A TECHNIQUE FOR ESTIMATING INCOME TRENDS FROM CURRENCY DATA AND AN APPLICATION TO NINETEENTH-CENTURY BRAZIL

BY NATHANIEL H. LEFF*

Columbia University, New York

This paper develops a method for estimating long-run trends in income growth from the data available on a country's currency stock. The method is applied to nineteenth-century Brazil. The results indicate that contrary to earlier beliefs, the country as a whole probably experienced only moderate growth in per-capita income during the nineteenth century. The approach may also be useful for other countries where data shortages preclude estimates of national income by conventional methods.

I

Analysis of the economic history of the less-developed countries has frequently been hampered by the absence of national income estimates before relatively recent dates, such as 1939 or 1920. Moreover, the underlying data which would be required to construct such estimates, such as figures for labor force distribution and sectoral output, are in many cases also not available. As a result, studies of the economic history of the less-developed countries have generally not had an adequate basis for dealing with such fundamental questions as long-run income trends, and the demarcation of different sub-periods in terms of varying income growth.

As a number of authors have suggested, however, monetary data—which are usually collected on a systematic basis before production data—can be used to provide approximate answers to these questions. The present paper is an application of this general approach based on the demand for real cash balances, with an additional algebraic manipulation to permit the estimation of long-term income trends from currency data. The analysis is applied to nineteenth-century Brazil: beginning in 1822 when, following Independence, the first data on the currency stock became available, and proceeding until 1913. Apart from providing an empirical basis for answering important substantive questions concerning income growth in nineteenth-century Brazil, the procedure applied may also be

*I am grateful to Michael Adler, Christopher Clague, Stanley Engerman, David Felix, Ronald Findlay, Raymond Goldsmith, Charles Tapiero, Buckner Wallingford, and Maurice Wilkinson for helpful discussion and comments on this project. They bear no responsibility for any errors. Bruce Phillips, Michael Sterling, and Shing Fung supplied research assistance far beyond the call of duty. Support of a grant from the Faculty Research Program of the Graduate School of Columbia University and from the Social Science Research Council is gratefully acknowledged.


355
methodologically relevant for the analysis of the economic history of other under-developed countries in the nineteenth century.

II

The procedure starts from the Quantity Theory, \( MV = PY \). Dividing by \( P \), \( Y = (M/P)V \). In growth terms, \( \dot{Y} = (M/P)\dot{V} \). Thus, in order to estimate trends in income, we require estimates of \( (M/P) \) and \( \dot{V} \). In the present case, however, we lack data on the money supply, \( M \), and we must go through an extra step to permit use of the currency supply figures which are available for nineteenth-century Brazil.

We begin with identities on the income velocity of circulation, \( V = Y/M \); and on the share of the currency stock in the total money supply, \( Z = CS/M \). Writing these in logarithmic form,

\[
\begin{align*}
\log V &= \log Y - \log M \\
\log Z &= \log CS - \log M.
\end{align*}
\]

Subtracting (2) from (1) and rearranging terms we have

\[
\log Y = \log CS - \log V - \log Z.
\]

Using dot notation and lower-case letters to denote annual percentage rates of change in the per-capita form of these variables,

\[
\hat{y} = \dot{CS} + \hat{v} - \hat{z}.
\]

In order to use expression (4) to draw conclusions from trends in the currency supply to long-term trends in income, we require knowledge concerning the magnitudes of \( \hat{v} \) and \( \hat{z} \) in nineteenth-century Brazil.

III

Cross-section and time series studies indicate that both \( Z \) and \( V \) are inversely related with \( Y \). At higher levels of income, currency constitutes a smaller fraction of the total money supply; and countries also have a lower ratio of income to money supply. In addition, as Professor Gurley has demonstrated, larger population size (as an indicator of an increased number of decentralized decision-

Sources for the data used in this paper are presented in Appendix II, below.


5Apart from the possibility that, particularly at low income levels, cash balances may be a luxury good, among the reasons usually adduced for the lower \( V \) are (1) the growing differentiation in the structure of the economy which interrupts the synchronization of production and requires growing cash balances, (2) relative decline in the share of production devoted to direct use by producers, for example, in agriculture, (3) disproportionate growth in purely financial transactions, (4) change in payment patterns, e.g., from daily to weekly payments.

356
makers who must hold cash balances to carry out their transactions) is also associated with a lower $V$. Consequently, even without rising per-capita income, growth in the scale of the economy and in the number of economic units would lead to a fall in velocity.

In the first decades of the nineteenth century, Brazil was not well developed financially. Hence it is reasonable to suppose that $Z$ was close to its upper limit of 1. In 1913, $Z$ was 0.41. If we assume that in 1822, $Z$ was equal to 0.9, this would imply $\dot{z} = -0.9$ percent. Choosing the initial value of $Z$ close to its upper limit, of course, increases the magnitude of $\dot{z}$, and in accordance with (4), to this extent, may bias upward our estimate for $\ddot{y}$.

