CONCEPTS AND PROBLEMS OF CONSISTENCY IN TERMS-OF-TRADE ANALYSIS*

BY ANDERS ØLGAARD

University of Copenhagen

The debate on how to deal with changes of relative prices in national accounts has, so far, remained inconclusive, especially with regard to the question of how to measure gains from changes of terms of trade. Keeping the experiences of the 1970s in mind (i.e. substantial changes of relative prices sparked off by increased oil prices), this state of affairs is not considered tenable.

On this background, the paper takes up the old debate on how to deflate figures of domestic product, total as well as by industries. It tries to argue that deflated figures should be presented not only as real product figures by industries (using the double deflation method), but also as real income figures, obtained by deflating the current-prices figures of a certain year by the same general price index. When this is done according to procedures spelled out in detail, gains/losses from changes of the terms of trade in foreign trade will show up as an integral part of the framework.

In the paper, special attention is given to the concept of industry terms of trade. On the basis of simplifying assumptions (which are, however, relaxed in the final part of the paper), it is shown how the ratio of real income divided by real product of a certain industry will be proportionate to the terms of trade of the industry concerned, when the latter concept is defined in the appropriate way. Furthermore, the sum of the industry gains/losses from changes of their terms of trade will be equal to the gain/loss of the economy taken as a whole from changes of the terms of trade in foreign trade.

INTRODUCTION

Over the last decade, substantial progress has been made in making national accounts statistics a consistent summary of basic statistics originating from almost any field, input-output tables providing one of the key tools in this activity. The routine publication of these results by the various Central Statistical Offices is, however, usually limited to figures partly at current prices and partly at constant prices, the latter being obtained by presenting current quantities at prices of the base year.

The problems with this way of presentation are becoming increasingly obvious during periods of sharply fluctuating rates of overall inflation, as have been experienced over the last decade. The changes of overall rates of inflation are dominating the figures at current prices at the expense of quantity changes, hence figures at constant prices are used to an increasing extent. But the main weakness of the latter figures is that, by definition, they assume *relative* prices to have remained *constant*, i.e. equal to relative prices of the base year, the time series at constant prices (quantity indices) usually being of the Laspeyres type.

However, relative prices do of course change over time. To the extent that these changes reflect differences of rates of increase of *long-term* productivity in different areas, these changes may even take place in a systematic way over time,

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relative prices continuously changing in the same direction. But this aspect is not reflected in the traditional figures at constant prices. This deficiency will be the more serious, the longer are the periods subject to empirical analysis.

Furthermore, to the extent that relative prices show abrupt and substantial *short-term* changes, as has been experienced during the 1970s especially for oil prices, these changes are, of course, not reflected in the traditional figures at constant prices. Hence the latter figures by definition exclude the possibility of taking into account the effects of changes of the terms of trade in foreign trade.

In the main part of the present paper, these problems are discussed with special reference to figures of *domestic product*, total as well as by industries.

The Concept of Real Product

The traditional domestic product figures by industries at constant prices may conveniently be labelled *real product* figures. They express *quantity changes exclusively*, assuming relative prices to have remained constant. Hence they are useful for some purposes, mainly related to analysis which takes the starting point on the production side, e.g. traditional analysis of productivity.

But when it comes to analysis of changes of the industrial pattern over time, i.e. of the (shares of) *total income earned by the various industries* over time, the real product figures are not very useful. They may even be claimed to be misleading, since such income figures ought to reflect not only changes of quantities, but also changes of *relative* prices. In the following, such figures will be labelled *real income* figures.[1]

It is one of the contentions of the present paper that it ought to be a standard procedure for the Central Statistical Offices to publish time series of real income figures by industries in addition to (the traditional) real product figures.

The Concept of Real Income

How, then, should the real income figures be arrived at? One basic feature would be that, when real income figures by industries in a certain year are calculated, the *same deflator should be used for all the various industries in that particular year.* In this way, the effect of "overall inflation" is eliminated, but the effects of changes of relative prices is retained in the figures. Note that, when this procedure is applied, the percentage distribution by industries of total real income is bound to be exactly equal to that of total domestic product at current prices.

In a closed economy, the obvious choice with respect to the "overall deflator" to be applied to all the domestic products by industries at current prices in a certain year would be the implicit deflator of total real product (= total domestic product at current prices divided by total domestic real product). This implies that, in a closed economy, *total* real income will equal *total* real product.

However, in an open economy the suggested procedure could serve an additional purpose, i.e. to *take into account the effects of changes of the terms of*

[1]Hence the label of "income" is used in a way different from the traditional one. According to traditional definitions, the difference between "product" concepts and "income" concepts relates to net foreign transfers. In the present context such transfers are assumed not to exist.

trade in foreign trade. Such effects, by definition, cannot be reflected in national accounts figures deflated according to traditional procedures. Here, all items are expressed at prices of the base year, hence the possibility of including the effects from changes of relative prices in foreign trade is ruled out.

When the effect (the gain or the loss) from changes of the terms of trade in foreign trade is to be calculated, a few aspects should be kept in mind in order to provide for consistency to the largest possible extent. First, the *appropriate index of terms of trade in foreign trade* should be calculated on the basis of the figures already included in the national accounts. Thus, the index of export prices should be determined as the implicit deflator of exports of goods and services, and import price indices determined in a similar way. Note that, when this procedure is applied, the terms of trade index will, by necessity, be of the Paasche type, assuming—as we did—that the time series of exports and imports at constant prices are of the Laspeyres type.

Having in this way determined the index of terms of trade in foreign trade, the *gain* (if the terms of trade have improved) or the *loss* (if the terms of trade have deteriorated) from changes of the terms of trade in foreign trade *is obtained by multiplying total exports or total imports by the percentage change of the terms of trade in foreign trade.* (How this should be done more precisely in order to obtain consistency to the largest possible extent is discussed below.)

Finally, *total* real income is obtained by adding the gain (subtracting the loss) from changes of the terms of trade in foreign trade to (from) total real product. Obviously, if the terms of trade in foreign trade have remained constant, total real income will equal total real product.

Now we have arrived at the deflator to be used for domestic products at current prices for all the industries of a given year, cf. above, in order to obtain real incomes by industries of that particular year. The deflator is simply equal to *total* domestic product at current prices divided by *total* real income, the latter to be obtained by adjusting total real product for the gain (or loss) from changes of the terms of trade in foreign trade.

An Old Battlefield

The problems and suggestions briefly indicated above are related to subjects on which many battles were fought—to a large extent at the conferences of the IARIW—during the 1950s. However, the purpose of this paper is not to survey these fights. On this issue, reference is made to a recent OECD paper, OECD (1979). A few comments on these battles may, however, be appropriate.

Two main, but interrelated, subjects were at stake. The first one was how to deflate national accounts. To quote the OECD paper: "In the 1930s, the emphasis was on the calculation of real national income by deflating the current-value series by a general price index. A number of statisticians thought that the same price index should be applied to all the items in the accounts. . . . After the war, however, the majority held another view—they claimed that each product flow should be deflated by its own price index" (the Geary method or the double deflation method). As it turned out, "the protagonists of the old school (the purchasing power concept) practically lost their case."

The second subject was the measurement of gains or losses from the terms of trade in foreign trade. Here the debate covered a variety of different problems. It became rather confused—and unfortunately inconclusive. One of the main reasons for this depressing outcome probably was that, since the protagonists of the Geary method won the first battle, the concept defended by "the losers," i.e. that of real income, was not given much attention in the form of independent definition and interpretation.

It is the contention of the present paper that this outcome of the battle was unfortunate. The question of real product versus real income should not, as was usually the case during the 1950s, be discussed to be "either/or." Instead, the answer should be similar to that of Winnie-the-Pooh who, when asked by Rabbit whether he preferred honey or condensed milk, excitedly said: "Both."[2] In other words, what is required is not only time series of real product figures, but also real income figures, total as well as by industries.

When this is done, procedures should be chosen which will, at the same time, allow for the introduction, in a consistent way, of the concept of gains/losses from the terms of trade in foreign trade. In addition, the concept of gains/losses from the terms of trade *of each individual industry* should be introduced. If this is done in the appropriate way, the sum of the gains of the individual industries will be exactly equal to the gain from the terms of trade *in foreign trade.*

The question may be asked why real income figures are not already published as a matter of routine as a supplement to the real product figures. Several explanations of this sin of omission suggest themselves.

The first is that most of the concepts and procedures mentioned above may be interpreted or defined in a great many ways. The concept of "the gain from changes of the terms of trade in foreign trade" is a case in point. However, if the rules of the game are to be adhered to, e.g. with respect to types of indices to be used, and if, furthermore, one insists on making the concepts and the analysis consistent, there really are not many degrees of freedom left, as will be shown in the following sections.

