

[2]Kendrick and Grossman [1980], Jack Faucett [1975], Mohr [1980], and Denison [1979]. It was used by the bureau of Labor Statistics in their capital stock series (U.S. Bureau of Labor Statistics [1979]). The authors of the official U.S. estimates use a gross concept (Young and Musgrave [1980, p. 31]). The new U.S. total factor productivity appears to use a gross or capacity to produce output concepts rather than an ability to produce income concept. [U.S. Bureau of Labor Statistics 1983.]

into the stock at its constant dollar cost and that it retain this value for its estimated life. It is usually recognized that capital equipment's ability to provide services declines over time due to wear and tear, and some adjustment is made for this.[3]

For studies of the determinants of investment, a surrogate for the new cost of desired capacity is wanted, and a gross measure of capital is most appropriate. Much confusion has been caused by failing to distinguish between capital as a surrogate for capacity and capital as the ability to contribute to output, measured by quasi-rents. In standard production functions (Cobb Douglas, constant elasticity of substitution, trans-log) which incorporate an assumption of perfectly malleable capital, there is no distinction between decline in output-producing capacity and ability to generate quasi-rents. However, in the real world there is such a distinction for fixed capital. Gross capital (sum of constant dollar cost) is usually a good surrogate for capacity to produce or just capacity. Capacity is not reduced (except following retirement) by the introduction of improved capital. Thus gross capital is appropriate in investment studies and growth models (Harrod–Domar type), where the product of required capacity and capacity cost is needed. It is also appropriate for factor substitution studies.

The basic measurement problem with the gross capital approach is simple. Different vintages will have equal weights which do not reflect the varying services of different vintages. As will be discussed further later, this statement of the problem implies that there has been embodied technical progress which was not picked up in the price deflation procedure. This implication is true for current deflation procedures and procedures that are practical enough to be adopted. It is not sufficient merely to call for improved price indices because the practical difficulties in obtaining such indices (not to mention the conceptual ones) are such that a solution is not to be expected. The gross capital approach implicitly assumes equal marginal products while with embodied technical progress, the more recent vintages will have higher marginal products (given cost-based price indices). This change in marginal product of the old capital stock from embodied technical progress in the new is obsolescence. That a gross capital concept does not measure changes in capital's service price or ability to produce value added was recognized by Denison [1972, p. 101; 1967, p. 135], although he apparently did not realize how this destroys the logical basis for his estimates of capital growth's contribution to output growth.

Suppose an aggregate production function is estimated using a gross capital concept. The marginal product of capital is then calculated from the production function. How is this to be interpreted? It is the increase in output from a proportional increase in all the components of capital stock, both the current vintage and the earlier vintages (see Green [1964], p. 83, Scott [1981]). To use the earlier example, the output increase would include the same expansion of old mechanical calculator services as it did of modern computer services. Since any realistic investment program would concentrate its new investment on the current vintage technology, the returns to investment would be seriously understated.

[3]Kendrick [1973] views this as insignificant and makes no adjustment. Denison [1967, p. 69, 1974, 1979] views it as significant, computing weighted averages of gross and net stocks.

Equiproportional expansion of all vintages creates no major problem *if* the value of services from a unit (dollar) of calculators equals that from a unit of computers. Then if investment is only in computers, the added output is still correctly estimated. With embodied technical progress the marginal product of the current vintage investment exceeds the marginal product of the existing gross capital stock, and the added output is underestimated. This problem has affected the work of most gross capital oriented authors.[4]

It will be argued that this difference in capital services is roughly proportional to that between a gross stock of capital and one net of straight-line depreciation. To indicate the importance of the issue, in 1975 total U.S. tangible business capital was 2.2 trillion 1972 dollars on a net basis but 3.4 trillion (Young and Musgrave [1980] p. 47) on a gross basis. A policy measure that generates an extra 50 billion dollars in investment expands the net capital stock by 2.3 percent, while it expands the gross capital stock by only 1.5 percent, producing a similar change in output growth. Thus whether the appropriate capital measure is closer to that gross or net of depreciation is important.[5]

The Logic of Growth Accounting

A standard procedure estimates the part of total growth attributable to capital as the product of the growth rate of capital and the share of income of capital.[6] Let us look at the economic logic behind this procedure (Wonnacott [1978, p. 481]). Total income from capital is total income multiplied by the share of capital in income. The price (or a measure of gross rate of return) of capital services is total income divided by the stock of capital. This is assumed to be equal to the marginal product of capital. For small changes in capital, this marginal product is assumed constant. The income increment is this marginal product multiplied by the quantity of capital added. The key to the procedure is the estimated marginal product of capital (or the gross rate of return). The concept of capital used should be one which when divided into income from capital gives a resonable rate of return estimate. Dividing by the original value of obsolete capital does not.