Observations for 1920 and 1925 indicate that in those years, $V$ averaged 3.4 in Brazil. Data for the earlier years are not available, but statistics from other countries suggest possible lower-bound, upper-bound, and intermediate figures for $V$ in Brazil in 1822. In the first half of the nineteenth century, $V$ averaged approximately 10 in the United States. This is probably an extreme lower limit for $V_{1822}$ in Brazil. First, all indications are that in 1822, per-capita income was lower in Brazil than in the United States. In addition, Brazil had a proportionately larger slave labor-force, which did not require the holding of cash balances for wage payments. Since the slaves were concentrated in many of the economy’s highest productivity activities, the demand for money in a relatively large percentage of the economy’s transactions was relatively low. Brazil also had a much larger quasi-subsistence sector than the United States. Moreover, the population was overwhelmingly rural—as late as 1890, only 9 percent of the population resided in the urban areas, the counties of the national capital and of the state capitals. As mentioned, the banking system seems to have been extremely primitive. Finally, the share of the export sector in aggregate output in 1822 was much smaller than has commonly been assumed. All of these conditions would lower the demand for money in Brazil, and lead to a $V_{1822}$ higher than in the United States during the first half of the nineteenth century.

Data from other countries suggest 20 as an upper-bound assumption for $V_{1822}$ in Brazil; and 15 an intermediate figure. (Alternative assumptions concerning $V_{1822}$ which the reader may find more plausible can of course readily be

---

7For material on the Brazilian banking system in this period, see J. Pandia Calógeras, A Política Monetária do Brasil (São Paulo, 1960) (trans. by Thomaz Newlands Neto from the 1910 edition of La Politique Monétaire de Brésil), Chapters II and III.
8I owe this information to a personal communication from Professor Raymond Goldsmith. He bears no responsibility for the present discussion.
11In Mexico during the years 1895–1897, $V$ averaged 17. See Leopoldo Solís, “La Evolución Económica de Mexico a Partir de la Revolucion de 1910,” Demografía y Economía, III, 1 (1969), Cuadro 2. More generally, see the estimates presented in Gurley, “Repercusión del Desarrollo Económico,” pp. 124–127. The following considerations, however, lead me to select 20 as an upper-bound for $V_{1822}$ even though this is higher than the figures computed by Gurley for (contemporary) underdeveloped countries. As noted below, as late as 1920–1925, per-capita income in Brazil was only 98 dollars. Consequently, the observations of $V$ most relevant in Gurley’s data are those for the countries with per-capita income below 100 dollars. Within this sub-set of countries, because of the association (demonstrated by Gurley) of large population
incorporated with the data provided in the paper to permit his own conclusions on the rate of income growth.) The annual figures for \( \dot{v} \) which these upper-bound, intermediate, and lower-bound assumptions imply are, respectively: -2.2, -1.8, and -1.1 percent.\(^1\) As these figures indicate, large differences in the initial \( V \) are associated with less than proportionate variations in \( \dot{v} \). Consequently, the final estimates of \( \dot{v} \) are not as sensitive as might have been expected to the assumptions concerning \( V_{1822} \).

We cannot, however, use these materials and equation (4) to estimate upper-bound, lower-bound, and intermediate rates of income growth without a price deflator for the nominal Brazilian currency stock. This is considered in the next section. Before we proceed to that discussion, however, we should note at the outset that even the data on the nominal currency stock do not suggest a rapid increase in Brazilian per-capita income during this period.

Between 1822 and 1913, the nominal Brazilian currency stock increased at an annual trend rate of 3.8 percent.\(^2\) On a per-capita basis, however, the growth in the currency supply was more modest, 2.0 percent. Much of this increase in the currency supply went to sustain increased demand for cash balances due to increased monetization of the economy (as reflected in any of our assumptions for \( \dot{v} \)). In terms of equation (4), the figure for \( \ddot{z} \) and the intermediate \( \dot{v} \), for example, the ratio \( y/cs \) declined at an annual trend rate of 0.6 percent, absorbing almost a third of the increase in the nominal per-capita currency stock. Nevertheless, the increase, such as it was, in the per-capita currency stock was sufficient to generate chronic inflation.\(^3\) As an indication of this inflation, despite generally favorable movements in the country's terms of trade, Brazil's exchange rate, which was free to float during most of this period, depreciated at an annual trend rate of 1.4 percent between 1822 and 1913, in a movement which was closely correlated with the annual changes in the currency stock.\(^4\) The existence of sustained size with low \( V \), the figures for \( V \) which are most applicable for Brazil in 1822 are those for the countries with a relatively small population. In 1822, the population of Brazil was only 4.7 million. By increasing the costs of holding cash balances, Brazil's price inflation at the beginning of this period would also have led to a relatively high value for \( V_{1822} \). Finally, Gurley's observations relate to non-slave economies, a condition which, as noted in the text, would lead to a lower \( V \) than for an economy in which slaves were a substantial portion of the labor force.

\(^{13}\)It has sometimes been suggested that the abolition, in 1888, of slavery in Brazil led to an abrupt shift in velocity, due to the sudden need to hold cash balances to pay wages to the labor force. In fact, however, the percentage of slaves in the Brazilian population declined steadily during the century, and by the time Emancipation came, the slaves constituted only some 4 percent of the population.

