STOCHASTIC PROCESS MODELS AND THE DISTRIBUTION OF EARNINGS*

BY LARS OSBERG

Dalhousie University, Halifax, N.S.

This article examines several hypotheses concerning the stochastic nature of year to year variations in individual incomes in light of newly available microdata on individual earnings. In particular, the models of Solow (1951), Champernowne (1953), and Rutherford (1955) are examined in some detail, and their predictions as to changes to be expected in the distribution of individual incomes are tested. The author concludes that the distributions arrived at using these models are not very similar either to each other or to the actual distribution of earnings. Thus, he believes that as an "explanation" of earnings dynamics stochastic process models are unsatisfactory. He further criticizes these models on the grounds that they foster a bias toward the belief in the inevitability, and perhaps desirability, of the current distribution of earnings.

1. INTRODUCTION AND SUMMARY

The current availability of longitudinal microdata on individual earnings (i.e., the LEED[1] file) enables direct examination of year to year changes in individual earnings and the testing of such models as Solow's (1951), Champernowne's (1953) and Rutherford's (1955) which assume that such changes can be viewed as a stochastic process. In particular Solow's conclusion as to the similarity of the ergodic distributions corresponding to Markov processes whose transition probabilities are the year to year changes in deflated earnings can now be examined directly, for 10 instead of 2 cases. It appears that these distributions are not very similar either to each other or to the actual distribution of deflated earnings. Neither is Solow's conclusion as to the normality of the distribution of cumulative earnings supported by the data. Champernowne's description of the sort of stochastic process which can produce a Paretian distribution can also be examined, and is found to be a poor approximation to actual year to year changes in arnings. (The hypothesis of a Paretian distribution of earnings has been extensively examined elsewhere—notably in Lydall (1968).) Rutherford's implication that the dispersion of earnings increases with age is also examined at the micro-economy level and seen to be not universally true.[2]

*Acknowledgements: This paper is a condensed version of part of Chapter 1 of my Ph.D. thesis, "A Structural Approach to the Distributions of Earnings"; (Yale, 1975) which benefitted greatly from the comments of G. H. Orcutt, R. Ruggles and M. Bailey. Additional thanks go to K. Peck and T. Hartge—remaining errors are solely my responsibility.

[1]Longitudinal Employer–Employee Data, 1957–1969 (Social Security Administration).

[2]Of course, these authors were not the earliest to use the analogy of a random process when considering the distribution of earnings. Kapetyn (1903) had been the first to suggest, but Gibrat (1931) the first to popularize, the idea of pure chance as a major determinant of the distribution of income. Inequality among occupational earnings as a result of risk preference is an idea that dates back at least to Adam Smith but there was little idea that, for individual earnings, chance was but an occasionally important factor. Early writers were too deterministic in their outlook to ascribe earnings differentials directly to random factors—even Gibrat hedged on the role of chance and suggested that it operated, at least partially, via the distribution of abilities.

These formulations of the stochastic process hypothesis do not therefore appear to be supported by available data. More complicated variants, such as the use of a higher order Markov-process (see Shorrocks, 1976), can always be proposed as remedies for this deficiency but the economic interpretation and the utility of these variants can become problematic. In short, as an "explanation" of earnings dynamics stochastic process models appear unsatisfactory. In addition one can argue that they foster a bias towards a belief in the inevitability, and perhaps desirability, of the current distribution of earnings.

2. Solow and the Ergodic Distribution

Solow's Ph.D. thesis (1951a) and paper (1951b) argued that chance was a determining mechanism in earnings distribution. Income distribution was viewed as a Markov process and since "a well-behaved Markov chain if left to itself will eventually transform any arbitrary initial distribution of income into a particular stationary or equilibrium (ergodic) distribution which depends only on the transition probabilities;"[3] a complete description of such a process is therefore contained in its matrix of transition probabilities. The basic empirical assumption is that "two people with the same income this year are assumed to face the same chances for next year, *ceteris paribus*, of course, with respect to such things as sex, age, etc., regardless of how they fared last year or earlier."[4] Perhaps because of limitations in his data source (the Continuous Work History Panel of the Bureau of Old Age and Survivors Insurance for the years 1944 and 1937 to 1941) Solow did not in fact estimate disaggregated transition probability matrices and therefore assumed away the existence of non-competing groups.