The impact of capital embodied technical progress is not so much on the size of increments to the capital stock as it is on the estimated marginal product of this increment (through the size of the capital stock used as a divisor). Failure to comprehend this led Denison [1964] to the incorrect conclusion that the

[4]Massell [1962], Denison [1967, 1974, 1979], Kendrick and Grossman [1980] and others.

[5]Fortunately, the U.S. rate of growth until recently was close enough to constant so that gross capital stocks and various net stocks have grown at similar rates (Young and Musgrave [1980, p. 47]), although the differences in other countries appear to have been larger (Denison [1967, p. 136–139]). The reason for the similarity of gross and net stocks is that in a growing economy the net capital calculations (deducting capital consumption evenly over its life) involve greater deductions for capital consumption than the gross capital calculations (which remove capital from the stock only when the capital is retired). For steady state growth, the differences in net investment (gross investment minus capital consumption) roughly offset the differences in the size of the capital stock to which they are being added, leaving the rate of growth relatively unaffected by capital stock definition.

[6]Solow [1957], Denison [1979], Kendrick and Grossman [1980], Jorgenson [1980].

embodiment question was unimportant. Failure to allow for embodied technical change (obsolescence) leads to an underestimate of the marginal product of all new investment, even that which itself has not benefited from technical change.

POSSIBLE SOLUTIONS

Logically, the inequality of the marginal products of a dollar of new investment and that of a dollar of gross capital stock could be corrected in two ways. One is to revalue the services of the existing capital stock by adjusting for its obsolescence following the introduction of new types of higher quality capital. This approach replaces the gross capital concept with one that is net of changes in the value of capital services due to obsolescence. The other approach is to adjust current investment for embodied technical change.

Some have recognized that the two approaches are similar conceptually.[7] However, Denison [1969, p. 15] argues that:

> "the choice of depreciation or replacement formula appropriate for measurement of changes in capital input has nothing to do with 'vintages,' that is, with the way one wishes to treat quality differences in capital goods that do not reflect a difference in costs and that result in 'unmeasured' quality change (or embodied 'technical progress') as time goes on. Use of a fast depreciation formula is not a method of making an allowance for unmeasured quality change."

Why he is wrong will become clear.

PROBLEMS WITH QUALITY ADJUSTING ONLY NEW CAPITAL

Let us start by considering the procedure of adjusting upwards the quality of the current vintage capital for the technical improvements embodied in new investments. This approach has been developed by Solow [1960, 1962, 1963, 1964]. Each item of capital continues to provide the value of services it provided when placed in service. However, when improved machines come into use they are considered as providing more capital services. The requirement that the marginal product of new investment equal the marginal product of the old capital stock is met by raising the value of new investment rather than lowering the value of old capital. Implementation of this approach requires estimates of the increases in the marginal product of new items of capital. These might be estimated from aggregate data.[8] They could be estimated for each item of capital

[7]Hickman [1965] and Hickman and Coen [1976] rationalized the use of a net capital estimate (unfortunately an exponential one which will be argued to be incorrect) as adjusting for improvements in the productivity of later vintages of capital. Sato [1975, p. 20] has argued for an efficiency corrected capital stock, and throughout the remainder of his discussion made clear that a major reason why firms differ in efficiency is technological progress embodied in capital. Tice [1967], Feldstein and Rothschild [1974], Boddy and Gort [1974], and Hall [1968] recognized the similarity between raising the value of newer vintages of capital for embodied technical change and lowering the value of old capital when embodied technical change makes the old obsolete. Bailey [1981, p. 30] draws a distinction between technical change embodied in new technology and obsolescence which he applies only to the product produced by the capital.