\(^{14}\)All trend rates of growth cited were computed by regressing the logarithm of the series against a trend variable, the latter specified with mean zero. Unless otherwise stated, the trend rates cited in the paper were all statistically significant at the 5 percent level or above.


Annual observations of the logarithm of the exchange rate were regressed against annual observations of the logarithm of the nominal Brazilian currency stock for the years 1822-1913. Because of the presence of strong time trends in both series, a trend term, \( r \), was also specified. The regression coefficient of the currency stock term in this specification shows the correlation between annual deviations of the exchange rate and the currency stock from their trend values. The estimated equation, with \( t \)-ratios in parenthesis, is:

\[
\log ER_t = -2.02 + 0.82 \log CS_t - 0.017r \\
R^2 = 0.76
\]

(2.5) (5.6) (3.1)

The coefficient of the log \( CS_t \) term indicates an elasticity not too far from unity for the deviations, while the \( t \)-ratio shows this to be significant above the 1-percent level.
inflation, despite the relatively modest rate of growth of the per-capita currency stock and the great increase in the monetization of the economy, suggests that the level of real transactions per capita cannot have been rising at a very high rate.

IV

A Brazilian price index to deflate the annual observations of the currency stock is not available. However, following a purchasing-power-parity hypothesis, the Brazilian exchange rate may be used to help give a rough indication of the long-term relative movement of Brazilian prices. The exchange rate was free to float during most of this period, and its movements were, as seen, closely correlated with changes in the Brazilian currency stock. In support of a PPP approach, we should note that most of Brazilian GNP was generated within agriculture, and all exports were agricultural commodities. Hence, given the possibilities for shifting resources within agriculture, price movements within the economy as a whole would be reflected in the export sector.

We begin, then, with the "relative-movement" version of the PPP hypothesis. Thus, changes in the ratio of milreis prices to prices in the United Kingdom, Brazil's principal foreign supplier, would be approximately equal to changes in the milreis-sterling exchange rate.

\[ \frac{\Delta \text{ER}}{\Delta \text{P}_{\text{Brazil}}} \approx \left( \frac{\text{P}_{\text{Brazil}}}{\text{P}_{\text{UK}}} \right) \]

Rearranging terms,

\[ \text{P}_{\text{Brazil}} \approx \text{ER} + \text{P}_{\text{UK}} \]

That is, changes in the Brazilian price level would be roughly equal to the change in the milreis/sterling exchange rate plus the change in United Kingdom prices. Following this reasoning, a series for deflating the annual observations of the Brazilian currency stock was constructed by forming the product of the annual

---


17 See Bela Balassa, "The Purchasing-Power-Parity Doctrine: A Reappraisal," *Journal of Political Economy* (December, 1964). The ratio of the exchange rate to the PPP ratio might of course change over time due to shifts in the demand and supply curves for foreign exchange in the course of the economy's expansion. In nineteenth-century Brazil, such shifts were at least partly offsetting. Demand may have been biased toward import goods. At the same time, however, the expansion path of supply was heavily biased toward exports. On the latter point, see my "Tropical Exports and Development in the Nineteenth Century: The Brazilian Case," forthcoming, *The Journal of Political Economy*.

18 One might expect that the PPP rate would also be affected by changes in Brazil's terms of trade (which generally improved during the century) and by the balance of trade (which was usually small but negative). In order to estimate a model which would indicate the exact effects of these variables on the exchange market, a series for Brazilian national income would be required. This is not available. However, on *a priori* grounds, the effects of improving terms of trade and of a negative trade balance on the PPP rate might be expected to be in opposite directions and hence at least partially offsetting. The PPP hypothesis, which is in any case only a crude approximation, is used only to provide a rough order of magnitude for the long-term movement of Brazilian prices.
milreis-sterling exchange rate and the British wholesale price index. Trend rates of growth were then computed from the series for the deflated currency stock.

This procedure gives 1.0 percent per annum as the implicit rate of long-term price inflation between 1822 and 1913. This estimate may appear very low in light of Brazil’s reputation as an inflationary country during the nineteenth century. Still, an increase of 1.0 per cent per annum over 92 years implies a rise from a base of 100 to an index of 260. The rising trend of Brazil’s prices may have appeared especially noteworthy to contemporary observers since it occurred against a background of world prices which were falling during much of the century. In addition, Brazil may have gained notoriety because of two spectacular episodes of very high rates of inflation which occurred in the 1860’s (during the Paraguayan War) and in the 1890’s (during the “encilhamento”). In any case, in the framework used here, the rate of real income growth is equal to the rate of monetary expansion plus the change in velocity minus the rate of price inflation. Consequently, if the long-term rate of inflation was higher than our estimate, the principal consequence in the present context is that the rate of real income growth was even lower than the figures presented below.

