A COMPARISON OF DISTRIBUTIONS OF ANNUAL

BY N. S. BLOMQUIST*

Stockholm, Sweden

The concern with income distribution has always mainly existed because of a concern with individuals' economic welfare. In recent years, the question has arisen whether the distribution of annual income—the distribution most often studied—is the best proxy for the distribution of economic welfare. Other measures, such as lifetime income, have been proposed instead.

The paper starts with a discussion of how to define and measure the distribution of lifetime income. By using a simulation model, which partly consists of estimated functions and partly of tax functions taken directly from tax laws, distributions of lifetime income, variously defined, are then constructed. These distributions are compared with each other, and with distributions of annual income. The simulations indicate that the distribution of lifetime income is considerably less unequal than the distribution of annual income. Whether inheritances are included or not seems to be of no importance for the inequality of lifetime income. If, on the other hand, we include the value of leisure time in lifetime income, inequality increases by about 10–15 percent. Distributions of income after tax have Gini coefficients which are approximately 25 percent less than the Ginis for the before-tax distributions. We thus find that the picture of inequality we get is very much dependent on which income concept we use.

For quite a long time many income distribution researchers have been dissatisfied with annual income as a measure of economic welfare. They argue that a large part of observed inequality in annual incomes depends on people being at different stages in their life cycles, and that people have different time preferences. Inequality due to these factors washes out if a longer time horizon is used. Lifetime income has therefore been suggested as a better measure of economic welfare, as this is an index of people’s long run opportunity sets. We will not discuss in this paper whether lifetime income really is a better measure of economic welfare than annual income. Instead we will investigate whether the picture of inequality differs much between the two concepts. After all, if the picture of inequality does not differ much between the two concepts, then it does not matter much which concept we use.

The first attempt to calculate something that can be called lifetime income was done by Farr (1853). The purpose of his study was to put a money value on a man; these calculations were used by the life insurance companies. Most of the early lifetime income studies, like Walsh (1935), Clark (1937), Friedman and Kuznets (1945), and Miller (1960), calculated lifetime income for various

*University of Stockholm and the Economic Research Institute at the Stockholm School of Economics. This paper builds on chapter 7 of my Ph.D. dissertation. I am indebted to an anonymous referee for helpful comments.

1See, for example, Weizsäcker (1978). The discussion of Weizsäcker’s paper, in Krelle and Shorrocks (1978) shows that there are also opponents to this view.

2A number of studies, such as Taussig (1973), Nordhaus (1973), Blinder (1974), Browning (1976), Habib et al. (1977), Shorrocks (1978) and Wolfson (1979), have studied various aspects of how inequality measures change as the income concept is varied.
socio-economic groups. Summers (1956), Solton (1965), Nordhaus (1973), Blinder (1974), Layard (1977), and Lillard (1977) are examples of recent studies where size distributions of lifetime income are calculated.3

In section I of this paper we discuss how to define lifetime income, and briefly describe two methods to calculate distributions of lifetime income. The method used in this study is to simulate income paths for a sample of individuals. The structure of the model used for this purpose is presented in section II. In section III we give detailed definitions of the lifetime income concepts we use and report the simulated distributions. The distribution of annual income and the validity of the simulation model is discussed in section IV. In section V we compare the various distributions simulated. We conclude the study in section VI. The symbols used in the paper are explained in appendix A, and appendix B presents the estimated functions that form part of the simulation model.

I. How to Define and Measure the Distribution of Lifetime Income

The “ordinary” way to define lifetime income is

\[ L0 = \sum_{t=1}^{T} a_t e^{-rt} \]

where \( a_t \) is annual income (capital income included), \( r \) is rate of interest and \( t \) an index for time. If we want our measure to index the opportunity set (the lifetime resources), then this income concept can be improved upon in several respects. First, capital income should not be included in the index. Saving, which results in capital income at a later date, is just a means of redistributing consumption possibilities over the lifetime and does not increase the lifetime resources.4 To include capital income, as done in \( L0 \), is therefore a form of double counting. Second, in \( L0 \) each period’s income is dependent on the individual’s choice between leisure and consumption. Some full income concept should be used instead, so that the value of leisure time is included in the income measure. Third, the present value of inheritances and gifts ought to be included in a measure of lifetime resources.5

The type of income concept we will use in the present study is

\[ L1 = \sum_{t=1}^{T} w_t \cdot 365 \cdot h \cdot e^{-rt} + B \]

where \( w \) is the hourly wage rate, \( h \) the average hours per day that can be spent on market and leisure activities, and \( B \) the present value of inheritances. Since the value assumed for \( h \) might be crucial for our empirical work, we will try

3Articles like Creedy (1977), Moss (1978), Creedy and Hart (1979) and Graham and Webb (1979) also contain valuable discussions of lifetime income issues.

4This statement is true if we have perfect capital markets. If capital markets are imperfect, so that the interest rate varies with the wealth of an individual, then a more complicated index than the one used in this paper is needed.

5See Blomquist (1977) or Archibald and Donaldson (1977) for a discussion of how to index an individual’s opportunity set.
several alternative values. In most cases we will assume that \( h \) equals 16. This implies that we assume that the individual must sleep and take care of his body 8 hours a day in order to be able to work and consume other forms of leisure efficiently, and that “maintenance” leisure gives no utility. The measure \( L_1 \) is the analogue to Becker’s (1965) full income concept and represents the maximum present value the individual can spend on consumption goods.

The measure \( L_1 \) takes account of the three types of desired improvements mentioned above and should therefore constitute a better measure of economic resources than \( L_0 \). The concept \( L_0 \) is dependent on both the individual’s resources and his preferences, so people with equal resources might have very different \( L_0 \)’s and people with very different resources might have the same \( L_0 \)’s. The concept \( L_1 \) is only dependent on the individual’s resources, as defined by wage rate path and inheritance.\(^6\)

We now know what type of income concept we would like to study. The question then arises how the distribution of income, using a lifetime income concept, can be calculated. There are, in principle, two possible methods one can use. One is to collect historical data, tracing individuals’ income paths for a sample of persons, and from these data compute a distribution. This method implies great difficulties in obtaining data. Another method is to construct a model of the income generating mechanism and use this model to simulate lifetime incomes for a sample of individuals. Most earlier studies have used the latter method, and it is the method adopted in this study.