[8]As Solow [1962, 1963, 1964] and Intriligator [1965] did or from historical evidence on specific assets (as depreciation rates and lengths of life are).

directly, or estimated for a few items which would be used to compute a "quality adjusted" price index and a quality adjusted investment.

There are frequent calls in the literature for measures of capital goods that incorporate quality improvements. A new machine that provided twice the capital services of the older machine would be treated as representing twice as much capital (usually to be implemented through a decrease in the price index of capital goods). The example used by Gordon [1982] is a good one. The introduction of the jet plane might have been (but was not) treated as a reduction in the price of aircraft. Instead price indices for different models of planes were chained together on the basis of costs showing a price increase of 2.5 percent annually from 1957 to 1972 for aircraft. Gordon shows that incorporating quality adjustments changes this to a 7.1 percent annual decline. Let us look at the implications of successfully adjusting new capital for quality.

The major point of this paper is that estimating the contribution to output of investment is a systems problem in which choice of production function, choice of capital concept, choice of method of adjusting capital prices for quality changes, treatment of obsolescence, and methods of estimating service decline (i.e. depreciation) must be consistent. These problems are all too often discussed separately in the literature and solutions adopted that imply inconsistent treatments of obsolescence.

If capital quality is incorporated in the price indices, the concept of capital used must be a gross one excluding obsolescence. Service decline rates must exclude the obsolescence included in the usual depreciation formulas based on historical length of life data. Likewise, obsolescence must be incorporated in service decline rates when not in price deflators (the usual case). Service decline rates and price deflators may not be consistent if estimated separately or taken from different sources. One can either spread the decline in capital services over the estimated life, incorporating embodied technical progress (usually assumed to continue at the historical rate), or directly estimate embodied technical progress and incorporate it in additions to the stock. One cannot do both. To reduce the old capital quantity while raising that of the new is to make the adjustment for technological change twice.[9]

It has often been presumed that since current capital goods price indices do not include many quality improvements, the incorporation of such improvements should raise the growth rate of capital. This is so only if the obsolescence assumptions for old capital are not changed. Logically, adjustment for new capital's embodied technical progress should eliminate the obsolescence deductions for old capital, leaving the direction of the change unknown. (For an example of changing the capital price index without making the required changes in the service decline rates see Jorgenson and Griliches [1967]).

This is a powerful argument against making partial adjustments for quality change in capital goods price indices. To do so partially leaves one unable to

[9]This appears to be a problem with Jorgenson and Griliches' [1967] early work. They used a double declining balance capital stock which would normally yield a net capital stock. However, they also adjusted for improved capital quality with a price index that incorporated embodied technical progress (which they argued might be the durable goods part of the consumer price index). This procedure counts embodied technical progress twice. This permitted them to explain most economic growth as due to capital accumulation.

adjust the old capital stock for obsolescence, since it is not known how much of the potential obsolescence adjustment is already in the quality adjustment.

To estimate a capital stock using an arbitrarily chosen rate of embodied technical progress which is then used to estimate the rate of technical progress is to expose oneself to the charge of assuming what is to be estimated, namely the rate of technical progress. Yet the same assumption of a rate of technical progress is made when a capital stock is computed assuming a continuation of the historical rate of obsolescence (which often takes the form of assuming that the sum of obsolescence and wear and tear occurs at the historical rate or even the rate built into tables of tax lives).

There is a fundamental uncertainty principle in the study of technological progress (with the standard "residual" methods) in which estimates of technical change require estimates of the capital stock, but determining the contribution to output of the old items of capital requires knowing how rapid technical progress has been and how much is incorporated in the new items of capital. Capital stock cannot be measured without knowing technical progress and technical progress cannot be measured without knowing the capital stock. (For more detail on this argument see Miller [1983a, 1983b]).

Obsolescence and Changing Weights for Old Capital

Most proponents of adjusting new capital for embodied technical change have implicitly assumed that the relative weights of old capital could be left unchanged. Unfortunately, only in limited circumstances can a machine earn the same rent as technology changes. A successful innovation lowers production costs. Under most realistic circumstances (including the competitive or cost minimizing assumptions usually employed in the theory of production), lowering production costs either lowers the price of the product or bids up wages and other input prices. Some growth theorists prefer to use models where wages rise steadily (Solow [1970], Chapter 3). Where there are many sectors each with different rates of technical progress, much of the adjustment must occur through relative prices if wages are to remain the same in different sectors. Salter [1960] has shown that adjustment normally occurs through prices. Thus, this exposition will work with a single industry in which technological progress lowers product prices leaving factor costs unchanged.