V

The deflated Brazilian currency stock grew at an annual trend rate of 2.8 percent between 1822 and 1913. On a per-capita basis, the annual trend rate of growth was 1.0 percent. Applying equation (4), the figure for $\hat{z}$ and the alternative assumptions for $V_{1822}$ and $\hat{v}$, we obtain the upper-bound, intermediate and lower-bound estimates for the long-term rate of growth of monetized per-capita income in Brazil between 1822 and 1913. Considering the rough nature of the model used, the approximate nature of these estimates must be stressed, though the bounding procedure applied to $\hat{v}$ reduces the probability of gross error. (Also, as noted earlier, if anything, the upper limit selected for $Z_{1822}$ imparts an upward bias to our estimates for $\hat{y}$.) In any case, it is reassuring to see that the estimates for $\hat{y}$

| TABLE 1 |
| ALTERNATIVE PROJECTIONS FOR THE ANNUAL LONG-TERM RATE OF GROWTH OF MONETIZED PER-CAPITA INCOME IN BRAZIL, 1822–1913 |

<table>
<thead>
<tr>
<th>Assumed Value for $V$, 1822</th>
<th>Implied Value for $\hat{v}$</th>
<th>Implied Rate of Growth of Monetized Per-Capita Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-1.8%</td>
<td>0.1%</td>
</tr>
<tr>
<td>15</td>
<td>-1.5%</td>
<td>0.4%</td>
</tr>
<tr>
<td>10</td>
<td>-1.1%</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

Another possibility would be to use the British export price index. This index was not used because the PPP hypothesis has usually been formulated in terms of the ratios of general price levels, which is better approximated by the wholesale price index. If, however, the British export price index is used in the deflator, the estimates, presented below, for $\hat{y}$ in Brazil are raised by 0.4 percent. The time-series for the nominal and deflated currency stock are presented in Appendix III of this paper.
are well within the bounds of, say ±2 percent, which might have been set on purely a priori considerations.

The picture indicated by the intermediate figure of Table I, which is in the most likely range of estimates, is one of relatively moderate growth in Brazilian per-capita income during the nineteenth century. That is, although output was able to exceed the high rate of population increase—1.8 percent per annum—per-capita income does not seem to have grown at a high rate. In particular, Brazil seems to have offered a notable contrast with the United States, where per-capita income is estimated to have grown at an annual long-term rate of approximately 1.5 percent during the nineteenth century. Over 92 years, a 1.5 percent annual rate of growth leads to an almost fourfold increase in the level of per-capita income. By contrast, a 0.4 percent annual growth rate, for example, cumulates to a total increase of only 44 percent.

As noted below, this view of the nineteenth century as being one of only slow growth for Brazil goes counter to earlier discussions of the subject. The existence of a long period of sustained rapid growth, however, would not be consistent with the low income levels observed in the country toward the end of this period. In 1920–1925, the years for which the first national income estimates are available for Brazil, per-capita income averaged only 98 dollars (1950 prices). In addition, as discussed in Appendix I of this paper, the results of Table I are closely supported by estimates of Brazilian income growth which were derived using another economic indicator, the data on exports. Finally, the figures of Table I may even over-state the rate of growth of per-capita income in nineteenth-century Brazil. The estimates of Table I relate only to monetized per-capita income. Non-monetized per-capita income grew at a lower rate during the century. Consequently, the rate of growth of total real product per capita in the economy was even lower than the estimates of Table I. In light of these

20Robert E. Gallman, "Gross National Product in the United States, 1834–1909," in Output, Employment, and Productivity in the United States After 1800, National Bureau of Economic Research (New York, 1966), pp. 9–10; and Paul A. David, "The Growth of Real Product in the United States before 1840: Some Controlled Conjectures," The Journal of Economic History, XXVII (June, 1967), p. 155. It may be objected that declines in Brazil's velocity of the magnitude assumed in Table I are not consistent with a low rate of per-capita income growth. As Gurley has demonstrated, however, a large increase in the scale of an economy would in itself lead to a substantial rise in the demand for cash balances and a fall in velocity. Such an increase in scale did occur in nineteenth-century Brazil, where the population increased fourfold, from approximately 4.7 millions in 1822 to some 23.7 millions in 1913.

21The percentage of the Brazilian labor force in the monetized sector increased during the nineteenth century. This shift in the allocation of the labor force would imply that the value of per-capita output was growing at a higher rate in the monetized sector than in the non-monetized sector of the economy. The estimates of Table I relate only to the growth of monetized income, excluding the growth of income in the non-monetized and subsistence part of the economy. Consequently, the discussion that follows in the text and in the next footnote is necessary to clarify the relation between the rate of growth of total income in the economy and that of the monetized sector. The Brazilian national income estimates cited above for 1920–25 do not include the subsistence sector.