An important question is for what sample of people distributions of lifetime income should be constructed. Should one or several birth-year cohorts be used? If several cohorts are used the overall distribution of lifetime income will be dependent on the age distribution of the sample. For this reason, but also to simplify the analysis, I have chosen to use one cohort only. The simulations are thus done for a representative sample of 381 Swedish males born around 1945.

II. THE SIMULATION MODEL\(^7\)

To simulate the paths of incomes I have used a model of how individual incomes are generated. To construct the model a wage rate function, a labor supply function, an asset function, and an inheritance function have been estimated.\(^8\) Together with tax functions based on prevailing tax laws these functions constitute the income model. Given the vector of exogenous variables characterizing an individual, this model can be used to simulate annual and lifetime incomes.

\(^6\)If the wage rate path to a large extent is determined by the individual’s tastes, the measure \( L_1 \) should be further refined. It might, however, be that the individual has little influence on variables such as innate ability and social background variables which determine the wage rate. If the schooling system works as a filter so that a person continues in school till he is sorted out, one might argue that even the educational variables are to a large extent determined by factors exogenous to the individual.

\(^7\)The description of the simulation model is very brief. Readers wanting a more detailed description are referred to Blomquist (1976).

\(^8\)These functions are reported in appendix B. A detailed description of the functions can be found in Blomquist (1976). The inheritance function is also described in Blomquist (1979a) and the wage rate function in Blomquist (1979b).
income defined in various ways. Exogenous variables in the model are such variables as education, physical age, number of years of on-the-job training, and social background variables. Since the model is used to simulate income paths stretching well over 50 years into the future the exogenous variables must all be such that they are constant over time or change in an easily predictable way as, for example, physical age does.

A. Estimation of the Income Model

To estimate the model, which is partly non-linear in the parameters, the maximum likelihood method has been used. The model segments into four parts in such a way that each function can be estimated separately. The model was purposely specified in this way, since computer costs otherwise would have been prohibitively high. For three of the functions the method used is equivalent to GLS-estimation. For the fourth—the inheritance function—a Tobit model is used.

Two cross-sections, 6 years apart in time, have been used to estimate the functions. The two cross-sections consist of the same individuals. The data originates from two Swedish surveys known as the “Level of Living Surveys,” conducted in 1968 and 1974. This data source contains data for approximately 6,000 people, both males and females, and is designed so as to be representative of the Swedish population. The functions are estimated for employed males only, which reduces the sample size. Due to missing variables for some observations the actual number of observations used differ between functions, but are for most functions between 1,000–1,500. (Exact figures are given in appendix B).

The model is used to predict income paths stretching well over 50 years into the future. This means that the paths must be interpreted as predictions of how the paths would be if the structure of the Swedish economy would remain as it was around 1970. The predictions are, however, not simple cross-section extrapolations. Since two cross-sections have been used for the estimation, “smooth” changes of the economic structure should have been captured by the model. The wage rate function, for example, contains both age (cohort), experience (on-the-job training) and time productivity effects. The last effect could not have been estimated had only one cross-section been used. It has also been possible to estimate the structure of the random terms in a more detailed way than is possible with cross-section data from only one point in time. When predicting income paths it is important to know as much as possible about the structure of the random terms.

B. Structure of the Random Terms

A typical estimated function can be written as

$$y^i_t = f(X^i, u^i_t),$$

where $y$ is the dependent variable, $X$ a vector of independent variables and $u$ a random term. The index $i$ runs over all individuals and the index $t$ takes on

---

9See Blomquist (1976) or Johansson (1970) for a detailed description of this data source.
two values, say 1 for 1968 and 7 for 1974. The superscript $j$ denotes function. The random terms are assumed to be normally distributed and, in general, uncorrelated with each other. However, it is assumed that

$$E(u^j_{it} \cdot u^j_{it}) = \sigma^2_j$$

and

$$E(u^j_{i1} \cdot u^j_{i7}) = k_j.$$

The correlation between $u^j_{i1}$ and $u^j_{i7}$ can result from several possible random mechanisms. We will here consider two possible ones. For convenience we omit the superscript $j$. Consider the two specifications (1) and (2).

(1) $u_{it} = v_i + \varepsilon_{it}$

where $v_i$ and $\varepsilon_{it}$ are independently distributed, and $E(\varepsilon_{it} \cdot \varepsilon_{ir}) = 0$ for $i \neq j$, or $t \neq \tau$, or both.

(2) $u_{it} = \rho \cdot u_{i,t-1} + \lambda_{it}, \quad E(\lambda_{it} \cdot \lambda_{ir}) = 0$

for $i \neq j$ or $t \neq \tau$, or both.

Both models can be interpreted to mean that the random term contains a component specific to the individual, reflecting some personal traits. In model (1) where this component, $v_i$, stays the same for all $t$, the component might, for example, reflect great industriousness, optimism, or some other deep seated personal trait. In (2) the component specific to the individual changes over time. This can be interpreted to mean that the effect of certain personal traits tapers off over time. Meanwhile new personal traits are developed.

For both models above we know that $E(u^j_{it} \cdot u^j_{i,t+\sigma}) \neq 0$. With the data at hand we cannot, however, discriminate between the two models. In the simulations we will therefore try both. As will be seen in the next section, the choice of model is of importance for the inequality of the simulated distributions. Most of the reported simulations are done with random terms generated according to (2).

C. Overview of Simulated Income Concepts

The income model can be used to simulate both annual and lifetime income. The various income concepts simulated are categorized in table 1. Since we are interested in the inequality of income, these distributions will be characterized by their Gini coefficients and coefficients of variation. Most of our comparisons of the distributions will be in terms of the Gini coefficients.

10 Both models can be considered special cases of a more general model. The parameters of this model can, however, not be estimated with only data from two points in time. See Blomquist (1976) pp. 125–128 for a detailed explanation of this.
of variation. Most of our comparisons of the distributions will be in terms of the Gini coefficients.