Consider a simple example where the current vintage machine (let us call it A) has total costs of $10 with variable costs of $7. An earlier vintage (B) has variable costs of $8. The rents will be $3 for the new vintage and $2 for the old. The correct aggregation is to treat the old capital as two units and the new capital as three units. With this weighting the marginal product of new investment will be equal to the marginal product of old capital.

Suppose newer technology causes the price to drop to $8.25 (the new marginal cost). This causes the older machine's rent (B) to drop to $.25 and that of A to $1.25. The ratio must be one to five (instead of the old two to three) for the aggregated capital's marginal product to equal the marginal product of the newest capital (which is the policy relevant type). A system based on time of introduction rents (or the closely related original costs) gives correct conclusions only by chance when technology changes. If technology changes required inputs, one cannot aggregate different vintages with fixed weights.

If the reader prefers not to imagine product prices changing, similar results occur in a process model where the addition of capital using a new process lowers the shadow prices of the old capital.

The problem shown above is an illustration of the result from aggregation theory[10] that different items of capital can be aggregated only if all other prices remain constant when one item of capital is substituted for the other. The problem is not merely that prices normally change over time and that the required assumption will be false. If that were the only problem, one might hope that price changes would be small enough for fixed weight aggregation to be reasonable, even if not precisely correct.

Unfortunately, as the simple example illustrates, the very process of technical change lowers the prices of the products produced (and sometimes changes the prices of labor, land, and other cooperating factors) in such a way as to prevent the use of constant weights for the old vintages of capital.

Improved machines that only save (augment) capital are rare. Most technical change saves variable inputs such as labor and energy. This is the seldom discussed (because mathematically awkward) case of capital embodied but labor or energy augmenting change. This author cannot think of even a single important purely capital augmenting change. Any such innovation would have the same outputs and inputs as an earlier model but less capital. It would have the same effects as a price reduction (Jorgenson [1966]).

Even capital augmenting technical change between the last two years may not permit fixed weight aggregation if there have been previous generations. Any cost reducing technical change reduces product prices, creating the above difficulty. It is just as possible for a capital saving innovation to reduce cost as it is for a labor saving one to do so. One might imagine a third generation innovation that permitted a capital good to be made from cheap plastic instead of the metal used in the second generation, but which was otherwise identical. This would be considered the same quantity of capital as the previous vintage. However, it would lower the price of the final product and change the relative weighting of the earlier vintages that are aggregated into the capital stock. The first generation might even fall to a negative rent and be dropped from the capital stock, going from a postive to a zero weight. Fixed weight aggregation is correct only if all technical change has been capital augmenting from the oldest machine's introduction.

This is not to say that a capital stock cannot be constructed using a fixed weight system (or even that such a measure may not be useful in some way[11]),

[10]Hall [1968] has reexpressed the aggregation conditions to require that relative rents be independent of the wage. His wage is measured in consumption goods, letting us restate his condition as requiring a constant price-wage ratio. Although one might hope that dollar wages in an industry would remain approximately constant as technical change occurs, such technical changes will change dollar prices destroying the constancy of the price-wage ratio. The inability to use fixed weights in aggregating capital is not surprising since capital aggregates exist only if capital embodied technical change affects all old capital proportionately (Green [1964, p. 92]) or if all technical change is capital augmenting (Fisher [1965], Diamond [1965]).

[11]Denison's [1957] approach evolved as a long run index number and is appropriate for that even if not for growth accounting.

but that the value of marginal products of different items in the stock will no longer be equal to each other or to the returns from new investments. This limits the usefulness of the capital stock for predicting the return to new investment or estimating the amount of growth that resulted from investment in any particular year (as is done in growth accounting exercises).