22Writing \( Y \) for income and \( N \) for population, and using subscripts 1 and 2 to denote the monetized and non-monetized sectors, respectively, the rate of growth for per-capita income in the economy as a whole can be written as

\[
\left( \frac{\dot{Y}_N}{N} \right) = \left( \frac{\dot{Y}_1 + \dot{Y}_2}{N_1 + N_2} \right) = \left( \frac{\dot{Y}_1}{N_1 + N_2} \right) + \left( \frac{\dot{Y}_2}{N_1 + N_2} \right).
\]

[continued at foot of next page]
considerations, the general conclusion that Brazil experienced only moderate
growth of per-capita incomes during this period seems warranted. As noted
earlier, this conclusion is reinforced to the extent that the price deflation tech-
nique used here understates the actual rate of price inflation in nineteenth-
century Brazil.

It should be emphasized, however, that our estimates relate to per-capita
income for the economy considered as a whole. As such, the aggregate estimates
reflect the different experiences of the economy’s various sectors and regions.
There is evidence that these fared very differently over the century. The South-
eastern coffee region did experience considerable economic progress.23 The large
North-east region, however, did poorly and may even have had a decline in
per-capita income levels.24 And in the domestic agricultural sector, which em-
ployed a large portion of the country’s labor force, the value of per-capita output
was probably stagnant until the beginning of large-scale railroad construction
forward toward the turn of the century.25

VI

The figures of Table 1 can also be extrapolated backward to form a range
of estimates for the level of monetized Brazilian per-capita income in 1822.
Although the index-number problems inherent in such an exercise preclude
welfare comparisons, it may be of interest to see in rough terms how monetized
Brazilian per-capita income stood in relation with that of the United States.
In 1822, per-capita income in the United States was approximately 253 dollars
(1950 prices).26 By contrast, taken together with the estimate of 98 dollars per-
capita in 1920–1925, the upper-bound, intermediate, and lower-bound growth
estimates for nineteenth-century Brazil imply 44, 66, and 89 dollars (1950 prices),
respectively, for the level of monetized per-capita income in the country in

The numbers given in Table 1 for the growth of monetized per-capita income relate to
\[
\frac{Y_1}{N_1+N_2}.
\]
This is less than \( (Y_2/N_2) \) because, due to reallocation effects over the century,
population in the monetized sector was growing at a rate higher than total population. The
usual expectation for subsistence agriculture is of stagnant per-capita output; i.e., \( (Y_2/N_2) \approx 0 \).
Because of the shift in population, population in the non-monetized sector was
increasing at a rate lower than total population. That is, \( (N_2) < (N_1 + N_3) \), and therefore,
\[
(Y_2/(N_1 + N_3)) < (Y_2/N_2).
\]
Consequently, the last term on the right-hand side of (5) is negative. Thus the rate of growth of per-capita income in the economy as a whole is even less than the
figures presented in Table 1, which relate only to monetized income.

23 Within the Southeast, however, there were important intra-regional shifts. The effects on
aggregate income growth of the new producing areas such as São Paulo were partly offset by
the decline of the older coffee areas within the Southeast. On this decline, see Stanley J. Stein,
24See Nathaniel H. Leff, “Development and Regional Inequality in Brazil,” The Quarterly
Journal of Economics, LXXXVI, 2; May, 1972, pp. 243–262. In light of the very different
economic growth experience of the Southeast and of the Northeast during the nineteenth
century, the estimates of Table 1 could be cited as an example of the irrelevance of aggregate
economic growth, or of the country as a unit of study. Still, from the viewpoint of some import-
ant issues in economic development, per-capita income growth in the country as a whole is an
important question, and it is in that context that those estimates are presented.
25See “Economic Retardation,” section VII.
26Computed from data in Simon Kuznets, Economic Growth and Structure (New York,
Thus as early as that date a sizable gap in monetized per-capita income seems to have existed as between Brazil and the United States. In addition, the picture conveyed by these backward extrapolations is also relevant for views concerning the level and growth of Brazilian income during the eighteenth century.

The data on the currency stock can also be used to derive some impressions concerning the existence of distinct periods in nineteenth-century Brazilian income growth.

Not surprisingly, the trend equation for the Brazilian currency stock in constant prices during the years 1822–1913 showed large residuals in individual years. Moreover, these residuals were distributed above or below the trend line in fairly discrete patterns, indicating that in those sub-periods conditions were present which led to growth of the Brazilian currency stock at a rate above or below the long-term trend value. This suggests a possible approach to the question of the periodization of Brazilian income growth in the century before 1913. After inspection of the pattern of the residuals, trend equations were fitted for different sub-periods, in an effort to identify relatively homogeneous sub-periods. Too much emphasis cannot be put on such variations in the rate of growth. Still, if the demand for real balances varied substantially in distinct periods which are sufficiently long for the short-term objections to the PPP deflation procedure to be mitigated, this fact should be mentioned. Material which would not be worth noting for a well-charted terrain is of interest as a suggestion for further investigation in what is now virtually a terra incognita.

Table 2 shows the statistical results for the sub-period equations which after experimentation, gave the best fits. The annual trend rate of growth of the deflated currency stock is denoted by $g_1$; and the growth of the per-capita deflated currency stock, by $g_2$.