Viewing earnings distribution as a many times repeated Markov process may lead one to believe that the current distribution of earnings should resemble the ergodic distribution of such processes. Unfortunately, the ergodic distributions corresponding to the transition probabilities between dollar income ranges for the years 1937 to 1940 "all differ substantially from each other and from the observed distributions tabulated."[5] The income ranges were therefore deflated and new ergodic distributions and actual distributions compared by visual inspection of their cumulative income graphs (i.e., seven points on each of four functions). "Neglecting the observed and ergodic distributions for the war years 1943–44 . . . the overpowering impression is that all the reduced distributions are similar"[6]—but one would think this method to be a very blunt tool for discriminating among distributions.

Using the property of non-degenerate Markov processes that the Central Limit Theorem may apply to individual earnings, Solow then conjectured that an individual's earnings, considered over a period of years, will be normally distributed; using nominal dollar earnings for continuous workers for the period 1937–45 he tested this hypothesis and concluded 10 years is a long enough time to estimate such a distribution.

The similarity of ergodic distributions is the crucial empirical point in Solow's work. The assumption of a simple Markov process is not, to Solow, methodologi-

[3]Solow (1951a, p. 3)—when in Feller's (1968, p. 385) terminology, the system forms an irreducible chain.
[4]*Ibid.*, (p. 2).
[5]*Ibid.*, (p. 89).
[6]*Ibid.*, (p. 92).

cally central—he emphasizes that the hypothesis of such a process has as its chief virtue analytical simplicity and one could in principle construct more complex Markov matrices where current income was dependent on several years previous income, or other factors. One might conclude that very complex Markov processes were too cumbersome to be of practical use but the use of Markov processes in a theory is much like the use of algebra[7]—it can be seen as a mathematical technique whose formulation in a particular instance may be inappropriate but which can in principle be elaborated to handle any eventuality—and therefore is not really possible to reject as a general approach.

In Solow's view, instability of the transition matrices would not disqualify the theory—the two matrices compared (1937/38 and 1939/40) do in fact vary fairly extensively.[8] In fact no real explanation is given for the presumed stability of the ergodic distribution—"It seems possible at least that the relative similarity of the ergodic distributions must in large part be due to other structural characteristics of the process . . . the general shape of the transition kernels may be such that . . . the changes in the transition mechanism tend to be ironed out and result in only small changes in the relative distribution of wage incomes."[9]

Solow was, of course, greatly constrained in his work by the presentation of BOASI data in tabular form for only a few years. Now, however, the Social Security Administration makes available microdata—The Longitudinal Employer/Employee Data Set (LEED)—which covers not only more years (1957 to 1969) but also with far greater possibilities for data manipulation. Where Solow, for example, was forced (in order to "remove" the effect of changing aggregate economic conditions), to deflate income interval classes by deflating the class endpoints [which in turn made tests of cumulative similarity appropriate] with microdata it is a relatively simple procedure to deflate individual earnings in any given year by the ratio of that year's mean income (of all workers) to the base year's mean. This is a much purer method of accomplishing Solow's aim of concentrating on relative movements of earnings and has the advantage that one can estimate directly comparable series of transition probabilities.

Simple cross-tabulation of year 1 deflated earnings by year 2's provides an unbiased estimate of transition probabilities and, using the resultant matrices of estimated transition probabilities, one can then compute the corresponding ergodic distributions of earnings. If Solow's hypotheses are correct, these should be fairly similar to each other and to the actual distributions of earnings in these years (see Table 1).[10]

[7] I owe this point to G. H. Orcutt.