III. The Distribution of Lifetime Income

A. Distributions of Lifetime Income before and after Tax

We start by defining the lifetime income concept before tax where the value of leisure time is included. We define this as

\[ LB1 = \sum_{\tau = 16}^{q} e^{-\tau} w_\tau \cdot (z_\tau - 2,344) + \sum_{\tau = q+1}^{78} e^{-\tau} \cdot w_\tau \cdot z_\tau \]

where \( w_\tau \) is the hourly wage rate in year \( \tau \) and \( r \) is taken to be the real market rate of interest, which we assume is 3 percent a year. The index \( \tau \) can here be interpreted as the person's physical age, \( z_\tau \) is the sum of leisure and working hours per year, and \( q \) is the point of time when the individual quits school. Except for the exclusion of inheritance, lifetime income defined in this way corresponds to index \( L1 \) described and discussed on p. 244 above.

We start to sum the incomes from the individual's 16th year of age, because that is the age at which the first individuals start to earn income. We end the summation at age 78, because that was the life expectancy for males 25 years old in 1970. Some people do not start to work at age 16, but study instead. The age at which people start to work is estimated as 7, the age at which people normally start school, plus the number of years of education they reported in 1974, plus one year for the military service. From the date at which the individual starts to work the wage rate is calculated by the estimated wage rate function.

In the original study, on which this paper builds, income concepts including the value of inheritance were studied. It was found that including inheritance in the lifetime income concept had a very small impact on the degree of inequality. In order to simplify the exposition in this paper I have therefore chosen not to report distributions of lifetime income where inheritances are included. Some of the major findings about these distributions will, however, be reported in the two concluding sections.

In Blomquist (1976, 1977) the distributions are reported in more detail. The decile shares, and the shares for the top five and one percent of the income holders are reported.

The Gini coefficient has been adopted in this study mainly because it for a long time has been the most frequently used measure of inequality. Using the Gini coefficient facilitates comparisons with other studies.

The role of the choice of interest is discussed on p. 251 below.

To be exact the expected remaining lifetime for 25 year old males was 53.38 years in 1970 according to the Swedish National Central Bureau of Statistics.
Before that date we impute to the individual the average wage rate of those of the same age who are working. Ideally one would like to use estimates of the individual’s shadow price of time \( (w^*) \) instead. However, since the schooling decision for many individuals is determined by their parents the shadow price of time may be less than, or greater than, the individual’s market wage. Thus it is very hard, if not impossible, to get unbiased estimates for \( w^* \). For convenience I therefore use the average wage rate of those working.\(^{16}\)

In most of the simulations I set \( z_r = 5,844.17 \) I have arrived at this arbitrary constant by assuming that the individual must sleep and take care of his body eight hours a day in order to be able to work and consume leisure efficiently. This leaves 16 hours a day for work and leisure activities. If we take the average number of days per year to be 365.25, this gives us the stated value for \( z_r \). I assume that education takes 2,344 hours a year and gives no present income or utility. Thus during years of schooling we deduct 2,344 from \( z_r \).

Lifetime income exclusive of the value of leisure time is defined as

\[
LB2 = \sum_{\tau = 16}^{78} e^{-r\tau} \cdot w_\tau \cdot K_\tau
\]

where \( K_\tau \) is obtained from the estimated labor supply function. The wage rate is set at zero during the schooling period and is estimated by the wage rate function for the other years. \( LB2 \) is thus the present value of the before tax earnings during the lifetime.

We next define lifetime income after tax. We start by defining the concept \( LA2 \), i.e. lifetime income where the value of leisure time is excluded. \( LA2 \) is defined as

\[
LA2 = \sum_{\tau = 16}^{78} e^{-r\tau} \cdot \bar{y}_\tau
\]

where \( \bar{y}_\tau \) is earnings after tax in year \( \tau \). Earnings after tax is among other things dependent on gross annual income in year \( \tau \), which is defined as

\[
a_\tau = r \cdot A_\tau + w_\tau \cdot K_\tau
\]

where \( A \) denotes financial assets. To get \( \bar{y}_\tau \) we compute \( a_\tau \) and \( \bar{a}_\tau \), the after tax income, by using the estimated functions and the tax functions. Earnings after tax are then taken to be \( a \cdot y \), where \( a \) is the ratio \( \bar{a}(\tau)/a(\tau) \). That is, we assume that the average tax rate is the same for earnings as for total income.

After tax lifetime income where the value of leisure time is included is defined as

\[
LA1 = \sum_{\tau = 16}^{a} e^{-r\tau} \cdot \bar{w}_\tau \cdot (z_\tau - 2,344) + \sum_{\tau = a + 1}^{78} e^{-r\tau} [\bar{y}_\tau + (z_\tau - K_\tau) \cdot \bar{w}_\tau]
\]

where \( \bar{w} \) is the marginal wage rate after tax. During the schooling period \( \bar{w}_\tau \) is set equal to the average wage rate after tax of those who are working.

\(^{16}\)The question how leisure time should be evaluated is one much discussed. See, for example, Nordhaus and Tobin (1972), Adler and Hawrylyshyn (1978), Eisner (1978), and Murphy (1978) for various approaches, within the national accounting framework, to how non-market activities, such as household work, should be evaluated.

\(^{17}\)In the next subsection we will see how sensitive the results are to this choice of \( z_r \).
In table 2 the Gini coefficient and the coefficient of variation are shown for distributions of lifetime income before and after tax, defined as above. We will later compare these figures with the corresponding figures for annual income. Here we limit the discussion to a comparison of the lifetime income distributions with one another. Comparing $LB_1$ and $LB_2$ we see that including the value of leisure time in the lifetime income measure increases inequality by about 15 percent. A comparison of $LA_1$ and $LA_2$, the corresponding after tax income concepts, shows also that inequality is about 15 percent higher for the income concept where the value of leisure time is included.

Comparing the distributions of lifetime income before tax with the distributions of lifetime income after tax we see that the Gini for $LA_1$ is 27 percent lower than the Gini for $LB_1$, and the Gini for $LA_2$ is 26 percent lower than the Gini for $LB_2$. The after tax distributions are considerably more equal than the before tax distributions.