Table 2 suggests a relatively high rate of growth between 1895 and 1913. This is consistent with the data on exports, central-government expenditure,

<table>
<thead>
<tr>
<th>Period</th>
<th>$g_1$</th>
<th>$P^2$ of Trend Equation</th>
<th>$g_2$</th>
<th>$P^2$ of Trend Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1822–1869</td>
<td>2.7 (20.1)</td>
<td>0.90</td>
<td>1.2 (8.4)</td>
<td>0.60</td>
</tr>
<tr>
<td>1870–1894</td>
<td>2.1 (4.5)</td>
<td>0.44</td>
<td>0.3 (0.6)</td>
<td>-0.03</td>
</tr>
<tr>
<td>1895–1913</td>
<td>4.3 (7.7)</td>
<td>0.77</td>
<td>2.2 (3.9)</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Total per-capita income, including the non-monetized sector, was higher than these figures. Using the notation of footnote 22, above, $(Y/N)_{1822} = [Y_1/(N_1 + N_2)] + [Y_2/(N_1 + N_2)]$. The second term on the right-hand side would raise the total figure.
and foreign investment, all of which show a sudden spurt around the turn of the century. Data on energy consumption between 1901 and 1920 also indicate rapid growth during this period. The data on the rate of growth of the per-capita, currency stock also suggest a period of perceptible growth between 1822 and 1869. A noteworthy feature of Table 2, however, is the quarter century between 1870 and 1894 during which the per-capita currency stock shows no statistically significant trend. Again, this aggregate picture probably reflects the poor experience of the large Northeast region, which suffered from the decline of the sugar industry and from a number of catastrophic droughts. Further research is clearly needed, however, to substantiate this suggested periodization, and to analyze the causes of the Brazilian economy's relatively modest growth performance during much of the nineteenth century.

VIII

As emphasized above, this periodization and the range of estimates implied for the growth of per-capita income during the century can only be considered as tentative and rough approximates. We should note, however, that the picture which they suggest of nineteenth-century Brazilian growth differs considerably from the earlier view, frequently cited in the literature, which has its origin in the pioneering work of Celso Furtado. Furtado estimated that from 1850 to the end of the nineteenth century, per-capita income in Brazil increased at an annual rate of approximately 1.5 percent. The methodology underlying Furtado's estimate is not completely clear. Moreover, he seems to have used the export statistics as a macroeconomic indicator without taking account of the changing share of exports in Brazilian national product during the nineteenth century. The latter shift is apparent when one considers the export statistics in conjunction with the data on the currency stock, which reflect conditions in the domestic sector of the economy as well. Consequently, even considering the rough nature of the estimates presented in Tables 1 and 2, they suggest the need for an entirely new view of income growth in nineteenth-century Brazil. At the least, the evidence from both the nominal and the "real" currency stock is inconsistent

---


31"Tropical Exports and Development," section VII, and Appendix I, below.
with the alternative picture of Brazil experiencing substantial growth in per-capita income during the last half of the nineteenth century. Thus, if only as a corrective to macroeconomic impressions given by partial indicators, the technique used in this paper may be useful in the study of the economic history of the less-developed countries.

IX

This paper has used a model based on the demand for real cash balances to derive a range of estimates for the rate of income growth in nineteenth-century Brazil. Because of the limitations of the data available, the principal conclusion of this "quantitative" analysis of Brazilian economic history is "qualitative": namely, that Brazil did not experience a substantial increase in aggregate per-capita income during the nineteenth century. No other conclusion is consistent with the rate of growth of the country's currency stock, either nominal or deflated. In its slow rate of aggregate per-capita income growth during most of the nineteenth century, Brazil's experience seems to have been similar to that of some other tropical countries, which also seem to have experienced only modest progress during this period. Finally, because this conclusion concerning Brazil contrasts sharply with earlier views, it suggests the need for a new perspective in research on the country's economic history in the nineteenth century. One's general attitude and the questions to be answered are clearly different for an economy in which per-capita income was growing at a very low rate as compared with an economy whose per-capita income is believed to have been growing for half a century at an annual rate of 1.5 percent.

Brazil's slow aggregate growth during most of the nineteenth century also sheds light on the origins of the income gap which now separates Brazil from the economically advanced countries. As we have seen, as early as 1822 there was a substantial difference in monetized per-capita income levels as between Brazil and the United States. Even if Brazil had grown at the same rate as the United States during the nineteenth century, the absolute difference would have increased. As it was, the rate of income growth in the United States was much greater than in Brazil, so that the size of the income gap increased considerably. During the twentieth century, the long-term rate of per-capita income growth in Brazil has not been below that of the United States and the other more developed countries. However, the long period in the nineteenth century when Brazilian per-capita income grew at a rate markedly lower than that of the United States led to the widening of the income gap. Consequently, Brazil began its modern economic growth with a substantial difference in income levels.