[8] *Ibid.*, (pp. 97–99); Solow ascribes this to the business cycle but clearly this is "post hoc ergo propter hoc."

[9] *Ibid.*, (p. 100).

[10] As Solow noted, his use of only continuously employed workers excludes labour force entrants and retirees. The resulting age bias in the sample would not be crucial for the analysis of a relatively short period (such as Solow's 1937 to 1940) but clearly becomes more important when a sample is followed for 13 years. The estimation of year to year transitions and calculation of corresponding ergodic distributions has therefore been done for three separate (but intersecting) sets of white males from the 1/10,000 LEED file—those continuously employed 1959 to 1969, those ever employed 1957 to 1969, and those employed in each of the pair of years under study. All these results are presented in Osberg (1975). The last is presented here as it seems most relevant if the intention is to model the earnings distribution of the entire labour force. Without some theory from which the Markov approach is derived, however, one is unsure as to whether it ought to be applicable to other (or all?) consistent partitionings of the labour force. In fact, it appeared applicable to none.

TABLE 1

ACTUAL AND COMPUTED ERGODIC DISTRIBUTIONS OF DEFLATED WAGES OF PAIRWISE EMPLOYED WHITE MALE MEMBERS 1/10,000 LEED FILE

	Ergodic							Actual						
	0–500	500–1,500	1,500–3,000	3,000–4,500	4,500–6,000	6,000–7,500	7,500+	0–500	500–1,500	1,500–3,000	3,000–4,500	4,500–6,000	6,000–7,500	7,500+
59/60	0.0741	0.1179	0.1707	0.2439	0.2070	0.0989	0.0872	0.115	0.141	0.171	0.213	0.181	0.092	0.087
60/61	0.0749	0.1277	0.1893	0.2058	0.1673	0.1062	0.1284	0.081	0.133	0.187	0.217	0.182	0.101	0.099
61/62	0.0501	0.1112	0.1902	0.2399	0.2061	0.0940	0.1070	0.075	0.138	0.191	0.219	0.187	0.093	0.098
62/63	0.0502	0.1205	0.1720	0.2454	0.2039	0.1253	0.0805	0.072	0.147	0.184	0.225	0.180	0.103	0.088
63/64	0.0555	0.1132	0.1848	0.2693	0.2073	0.0911	0.0784	0.076	0.139	0.192	0.236	0.182	0.093	0.083
64/65	0.0571	0.1109	0.1916	0.3065	0.2125	0.0864	0.0346	0.076	0.137	0.194	0.250	0.183	0.095	0.066
65/66	0.0487	0.1106	0.2200	0.3031	0.1987	0.0544	0.0641	0.079	0.139	0.205	0.255	0.185	0.066	0.069
66/67	0.0746	0.1536	0.2289	0.2823	0.1435	0.0592	0.0576	0.089	0.153	0.205	0.261	0.161	0.066	0.065
67/68	0.0532	0.1259	0.2211	0.2881	0.1760	0.0654	0.0699	0.083	0.157	0.212	0.262	0.160	0.062	0.064
68/69	0.0624	0.1312	0.2280	0.3124	0.1550	0.0637	0.0467	0.087	0.156	0.218	0.270	0.149	0.065	0.056

It appears that even these distributions of deflated earnings "differ substantially from each other and from the observed distributions."

Furthermore, the distribution of both cumulative nominal wages and cumulative deflated wages [which seems the more appropriate measure] of continuously employed workers show significant kurtosis and skewness, leading one to doubt[11] the hypothesis of normal distribution. [Indeed, their distribution is more nearly log-normal—see Table 2.]