Another explanation may be that, although time series of the ratio of real income over real product of a certain industry may be taken somehow to express the terms of trade of the industry concerned, it is not always clear exactly how these figures should be interpreted, i.e. which are the elements of "trade," the "terms" of which are reflected in the figures for a certain industry. This issue will also be discussed below.

Although the battlefield is old, the present paper tries to attack the problems from a different angle as compared with most earlier contributions (including Stuvel's article as well as the OECD paper mentioned above). In my own view, the reason why the discussion during the 1950s of the measurement of gains from terms of trade was rather confused reflects the fact that difficulties are bound to arise when there are surpluses or deficits on the balance of payments, when factor prices are not equal to market prices etc. In most of the following, such problems are simply assumed away. Thus a price is paid in the form of *simplifying*

[2]This view was also expressed during the 1950s, see e.g. Stuvel (1959), pp. 282–287.

assumptions. Hopefully, a corresponding gain is made in the form of consistent results, easy to interpret.

Before we turn to the simplifying assumptions, a word should be said on the importance of the subjects dealt with. The intensive debate during the 1950s partly reflected the substantial change of relative prices during the Korean war. However, during the 1960s the pattern of relative prices did not show abrupt changes. But in 1974 and 1979, relative prices again changed substantially, and additional future changes cannot be ruled out. Furthermore, the future rate of overall inflation is very unpredictable. These facts call for renewed attention to questions of how to handle price problems in national accounts. It is as a possible framework for the debate on these issues that the present paper should be seen.

The Balanced-Flows Assumption

The most basic assumption, which must be made in order to provide rigorous proofs of the various relationships discussed below, is that *total purchases of each industry in the base year must equal total sales of the same industry.* Obviously, this assumption among other things implies that, in the base year, exports (of goods and services) must equal imports (of goods and services), in other words that, in the base year, there is *equilibrium on the balance of payments.* We may call this assumption the "balanced-flows assumption".

The balanced-flows assumption implies that the *gain* from changes of the terms of trade in foreign trade will be proportionate to the *percentage change* of the terms of trade in foreign trade. It may be recalled that, for the various gain concepts which have been suggested from time to time, this relationship does not always hold. It will be beyond the scope of the present paper to discuss this issue in detail, but it should be recalled that most of the gain concepts that have been suggested may be split up into a *terms-of-trade effect* and a *price level effect.* The latter may be exemplified as follows: Suppose that an economy has a permanent balance-of-payments surplus and that prices in foreign trade increase at constant terms of trade in foreign trade. In that case, some gain concepts[3] will show positive gains in spite of constant terms of trade in foreign trade. Such properties complicate the interpretation of results, based on gain concepts which may be affected by the price level effect. However, the balanced-flows assumption rules out the possibility of the price level effect being able to influence the gains.[4]

A fundamental consequence of the balanced-flows assumption is that, by definition, *savings in the form of acquiring net financial assets by a sector* (a country or an industry) *from the rest of the world is ruled out.* Hence, by assumption, the question of how to deflate savings is avoided—unless savings take place in the form of flows of (real) investment. In the latter case, no problems are of course

[3]E.g. the gain defined as the current balance-of-payments surplus at current prices minus the current balance-of-payments surplus at base year prices.

[4]This issue is elaborated in Ølgaard (1966), Chapter 13, pp. 244–256. Here, twelve different possible gain concepts are compared, six of them being of the Laspeyres type and six of the Paasche type.

involved, because the appropriate deflator of savings will be the deflator of the corresponding investment.

Questions related to the possibilities of deflating savings will not be discussed in the present paper. In passing it should, however, be noted that according to the framework of the present analysis the proper starting point for discussions of these issues is balance sheets (a stock approach), indicating the outcome of accumulated savings from the past.

Be this as it may, in the present paper financial assets and liabilities are assumed away, even at the industry level. However, this rigid position of course has implications in the form of constraints as to how to define the "sectors" into which the economy is divided. For the analysis to be relevant for empirical purposes, the disaggregation into sectors must be undertaken in such a way that the assumption of no financial savings does not give rise to basic contradictions with respect to the sectors and concepts involved. As far as the balance of payments is concerned, according to the reasoning behind the present paper such basic contradictions are not necessarily involved. After all, surpluses and deficits on the balance of payments usually amount only to a few percent of total foreign sales or purchases.[5]

When it comes to analysis *at the industry level,* the balanced-flows assumption leads to the immediate observation that the analysis cannot be limited to flows of output and flows of inputs of raw materials and other intermediate goods. If that had been the case, the implication would have been that flows of output of each industry (in the base year) would equal flows of inputs of intermediate goods in the same industry and thus that no value added would originate. Obviously, such an implication would be nonsensical.

But the way out of this dilemma is obvious—and consistent with the assumption of balance-of-payments equilibrium. It is assumed that, in the base year, the *value added originating in each industry* (be it in the form of wages or profits—this distinction will not be used in the following) *is exactly equal to the total final demand of the industry concerned.* In other words, the value of output of each industry exactly corresponds to the value of inputs of intermediate goods *plus* the value of final demand purchased by those who have earned the income (equal to the value added) of the sector concerned. In this way, the balanced-flows assumption is fulfilled.

More important, this approach makes sense when the basic issue is to which extent a certain industry is affected by changes of relative prices (= changes of industry terms of trade). Obviously, real income of a certain industry will be affected by changes of prices of its output and of its inputs of intermediary products. But real income of the industry concerned will also be affected by price changes of its final demand. We shall not go into details, but only notice that increased prices of consumers goods, as, e.g. agricultural products or fuel, are bound to affect real income in a downward direction for industries not producing these components of final demand.

[5]When it comes to practical application of the suggestions in the present paper, some *ad-hoc* adjustments must of course be made in order to allow for actual surpluses and deficits on the balance of payments.

THE FRAMEWORK OF FORMAL ANALYSIS

In the following sections, the above arguments are formalized and conclusions are derived on the basis of input-output tables of a certain economy for two years, year 1 and year 2. The economy has three industrial sectors. First the economy is assumed to be a *closed* one, subsequently allowance is made for foreign trade, i.e. the economy becomes *open*, exports and imports being explicitly shown in the tables. For convenience, it is assumed that all imports consist of intermediate goods. Furthermore, only one type of domestic final demand (including consumption as well as investment goods) is indicated, the implication being that all the three industries use their value added in order to buy final demand in the same proportion.[6] As was mentioned above, the final demand of each industry is assumed to be exactly equal to its value added. (Hence financial savings are ruled out, even at the industry level.)

In accordance with traditional assumptions, output by a certain industry is sold at the same price, independently of who is the purchaser. In addition, the *sales price* by the producer of a certain output is equal to the price at which the purchaser buys it. In other words, commodity taxes are assumed away. Basically it is assumed that *no government sector exists* (or that it behaves like a market industry, if it exists). This assumption will, however, be relaxed in the final part of the paper.

A CLOSED ECONOMY

On these assumptions, a closed economy in year 1 is described by the input-output table shown in Table 1. For convenience, all the prices in year 1 are

TABLE 1

INTER-SECTORAL FLOWS IN A CLOSED ECONOMY, YEAR 1

(All prices = 1)

	Intermediate Flows to Sectors				
	1	2	3	Final Demand	Total Output
Output from sector					
1	—	X_{12}	X_{13}	X_{1D}	x_1
2	X_{21}	—	X_{23}	X_{2D}	x_2
3	X_{31}	X_{32}	—	X_{3D}	x_3
Value added	X_{E1}	X_{E2}	X_{E3}	—	x_E
Total	x_1	x_2	x_3	x_D	

Source: See Table 14.1, p. 262, in Ølgaard (1966).

[6]This assumption is not crucial and could be relaxed. This, of course, would require additional information on the different patterns of final demand of the various industries. However, the formal presentation will be much more complicated if expenditure patterns of final demand by the various industries differ. In the following, the analysis will always assume the same final demand pattern for all the three industries.

assumed to be equal to unity.[7] The symbols should largely be self-explaining. Total outputs (quantities) are indicated by small x's; intermediate or final deliveries (quantities) by capital X's. Capital X's always have two subscripts, the first indicating the sector of origin, the last the sector buying the flow of goods. Subscript D indicates final demand, total final demand being denoted x_D.

Value-added figures by industries, X_{Ei} $(i = 1, 2, 3)$, are obtained by deducting purchases of intermediate goods from total output of the industry concerned. Total value added (= total domestic product) is denoted x_E.[8] Hence

$$(1) \qquad \begin{aligned} x_E &= X_{E1} + X_{E2} + X_{E3} \\ &= X_{1D} + X_{2D} + X_{3D} \\ &= x_D. \end{aligned}$$

Note that, according to the balanced-flows assumption and the assumption of the same final demand pattern for the three industries, final demand by, say, sector 1 of output from sector 2 is equal to

$$\frac{X_{E1}}{x_E} X_{2D}.$$

From year 1 to year 2, prices as well as quantities change. The flows of year 2 at current prices are indicated in Table 2. The output price (or price index) of sector i is denoted p_i. Note that as soon as assumptions with respect to p_1, p_2 and p_3 have been made, the *implicit real-product deflators* p_{Ei} have been determined, and so has the implicit deflator of *total* real product (p_E), the latter being equal to the implicit deflator of *total* real income (p_D).