34Cf. Furtado, loc. cit., for a similar view of the source of Brazil's income gap. The material presented above, which suggests that sustained Brazilian growth began approximately half a century later than proposed by Furtado, however, increases the poignancy of the opportunities missed during the nineteenth century.
APPENDIX I: THE EVIDENCE FROM EXPORTS ON INCOME GROWTH IN BRAZIL: 1822–1913

We begin with the identity

\[ \frac{X}{Y} = \frac{X}{CS} \cdot \frac{CS}{Y}. \]

Using dot notation and lower-case letters to denote the annual percentage rates of change in the per-capita form of these variables:

\[ \dot{x} - \dot{y} = (x/c)s + (cs/\dot{y}) \]
\[ \dot{y} = \dot{x} - (x/c)s - (cs/\dot{y}). \]

The data on the (nominal) per-capita milreis value of exports show \( \dot{x} \) and \((x/c)s\) to have been 2.5 and 0.5 percent, respectively. Using the value for \( \dot{x} \) and the intermediate value for \( (cs/\dot{y}) \), as discussed in section III above, \((cs/\dot{y}) = 0.6 \) percent. Hence, in nominal milreis,

\[ \dot{y} = 2.5 - 0.5 - 0.6 = 1.4\% \]

Since the annual trend rate of growth of the PPP price deflator discussed in Section IV was 1.0 percent, equation (8) implies that aggregate “real” per-capita income grew at an annual trend rate of 0.4 percent over the period as a whole. This is the same as the estimate of 0.4 percent which in section V was derived from the currency stock, using, as with the present procedure, the intermediate figure for \( \dot{y} \).

