INCOME INEQUALITY AND POVERTY: SOME PROBLEMS

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In an important recent book dealing with the measurement of income inequality with particular reference to poverty,[1] Prof. N. Kakwani derives several poverty indices, investigates the effect of negative income tax schemes with the help of those indices and gives a numerical illustration based on Malaysian data.

The aim of this note is to point out some logical flaws in his argument. Some of the ideas expressed in the part of his book we are concerned with have been disseminated for some time now[2] and referred to in subsequent literature;[3] yet their shortcomings do not seem to have attracted anyone's attention. The introductory section gives a concise presentation of the relevant part of Kakwani's contribution. The next two sections deal with some problems with his approach.

1. INTRODUCTION: PROF. KAKWANI'S POVERTY INDICES

Suppose[4] that the income of a person is a random variable x with *p.d.f.* $F(x)$. Given x^*, the poverty line, $F(x^*)=q/n$ where q is the number of people below the poverty line and n is the total population. μ is the mean income of the whole distribution and μ^* is the mean income of the poor. As a starting point, Kakwani proposes:

$$P=F(x^*)\frac{(x^*-\mu^*)}{\mu},$$

which is interpreted as the percentage of total income that must be transferred from the nonpoor to the poor so that the income of everyone below the poverty line may be raised to x^*.

As a poverty index, P has the defect of being insensitive to the spread of the income of the poor. Taking G^*, the Gini coefficient of the poor as a measure of income inequality among the poor, Kakwani proposes that a poverty index, $\tilde{P}$, should satisfy:

$$\text{if } G^*=0, \tilde{P}=F(x^*)\frac{(x^*-\mu^*)}{\mu} \tag{1}$$

$$\tilde{P}\leq F(x^*)\frac{x^*}{\mu} \tag{2}$$

$$\frac{\delta P}{\delta G^*}>0, \text{ for all } G^*. \tag{3}$$

[1][3] N. Kakwani, *Income Inequality and Poverty: Methods of Estimation and Policy Implications*, Oxford University Press (A World Bank Research Publication), New York (1980).

[2]They were originally presented in [1].

[3]E.g. in [2] and [6].

[4]See [3], pp 327–350.

(4) "Any poverty index that satisfies the above conditions will necessarily satisfy both axioms 15.1 and 15.2"[5]

The axioms are two requirements for a poverty measure, originally formulated by Sen in [4]:

Axiom 15.1. "Other things remaining the same, a reduction in income of a person below the poverty line must increase the poverty measure."

Axiom 15.2. "Other things remaining the same, a transfer of income from a person below the poverty line to anyone who is richer must increase the poverty measure."

"In addition, a general class of poverty measures satisfying these conditions ((1)–(3)) can be written as:

$$P_g = \frac{F(x^*)}{\mu}[x^* - \mu^* g(G^*)], \tag{5}$$

where $g(G^*)$ is a monotonic function of G^* such that $0 \leq g(G^*) \leq 1$; $g(G^*) = 1$ if $G^* = 0$; $g'(G^*) < 0$."[6]

Kakwani presents and uses in his empirical illustration two particular cases of P_g:

$$P_1 = \frac{F(x^*)}{\mu}[x^* - \mu^*(1 - G^*)] \quad \text{and} \tag{6}$$

$$P_2 = \frac{F(x^*)}{\mu}[x^* - \mu^*/(1 + G^*)], \tag{7}$$

with $g(G^*)$ taken to be $(1 - G^*)$ and $1/(1 + G^*)$, respectively.

In the two following sections, we show that:

(i) Statement (4) is false. A poverty index satisfying (1)–(3) does not necessarily satisfy axioms 15.1 and 15.2. In particular both P_1 and P_2 violate axiom 15.2.

(ii) The reaction of the indices P_1 and P_2 to the imposition of some negative income tax schemes is somewhat perverse.

2. Properties of the Poverty Indices

The claim that a poverty index satisfying (1)–(3) can violate axioms 15.1 or 15.2 is now established.

The index:

$$P_3 = \frac{F(x^*)}{\mu}[x^* - \mu^*(1 - G^*)^4]$$

can easily be checked to satisfy conditions (1)–(3). Yet it violates axiom 15.1. Example: Suppose $X = (2,2,4,10,10,10)$, $X' = (2,2,2,10,10,10)$ and $x^* = 6$. X'can be reached from X by a reduction in the income of a poor person. Yet $P_3(X) =$

[5] See [3], p 331.
[6] See [3], p 331.