TABLE 2

INTER-SECTORAL FLOWS IN A CLOSED ECONOMY, YEAR 2, CURRENT PRICES

	Intermediate Flows to Sectors				
	1	2	3	Final Demand	Total Output
Output from Sector					
1	—	$p_1\mathbf{X}_{12}$	$p_1\mathbf{X}_{13}$	$p_1\mathbf{X}_{1D}$	$p_1\mathbf{x}_1$
2	$p_2\mathbf{X}_{21}$	—	$p_2\mathbf{X}_{23}$	$p_2\mathbf{X}_{2D}$	$p_2\mathbf{x}_2$
3	$p_3\mathbf{X}_{31}$	$p_3\mathbf{X}_{32}$	—	$p_3\mathbf{X}_{3D}$	$p_3\mathbf{x}_3$
Value added	$p_{E1}\mathbf{X}_{E1}$	$p_{E2}\mathbf{X}_{E2}$	$p_{E3}\mathbf{X}_{E3}$	—	$p_E\mathbf{x}_E$
Total	$p_1\mathbf{x}_1$	$p_2\mathbf{x}_2$	$p_3\mathbf{x}_3$	$p_D\mathbf{x}_D$	

Source: See Table 14.2, p. 263, in Ølgaard (1966).

[7]Of course this assumption is not crucial. It only implies certain rules with respect to the units of measurement of quantities of the various outputs.

[8]The symbols are those used in Ølgaard (1966). They originate from P. Nørregaard Rasmussen (1956).

That quantities in Table 2 refer to year 2 is indicated by using bold types: $\mathbf{X}$ and $\mathbf{x}$. $\mathbf{X}_{Ei}$ thus indicates the value added of industry i which would have been obtained by industry i *if* prices of year 1 had prevailed. In other words, $\mathbf{X}_{Ei}$ indicates *real product* in year 2 of industry i. For instance, real product of industry 1 in year 2 is equal to

$$\mathbf{X}_{E1} = \mathbf{x}_1 - \mathbf{X}_{21} - \mathbf{X}_{31} \tag{2}$$

recalling that all prices of year 1 were assumed to be equal to unity. The corresponding implicit deflator, p_{E1}, will be equal to

$$p_{E1} = \frac{p_1\mathbf{x}_1 - p_2\mathbf{X}_{21} - p_3\mathbf{X}_{31}}{\mathbf{x}_1 - \mathbf{X}_{21} - \mathbf{X}_{31}}.$$

Obviously, *total* real product in year 2 equals the sum of real products by industries:[9]

$$\mathbf{x}_E = \mathbf{X}_{E1} + \mathbf{X}_{E2} + \mathbf{X}_{E3}. \tag{3}$$

Real income of industry i in year 2, denoted by $\mathbf{Y}_i$, is equal to the value added of industry i in year 2 at current prices, divided by an index of prices of final demand. The latter price index, p_D, will be the same for all industries. It becomes an index of the Paasche type:

$$p_D = \frac{p_1\mathbf{X}_{1D} + p_2\mathbf{X}_{2D} + p_3\mathbf{X}_{3D}}{\mathbf{X}_{1D} + \mathbf{X}_{2D} + \mathbf{X}_{3D}}. \tag{4}$$

Hence real income in year 2 of industry i is equal to

$$\mathbf{Y}_i = \frac{p_{Ei}\mathbf{X}_{Ei}}{p_D}. \tag{5}$$

Since the economy is assumed to be closed ($\mathbf{x}_E$ being equal to $\mathbf{x}_D$) we have that

$$p_D = p_E \tag{6}$$

i.e. that the deflator of final demand (equal to that of real income) is equal to the deflator of real product.

Total real income being denoted by $\mathbf{y}$, it follows that

$$\begin{aligned} \mathbf{y} &= \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 \\ &= \mathbf{x}_E \\ &= \mathbf{x}_D. \end{aligned} \tag{7}$$

[9]The real product expressions are based on prices of year 1, hence they correspond to Laspeyres quantity indices. On this background it is not surprising that the appropriate price indices developed below will be of the Paasche type. In fact, almost all the expressions in the following will contain x's which are bold.

As was mentioned above, quite a few suggestions have been made during recent decades as to how to measure the gain from changes of the terms of trade in foreign trade. However, when it comes to the question of how to measure the *industry gains from changes of inter-industry terms of trade*, suggestions have been even more numerous. In the present context we will, however, simply define the gain by industry i from changes of inter-industry terms of trade, G_i, as

$$G_i = \mathbf{Y}_i - \mathbf{X}_{Ei} \tag{8}$$

i.e. as its *real income minus its real product.* Defining the gains in this way,[10] it follows immediately that, in a closed economy,

$$G_1 + G_2 + G_3 = 0. \tag{9}$$

Now, at long last, we have arrived at the interesting basic question, namely whether the industry gains G_i can be shown to be proportionate to the terms of trade of the industry concerned and—if this is the case—what the factor of proportionality is. As it turns out, the answer to the first question is affirmative. In fact, e.g. for industry 1 it can be shown that

$$G_1 = \mathbf{x}_1 \, ({}_1T_{\text{total}} - 1). \tag{10}$$

The concept of ${}_1T_{\text{total}}$ represents the terms-of-trade index of industry 1, the appropriate index of *sales* prices of course being equal to p_1. Furthermore, in order for (10) to hold, the corresponding price index for the *purchases* of the sector should cover *total* purchases of the industry, intermediate goods as well as final demand. Recalling that each industry uses its value added (at current prices) in order to buy final goods at the same proportions as the other industries, the price index of total purchases of, e.g. industry 1 in year 2 will be equal to the following Paasche price index:

$$\frac{p_2\mathbf{X}_{21} + p_3\mathbf{X}_{31} + (p_{E1}\mathbf{X}_{E1}/p_E\mathbf{x}_E)(p_1\mathbf{X}_{1D} + p_2\mathbf{X}_{2D} + p_3\mathbf{X}_{3D})}{\mathbf{X}_{21} + \mathbf{X}_{31} + (p_{E1}\mathbf{X}_{E1}/p_E\mathbf{x}_E)(\mathbf{X}_{1D} + \mathbf{X}_{2D} + \mathbf{X}_{3D})}.$$

This expression can easily be reduced to

$$\frac{p_2\mathbf{X}_{21} + p_3\mathbf{X}_{31} + p_{E1}\mathbf{X}_{E1}}{\mathbf{X}_{21} + \mathbf{X}_{31} + (p_{E1}\mathbf{X}_{E1}/p_E)}$$

equal to

$$\frac{p_1\mathbf{x}_1}{\mathbf{X}_{21} + \mathbf{X}_{31} + (p_{E1}\mathbf{X}_{E1}/p_E)}.$$

[10] At this point, the present paper departs definitely from the exposition in Chapter 14 of Ølgaard (1966), where a substantial number of gain concepts are defined—and redefined. The latter way of presentation may, however, give rise to a substantial amount of confusion. Hopefully, an advantage is to be gained from the present simplified way of presentation with respect to the fundamental issues which are involved.

The terms of trade index of industry 1, ${}_1T_{\text{total}}$, is equal to the index of its sales prices, p_1, divided by the latter expression:

$$(11) \qquad {}_1T_{\text{total}} = \frac{p_1(\mathbf{X}_{21}+\mathbf{X}_{31}+(p_{E1}\mathbf{X}_{E1}/p_E))}{p_1\mathbf{x}_1}$$

$$= \frac{p_E\mathbf{X}_{21}+p_E\mathbf{X}_{31}+p_{E1}\mathbf{X}_{E1}}{p_E\mathbf{x}_1}$$

$$= \frac{p_E\mathbf{x}_1-p_E\mathbf{X}_{E1}+p_{E1}\mathbf{X}_{E1}}{p_E\mathbf{x}_1}.$$

Recalling (8) and (11), the fundamental relationship (10) can be proved after some rearranging.