B. Sensitivity Analysis

The results presented above depend on how the simulation model is specified. In this subsection we will see how the results change as we vary some key assumptions. We will do three types of variations. Firstly, we will vary $z_r$, i.e. the amount of time used to compute “full” lifetime income. Secondly, we will make variations in the rate of interest. Thirdly, we will change the random mechanism, which generates the random terms.

In table 3 are shown the Gini coefficient and the coefficient of variation for the income concepts $LB_1$ and $LA_1$ for three values of $z_r$. From the table we

<table>
<thead>
<tr>
<th>Income Concept</th>
<th>Gini Coefficient</th>
<th>Coefficient of Variation</th>
<th>Mean Value in Skr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB1 ($z = 2922$)</td>
<td>0.133</td>
<td>0.241</td>
<td>1,666,125</td>
</tr>
<tr>
<td>LB1 ($z = 5844$)</td>
<td>0.139</td>
<td>0.251</td>
<td>3,402,680</td>
</tr>
<tr>
<td>LB1 ($z = 8764$)</td>
<td>0.141</td>
<td>0.255</td>
<td>5,139,266</td>
</tr>
<tr>
<td>LA1 ($z = 2922$)</td>
<td>0.096</td>
<td>0.173</td>
<td>1,001,589</td>
</tr>
<tr>
<td>LA1 ($z = 5844$)</td>
<td>0.102</td>
<td>0.184</td>
<td>1,903,470</td>
</tr>
<tr>
<td>LA1 ($z = 8766$)</td>
<td>0.105</td>
<td>0.188</td>
<td>2,805,378</td>
</tr>
</tbody>
</table>
see that the mean value of lifetime income increases quite rapidly as \( z_r \) is increased. However, the inequality of the distributions does not change much. We see that whether \( z_r \) is 5,844 or 8,766 does not matter much for our measures of inequality. When \( z_r \) is 2,922 the measures of inequality for \( LA_1 \) are about 6 percent lower than when \( z_r \) is 5,844. For \( LB_1 \) the inequality measures are 4–5 percent lower when \( z_r \) is 2,922. Since inequality does not vary much even when we make rather large variations of \( z_r \), I report in the rest of the paper only distributions with \( z_r = 5,844 \).

For the simulations presented in section III.A I have used an interest rate of 3 percent. This is an approximate unweighted average of the historical real rates of return in Sweden for shares and bank accounts.\(^1\)\(^8\) To check how sensitive the simulated distributions are to this choice of interest rate, I have also done simulations with the interest rate set at zero and six percent, respectively. The results of these simulations are presented in table 4. Only the Gini coefficient

<table>
<thead>
<tr>
<th>TABLE 4</th>
<th>DISTRIBUTIONS OF LIFETIME INCOME AT ALTERNATIVE RATES OF INTEREST (( r ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income Concept</td>
<td>( r = 0.00 )</td>
</tr>
<tr>
<td>LB1</td>
<td>0.158</td>
</tr>
<tr>
<td>LB2</td>
<td>0.140</td>
</tr>
<tr>
<td>LA1</td>
<td>0.120</td>
</tr>
<tr>
<td>LA2</td>
<td>0.106</td>
</tr>
</tbody>
</table>

is reported. As seen from the table, the choice of interest rate does have a significant effect. In general the Gini coefficients are about 15 percent higher for the distributions where we use a 3 percent rate of interest than for the distributions where we use a 6 percent rate of interest, and are still an additional 15 percent higher when we use a zero rate of interest. However, if we compare the relative inequality of \( LB_1, LB_2, LA_1 \) and \( LA_2 \) for a given rate of interest, we get roughly the same results independently of what interest rate we use.

In the simulations reported so far we have assumed that the random terms are generated according to model (2), presented on p. 247. Since we have no information whether random mechanism (2) is closer to reality than (1), it is of interest to see how our results change if we use model (1) instead of (2) when generating the random terms in the simulations. The distributions obtained when we use (1) to generate the random terms are given in table 5. To facilitate comparisons, the corresponding distributions using (2) are reproduced as the last three columns in the table.

The distributions generated according to (1) seem to be somewhat more unequal than those generated according to (2). The qualitative conclusions we

\(^1\)\(^8\)See Blomquist (1974) p. 22.
TABLE 5

DISTRIBUTIONS OF LIFETIME INCOME UNDER ALTERNATIVE ASSUMPTIONS OF THE STRUCTURE OF RANDOM TERMS

<table>
<thead>
<tr>
<th>Income Concept</th>
<th>Structure of Random Terms According to Model (1)</th>
<th>Structure of Random Terms According to Model (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB1</td>
<td>0.152 0.281 3,354,961</td>
<td>0.139 0.251 3,402,680</td>
</tr>
<tr>
<td>LB2</td>
<td>0.139 0.266 1,082,505</td>
<td>0.121 0.222 1,101,778</td>
</tr>
<tr>
<td>LA1</td>
<td>0.113 0.205 1,882,738</td>
<td>0.102 0.184 1,903,470</td>
</tr>
<tr>
<td>LA2</td>
<td>0.103 0.192 694,801</td>
<td>0.089 0.161 705,494</td>
</tr>
</tbody>
</table>

can draw from the simulations are, however, the same as those obtained when we use model (1). The choice of random mechanism influences the quantitative results, but the magnitude of the results is the same for both specifications.

The analysis in this subsection shows that it does matter how \( z, r \), the rate of interest, and the random mechanism are specified. However, in no case do results change dramatically as we vary the specification. All qualitative results (as rankings of the distributions in terms of inequality) are the same for all the specifications. When comparing the distributions of lifetime income with distributions of annual income we will use the distributions obtained when \( z = 5,844, r = 0.03, \) and random mechanism (2) is used. Readers thinking this specification is unreasonable should be able to make their own comparisons using the results presented in this subsection.

IV. THE DISTRIBUTION OF ANNUAL INCOME AND THE VALIDITY OF THE SIMULATION MODEL

The simulation model can also be used to simulate annual income. There are two good reasons for doing so. Firstly, the distributions of annual income we simulate can be used to make comparisons with the simulated distributions of lifetime income. Secondly, one way to validate the model to see how well it can imitate the Swedish economy, with respect to income distribution, is to see how close the actual and simulated distributions of annual income are.