TABLE 2
MOMENTS OF DISTRIBUTION
CUMULATIVE WAGES OF CONTINUOUSLY EMPLOYED WHITE MALE WORKERS

	Mean	Std. Dev.	Skew	Kurtosis
(1) Sum Nominal Earnings 57/69	83,708	56,036	3.480	20.593
(2) Sum Deflated Earnings 57/69	58,498	40,323	3.628	23.333
(3) Natural Log (1)	11.166	0.594	−0.529	2.504
(4) Natural Log (2)	10.797	0.615	−0.530	2.143

One should note that most of the ways in which the LEED sample deviates from the B.O.A.S.I. data should tend to increase the reliability of the current estimates. Social Security coverage has increased as a percentage of the labour force over the years, increasing the representativeness of the sample. The use of an imputation procedure for workers whose wages exceed the social security ceiling should in most cases increase the accuracy of estimation of the upper tail of the distribution.[12] The fact that LEED is a running 1/10,000 sample opens the possibility of examining a population inclusive of entrants and retirees—which is again more representative. In addition, since attention has been restricted to white males, and it is well known that race and sex are two of "the characteristics known to have an effect on earnings," the greater homogeneity of the population makes the hypothesis of a uniform Markov process more credible.

One can therefore have reasonable grounds for believing that if the "structural characteristics" of random processes were really responsible for the stability of earnings distributions, such stability would be revealed in Table 1. Instead one observes that the relative frequency of membership in each *deflated* income class of the ergodic distributions varies in several cases by 50% or more.

A follower of Solow would predict, for example, if he used 1964/65 data, 3.5 percent to earn over $7,500 (deflated) while in actuality, 6.6 percent lay in this class; using 1960/61 data, he would predict 12.8 percent of workers to earn this much when actually 9.9 percent did. Looking only at cumulative graphs, one would likely not be able to notice these differences but they do imply quite different sorts of earnings distributions. The computed (ergodic) estimate of high earnings workers varied beween 12.8 percent and 3.4 percent while actual

[11]Shapiro *et al.* (1968, p. 1343).

[12]See Osberg (1975, pp. 227–236).

variance was from 9.9 percent to 6.6 percent. It appears[13] that the ergodic distributions corresponding to one-step transition matrices are neither particularly stable nor similar to one another or to the actual distributions of earnings.

3. Champernowne and Proportionate Shifts

Champernowne's 1953 paper adds to Solow's theoretical formulation the specification that if one constructs income ranges which are defined proportionately and assumes that "prospects of shifts upward and downward along the ladder of income are distributed in a manner independent of present income"[14] then the ergodic distribution approaches the exact Pareto distribution. To handle the lower tail of the distribution he hypothesizes that an analogous but different process also operates in income ranges less than some modal value, thereby generating a distribution with two Paretian tails. Many of the same objections can be made of Champernowne's as of Solow's model but it does have greater empirical content—one can test the Paretian hypothesis and also the hypothesis that transition probabilities (between proportionate ranges) are invariant with respect to current income. Empirically, however, there seems to be little support for Champernowne's conjectures, at least with respect to earnings. Using the 1/10,000 sample of the LEED data base and proportionately-defined earnings intervals, the year by year transition matrices set out in Tables 3 and 4[15] were computed. If Champernowne's type of stochastic process were at work for earnings, one would expect elements on downward-sloping diagonals to be roughly equal (i.e., $P_{ij} \simeq P_{i+x,j+x}$ for all i, j, for at least $x = 1$)—and such does not appear to be the case. Looking, for example, at 1960/61 data, Champernowne would have predicted the probability of moving up one income range to be fairly constant but Table 4 reveals transition probabilities of 0.114, 0.176, 0.170, 0.359, 0.370, 0.289, 0.132, 0.031, and 0.129. The wide variation in these transition probabilities clearly offers no support for Champernowne's hypothesis.[16] This is not a direct test of Champernowne's hypothesis as he referred to total income, not earnings, but it does discredit one elaboration of it (Mandelbrot, 1961) and implies that rather elaborate theories regarding the counterbalancing effects of year to year variations in capital income would be required in order to rescue the hypothesis.