Summarizing these exercises, we have defined the gain G_i by industry i from changes of its terms of trade as its real income minus its real product, cf. (8), and we have proved that this gain will be proportionate to the relative (percentage) change of its total terms of trade $({}_iT_{\text{total}}-1)$, the factor of proportionality being equal to $\mathbf{x}_i$:

$$(12) \qquad G_i = \mathbf{x}_i\,({}_iT_{\text{total}}-1).$$

It follows from (12), recalling (8), that the *relative* change of the ratio of (an index of) real income of a certain industry divided by the corresponding real product (index) is proportionate to the relative change of the total terms of trade index of the sector concerned, the factor of proportionality being equal to the ratio of the real output of the industry over its real product:

$$(13) \qquad \left(\frac{\mathbf{Y}_i}{\mathbf{X}_{Ei}}-1\right) = \frac{\mathbf{x}_i}{\mathbf{X}_{Ei}}\,({}_iT_{\text{total}}-1).$$

Hence (13) shows explicitly what is involved when, in a closed economy, real income figures by industries are divided by corresponding real product figures, as is sometimes done in studies of industry terms of trade.[11]

[11]Perhaps an additional comment could be added on the "factors of proportionality" according to (12) and (13). As has been mentioned above, the quantity indices involved, cf. e.g. (2), are of the Laspeyres type, hence the corresponding price indices will be of the Paasche type with weights from year 2, cf. e.g. (11). Since ${}_iT_{\text{total}}$ refers to total sales as well as total purchases of sector i, it is hardly surprising that the factor of proportionality according to (12) is equal to total sales, $\mathbf{x}_i$. That these sales are taken from year 2 is in accordance with the fact that the corresponding price indices are of the Paasche type. Furthermore, since the sales are to be multiplied by an expression of price changes, i.e. by $({}_iT_{\text{total}}-1)$, it is not surprising either that $\mathbf{x}_i$ as the factor of proportionality is measured at prices of year 1.

The result (13) follows immediately from (12), hence additional interpretation of (13) is hardly needed. Still it should be mentioned that while the numerator of the factor of proportionality, i.e. $\mathbf{x}_i$, reflects the expression (12) and makes the gain a concept "in absolute terms," division by $\mathbf{X}_{Ei}$ relates the gain to the value added of the industry concerned. Given a certain order of magnitude of the relative change of ${}_iT_{\text{total}}$, the relative gain as expressed by (13) will be higher, the larger is total output by the industry concerned as compared with its value added, $\mathbf{X}_{Ei}$. In other words, the *absolute* gain G_i according to (12) is determined by $\mathbf{x}_i$ and by ${}_iT_{\text{total}}$ exclusively. However, in order to find the *relative* gain, the absolute gain must be related to the real product of industry i. Hence real product $\mathbf{X}_{Ei}$ is used as denominator ("numeraire") in (13).

Concluding this section, it should be briefly mentioned that the industrial gain G_i according to (12) can also be expressed as

$$G_i = \mathbf{X}_{Ei}\left(\frac{p_{Ei}}{p_E} - 1\right). \tag{14}$$

Hence the gain will be larger, the higher is the implicit real product deflator of the sector concerned (p_{Ei}) as compared with the deflator of total real *product* (p_E). By the same token—but interpreted from the *income* side—

$$G_i = \mathbf{X}_{Ei}\left(\frac{p_{Ei}}{P_D} - 1\right) \tag{15}$$

where p_D is the implicit deflator of total final demand, cf. (4). In a closed economy, p_E will equal p_D, cf. (6), and (14) and (15) will, of course, lead to the same result.

An Open Economy

In the following, the analysis will be repeated, but on the assumption that the economy is an open one. Table 3 indicates inter-sectoral flows in year 1. As in

TABLE 3
Inter-Sectoral Flows in an Open Economy, Year 1
(All prices = 1)

	Intermediate Flows to Sectors			Final Demand		Total Output
	1	2	3	Domestic	Exports	
Output from Sector						
1	—	X_{12}	X_{13}	X_{1D}	X_{1A}	x_1
2	X_{21}	—	X_{23}	X_{2D}	X_{2A}	x_2
3	X_{31}	X_{32}	—	X_{3D}	X_{3A}	x_3
Imports	X_{B1}	X_{B2}	X_{B3}	—	—	x_B
Value added	X_{E1}	X_{E2}	X_{E3}	—	—	x_E
Total	x_1	x_2	x_3	x_D	x_A	

Table 1, all prices in year 1 are assumed to be equal to unity.

There are still three industries, but now they have an additional source of final demand, i.e. exports. Exports are indicated by subscript A. Export prices of a certain industry are equal to domestic prices for the same output.

For convenience, imports (denoted by subscript B) are assumed only to consist of intermediate goods. Total imports are denoted by x_B. The assumption of balance-of-payments equilibrium will not be introduced until a later stage.

Table 4 indicates corresponding inter-sectoral flows in year 2. Output prices (for intermediate as well as final deliveries) of the three industries in year 2 are

TABLE 4

INTER-SECTORAL FLOWS IN AN OPEN ECONOMY, YEAR 2, CURRENT PRICES

	Intermediate Flows to Sectors			Final Demand		Total Output
	1	2	3	Domestic	Exports	
Output from Sector						
1	—	$p_1\mathbf{X}_{12}$	$p_1\mathbf{X}_{13}$	$p_1\mathbf{X}_{1D}$	$p_1\mathbf{X}_{1A}$	$p_1\mathbf{x}_1$
2	$p_2\mathbf{X}_{21}$	—	$p_2\mathbf{X}_{23}$	$p_2\mathbf{X}_{2D}$	$p_2\mathbf{X}_{2A}$	$p_2\mathbf{x}_2$
3	$p_3\mathbf{X}_{31}$	$p_3\mathbf{X}_{32}$	—	$p_3\mathbf{X}_{3D}$	$p_3\mathbf{X}_{3A}$	$p_3\mathbf{x}_3$
Imports	$p_{B1}\mathbf{X}_{B1}$	$p_{B2}\mathbf{X}_{B2}$	$p_{B3}\mathbf{X}_{B3}$	—	—	$p_B\mathbf{x}_B$
Value added	$p_{E1}\mathbf{X}_{E1}$	$p_{E2}\mathbf{X}_{E2}$	$p_{E3}\mathbf{X}_{E3}$	—	—	$p_E\mathbf{x}_E$
Total	$p_1\mathbf{x}_1$	$p_2\mathbf{x}_2$	$p_3\mathbf{x}_3$	$p_D\mathbf{x}_D$	$p_A\mathbf{x}_A$	

Source: See Table 14.3, p. 268, in Ølgaard (1966).

denoted p_1, p_2 and p_3, respectively. Import prices may change from industry to industry; they are denoted p_{B1}, p_{B2} and p_{B3}. The remaining "price expressions" in Table 4 are price indices or deflators. As in Table 2, they are given as soon as assumptions with respect to the just mentioned prices have been made.

Let us first consider the open economy as a whole, in year 2 at current prices. Total value added is equal to

$$p_E\mathbf{x}_E = p_D\mathbf{x}_D + p_A\mathbf{x}_A - p_B\mathbf{x}_B \tag{16}$$

where

$$p_E = \frac{p_{E1}\mathbf{X}_{E1} + p_{E2}\mathbf{X}_{E2} + p_{E3}\mathbf{X}_{E3}}{\mathbf{X}_{E1} + \mathbf{X}_{E2} + \mathbf{X}_{E3}}. \tag{17}$$

Furthermore,

$$p_D = \frac{p_1\mathbf{X}_{1D} + p_2\mathbf{X}_{2D} + p_3\mathbf{X}_{3D}}{\mathbf{X}_{1D} + \mathbf{X}_{2D} + \mathbf{X}_{3D}}. \tag{18}$$

The index of export prices equals

$$p_A = \frac{p_1\mathbf{X}_{1A} + p_2\mathbf{X}_{2A} + p_3\mathbf{X}_{3A}}{\mathbf{X}_{1A} + \mathbf{X}_{2A} + \mathbf{X}_{3A}} \tag{19}$$

and the index of import prices equals

$$p_B = \frac{p_{B1}\mathbf{X}_{B1} + p_{B2}\mathbf{X}_{B2} + p_{B3}\mathbf{X}_{B3}}{\mathbf{X}_{B1} + \mathbf{X}_{B2} + \mathbf{X}_{B3}}. \tag{20}$$

Note that all the price indices (17)–(20) are of the Paasche type.

Total *real product* will be equal to

$$\mathbf{x}_E = \mathbf{X}_{E1} + \mathbf{X}_{E2} + \mathbf{X}_{E3} \tag{21}$$
$$= \mathbf{x}_D + \mathbf{x}_A - \mathbf{x}_B$$

and total *real income* will be equal to

$$\mathbf{y} = \frac{p_E \mathbf{x}_E}{p_D} \tag{22}$$
$$= \mathbf{x}_D + \frac{p_A \mathbf{x}_A - p_B \mathbf{x}_B}{p_D}.$$

Note that, contrary to the closed economy, cf. (7), $\mathbf{y}$ will not necessarily be equal to $\mathbf{x}_D$ anymore. In this connection it should be recalled that the balanced-flows assumption has not yet been introduced for the case of the open economy.