In this section we will first define the four concepts of annual income used in this paper. Since we are not interested in the distribution of annual income per se we will, however, not report the simulated distributions in detail. In this section we will try to validate the simulation model and will hence only present the results that are relevant for this purpose. The figures we will use for the comparison between the distributions of annual and lifetime income will be presented in section V.

252
Annual income before tax is the one of our income concepts that is closest to the income concept used in studies with actual data. We define this concept as

\[ AB2 = w \cdot K + r \cdot A = a. \]

Annual income after tax is defined as

\[ AA2 = a - T(a) = \tilde{a} \]

where \( T(\cdot) \) denotes the tax function. If we include leisure in the income concept we obtain

\[ AB1 = w \cdot 5,844 + r \cdot A \]

and

\[ AA1 = AA2 + \tilde{w} \cdot (5,844 - K). \]

In the rest of this section we will study how similar the simulated and actual distributions of annual income are. Since income defined as \( AB2 \) is the concept closest to the one used in studies of actual income distributions, we will limit the comparison to this income concept. When making the comparisons, we must be careful that the distributions we compare are analogously defined. Let us therefore define some concepts more precisely.

Let there be \( n \) individuals. Let \( a \) denote annual income and \( b \) the physical age and let \( \phi(a, b) \) denote the joint relative frequency distribution of \( a \) and \( b \). What is normally studied is the marginal distribution

\[ f(a) = \sum_{b \in B} \phi(a, b) \]

where \( B \) in this context denotes the index set for \( b \). Another type of distribution is the conditional distributions

\[ g(a) = \frac{\phi(a, \tilde{b})}{\phi(\tilde{b})} \]

where \( \tilde{b} \) is set at a fixed value.

The marginal distribution is dependent on the age distribution, which is not the case for a conditional distribution. We will use both types of distributions to try to validate the model. We start with a study of the marginal distribution.

A. Marginal Distribution

I have had access to a subset of the Level of Living Survey, consisting of data for males between 21 and 75 years of age in 1974. For all employed males in this sample with valid measures on the variables needed to compute \( AB2 \) (it turned out to be 1,426 persons), this income concept was simulated for the year 1974. The Gini coefficient was found to be 0.233. In the data source there is also a measure of assessed income in 1973. The Gini coefficient for this variable is 0.239. The income concept \( AB2 \) and assessed income are not completely synonymous. However, as a rough approximation we could say that they are proportional to each other. We would therefore expect that the Lorenz Curves
Figure 1. Lorenz Curves for the Distribution of Simulated and Actual Annual Income

(and Gini coefficients) would be approximately the same for the two income concepts. The Lorenz Curves for the simulated distribution and the actual distribution of annual income are shown in Figure 1. As seen from the figure the Lorenz Curves for the two distributions are very close together, as are the Gini coefficients.

B. Conditional Distributions

In order to validate our income model I have also used the sample of 381 males born around 1945 to simulate conditional distributions for the ages 25, 30 and 50. The Gini coefficients for AB2 were 0.259, 0.218 and 0.222, respectively. We would like to compare these figures with official statistical figures from the Swedish National Central Bureau of Statistics. Unfortunately there are no figures available for employed males classified into these age groups. There are figures available for all males in age groups close to the ones used in our simulations. Both Blomquist (1976) and Spånt (1976) report Gini coefficients for employed males and all males. Using these figures one can transform the official figures available for all males, and make an estimate of what the corresponding figures would be for employed males. We then obtain the figures 0.26, 0.22 and 0.25. At ages 25 and 30 the estimated Gini coefficients for the actual distributions are very close to the Gini coefficients for our simulated distributions. At age 50 the Gini coefficient for our simulated distribution seems to be a little bit low.

19To avoid the influences of sampling errors, these figures are based on averages of 25 simulations.
20One explanation for this might be that the distribution of education and hence the distribution of wage rates is more equal for the sample used in our simulations than the actual distribution of education for 50 year old males in 1972.
The examples just given cannot, of course, be taken as a proof that our income model can predict the distribution of income in a correct way. The comparisons between simulated and actual conditional distributions, together with the comparison between the simulated and actual marginal distributions, indicate, however, that the model can imitate the Swedish economy, in generating a distribution of annual income, in a satisfactory way.

V. COMPARISON OF THE DISTRIBUTIONS

The distributions described in section III.A above all come from simulations in which the same set of random terms has been used. If we made another drawing of random terms and used these in the simulations we would probably get somewhat different results. It might so happen that the difference in distribution between two income concepts is accentuated (or diminished) for the particular set of random terms used. To avoid this we will in this section use mean Gini coefficients for the comparison of our distributions. These coefficients are the means calculated from 25 different simulations and are shown in table 6. In the table are also shown the sampling standard deviation and the highest and lowest Gini coefficient for the 25 runs. As seen from the table the sampling variance for the distributions of lifetime income is quite small. Moreover, the Gini coefficients for the simulations presented in section III.A seem to be quite

<table>
<thead>
<tr>
<th>TABLE 6</th>
<th>SAMPLING STATISTICS FOR THE GINI COEFFICIENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income Concept</td>
<td>Mean</td>
</tr>
<tr>
<td>LA1</td>
<td>0.101</td>
</tr>
<tr>
<td>LA2</td>
<td>0.088</td>
</tr>
<tr>
<td>LB1</td>
<td>0.137</td>
</tr>
<tr>
<td>LB2</td>
<td>0.121</td>
</tr>
<tr>
<td>Annual income at age 25</td>
<td></td>
</tr>
<tr>
<td>AA1</td>
<td>0.201</td>
</tr>
<tr>
<td>AA2</td>
<td>0.225</td>
</tr>
<tr>
<td>AB1</td>
<td>0.238</td>
</tr>
<tr>
<td>AB2</td>
<td>0.259</td>
</tr>
<tr>
<td>Annual income at age 30</td>
<td></td>
</tr>
<tr>
<td>AA1</td>
<td>0.158</td>
</tr>
<tr>
<td>AA2</td>
<td>0.177</td>
</tr>
<tr>
<td>AB1</td>
<td>0.202</td>
</tr>
<tr>
<td>AB2</td>
<td>0.218</td>
</tr>
<tr>
<td>Annual income at age 50</td>
<td></td>
</tr>
<tr>
<td>AA1</td>
<td>0.167</td>
</tr>
<tr>
<td>AA2</td>
<td>0.175</td>
</tr>
<tr>
<td>AB1</td>
<td>0.208</td>
</tr>
<tr>
<td>AB2</td>
<td>0.222</td>
</tr>
<tr>
<td>Annual income at age 75</td>
<td></td>
</tr>
<tr>
<td>AA1</td>
<td>0.188</td>
</tr>
<tr>
<td>AA2</td>
<td>0.207</td>
</tr>
<tr>
<td>AB1</td>
<td>0.232</td>
</tr>
<tr>
<td>AB2</td>
<td>0.258</td>
</tr>
</tbody>
</table>