THE NEED TO REVALUE OLD CAPITAL FOR OBSOLESCENCE

Fortunately, there is no logical need to construct fixed weight capital stock measures, or even to compare the quantities of capital at different times in estimating the returns to new investment or in growth accounting. The contribution to output of incremental investment at any given time and technology requires only estimating the marginal product of capital and the quantity of investment. As long as investment is small enough to assume constant prices, the different items of capital can be aggregated using their marginal products (which imples using the current technologies, relative prices, and presence of cooperating or competing items of capital) and a meaningful contribution of investment to growth calculated. Any observed output growth in excess of capital growth's contribution can be attributed to the residual or technological change. While this can be done for any year,[12] and the allocations compared, the relative weights given to different vintages will be different each year reflecting the relative price changes from introduction of improved capital equipment. One must revalue old capital for obsolescence.[13] The intuitively attractive idea of thinking of a particular piece of equipment as always representing the same quantity of capital must be abandoned.

There is one formula that keeps relative weights constant. This is exponential decay, or quality improvement where each vintage is treated as being a constant proportion of the later vintage. It has been much used in the growth literature[14] and in the productivity literature, especially by Jorgenson and his associates.[15] It obviously does not estimate the gross capital stock (i.e. one not adjusted to incorporate obsolescence) required if capital embodied technical progress is to be incorporated via quality adjusted price indices. Unfortunately, as will be seen, the exponential formula is not generally correct where normal capital embodied labor (or other variable cost) saving technical progress occurs, even if exponential growth is occurring.

[12]In continuous growth models one can make the additions to capital arbitrarily small by making the time period arbitrarily small; one can make the difference in beginning and ending period prices arbitrarily small by choosing a sufficiently short period. With no indivisibilities, the technique proposed works precisely. Where investments are made in large discrete lumps, appreciable price differences before and after the investment may exist, leaving a capital index number problem.

[13]A similar proposal has been made by Rymes [1971, 1980, p. 59]. Using a somewhat different argument, Hicks [1965, 1973] has argued that technical progress forces a revaluation of the older vintages of capital, a conclusion which he views as inconsistent with the neoclassical idea of capital.

[14]Solow [1960, 1962, 1963, 1964], Samuelson [1961].

[15]Jorgenson [1980], Christensen, Cummings, and Jorgenson [1980], Fraumeni and Jorgenson [1980], Gollop and Jorgenson [1980], Jorgenson and Griliches [1967, 1972].

It has been argued that obsolescence logically must be deducted in calculating the size of the capital stock especially with the available price indices, which are not quality adjusted. The next question is the shape of the obsolescence curve. Factor prices will be assumed to be constant, implying that labor and materials are used in constant proportions. This is made even more plausible by the fact that most machines have roughly constant labor and materials requirements, and these cannot vary much even when factor prices change (putty-clay technology). This implies fixed variable costs V.

Let P be the price of output. This is not fixed but is set in a competitive market where the price is equal to the marginal cost (including a normal return on capital) of production by the latest vintage machine (Salter [1960] gives a good exposition). This price declines steadily as new and improved machines are introduced. The quasi-rent R is equal to $Q(P-V)$ where Q is output. Scrapping occurs when the quasi-rent falls to a normal return on scrap value. For simplicity, in the remainder of the discussion scrap value will be taken to be zero, causing scrapping at zero quasi-rent.

The problem is to distribute this quasi-rent decline over the machine life. Economic theory gives us two points. At purchase the quasi-rent is equal to a normal return on the purchase price plus depreciation. On scrapping it is zero. With no technological regress and a growing economy, there will normally be steady quasi-rent decline, requiring a monotonically declining function.

One possibility is the simplest possible function, connecting the two known points with a straight line. This gives a value for quasi-rent proportional to the machine value using straight line depreciation. This is a great labor saver since the Bureau of Economic Analysis publishes capital stock estimates incorporating straight line depreciation and quite elaborate calculations have been done using different service lives and retirement rates for different items, beyond what most outside investigators could accomplish (Young and Musgrave [1980]). Thus, a researcher using U.S. government capital stock figures should use the net, not the gross capital stock.[16]

It should be noted that use of net figures calculated with straight line depreciation is a computational convenience; the argument does not depend on any assertion that depreciation is necessarily straight line. Indeed, if quasi-rent declines in a straight line manner, depreciation will not be straight line, since the value of the machine is the present value of its future quasi-rents. Straight line service decline and net income calculation using straight line depreciation are inconsistent. Those doing growth studies and seeking to achieve perfect consistency will have to deal with this inconsistency somehow. (See Jorgenson and Griliches' [1972] comments on Denison's work.)