The (index of) *terms of trade in foreign trade* will be equal to

$$T_{\text{foreign}} = \frac{p_A}{p_B} \tag{23}$$

where p_A and p_B are defined according to (19) and (20). Finally, let us define the *gain* from the terms of trade in foreign trade as

$$G_{\text{foreign}} = \mathbf{y} - \mathbf{x}_E \tag{24}$$
$$= \frac{p_A \mathbf{x}_A - p_B \mathbf{x}_B}{p_D} - (\mathbf{x}_E - \mathbf{x}_D)$$
$$= \frac{p_A \mathbf{x}_A - p_B \mathbf{x}_B}{p_D} - (\mathbf{x}_A - \mathbf{x}_B).$$

Now the time has come to *introduce the balanced-flows assumption*, implying balance-of-payments equilibrium in the base year (year 2):

$$p_A \mathbf{x}_A = p_B \mathbf{x}_B. \tag{25}$$

The first thing to observe is that, under this additional assumption, total real income will equal total real final domestic demand. Equations (22) and (25) give the result

$$\mathbf{y} = \mathbf{x}_D. \tag{26}$$

Furthermore, the gain of the total economy from the changes of terms of trade in foreign trade, i.e. G_{foreign} according to (24), can now be written as

$$G_{\text{foreign}} = \mathbf{x}_A \left(\frac{\mathbf{x}_B}{\mathbf{x}_A} - 1 \right). \tag{27}$$

From (25) it follows that

$$\frac{\mathbf{x}_B}{\mathbf{x}_A} = \frac{p_A}{p_B}. \tag{28}$$

Hence[12]

$$G_{\text{foreign}} = \mathbf{x}_A\left(\frac{p_A}{p_B} - 1\right) \tag{29}$$

or

$$G_{\text{foreign}} = \mathbf{x}_A(T_{\text{foreign}} - 1). \tag{30}$$

Recalling (26), (24) and (30), it should be noted in passing that, under the balanced-flows assumption, total real income can be determined in two different ways, both leading to the same result:

$$\begin{aligned} \mathbf{y} &= \mathbf{x}_D \\ &= \mathbf{x}_E + \mathbf{x}_A(T_{\text{foreign}} - 1). \end{aligned} \tag{31}$$

Equation (30) describes, for an open economy with balance-of-payments equilibrium, a basic relationship corresponding to that according to (12) for an industry in a closed economy. Keeping in mind that the gain of the economy as a whole from changes of the terms of trade in foreign trade is equal to total real income minus total real product, cf. (24), it follows from (30) that the (absolute) gain will be proportionate to the relative change of the terms of trade in foreign trade, the factor of proportionality being equal to total foreign sales of year 2 at prices of year 1.[13]

Corresponding to the result (13) for an industry in a closed economy, it follows from (24) and (30) that, for an open economy with balance-of-payments equilibrium,

$$\left(\frac{\mathbf{y}}{\mathbf{x}_E} - 1\right) = \frac{\mathbf{x}_A}{\mathbf{x}_E}(T_{\text{foreign}} - 1). \tag{32}$$

The *relative* change of the ratio of (an index of) total real income over (an index of) total real product is proportionate to the relative change of the terms of trade in foreign trade, the factor of proportionality being equal to the ratio of total exports (in year 2 at prices of year 1) over total real product.[14]

Obviously, according to (30) as well as to (32), total real income will exceed total real product if the terms of trade in foreign trade have improved, while the opposite will be true if the terms of trade in foreign trade have deteriorated. If the terms of trade in foreign trade have remained constant, total real income will equal total real product.

[12]Note that G_{foreign} according to (29) can also be written as

$$p_A\mathbf{x}_A\left(\frac{1}{p_B} - \frac{1}{p_A}\right). \tag{29a}$$

This expression is frequently found in the literature on these issues. If an expression of the form of (29a) is used, total exports should be measured at *current* prices.

[13]On the interpretation of this factor of proportionality, see the footnote comments to (12) above. Exactly the same way of reasoning holds for the factor of proportionality, $\mathbf{x}_A$, according to (30).

[14]On the interpretation of this factor of proportionality, see the footnote comments to (13) above.

It now only remains to study the *individual industries* in an open economy, still assuming the balanced-flows assumption to hold. As it turns out, (12) and (13) still hold in an open economy. The proofs run along the same lines as those indicated for an industry in a closed economy, cf. the expressions preceding equation (11), and will not be repeated in the present context. It should be recalled that, for an industry in an open economy, the price index of total purchases of, e.g. industry 1 in year 2 will include import prices and hence be equal to

$$\frac{p_2\mathbf{X}_{21}+p_3\mathbf{X}_{31}+p_{B1}\mathbf{X}_{B1}+(p_{E1}\mathbf{X}_{E1}/p_E\mathbf{x}_E)(p_1\mathbf{X}_{1D}+p_2\mathbf{X}_{2D}+p_3\mathbf{X}_{3D})}{\mathbf{X}_{21}+\mathbf{X}_{31}+\mathbf{X}_{B1}+(p_{E1}\mathbf{X}_{E1}/p_E\mathbf{x}_E)(\mathbf{X}_{1D}+\mathbf{X}_{2D}+\mathbf{X}_{3D})}.$$

Recalling that (25) implies that $p_E\mathbf{x}_E = p_D\mathbf{x}_D$, cf. (16), the price index of total purchases can be reduced to

$$\frac{p_1\mathbf{x}_1}{\mathbf{x}_1-\mathbf{X}_{E1}+(p_{E1}\mathbf{X}_{E1}\mathbf{x}_D/p_E\mathbf{x}_E)}.$$

Hence the terms of trade of industry 1, ${}_1T_{\text{total}}$, will, in an open economy, be equal to

$$(33) \qquad {}_1T_{\text{total}} = \frac{p_E\mathbf{x}_E\mathbf{x}_1 - p_E\mathbf{x}_E\mathbf{X}_{E1} + p_{E1}\mathbf{X}_{E1}\mathbf{x}_D}{p_E\mathbf{x}_E\mathbf{x}_1}.$$

Note that if $\mathbf{x}_E = \mathbf{x}_D$, which will be the case in a closed economy, cf. (1), and in an open economy with constant terms of trade, cf. (31), (33) becomes equal to (11).

Hence we may conclude this exercise by explicitly stating that, for industry i in an open economy with balance-of-payments equilibrium in the base year (year 2)

$$(34) \qquad G_i = \mathbf{Y}_i - \mathbf{X}_{Ei}$$

where $\mathbf{Y}_i$ is still defined as in (5). Furthermore,

$$(35) \qquad G_i = \mathbf{x}_i\,({}_iT_{\text{total}} - 1)$$

and

$$(36) \qquad \left(\frac{\mathbf{Y}_i}{\mathbf{X}_{Ei}} - 1\right) = \frac{\mathbf{x}_i}{\mathbf{X}_{Ei}}({}_iT_{\text{total}} - 1).$$

Finally, it should be noted that, according to the definitions and assumptions made above,

$$(37) \qquad G_{\text{foreign}} = G_1 + G_2 + G_3$$

G_{foreign} being defined in (24) and G_i in (34). Inserting (30) and (35) in (37), it follows that

$$(38) \qquad \mathbf{x}_A(T_{\text{foreign}} - 1) = \mathbf{x}_1({}_1T_{\text{total}} - 1) + \mathbf{x}_2({}_2T_{\text{total}} - 1) + \mathbf{x}_3({}_3T_{\text{total}} - 1).$$

Hence, according to (38), the gain (of the economy as a whole) from changes of the terms of trade in foreign trade, G_{foreign}, is exactly distributed among the

various industries, the gain of each industry being equal to the relative change of its terms of trade (${}_iT_{\text{total}} - 1$) multiplied by its total sales, $\mathbf{x}_i$ in year 2 at prices of year 1. In turn, the latter expression is equal to the difference between real income and real product of the industry concerned.

Equation (38) emphasizes the fact that the gain from changes of the terms of trade in foreign trade will usually not be distributed evenly among the various industries. E.g. an increase of prices of agricultural products, including agricultural exports,[15] will improve the terms of trade in foreign trade—and the terms of trade of the agricultural sector as well. But, at the same time, the other industries may suffer a deterioration of their terms of trade, because prices of part of their final demand, i.e. food prices, will increase. In the numerical example below, such relationships are illustrated for an open three-sector economy.

The Real Income/Real Product Ratio by Industries. How to Do It Yourself!