[13]The literature on Markov processes (see Anderson/Goodman (1957) and bibliography in Billingsley (1961)) contains several alternative tests of the equality of transition probability matrices but I have not been able to locate a defined statistical test for the equality of several ergodic distributions. The above inelegant procedure has therefore been adopted by default.

[14]D. G. Champernowne, "A Model of Income Distribution," *Economic Journal*, (June 1953, p. 324); additional specifications are that the number of incomes remain constant and that no one shifts more than one income range at a time. If the latter assumption is relaxed for upward shifts nothing is changed, but for downward shifts the ergodic distribution becomes Paretian only in the upper income ranges.

[15]Similar tables were also calculated for the years 1961/62 through 1968/69 and are presented in Osberg (1975, pp. 44–51).

[16]That the transition probability matrices are not as a whole constant is confirmed by calculation of the Anderson-Goodman (1957, p. 98) modified χ^2 criterion of homogeneity. As these are the same populations as are involved with Table 1, one must follow Solow in focusing on the ergodic distributions if one is not to consider this evidence against a one step Markov approach to income distribution.

TABLE 3

TRANSITION PROBABILITIES BETWEEN INCOME RANGES, 1959–60

1959 \ 1960	0 to 100	100–200	200–400	400–800	800–1,600	1,600–3,200	3,200–6,400	6,400–12,800	12,800–25,600	25,600+
0 to 100	0.223	0.093	0.155	0.198	0.141	0.116	0.048	0.02	0.0	0.006
100 to 200	0.085	0.213	0.191	0.191	0.170	0.106	0.043	0.0	0.0	0.0
200 to 400	0.073	0.098	0.207	0.341	0.220	0.061	0.0	0.0	0.0	0.0
400 to 800	0.045	0.067	0.084	0.287	0.292	0.169	0.056	0.0	0.0	0.0
800 to 1,600	0.02	0.02	0.032	0.13	0.358	0.303	0.13	0.006	0.0	0.0
1,600 to 3,200	0.013	0.007	0.017	0.032	0.126	0.485	0.288	0.03	0.002	0.0
3,200 to 6,400	0.001	0.001	0.005	0.007	0.02	0.078	0.752	0.135	0.001	0.0
6,400 to 12,800	0.0	0.0	0.003	0.002	0.003	0.014	0.111	0.837	0.028	0.003
12,800 to 25,600	0.0	0.0	0.0	0.0	0.018	0.0	0.0	0.232	0.661	0.089
25,600+	0.0	0.0	0.0	0.0	0.0	0.03	0.0	0.0	0.152	0.818

Total Observations = 3,715.

[Tables 3 and 4 report transition probabilities between proportionately defined income ranges for workers employed in each of the two named years.]

TABLE 4

TRANSITION PROBABILITIES BETWEEN INCOME RANGES, 1960–61

1960 \ 1961	0–100	100–200	200–400	400–800	800–1,600	1,600–3,200	3,200 6,400	6,400–12,800	12,800–25,600	25,600+
0 to 100	0.243	0.114	0.443	0.229	0.186	0.043	0.029	0.014	0.0	0.0
100 to 200	0.157	0.137	0.176	0.275	0.176	0.078	0.0	0.0	0.0	0.0
200 to 400	0.102	0.148	0.159	0.170	0.250	0.091	0.068	0.0	0.011	0.0
400 to 800	0.026	0.042	0.089	0.271	0.359	0.177	0.031	0.005	0.0	0.0
800 to 1,600	0.016	0.028	0.05	0.088	0.345	0.370	0.094	0.009	0.0	0.0
1,600 to 3,200	0.011	0.007	0.02	0.048	0.116	0.486	0.289	0.023	0.0	0.0
3,200 to 6,400	0.004	0.001	0.008	0.011	0.017	0.084	0.744	0.132	0.001	0.0
6,400 to 12,800	0.0	0.001	0.0	0.001	0.004	0.016	0.123	0.822	0.031	0.001
12,800 to 25,600	0.0	0.0	0.0	0.0	0.0	0.0	0.081	0.129	0.661	0.129
25,600+	0.0	0.0	0.0	0.0	0.0	0.057	0.0	0.029	0.029	0.886

Total Observations = 3,469.