0.372 and $P_3(X') = 0.333$; i.e. such an income reduction decreases the value of the poverty measure and axiom 15.1 is violated.

That (1)–(3) is not sufficient for axiom 15.2 to be satisfied is now proven by showing that P_1 and P_2 themselves fail to satisfy the axiom. Consider: $X = (1,2,3,4)$, $x^* = 3.1$. Then $P_1(X) = 0.463$ and $P_2(X) = 0.439$. Now, $X' = (1,1.8,3.2,4)$ can be obtained from X by a transfer from a poor person to someone richer. $P_1(X') = 0.379$ and $P_2(X') = 0.374$. Both poverty indices have decreased, a violation of axiom 15.2 which requires a transfer such as the one considered to increase a poverty index.[7]

Any transfer from a poor person to a richer poor person who crosses the poverty line as a result of the transfer reduces the value of q; μ^* and G^* on the other hand may increase or decrease. The problem arising in the counter-example just given is that μ^* and G^* decrease and that the decrease in G^* is large enough to overcompensate for the downward effect of the decrease in q and μ^* on P_1 and P_2. In general, the net effect on P_1 and P_2 of the variation in q, μ^* and G^* resulting from a transfer such as considered by axiom 15.2 can be either a decrease or an increase and the axiom is thus not necessarily satisfied.

Condition (3) dictates the direction of the change in the index as a result of a "ceteris paribus" change in G^*. The ceteris paribus condition implicit in the definition of the partial derivative (3) is that μ, μ^* and $F(x^*)$ do not vary. This implies that the variation in G^* is caused by a transfer between two people who are both below the poverty line before and after the transfer. The statements of axioms 15.1 and 15.2 are not subject to such a qualification. Some transfers from a poor person to someone richer affect not only G^* but also μ^* and $F(x^*)$, such as a transfer from a poor person to a richer poor person who crosses the poverty line as a result of the transfer. A pure decrease in the income of a poor person affects not only G^* but also μ^* and μ. This explains why conditions (1)–(3) are not sufficient for the axioms to be satisfied.

3. Negative Income Tax Schemes and Poverty Measures

Kakwani applies his two indices to the measurement of the impact of two negative income tax schemes.[8] He first assumes that the people below the poverty line are subsidized by a fraction of the amounts by which their incomes fall short of the poverty line and the people above the poverty line are taxed by the same percentage of the excess of their incomes over the poverty line. In such a case, clearly the total pre-tax income is equal to the total post-tax income only by a fluke. The second case he considers is different in that the rate of taxation is not assumed to be equal to the rate of subsidy; instead they have been so calculated as to leave the total income unchanged. In both cases, the Gini of the poor and the Gini of the whole distribution, P_1 and P_2, decrease as a result of the tax.

Suppose now that the subsidy rate and the taxation rate are different and allow for some degree of net taxation. If one assumes temporarily for the sake of

[7]Sen's index, P_s (see [4]) which is related to P_1 by the relation $P_1 = P_s(x^*/\mu)$ (see [3], p 337) behaves in a similar way in this respect. As a result P_1's violation of 15.2 follows from Sen's later remark ([5], p. 77, n. 52) that P_s does not satisfy 15.2. See [6] and [7] for a discussion.

[8]See [3], p. 339–341.

the argument that all the poor people have the same income (then $G^* = 0$ before and after tax), then P_1 and P_2 alike reduce to:

$$P = F(x^*)\frac{(x^* - \mu^*)}{\mu}, \tag{11}$$

where $F(x^*)$ is invariant to the operation of the tax. Then if, for example, the nonpoor population is larger than the poor one and the rate of taxation is sufficiently higher than the rate of subsidy, it is easy to see from (11) that P might increase as a result of the tax by μ decreasing by enough to overcompensate for the decrease in $(x^* - \mu^*)$. The same will be true if $G^* \neq 0$ and does not decrease "too much" as a result of the tax. Example: if $X = (1,2,3,10,10,10,10,10)$, $x^* = 4$ and if a negative income tax scheme is put into effect, which subsidizes the poor at the rate of 5% of their income gaps and taxes the rich at the rate of 50% of their income in excess of the poverty line, then after tax the income vector is: $(1.15,2.10,3.05,7,7,7,7,7)$. P_1 and P_2 before tax are 0.131 and 0.127 and after tax they are 0.168 and 0.163, respectively.

One feels confident in arguing that it is unacceptable that a poverty index should increase in such circumstances as the effect of the tax is unambiguously to move inward the Lorenz curve of both the whole income distribution and the income distribution of the poor as well as to increase the income of every poor person.

References

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