As appears from (36), the ratio of real income over real product of a certain industry plays a decisive role in determining its industry terms of trade. It might be worth noting that such figures are easily arrived at on the basis of time series available in most countries. The *real income/real product ratio* of a certain industry will simply be roughly equal to the *share of total domestic product at current prices* of the industry concerned, *divided by the share of total real product* (= domestic product at constant prices) *of the same industry*. The ratio of these two shares is equal to

$$\frac{(p_{Ei}\mathbf{X}_{Ei}/p_E\mathbf{x}_E)}{\mathbf{X}_{Ei}/\mathbf{x}_E} = \frac{p_{Ei}}{p_E}. \tag{39}$$

Recalling (5), the real income/real product ratio of a certain industry is defined as

$$\frac{\mathbf{Y}_i}{\mathbf{X}_{Ei}} = \frac{p_{Ei}}{p_D}. \tag{40}$$

Hence (39) will be exactly equal to (40) if, in the base year, p_E is equal to p_D. Sufficient conditions for $p_E = p_D$ are that, in the base year, there is equilibrium on the balance of payments (implying $p_E\mathbf{x}_E = p_D\mathbf{x}_D$, cf. (16)) and that, furthermore, $\mathbf{x}_E$ is equal to $\mathbf{x}_D$, which will be the case if the terms of trade in foreign trade have remained constant, cf. (31). Of course, these conditions will rarely be fulfilled completely, but the quantitative effect of possible discrepancies will usually not be substantial, as long as attention is limited to terms of trade at the industry level, especially if foreign trade as a percent of total domestic product is comparatively small.[16]

[15] As was experienced when Denmark joined the EEC.

[16] Expressed in more general terms, the possible discrepancy between the ratios according to (39) and (40) will be related to gains from the terms of trade in *foreign* trade. (In a closed economy, $p_E = p_D$ by definition, cf. (6)). Such gains may simply reflect changes of the terms of trade in foreign trade as defined by (23), but they may also be due to a price level effect, cf. the section on "the balanced-flows assumption" above. However, the condition of equilibrium on the balance of payments rules out a possible price level effect.

Summary

The main conclusions to be drawn from the above analysis may be summarized as follows:

(a) As a matter of routine, traditional figures of domestic product by industries at constant prices (*real product* figures) ought to be supplemented by publication of *real income* figures, by industries (cf. equation (5)) and for the economy as a whole (cf. equations (22) and (31)). The real income figures will reflect changes of *relative* prices (as well as quantity changes). Furthermore, the difference between *total* real income and *total* real product will reflect changes of the terms of trade in *foreign trade*, cf. equations (24) and (30).

(b) Assuming the value of total sales of each industry in the base year to be equal to the value of its total purchases, of inputs as well as of final goods, the difference between real income and real product of a certain industry (or the ratio between these two concepts) will be *proportionate to the relative change of the terms of trade of the industry concerned*, cf. equations (35) and (36).

Furthermore, the sum of the gains by industries from changes of their terms of trade will be equal to the gain of the economy as a whole from changes of its terms of trade in foreign trade, cf. equation (38). Usually, a gain from changes of the terms of trade in foreign trade will not be distributed evenly among the various industries. The suggested concepts and procedures make it possible to indicate exactly how the gain is actually distributed.

(c) The advantages to be obtained from such real income figures should be appreciated from at least two different viewpoints:

First, the suggested figures of *total real income* will make it possible to take into account, in a systematic way, the changes of the terms of trade in *foreign trade*. This aspect will be of particular importance from a *short run* viewpoint, the effect from increasing fuel prices in 1974 and in 1979 being a case in point.

Secondly, to the extent that long term trends in sector productivity vary between industries—and that these differences are reflected in relative output prices—the result will be one, not only of real income figures by industries differing from corresponding real product figures, but of *systematic differences of long term patterns* of real product figures as compared with corresponding real income figures. Hence the latter are needed as a basis for analysis, not in order to replace the real product figures, but in order to supplement them.

A Numerical Example

In order to illustrate the relationships presented above in general terms, a numerical example may be appropriate. Recalling that quantities of year 1 did not enter into the price indices above, the latter being of the Paasche type, it would not add much to the following illustration to assume quantities to change from year 1 to year 2. Instead, it is assumed that they remain unchanged.[17]

[17]This, however, does not hold for the implicit distribution of final domestic demand by industries, cf. below.

TABLE 5

EXAMPLE. INPUT-OUTPUT TABLE OF YEAR 1

(All prices = 1)

	Intermediate Flows to Industry			Final Demand		Total Output
	A	B	C	Domestic	Exports	
Output from Industry						
A	—	—	20	100	50	170
B	10	—	—	60	10	80
C	40	10	—	60	10	120
Imports	70	20	30	—	—	120
Value added	50	50	70	—	—	170
Total	170	80	120	220	70	

As in Table 3 above, all prices are assumed to be equal to unity in year 1. Total value added is equal to 170, distributed between industries A, B and C[18] as 50, 50 and 70, respectively. Total imports equal 120 and total exports 70. Hence there is a balance-of-payments deficit in year 1—but year 1 is not the base year of the following price indices. Total final domestic demand equals total value added plus the balance-of-payments deficit, i.e. 220.

From year 1 to year 2, output prices of industry A increase from unity to 3, those of industry B increase to 2 while those of industry C remain constant. Note that by these assumptions, *export prices* have been fully described. For convenience, all *import prices* are assumed to increase at the same rate, namely by fifty percent.

In Table 6, the input–output table of year 2 is shown at current prices, all flows (except those of value added) being indicated as quantities multiplied by prices. Year 2 is the base year of the various Paasche price indices.[19] Note that, in year 2, total exports at current prices equal total imports at current prices (= 180). Hence total final domestic demand equals total value added (= 480).

Real income and real product figures of year 2 are indicated in Table 7. *Real product* figures follow immediately from Table 5, because quantities of output and input are assumed to have remained constant. These indices are of the Laspeyres type.

Real income figures of year 2 by industries are found by deflating value-added figures at current prices, cf. Table 6. The common deflator, used for *all*

[18]In order to facilitate the presentation, the industries have been renamed as compared with Tables 1–4. Note that, in the following, A and B are no longer indicating exports and imports.

[19]In the present example, quantities remain unchanged from year 1 to year 2. Hence Laspeyres and Paasche indices will usually lead to the same result. Note, however, that this is not true with respect to the implicit distribution of final domestic demand by industries. Thus, even in the example, price indices may differ according to type, and the proper price-index type to use is Paasche price indices.

TABLE 6

EXAMPLE. INPUT-OUTPUT TABLE OF YEAR 2, CURRENT PRICES

	Intermediate Flows to Industry			Final Demand		Total Output
	A	B	C	Domestic	Exports	
Output from Industry						
A	—	—	20×3	100×3	50×3	170×3
B	10×2	—	—	60×2	10×2	80×2
C	40×1	10×1	—	60×1	10×1	120×1
Imports	$70\times1\frac{1}{2}$	$20\times1\frac{1}{2}$	$30\times1\frac{1}{2}$	—	—	$120\times1\frac{1}{2}$
Value added	345	120	15	—	—	480
Total	510	160	120	480	180	—

industries, is that of total final domestic demand, equal to 480 divided by $220 = 24/11$.[20] As appears from Table 7, industry figures of real income and real product in year 2 may differ substantially. Output prices of *industry A* show the largest increase from year 1 to year 2, namely from 1 to 3. Hence the substantial gain from changes of relative prices obtained by industry A is not surprising.

At the other extreme, output prices of *industry C* remain constant from year 1 to year 2, while all other prices increase. In accordance with this pattern, industry C suffers a substantial loss due to a deterioration of its terms of trade.

TABLE 7

EXAMPLE. REAL INCOME, REAL PRODUCT ETC., YEAR 2

	Real Income (1)	Real Product (2)	Difference = (1) − (2)
Industry A	158.1	50.0	108.1
Industry B	55.0	50.0	5.0
Industry C	6.9	70.0	−63.1
Total	220.0	170.0	50.0

[20] An alternative way of determining the common deflator, leading to the same result, cf. equation (31) above, is the following:

In year 2, total real *product* equals 170. In order to arrive at total real *income*, an additional term must be added, cf. the final expression in (31). This term is exports (in year 2 at prices of year 1, i.e. 70) multiplied by the percentage change of the terms of trade in foreign trade, the latter being equal to 71.4 percent, cf. Table 8 below. Hence the gain from changes of the terms of trade in foreign trade is equal to 50 and total real income in year 2 equals $170 + 50 = 220$. Thus the real income deflator is equal to total value added of year 2 at current prices (480) divided by $220 = 24/11$.

Note, finally, that the ratio of total value added in year 2 divided by total value added in year 1 (480/170) represents the deflator of *total* real *product*—but not that of its components in terms of value added by industries.

Terms of trade figures for individual industries as well as for the total economy are shown in Table 8. According to the bottom row, export prices have increased by 157.1 percent, while import prices only increased by 50 percent.