APPENDIX II: DATA SOURCES

APPENDIX III: THE NOMINAL AND DEFLATED AGGREGATE BRAZILIAN CURRENCY STOCK, 1822–1913

<table>
<thead>
<tr>
<th>Year</th>
<th>Nominal Currency Stock (millions of milreis)</th>
<th>Deflated Currency Stock (thousands of milreis)</th>
<th>Year</th>
<th>Nominal Currency Stock (millions of milreis)</th>
<th>Deflated Currency Stock (thousands of milreis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1822</td>
<td>33.45</td>
<td>58.83</td>
<td>1859</td>
<td>185.55</td>
<td>167.80</td>
</tr>
<tr>
<td>1823</td>
<td>34.95</td>
<td>62.04</td>
<td>1860</td>
<td>179.85</td>
<td>161.86</td>
</tr>
<tr>
<td>1824</td>
<td>37.65</td>
<td>61.72</td>
<td>1861</td>
<td>175.05</td>
<td>161.35</td>
</tr>
<tr>
<td>1825</td>
<td>39.45</td>
<td>64.37</td>
<td>1862</td>
<td>173.15</td>
<td>158.72</td>
</tr>
<tr>
<td>1826</td>
<td>42.25</td>
<td>72.22</td>
<td>1863</td>
<td>176.85</td>
<td>166.62</td>
</tr>
<tr>
<td>1827</td>
<td>51.85</td>
<td>65.14</td>
<td>1864</td>
<td>196.15</td>
<td>182.96</td>
</tr>
<tr>
<td>1828</td>
<td>54.25</td>
<td>62.00</td>
<td>1865</td>
<td>199.95</td>
<td>177.73</td>
</tr>
<tr>
<td>1829</td>
<td>56.45</td>
<td>52.86</td>
<td>1866</td>
<td>215.85</td>
<td>181.67</td>
</tr>
<tr>
<td>1830</td>
<td>59.25</td>
<td>51.64</td>
<td>1867</td>
<td>221.15</td>
<td>174.30</td>
</tr>
<tr>
<td>1831</td>
<td>60.25</td>
<td>55.95</td>
<td>1868</td>
<td>229.85</td>
<td>141.91</td>
</tr>
<tr>
<td>1832</td>
<td>60.95</td>
<td>82.20</td>
<td>1869</td>
<td>288.85</td>
<td>210.56</td>
</tr>
<tr>
<td>1833</td>
<td>61.05</td>
<td>59.34</td>
<td>1870</td>
<td>298.55</td>
<td>249.70</td>
</tr>
<tr>
<td>1834</td>
<td>61.15</td>
<td>87.90</td>
<td>1871</td>
<td>298.85</td>
<td>259.87</td>
</tr>
<tr>
<td>1835</td>
<td>71.55</td>
<td>104.77</td>
<td>1872</td>
<td>296.45</td>
<td>240.87</td>
</tr>
<tr>
<td>1836</td>
<td>71.65</td>
<td>93.79</td>
<td>1873</td>
<td>293.15</td>
<td>251.60</td>
</tr>
<tr>
<td>1837</td>
<td>71.65</td>
<td>74.69</td>
<td>1874</td>
<td>291.55</td>
<td>260.23</td>
</tr>
<tr>
<td>1838</td>
<td>80.45</td>
<td>78.42</td>
<td>1875</td>
<td>290.75</td>
<td>283.29</td>
</tr>
<tr>
<td>1839</td>
<td>80.45</td>
<td>81.69</td>
<td>1876</td>
<td>288.45</td>
<td>263.37</td>
</tr>
<tr>
<td>1840</td>
<td>80.45</td>
<td>81.71</td>
<td>1877</td>
<td>288.75</td>
<td>267.75</td>
</tr>
<tr>
<td>1841</td>
<td>81.45</td>
<td>85.49</td>
<td>1878</td>
<td>318.55</td>
<td>299.63</td>
</tr>
<tr>
<td>1842</td>
<td>84.95</td>
<td>84.95</td>
<td>1879</td>
<td>326.85</td>
<td>296.83</td>
</tr>
<tr>
<td>1843</td>
<td>87.75</td>
<td>90.26</td>
<td>1880</td>
<td>325.95</td>
<td>293.99</td>
</tr>
<tr>
<td>1844</td>
<td>89.55</td>
<td>87.06</td>
<td>1881</td>
<td>322.85</td>
<td>296.76</td>
</tr>
<tr>
<td>1845</td>
<td>92.15</td>
<td>88.80</td>
<td>1882</td>
<td>323.15</td>
<td>281.56</td>
</tr>
<tr>
<td>1846</td>
<td>92.75</td>
<td>95.30</td>
<td>1883</td>
<td>322.35</td>
<td>287.24</td>
</tr>
<tr>
<td>1847</td>
<td>91.65</td>
<td>92.45</td>
<td>1884</td>
<td>321.35</td>
<td>290.91</td>
</tr>
<tr>
<td>1848</td>
<td>90.85</td>
<td>94.48</td>
<td>1885</td>
<td>320.15</td>
<td>283.77</td>
</tr>
<tr>
<td>1849</td>
<td>90.45</td>
<td>102.83</td>
<td>1886</td>
<td>326.65</td>
<td>306.97</td>
</tr>
<tr>
<td>1850</td>
<td>90.95</td>
<td>114.88</td>
<td>1887</td>
<td>317.85</td>
<td>231.52</td>
</tr>
<tr>
<td>1851</td>
<td>97.75</td>
<td>131.05</td>
<td>1888</td>
<td>323.15</td>
<td>196.20</td>
</tr>
<tr>
<td>1852</td>
<td>108.55</td>
<td>132.80</td>
<td>1889</td>
<td>330.55</td>
<td>432.86</td>
</tr>
<tr>
<td>1853</td>
<td>115.75</td>
<td>122.98</td>
<td>1890</td>
<td>418.95</td>
<td>452.66</td>
</tr>
<tr>
<td>1854</td>
<td>128.05</td>
<td>117.81</td>
<td>1891</td>
<td>570.95</td>
<td>411.61</td>
</tr>
<tr>
<td>1855</td>
<td>141.65</td>
<td>130.32</td>
<td>1892</td>
<td>647.35</td>
<td>394.72</td>
</tr>
<tr>
<td>1856</td>
<td>165.85</td>
<td>153.81</td>
<td>1893</td>
<td>755.95</td>
<td>442.51</td>
</tr>
<tr>
<td>1857</td>
<td>180.65</td>
<td>157.89</td>
<td>1894</td>
<td>837.15</td>
<td>475.15</td>
</tr>
<tr>
<td>1858</td>
<td>180.55</td>
<td>172.42</td>
<td>1895</td>
<td>803.95</td>
<td>457.80</td>
</tr>
<tr>
<td>Year</td>
<td>Nominal Currency Stock (millions of milreis)</td>
<td>Deflated Currency Stock (thousands of milreis)</td>
<td>Year</td>
<td>Nominal Currency Stock (millions of milreis)</td>
<td>Deflated Currency Stock (thousands of milreis)</td>
</tr>
<tr>
<td>------</td>
<td>---------------------------------</td>
<td>---------------------------------</td>
<td>------</td>
<td>---------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>1896</td>
<td>839.65</td>
<td>425.58</td>
<td>1905</td>
<td>799.85</td>
<td>604.54</td>
</tr>
<tr>
<td>1897</td>
<td>909.05</td>
<td>380.82</td>
<td>1906</td>
<td>833.55</td>
<td>591.55</td>
</tr>
<tr>
<td>1898</td>
<td>909.85</td>
<td>349.94</td>
<td>1907</td>
<td>883.05</td>
<td>573.53</td>
</tr>
<tr>
<td>1899</td>
<td>864.15</td>
<td>318.91</td>
<td>1908</td>
<td>869.25</td>
<td>629.46</td>
</tr>
<tr>
<td>1900</td>
<td>829.95</td>
<td>355.69</td>
<td>1909</td>
<td>999.85</td>
<td>692.20</td>
</tr>
<tr>
<td>1901</td>
<td>810.85</td>
<td>443.14</td>
<td>1910</td>
<td>1,074.85</td>
<td>742.42</td>
</tr>
<tr>
<td>1902</td>
<td>805.85</td>
<td>468.52</td>
<td>1911</td>
<td>1,138.45</td>
<td>747.80</td>
</tr>
<tr>
<td>1903</td>
<td>805.35</td>
<td>458.86</td>
<td>1912</td>
<td>1,163.75</td>
<td>749.72</td>
</tr>
<tr>
<td>1904</td>
<td>804.05</td>
<td>494.05</td>
<td>1913</td>
<td>1,060.25</td>
<td>599.00</td>
</tr>
</tbody>
</table>