An assumption much beloved by growth theorists and other economists is constant growth. Assume that technical progress is occurring at a steady rate such that the output of new machines (using the same amounts of all factors of production including capital) rises constantly at $x\%$ per year. With the competi-

[16]Similar net capital values are available for many foreign countries. See Hibbert, Griffin, and Walker [1977] for the United Kingdom and Blades [1980].

tive economy assumed, this implies that prices decline by $x\%$ per year. This gives immediately a formula for the quasi-rent of $Q\ (P_0(1-x^t)-V)$ where t is time, and P_0 is the price at the time of introduction of the machine. In exponential form the equation is $R=Q\ (P_0e^{-xt}-V)$. Notice that steady technical progress does not lead to exponential obsolescence of the capital stock. As long as V is positive, the above formula provides for a rate of decline of the quasi-rent that is faster than given by exponential decay. Thus, the theoretically neat and computationally convenient exponential formula lacks realism in a model with capital vintages. In neoclassical putty-putty models one can use exponential decay by assuming capital augmenting technical progress at a constant rate. However, this is inconsistent with labor augmenting but capital embodied technical progress (automation), or with the real world's limited factor substitution after a machine has been built.

The $Q(P_0e^{-xt}-V)$ formula gives a curve that is close to a straight line for situations where the capital costs of the machine are only a small fraction of the total costs. This is likely to be the typical situation for most machines. Even at an economy wide level, capital receives only about a quarter of total factor income after netting out intermediate products. For the typical industry, materials are the largest expense, making capital relatively minor. Within a particular firm, materials frequently flow from machine to machine making the capital costs an even smaller fraction of the costs of production for any particular machine. Thus, a straight line is frequently a reasonable approximation.

Choice of Base Year

Let K_{mv} be the number of machines of type m and vintage v. As was argued earlier, it is necessary that the marginal products of investment and capital be equal, which means that dY/dK_{mv} must be equal for all m and v. Let us separate this condition into two parts. One is that all vintages of the same type of machine have the same marginal product. This requires that $dY/dK_{m1}=dY/dK_{m2}=\ldots=dY/dK_{mv}$ for all m. This condition alone could be met either by treating the newest vintages as being more capital (lowering its price) or by treating the earlier vintages as being less capital by a revaluation for obsolescence.

The other necessary condition is that different types of investment goods of the current vintage (say airplanes and computers) have the same marginal product per dollar of capital. Otherwise, the growth increment from investment depends on the types of investment made. This is to say $dY/dK_{11}=dY/dK_{21}=\ldots=dY/dK_{m1}$. The 1 for vintage represents the current one, which would be used for any new investment. This condition requires that our measurement system equate the marginal products of *current* vintages for different machines (assuming the price of a dollar of capital to be one dollar). If earlier vintages are properly related to the current vintage for each machine type, earlier vintages of different machines will also have equal marginal products (per dollar of current value, not per dollar of original cost). By making the current year the base year one can satisfy both cost equality for currently produced capital, and marginal product equality between vintages.

National income accounting systems usually use a base year which is earlier than the current year. Everything is then expressed in the prices of that year (currently, in the U.S., 1972). This implies that in a world where each type of machinery and each industry undergo different rates of technological progress, all types of capital will not have marginal products proportional to price.

CONCLUSIONS

In estimating a capital stock for use in a production function or for use in productivity studies, the best measure will be net of both obsolescence and deterioration. In the presence of technical change, it will not be practical to construct measures of the capital stock using fixed weights for different vintages without violating the condition that marginal products be equal. This requires the keeping of separate accounts for each vintage, with the quantity of capital of each vintage decreasing as technical progress makes it obsolete. The unavailability of price indices reflecting embodied technical progress leaves reducing old capital for obsolescence as the only practical approach. The necessity for assuming a rate of embodied technical progress in order to calculate a capital stock to measure the rate of technical change is a serious problem. Exponential declines in value due to obsolescence will occur only by accident. Constant rates of technical progress lead to the value of the capital stock being $(1-x)^t - V$ where x is the percentage rate of embodied technical progress reflected in product prices, t is time, and V is the level of variable costs. Straight line service decline will often be a reasonable approximation.

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