TABLE 8

EXAMPLE. TERMS OF TRADE ETC., YEAR 2

	Price Index of					
	Sales (1)	Purchases (2)	t.o.t. = (1)/(2) (3)	$\frac{\text{t.o.t.}}{100} - 1$ (4)	Total Sales[a] (5)	Gain = (4) · (5) (6)
Industry A	300.0	183.4	163.6	0.6360	170	108.1
Industry B	200.0	188.2	106.3	0.0625	80	5.0
Industry C	100.0	211.0	47.4	−0.5260	120	−63.1
Total economy = foreign trade	257.1	150.0	171.4	0.7143	70	50.0

[a]in year 2 at prices of year 1.

Note: All figures are calculated before rounding of the basic data. Apart from price indices of purchases, the figures should be self-explaining. E.g. sales prices of the total economy, i.e. the index of export prices, equals 180/70 = 257.1. Correspondingly, the index of import prices is of course equal to 150.

The price index of purchases by industry B is determined as follows (those of industries A and C are found in a similar way): In year 2 at current prices, the value added of B equals one fourth (120/480) of total value added. Hence final domestic demand of B equals 25 units of the output of A, 15 units of the output of B and 15 units of output of C. Furthermore, B buys inputs equal to 10 units of the output of C and 20 units of imports, cf. Table 6. On the basis of all these quantities, the (Paasche) price index of total purchases by B is determined.

Hence the terms of trade improved by 71.4 percent. Multiplying this figure by total exports (in year 2 at prices of year 1) provides the gain of the economy from changes of the terms of trade *in foreign trade*. This gain is equal to 50 units. This figure, in turn, equals the difference between *total* real income and *total* real product according to Table 7.

Corresponding *terms of trade figures by industries* are calculated in the first rows of Table 8. Indices of sales prices by industries follow immediately; indices of prices of purchases are calculated according to a procedure described in the note to Table 8. From these indices, the terms of trade indices of the industries are easily found. When the percentage changes of these indices are multiplied by total sales (= total *output* of year 2 at prices of year 1) of the industry concerned, figures of gains (or losses) by industries due to changes of relative prices are finally arrived at. In accordance with equations (34) and (35), the industry gains according to Table 8 equal differences between real income and real product figures according to Table 7. Furthermore, the sum of the industry gains equals the gain from the terms of trade in foreign trade, cf. equations (37) and (38).

As appears from Table 8, the terms of trade in foreign trade improved by well over 70 percent from year 1 to year 2. This result is not surprising, recalling that

import prices only increased by 50 percent, while prices of the main part of exports either trebled (A) or doubled (B).

However, the gain from changes of the terms of trade in foreign trade was certainly not distributed evenly among industries. Industry A had a substantial gain, while C suffered a loss.

Immediately, the result obtained by *industry B*—a very minor gain at only 5 units—may be surprising. After all, the total gain of the economy from the improvement of terms of trade in foreign trade, to be distributed among the industries, was substantial (equal to 50 units). Furthermore, output prices of industry B doubled, increasing more than import prices. In addition, apart from imports, B only uses input from industry C, and the price of the latter remained unchanged.

However, the share of input in total output of B is rather modest, implying that the share of value added (and hence of final demand) in total output is comparatively high, as far as industry B is concerned. Furthermore, average prices of final demand have increased more than prices of B's output, the price index of final demand being equal roughly to 218 (= 480/220). Hence the modest gain by sector B in spite of low prices of its inputs in year 2 is explained by a substantial increase of the average price of final demand, the latter exceeding the price increase of B's own output.

The Balanced-Flows Assumption Reconsidered

As was pointed out in the introductory part of this paper, one of its purposes is to try to persuade Central Statistical Offices in the various countries to publish figures of real income by industries on a regular basis. The interpretation of such figures has been discussed at great length, but mainly under the balanced-flows assumption. However, actual current national accounts statistics will, of course, usually show surpluses or deficits on the balance of payments. Hence some *ad hoc* adjustments will be required. In the following, a few comments will be made on this problem. Attention will be limited to the relationship between *total* real income and *total* real product.[21]

On the balanced-flows assumption, total real income ($\mathbf{y}$) and total real final domestic demand ($\mathbf{x}_D$) are bound to be identical, even in an open economy, cf. (31). However, if flows are not balanced, this relationship will not hold, cf. (22). The appropriate procedure under these less restricted assumptions might be the following:

(a) The starting point should still be *total real product*, $\mathbf{x}_E$, cf. (21):

$$\mathbf{x}_E = \mathbf{X}_{E1} + \mathbf{X}_{E2} + \mathbf{X}_{E3}.$$

(b) In order to arrive at *total real income*, $\mathbf{y}$, the gain from foreign trade, G_{foreign} cf. (30), should be added to $\mathbf{x}_E$, cf. (31):

$$\mathbf{y}^* = \mathbf{x}_E + \mathbf{x}_A(T_{\text{foreign}} - 1)$$

the asterisk indicating that the expression only represents an approximation.

[21] In the following, the implicit assumption of the same final demand pattern by all industries is retained. In addition, the percentage distribution of real income is, of course, still equal to that of domestic product at *current* prices.

(c) In order to arrive at an expression roughly equal to *total real final domestic demand*, $\mathbf{x}_D$, it should be recalled that, according to (22) which is not restricted by the balanced-flows assumption,

$$\mathbf{x}_D = \mathbf{y} + \frac{p_B\mathbf{x}_B - p_A\mathbf{x}_A}{p_D}.$$

This suggests that an approximation of total real final domestic demand is best obtained by adding $(\mathbf{x}_B - \mathbf{x}_A)$ to $\mathbf{y}^*$:[22]

$$\mathbf{x}_D^* + \mathbf{y}^* + (\mathbf{x}_B - \mathbf{x}_A).$$

For practical purposes, this procedure will probably give results which are acceptable. Whether this will, in fact, be the case will depend on whether $\mathbf{x}_D^*$, determined according to this procedure, will be equal to—or at least close to—total final domestic demand at current prices deflated by p_D, p_D being determined from (18).[23]

However, at this point additional problems are usually bound to arise, because figures of final domestic demand will be indicated at market prices. This leads us to the final supplementary comment on the concepts involved.

Corresponding Problems of Deflation for Components of Domestic Demand

On the basis of the above analysis, an obvious question would be whether a corresponding procedure should not be followed with respect to components[24] of *final domestic demand.* Immediately, the answer is affirmative. Traditional figures published in this field at constant prices are conceptually similar to what was labelled "real product" above. But relative prices also change on the demand side, a comparative increase of the "price" of public consumption usually being the main example. However, in the following we will limit ourselves to a few comments on the subject.

The first may be considered trivial, namely how to "label" the two different concepts of final domestic demand at constant prices, corresponding to the concepts of "real product" and "real income," introduced above.

The second comment contains more basic substance. As was mentioned in the section on the framework of the formal analysis, it was assumed that sales prices would equal the prices to be paid by the purchasers. This implies that commodity taxes and subsidies were assumed away.

[22]Note that if $p_A = p_B = p_D$, the difference between $\mathbf{x}_D$ and $\mathbf{y}$ will exactly be equal to $(\mathbf{x}_B - \mathbf{x}_A)$.

[23]This suggestion is in accordance with the comments on how to arrive at the real income deflator made in connection with Table 7, although it should be kept in mind that, in the numerical example, flows were actually balanced in the base year (year 2). Real income figures were found, cf. the text, by deflating value added figures at current prices by the implicit deflator of total final domestic demand. The alternative procedure, corresponding to the suggestions in the present section, was described in a footnote.

[24]Note that, in the above analysis, *total* real final domestic demand, i.e. $\mathbf{x}_D$, has already been determined.

On this background, traditional demand figures at current market prices do not provide the obvious starting point for a procedure of deflation, according to which figures at current prices for all demand components in a certain year are deflated by the same deflator. If this procedure is followed, the resulting figures will not only reflect changes of relative prices, but also changes of rates of commodity taxes.

More fundamentally, following this procedure would imply that the rule according to which sales prices should equal purchase prices would, of course, not be adhered to. In order to "solve" this problem, an easy way out might be to assume that all commodity taxes (and subsidies) only affect the component of private consumption.[25] Hence the figure on total net commodity taxes could be deducted from total private consumption at current market prices. By introducing this adjustment, the figures would become consistent.

Clearly, this procedure is not very sophisticated. A superior solution might be to express all the flows, on the demand as well as on the supply side, at basic values.

Some Results, Based on Danish data

It has not been the purpose of the present paper to provide analyses of time series describing the concepts suggested. On this issue, reference is made to, e.g., Bjerke (1972). Still, just in order to provide an idea of the orders of magnitude involved, a few figures might be mentioned, cf. Ølgaard (1980). The figures, shown in Tables 9–12, cover the period 1950–73.