255
close to the mean Gini coefficients for the 25 runs. The remarks and conclusions made in that section, which are based only on one simulation for each type of distribution, are also valid if we consider the mean Gini coefficients.

We will now compare our simulated distributions and see if the simulation results support or refute the following hypotheses:

Hypothesis: The distribution of lifetime income is less unequal than the distribution of annual income.

Comparing the distribution of lifetime income before tax and with the value of leisure time included \((LB_1)\), with the distribution of annual income before tax and with leisure time included \((AB_1)\) we find that, depending at what age we study annual income, the inequality of lifetime income is 32–42 percent lower than the inequality of annual income. If we compare \(LB_2\) with \(AB_2\) we find that \(LB_2\) has a Gini coefficient which is 44–53 percent lower than the Gini for \(AB_2\). Comparing the distribution of \(LB_2\) with the Gini of actual income for all employed males in my subset of the Level of Living Survey, we find that lifetime income has a Gini coefficient which is 49 percent lower than the Gini coefficient for annual incomes. Comparing the distribution of \(LB_2\) with the simulated distribution of \(AB_2\) for all employed males in my subset of the Level of Living Survey we get almost the same result or 48 percent. If we compare say, \(LA_1\) and \(AA_1\), or \(LA_2\) and \(AA_2\), we obtain similar results. We thus conclude that our simulation results support the hypothesis. This implies that if the social welfare function has the distribution of lifetime income as argument, then the use of annual income as a proxy can be highly misleading.

Hypothesis: The distribution of lifetime income is more unequal if the value of leisure time is included in the income concept.

The hypothesis is not rejected by our results. The Gini coefficient for \(LB_1\) is about 13 percent higher than the Gini coefficient for \(LB_2\). Likewise the Gini coefficient for \(LA_1\) is about 15 percent higher than the Gini coefficient for \(LA_2\). Since, according to the estimated labour supply function, those with high wage rate paths work less than others this is not surprising. When simulating the distribution of \(LB_1\) and \(LA_1\) all individuals’ wage rates are weighted by the same number of hours. When simulating \(LB_2\) and \(LA_2\) those with a high wage rate, in general, get a lower amount of hours of work as a weight for the wage rate. The relative dispersion of incomes is thus smaller for \(LB_2\) and \(LA_2\) than for \(LB_1\) and \(LA_1\).

Even though I have not presented any results earlier from simulations of income concepts with the value of inheritance included, I want to report here that such simulations have been done. It was found that the inclusion of the value of inheritance in lifetime income had almost no impact at all on inequality. The policy conclusion we can draw from this is that we cannot reduce inequality of lifetime income much by constructing policies to further influence inheritances. Wealth in Sweden is probably transmitted between generations in another form than inheritances.
It may also be of interest to use the product moment correlation to see how similar the distributions are. A correlation matrix was therefore computed. It was found that the various lifetime income concepts were highly correlated. The highest correlations (0.99) were obtained for income concepts differing only in regard to whether the value of inheritance was included or not. The correlations between lifetime income concepts differing only in regard to whether the value of leisure time was included or not, were around 0.85–0.90.

The correlation between annual and lifetime income was much lower. Annual income at age 25 and lifetime income had the lowest correlation. The correlation between AA2 and LA2 being 0.29, and that between AB2 and LB2 0.28. This means that only 8 percent of the variance of the lifetime income concept can be “explained” by the variance of the annual income concept. Annual income at age 50 showed a higher correlation with lifetime income; the correlations corresponding to the income concepts used above being 0.63 and 0.62 respectively. This means that close to 40 percent of the variance of the lifetime income concept can be explained by the variance of the corresponding concept of annual income. From this we can draw two conclusions. Firstly, we conclude that annual income in a particular year in many cases is a poor predictor of lifetime income. Secondly, that this is particularly so for young people. This is because of more irregular working habits at young ages.21

VI. CONCLUDING COMMENTS

In this paper we have compared distributions of annual and lifetime income, variously defined, in order to see how dependent the distribution of income is on how the income concept is defined. To accomplish this, a model of the income generating mechanism was constructed and used to simulate annual and lifetime incomes for a representative sample of Swedish males born around 1945.

Lifetime income can be defined in many ways, and several definitions have been used in this study. However, some conceptual problems, which should be attacked in future research, have been neglected. Firstly, in the computations of lifetime income the same interest rate has been used for all individuals. That is, we have implicitly assumed the existence of perfect capital markets, which clearly is an assumption we would like to do without. Secondly, lifetime incomes are calculated without any attention to the fact that mortality rates vary both with economic and social background variables. Thirdly, attention has been paid to random terms when the income paths have been simulated. However, we have not taken into account in the definitions of lifetime income the fact that, if people are risk averse, the expected utility of an expected income path decreases as the variability of income around the path increases.

Keeping the qualifications above in mind, the conclusion of the study is that the picture of inequality we get is very much dependent on the income concept we use. Including leisure time in the income concept increases inequality, as

21The reason for the low correlation between annual income at age 25 and lifetime income is in our sample partly due to the fact that some persons in the sample had not finished their education at age 25, with very low annual incomes as a result.
measured by the Gini coefficient, by about 10–15 percent. Inequality is larger for distributions of lifetime income where a low interest rate is used for discounting. Using a 3 percent rate of interest instead of a 6 percent rate of interest raises the inequality measure by about 15 percent. Whether inheritances are included or not seems to be of little importance for the inequality of lifetime income. Distributions of income after tax have Gini coefficients which are approximately 25 percent less than the Ginis for the before tax distributions. Finally, the simulations show that the distribution of lifetime income is considerably more even than the distribution of annual income, the Gini coefficient for lifetime income being about 40–50 percent lower than for annual income.