4. Rutherford and Aging Cohorts

For Champernowne and Solow convergence to an "equilibrium" distribution from any arbitrary initial distribution was an important property; but Rutherford (1955), following Gibrat, was content to calculate the equilibrium distribution directly. Rutherford reintroduced Gibrat's "loi de l'effect proportionnel" but with the realistic complication that earners are in fact mortal. He hypothesized that cohorts enter the world with a given earnings distribution which is then subject to series of independent stochastic shocks (acting proportionately) in successive years. As each cohort ages, some of its members die so that, as proportionate effect implies, although the variance of cohort income increases uniformly as it ages, its weight in the aggregate distribution diminishes.

Rutherford used aggregative data for various years on a number of nations to test his model. He concluded that the data supported his model but was unable to test directly one of his principal conclusions—that intra-cohort dispersion increases with age. Such an effect, if it exists, should be discernible at the level of local labour markets. Earnings data for a sample of counties[17] were therefore abstracted from the 1/100 version of the LEED file and the standard deviation and coefficient of variation of earnings within 10-year birth cohorts calculated for each county. The averages across counties of these statistics for white male, "other" and all workers are presented in Tables 5 and 6. Rutherford explicitly abstracted from the difference in mean incomes among cohorts and would therefore have predicted the mean of coefficients of variation of earnings to increase monotonically with age in Table 5—which does not seem always to be the case.

Log-normality of cohort distributions, proportionate income changes at all income levels and the finding of a Standard Symmetrical Gram Charlier Type A for the aggregate distribution are other testable implications of this theory—but Rutherford cautions: "To remove the matter from the level of conjecture to one of more exact determination it is obviously necessary to have more knowledge about the nature of the year-to-year shock system."[18]

5. Implications

This admonition to specify more clearly just what is going on underscores an important sociological implication of stochastic process theories. Although these theories differ in the extent of their empirical predictions they agree in assigning a semi-inevitability to the existing distribution of earnings. Solow and Champernowne assume that real world earnings behaviour resembles a Markov process which has the property of convergence and that this process has converged. Deserting this assumption would imply that the existing distribution would still be under the influence of an unknown initial distribution—consequently their theories would lose predictive power. Gibrat, Simon (1955) and Rutherford derive directly their expected equilibrium distributions and for similar reasons must hypothesize that this is the current distribution. One might then ask (if this is

[17]See Osberg (1975, pp. 99–116).
[18]Rutherford (1955, p. 294).

TABLE 5

AVERAGE COEFFICIENT OF VARIATION WITHIN BIRTH COHORTS ACROSS COUNTIES

	1963			1969		
	Mean Coefficient of Variation of Earnings			Mean Coefficient of Variation		
Birth year	White Males	Others	All	White Males	Others	All
pre-1900	1.154	0.816	1.241	1.049	0.664	1.095
1900–10	0.970	0.752	1.024	1.065	0.782	1.119
1910–20	0.903	0.740	1.019	0.927	0.696	1.018
1920–30	0.838	0.769	0.972	0.878	0.745	1.029
1930–40	0.762	0.827	0.896	0.779	0.805	0.939
1940–50	1.134	0.998	1.117	0.850	0.808	0.920
1950+				1.206	1.051	1.180