In Table 9, year-by-year figures are indicated for gross domestic product at current prices, for total real product and for total real income.[26] From the viewpoint discussed in the present paper, the most interesting results may be those to be obtained by comparing the columns of year-by-year growth rates of total real product and of total real income, keeping the column of the index of terms of trade in foreign trade in mind. The percentage increase in a certain year (as compared with the preceding) of real product and real income may differ quite substantially. This is true in years where the terms of trade in foreign trade show large changes, cf. the first years illustrated in the table.[27]

In Table 10, traditional figures of domestic product by industries at constant prices are shown. For comparison, corresponding real income figures are shown in Table 11. As can be observed immediately, the changes of the industrial pattern according to the two tables are very different.

Finally, indices illustrating the ratio of real income over real product, by industries as well as for the total economy, are shown in Table 12. The figures describe the ratios $\mathbf{Y}_i/\mathbf{X}_{Ei}$ as well as $\mathbf{y}/\mathbf{x}_E$, cf. equations (36) and (32) above.

[25]This solution of course presupposes that the disaggregation into demand components is not very detailed.

[26]Real income figures have been determined by adjusting real product figures, $\mathbf{x}_E$, according to (31), i.e. adding G_{foreign} according to (29).

[27]When the terms of trade in foreign trade improve, the rate of increase of real income will, of course, always exceed that of real product and vice versa.

TABLE 9

TOTAL GDP, REAL PRODUCT, REAL INCOME ETC., DENMARK 1949–73

	GDP, current prices bill. kr.	GDP, deflator 1955 = 100	Terms of trade, f.t. 1955 = 100	Real product, 1955 prices		Real Income, 1955 prices	
				bill. kr.	% incr.	bill. kr.	% incr.
1949	18.9	78.6	107	24.0		24.4	
1950	21.6	83.6	98	25.8	7.6	25.7	5.3
1951	23.4	90.2	89	25.9	0.4	25.1	−2.3
1952	25.0	95.1	94	26.3	1.2	25.8	2.9
1953	26.5	95.3	97	27.8	5.9	27.6	6.8
1954	27.6	96.8	99	28.5	2.5	28.5	3.3
1955	28.7	100.0	100	28.7	0.1	28.7	0.8
1956	30.6	104.7	101	29.3	1.9	29.4	2.4
1957	32.7	106.0	98	30.8	5.4	30.6	4.2
1958	34.0	107.8	100	31.5	2.3	31.6	3.1
1959	37.4	111.9	106	33.4	6.1	34.2	8.4
1960	40.5	113.8	103	35.6	6.5	36.1	5.4
1961	45.4	120.7	102	37.6	5.6	37.8	4.8
1962	50.8	127.9	104	39.7	5.6	40.3	6.6
1963	53.5	134.2	105	39.9	0.4	40.6	0.9
1964	61.1	140.9	107	43.4	8.8	44.6	9.7
1965	68.3	150.0	107	45.5	5.0	47.0	5.3
1966	74.7	160.7	109	46.5	2.2	48.3	2.8
1967	81.9	169.1	107	48.4	4.1	49.8	3.2
1968	89.3	176.4	104	50.6	4.5	51.6	3.6
1969	101.1	184.5	106	54.8	8.2	56.3	9.1
1970	112.6	200.2	106	56.3	2.6	57.9	2.8
1971	123.5	211.9	106	58.3	3.6	60.0	3.6
1972	138.9	227.7	109	61.0	4.6	63.9	6.4
1973	159.1	253.7	110	62.7	2.8	66.1	3.5

Note: The terms-of-trade figures in the third column are calculated from the implicit deflators of exports and imports of goods and services according to national accounting data. On the whole, they are very similar to the unit-value indices published by Danmarks Statistik, the latter covering trade of goods only. The main exception is that the figures above show a smaller deterioration from 1949 to 1951 (terms of trade deteriorating from 107 to 89) than figures derived from the unit-value data (from 112 to 86).

It should be added that real-income figures are calculated before rounding of the terms-of-trade figures above. In fact the terms of trade improved slightly from 1964 to 1965 and from 1969 to 1970.

Source: See Appendix Table I.1, p. 32, in Ølgaard (1980).

TABLE 10

GROSS DOMESTIC PRODUCT AT FACTOR COSTS, 1955 PRICES (= REAL PRODUCT)

	1950		1958		1966		1973	
	billion kr.	percent	billion kr.	percent	billion kr.	percent	billion kr.	percent
Agriculture etc.	5.2	(20.1)	5.9	(18.7)	6.3	(13.5)	6.2	(9.9)
Manufacturing, handicraft etc.	7.5	(29.1)	9.0	(28.4)	15.1	(32.4)	22.1	(35.3)
of which								
Manufacturing	4.8	(18.5)	5.9	(18.6)	10.2	(22.0)	15.8	(25.3)
Handicraft	2.4	(9.4)	2.6	(8.3)	3.8	(8.1)	4.2	(6.7)
Electricity, gas, etc.	0.3	(1.2)	0.5	(1.5)	1.1	(2.3)	2.1	(3.3)
Building and construction	1.8	(7.1)	2.2	(6.9)	3.8	(8.3)	5.0	(7.9)
Commerce, banking, hotels etc.	4.9	(18.9)	6.0	(19.1)	9.3	(19.9)	12.7	(20.2)
Transport	2.2	(8.7)	3.1	(10.1)	4.6	(9.9)	6.4	(10.3)
Private services, including use of dwellings	2.1	(8.0)	2.5	(7.9)	3.0	(6.6)	3.5	(5.6)
Government services	2.1	(8.1)	2.8	(8.9)	4.4	(9.4)	6.8	(10.8)
Total real GDP = real product	25.8	(100.0)	31.5	(100.0)	46.5	(100.0)	62.7	(100.0)

Source: See Table I.6, p. 19, in Ølgaard (1980).

TABLE 11

REAL INCOME BY INDUSTRIES, 1955 PRICES

	1950		1958		1966		1973	
	billion kr.	percent	billion kr.	percent	billion kr.	percent	billion kr.	percent
Agriculture etc.	5.4	(21.2)	5.1	(16.2)	5.0	(10.3)	5.1	(7.8)
Manufacturing, handicraft etc.	7.5	(29.2)	9.4	(29.7)	14.8	(30.6)	19.2	(29.1)
of which:								
Manufacturing	4.9	(19.0)	6.0	(19.0)	9.3	(19.3)	12.8	(19.4)
Handicraft	2.2	(8.7)	2.8	(8.8)	4.6	(9.5)	5.5	(8.3)
Electricity, gas etc.	0.4	(1.5)	0.6	(1.9)	0.9	(1.8)	0.9	(1.4)
Building and construction	1.7	(6.6)	2.2	(7.0)	4.4	(9.1)	6.3	(9.5)
Commerce, banking, hotels etc.	5.1	(19.8)	5.9	(18.7)	9.0	(18.7)	11.8	(17.8)
Transport	2.1	(8.3)	3.2	(10.1)	4.7	(9.7)	6.2	(9.4)
Private services, including use of dwellings	2.0	(7.6)	2.7	(8.5)	4.0	(8.3)	4.9	(7.4)
Government services	1.9	(7.3)	3.1	(9.8)	6.4	(13.3)	12.6	(19.0)
Total real income	25.7	(100.0)	31.6	(100.0)	48.3	(100.0)	66.1	(100.0)

Source: See Table I.7, p. 20, in Ølgaard (1980).

TABLE 12

INDICES OF REAL INCOME, REAL PRODUCT ETC. BY INDUSTRY; 1958, 1966 AND 1973 (1950 = 100)

	1950	1958	1966	1973
Agriculture, etc.				
Real income	100	94	92	95
Real product	100	114	121	120
Real income/real product	100	83	76	79
Manufacturing				
Real income	100	123	191	263
Real product	100	123	215	332
Real income/real product	100	100	89	79
Handicraft				
Real income	100	124	205	245
Real product	100	108	155	174
Real income/real product	100	115	132	140
Electricity, gas, etc.				
Real income	100	152	221	228
Real product	100	160	360	680
Real income/real product	100	95	62	34
Building and construction				
Real income	100	131	259	369
Real product	100	119	209	272
Real income/real product	100	110	124	136
Commerce, banking, etc.				
Real income	100	116	178	231
Real product	100	123	190	259
Real income/real product	100	95	94	89
Transport				
Real income	100	148	220	289
Real product	100	141	204	285
Real income/real product	100	106	108	101
Private services				
Real income	100	137	205	252
Real product	100	120	147	168
Real income/real product	100	115	140	149
Government services				
Real income	100	166	343	670
Real product	100	133	208	322
Real income/real product	100	125	165	208
Total value added:				
Real income	100	123	188	257
Real product	100	122	180	243
Real income/real product	100	101	104	106

Source: See Appendix Table I.3, p. 34, in Ølgaard (1980).

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