Appendix A: Symbols

Below is stated the meaning of symbols used in the main article or in appendix B.

\[s = \text{years of schooling}\]
\[p = \text{years of on-the-job-training}\]
\[\text{AGE} = \text{physical age}\]
\[w = \text{wage rate per hour}\]
\[\bar{w} = \text{marginal wage rate per hour after tax}\]
\[w_{35} = \text{variable indicating the general level of the wage path}\]
\[(w-w_{35})/w_{35} = \text{relative wage rate}\]
\[K = \text{hours of work per year}\]
\[A = \text{assets}\]
\[y = \text{annual earnings before tax}\]
\[\bar{y} = \text{annual earnings after tax}\]
\[a = \text{annual income before tax}\]
\[\bar{a} = \text{annual income after tax}\]
\[\bar{B} = \text{present value of inheritances received up to present time}\]
\[B = \text{present value of all inheritances received during the complete lifetime}\]
\[\text{NS} = \text{number of siblings}\]

A number of variables are represented by dummy variables. We define these variables below.

The father's education is described by the dummy variables FEDUCI, \(I = 1, \ldots, 4\), and the mother's education by the analogously coded variables MEDUCI, \(I = 1, \ldots, 4\).

\[\text{FEDUC 1} = \text{Folkskola (roughly grammar school)}^{22}\]
\[\text{FEDUC 2} = \text{Vocational education for at least one year in addition to folkskola}\]
\[\text{FEDUC 3} = \text{Realskola}\]
\[\text{FEDUC 4} = \text{Studentexamen or a higher exam}\]

\(^{22}\)In Sweden people normally start school at the age of seven. Folkskola was normally 6–8 years of education. Realskola, grundskola, etc. is normally education for 9 years. Studentexamen normally means education for about 12 years.
The marital status of the individual is described by the dummy variables MRSTI, I = 1, 2, 3.

MRST 1 = never married
MRST 2 = divorced or widower
MRST 3 = married or living together as if married

The kind of community where the individual grew up is indicated by the dummy variables UPVRTI, I = 1, ..., 6.

UPVRT 1 = in the countryside
UPVRT 2 = in a community with at least 500 inhabitants
UPVRT 3 = in a small town with less than 10,000 inhabitants
UPVRT 4 = in an ordinary big town
UPVRT 5 = in a big city (i.e. Stockholm, Gothenburg or Malmö)
UPVRT 6 = abroad

The mother’s type of work is described by the dummy variable MWC, I = 1, ..., 5.

MWC 1 = mother stayed at home all the time
MWC 2 = blue collar work during the whole period of the individual’s upbringing
MWC 3 = blue collar work for some periods
MWC 4 = white collar work during the whole period of the individual’s growth
MWC 5 = white collar work for some periods

Serious conflicts in the family during upbringing are described by the dummy variables CONFLI, I = 1, ..., 3.

CONFL 1 = serious conflicts
CONFL 2 = small conflicts
CONFL 3 = no conflicts

The educational level of the individual is described by the dummy variables EDI, I = 1, ..., 7.

ED 1 = Folkskola, fortsättningsskola
ED 2 = Vocational education for at least one year in addition to folkskola
ED 3 = Realskola, grundskola, högre folkskola, flickskola, or folkhögskola (roughly junior high school)
ED 4 = Vocational education for at least one year in addition to realskola, grundskola, lägre folkskola, flickskola, or folkhögskola
ED 5 = Studentexamen (roughly junior college)
ED 6 = Vocational education for at least one year in addition to studentexamen
ED 7 = Degree from a university or a corresponding school

259
The type of family where the individual was brought up is described by the
dummy variables UPBRFI, \( I = 1, \ldots, 4 \).

- UPBRF 1 = brought up in a family where both biological parents were living
- UPBRF 2 = one or both of the parents dead before the individual was 16
  years old, or father away for long periods
- UPBRF 3 = parents divorced before the individual was 16 years old
- UPBRF 4 = born outside a marriage

The economic conditions in the family where the individual was brought
up are described by the dummy variables ECNCND.

- ECNCND = 0 = The family lived under hard economic constraints
- ECNCND = 1 = The family did not live under hard economic constraints

**Appendix B: Estimated Functions**

All relevant variables are measured in real prices, with 1967 as base year.
Inheritances are expressed as present values. All functions are described in detail
in Blomquist (1976). As the functions are estimated by the maximum likelihood
method on fairly large samples, the asymptotic properties of M-L estimates
should be valid. This implies that we can regard the estimates to be normally
distributed around the true values. The figures within parentheses below are
"\( z \)"-values, that is the estimated parameter divided by its standard-deviation.
Under the null hypothesis that the parameter is zero, \( z \) will be distributed \( n(0, 1) \).

**Wage Rate Function**

\[
\ln w_t = 1.515 + 0.0179 \cdot t + 0.052 \cdot s - 0.00058 \cdot s^2 + \\
(14.390) (10.867) (2.950) (-0.984)
+ 0.038 \cdot p - 0.00034 \cdot p^2 - 0.00020 \cdot \text{AGE}^2 + 0.029 \cdot \text{MRST2} + \\
(8.340) (-4.005) (-2.089) (0.870)
+ 0.132 \cdot \text{MRST3} - 0.0030 \cdot \text{MEDUC2} + 0.062 \cdot \text{MEDUC3} + \\
(7.797) (-0.92) (2.075)
+ 0.054 \cdot \text{MEDUC4} + 0.040 \cdot \text{UPVRT2} + 0.080 \cdot \text{UPVRT3} + \\
(1.072) (2.258) (2.847)
+ 0.051 \cdot \text{UPVRT4} + 0.104 \cdot \text{UPVRT5} - 0.117 \cdot \text{UPVRT6} - \\
(2.595) (4.736) (-3.419)
- 0.065 \cdot \text{MWC2} + 0.071 \cdot \text{MWC3} - 0.083 \cdot \text{MWC4} - \\
(2.332) (2.286) (-2.754)
- 0.020 \cdot \text{MWC5} - 0.089 \cdot \text{CONFL2} - 0.0068 \cdot \text{CONFL3} + \\
(-0.477) (-1.748) (-0.213)
+ 0.070 \cdot \text{ED2} + 0.025 \cdot \text{ED3} + 0.155 \cdot \text{ED4} + 0.228 \cdot \text{ED5} + \\
(3.412) (0.949) (5.278) (5.351)
\]
The function is estimated on a sample of 1,145 individuals all having a strictly positive wage rate and labor supply in both 1967 and 1973. The correlation between $w^u$ and $w^u_{t+6}$ was estimated to be 0.337. If the random terms are generated according to mechanism (2) on page 247, this implies that the correlation between $w^u_t$ and $w^u_{t+1}$ is 0.834.