TABLE 6

MEAN ACROSS COUNTIES OF STANDARD DEVIATION OF EARNINGS WITHIN COHORTS OF COUNTIES

	1963 Data		1969 Data	
Birth year	White Males	Others	White Males	Others
pre-1900	4,876	1,853	3,963	1,331
1900–10	5,802	2,346	8,549	3,019
1910–20	6,048	2,150	8,841	2,971
1920–30	5,526	2,076	9,013	2,982
1930–40	4,078	1,887	7,114	2,987
1940–50	2,234	1,378	4,676	2,448
1950+			1,709	1,132

an equilibrium distribution of earnings) is it the unique one and if not, how could it be changed? These questions have not been addressed by stochastic-process theorists and the issue remains shrouded in mystery—the "other structural characteristics" of Solow,[19] the "varied and complex forces" of Champernowne are unspecified and hence one cannot tell if they are alterable (indeed Solow's work suggests that the obvious idea of altering existing transition probabilities may not work).[20] And after all, in terms of what one can do about it, is there much difference between saying, à la Pareto, "This is the equilibrium distribution of earnings and its cause lies in the nature of things" and proclaiming "this is the equilibrium distribution of earnings and its cause lies in the nature of unknown

[19]Solow protests (p. 102) that he does not believe the existing distribution to be inevitable and puts the qualification of "constant institutional structure" in the preface but these comments are not developed and are not emphasized. Had they been, one might have seen some "radical" content to his analysis in that it implies that the *only* way to increased equality is via major institutional change.

[20]Clearly some changes in transition probabilities would necessarily "work", e.g. if the mean income interval were to become an absorbing state, this would certainly produce, in time, an increased level of equality—but without knowing what forces determine the transition probabilities one cannot know if less drastic alterations (even by fiat) to some probabilities may not simply cause balancing alterations elsewhere.

random processes"? As Lydall concludes, "Perhaps because of the cogency of mathematical logic, stochastic process models seem to create a bias towards believing that the existing distribution of income is inevitable and unchangeable."[21]

Furthermore, by assumption stochastic process models create a certain kind of equality—that of equality before the odds. Action to diminish inequality would be encouraged by a value system that said that inequality should not exist and cognitive perceptions that it does. Each of these three models assumes that the odds, depending only on Y_{t-1}, are the same for all individuals—hence inequality of initial "life-chances" does not exist. Many people share a belief in the desirability of "equality" but rarely can they define it rigorously—equal "life-chances" would satisfy at least some people's egalitarian urges. If they think such "equality" exists, clearly little change is needed in existing arrangements. Sociologically, one can see these theories as conservative in social impact.

In his 1953 article Friedman conjectured that a society where people differed in their preference for risk might find a set of lotteries with widely varying probability distributions Pareto-superior to a single lottery with certain pay-off [i.e., equality]. Adam Smith had speculated that preference of this sort drew young risk-loving men into law, the army and the merchant marine but viewing the entire distribution of earnings to be the result of risk preference is quite another matter. It is not clear how one can know that the current set of "lotteries" is an equilibrium set or an optimal equilibrium set. Neither is it specified what sort of lotteries are in mind, nor what decision agent "produces" them, nor to what such production is responsive so it cannot be said that logical foundation has been shown for the statement "the inequality of income in a society can be regarded in much the same way as the kind of goods that are produced, as at least in part—and perhaps in major part—a reflection of deliberate choice in accordance with tastes and preferences."[22] Furthermore, since these mechanisms have not been specified, their existence and operation cannot be tested. One is left with the assertion that the current distribution of income is "in part" dependent on the aggregate distribution of risk preference—which is, for all practical purposes, an unobservable. The degree and method of such dependence is not, however, specified so even if societal risk-aversion were an observable, the theory could not be refuted[23]—hence it has no empirical content.[24]

It is also interesting to contrast the qualified conclusion of the article that one "cannot rule out the possibility that the distribution of wealth is the result of conscious choice"[25] with the language of Friedman's more directly polemical writings: "much of the inequality of income produced by payment in accordance with product reflects "equalizing" differences or the satisfaction of men's taste for uncertainty."[26]

[21]Lydall (1968, p. 25).

[22]Friedman (1953, p. 278).

[23]Conceivably one could ascribe the income distribution's unimodality to people's risk-aversion and equality-preference if it turned out everybody was risk-averse; its dispersion could be "explained" if risk-lovers were found.

[24]In the sense of Hutchison (1960), pp. 26, 30.

[25]*Ibid.*, (p. 290).

[26]Friedman (1962, p. 148).

The interest in Friedman's approach therefore derives not from its scientific status but from its sociological impact; as he puts it "inequalities resulting from deliberate decisions to participate in a lottery clearly raise very different normative issues than do inequalities imposed on individuals from the outside."[27] If inequality does not "really" exist there is clearly no need to alter existing economic arrangements to eliminate it, and in this way stochastic approaches to earning distribution may assist in the sociological function[28] of reducing the social strain between institutionalized values of equality and the actual state of western economies:

6. Conclusion

The stochastic process models of Solow (1951), Champernowne (1953) and Rutherford (1955) yield implications which do not appear to be supported by analysis of actual year-to-year changes in individual earnings. It is conjectured that these theories create a bias towards belief in the inevitability, and perhaps desirability, of the current distribution of earnings.

References

[1] T. W. Anderson and Leo Goodman (1957), "Statistical Inference about Markov Chains," *Annals of Mathematical Statistics*, p. 89.
[2] P. Billingsley (1961), "Statistical Methods in Markov Chains," *Annals of Mathematical Statistics*, p. 12.
[3] D. G. Champernowne (1953), "A Model of Income Distribution," *Economic Journal*, Vol. 63.
[4] W. Feller (1968), *An Introduction to Probability Theory and Its Applications*, Vol. I, 3rd ed., John Wiley, New York.
[5] M. Friedman (1962), *Capitalism and Freedom*, Chicago, University of Chicago Press.
[6] ——— (1953), "Choice, Chance and the Personal Distribution of Income," *Journal of Political Economy*, Vol. LXI.
[7] R. Gibrat (1931), *Les Inégalités Economiques*, Sirey, Paris, pp. 25, 60.
[8] T. W. Hutchison (1960), *The Significance and Basic Postulates of Economic Theory*, A. M. Kelly, New York, pp. 26, 30.
[9] Kapetyn (1903), *Skew Frequency Curves in Biology and Statistics*, Groningen Nordhoff.
[10] Harold Lydall (1968), *The Structure of Earnings*, Clarendon Press.
[11] L. S. Osberg (1975), *A Structural Approach to the Distributions of Earnings*, Ph.D. Dissertation, Yale.
[12] T. Parsons (1967), "An Approach to the Sociology of Knowledge" in *Sociological Theory and Modern Society*, New York, The Free Press.
[13] Talcott Parsons (1952), *Economy and Society*, Glencoe, The Free Press.
[14] R. S. G. Rutherford (1955), "Income Distributions—A New Model," *Econometrica*, Vol. 23, p. 277.
[15] S. S. Shapiro, M. B. Wilk, and H. J. Chen (1968), "A Comparative Study of Various Tests for Normality," *Journal of the American Statistical Association*, Vol. 63, p. 1343.
[16] A. F. Shorrocks (1976), "Income Mobility and the Markov Assumption," *Economic Journal*, September, Vol. 86, p. 566.
[17] H. A. Simon (1955), "On a Class of Skew Distribution Functions," *Biometrika*, Vol. 42, December.
[18] Neil J. Smelser (1963), *The Sociology of Economic Life*, Prentice-Hall.
[19] R. M. Solow (1951), *On the Dynamics of Income Distribution*, Ph.D. Dissertation, Harvard.
[20] ——— (1951), "Some Long-Run Aspects of the Distribution of Wage Incomes," *Econometrica*, Vol. 19.

[27] Friedman (1953, p. 290).
[28] Parsons (1952, p. 48; 1967, p. 163); Smelser (1963, p. 43).