Labor Supply Function

$$K_{it} = 1892.44 + 34.638 \cdot \text{AGE} - 0.489 \cdot \text{AGE}^2 - 32.214 \cdot w35 - (11.489) (5.843) (-7.500) (-5.464)$$

$$- 26.610 \cdot (w-w35)/w35 + 1.748 \cdot A - 0.0042 \cdot A^2 + (-0.963) (3.028) (-3.094)$$

$$+ 39.795 \cdot \text{MRST2} + 110.181 \cdot \text{MRST3} + 33.112 \cdot \text{ED2} - 43.129 \cdot \text{ED3} (0.789) \quad (4.068) \quad (1.317) \quad (-1.207)$$

$$+ 40.217 \cdot \text{ED4} + 26.178 \cdot \text{ED5} + 26.401 \cdot \text{ED6} + 232.600 \cdot \text{ED7} + u^K (1.196) \quad (0.446) \quad (0.417) \quad (3.322)$$

The labor supply function is estimated on a sample of 1,093 individuals. The correlation between $u^K_t$ and $u^K_{t+6}$ was estimated to be 0.170. This implies that if the random terms are generated according to the random mechanism (2), then the correlation between $u^K_t$ and $u^K_{t+1}$ is 0.744.

The labor supply function is specified in an unorthodox way. Normally only the wage rate from the present period is included in the labor supply function. In the present study we have one term capturing the level of the whole lifetime wage path ($w35$), and also a variable measuring the present wage rate in relation to the wage path. The idea behind this specification is that life cycle models of labor supply indicate that a person's labor supply for a certain year is determined by the wage path, and not by the wage rate during that particular year. Since it is impossible to describe the wage path completely I have tried to approximate it by the two variables given above.\(^{23}\)

Asset Function

$$A_i = 19.675 - 2.109 \cdot \text{AGE} + 0.061 \cdot \text{AGE}^2 - 0.00038 \cdot \text{AGE}^3 - (0.676) (-0.951) (1.160) (-0.973)$$

$$- 12.962 \cdot \text{MRST2} - 5.936 \cdot \text{MRST3} - 2.222 \cdot \text{MEDUC2} + (-2.310) \quad (-1.972) \quad (-0.419)$$

\(^{23}\)See Blomquist (1976) for a fuller description of the specification of the labor supply function.
The asset function is estimated on a sample of 935 individuals, all having a positive asset position in 1967. Due to deficiencies in the data source only data from 1967 have been used. This implies that we have been unable to estimate the covariance structure for the asset function. In the simulations we have, when random mechanism (2) has been used, assumed that the correlation between $u_{it}$ and $u_{it+1}$ is 0.9.

In the function both $A$ and $B$ are measured in thousands of crowns, while $w$ is measured in crowns.

**Inheritance Function**

Many people have no inheritance. This implies that a linear specification of the inheritance function is not appropriate. I have instead assumed that the data have been generated according to a Tobit model. Let us define an index $I = \beta X$, where $X$ is a vector of independent variables and $\beta$ a vector of parameters. The inheritance, $\delta$, is then assumed to be determined as

$$
\delta = \beta^T X + \tilde{B} + u_t^A
$$

where $\tilde{B}$ is a normally distributed variable with zero expectation and variance $\sigma^2$. To estimate the parameters for this model the M-L-method was used. The estimated parameters gives the index

$$
I = -219.29 + 5.654 \cdot AGE - 0.044 \cdot AGE^2 + 14.113 \cdot MEDUC2 + \\
- 11.890 (8.699) (-6.230) (1.545) \\
+ 24.352 \cdot MEDUC3 + 6.329 \cdot MEDUC4 - 12.082 \cdot CONFL2 + \\
3.028 (0.520) (-0.884) \\
+ 0.194 \cdot CONFL3 + 7.714 \cdot UPBRF2 - 1.383 \cdot UPBRF3 - \\
(0.019) (1.298) (-0.108) \\
- 25.655 \cdot UPBRF4 + 16.912 \cdot FEDUC2 + 12.858 \cdot FEDUC3 + \\
(1.954) (2.698) (1.802) \\
+ 40.058 \cdot FEDUC4 + 27.596 \cdot ECNCND - 2.982 \cdot NS \\
4.900 (6.793) (-4.220)
$$

The variance of $I^*$ was estimated to be 4,459.37. The standard deviation of this estimate is 239.43. The function was estimated on a sample of 2863
persons. Only data from 1967 was used. In the simulations it has been assumed that each person has the same random term in all years. There was thus no need to estimate the autocorrelation for this random term. We see that the index $I$ is a quadratic function of age. The estimated coefficients imply that $I$, and hence $\hat{B}$, increases up to age 65 and then decreases. It is obvious that a person’s cumulated inheritances over life can never decrease. In the simulations we use the value of $\hat{B}$ at age 65 as predictor of the individual’s total inheritances during his lifetime. We denote this value by $B$.

**References**


———, The Distribution of Lifetime Income, Discussion Paper no. 77-34, Department of Economics, University of British Columbia, 1977.


---

24 Without this assumption we obtain the absurd result that a person’s cumulated inheritances are a non-monotonic stochastic series over time. In real life cumulated inheritances are, of course, always non-decreasing over time.

25 When we approximate a general function by a finite polynomial we, of course, have to accept that the approximation will differ from the true curve.


