NOTES AND MEMORANDA

TERMS OF TRADE EFFECT, PRODUCTIVITY CHANGE AND NATIONAL ACCOUNTS IN CONSTANT PRICES—REPLY AND FURTHER COMMENT

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1. I thank Mr. Courbis for his comments on my paper [1], showing that the formulation of the terms of trade effect attempted in my paper virtually reduces to the same solution as he has accomplished in his elaborate papers [2] and [3] and, thus, throwing new light on the construction of national accounts in constant prices.

When I wrote my paper, I had been aware of the fact that the formulation of p_N (the common deflator of both net factor income from abroad (P_r) and net lending to the rest of the world after the adjustment of net flow of transfers (N)) according to my rule 3 produces same result as is derived from Courbis' formulation, i.e., as he correctly points out,

$$p_N^{(K)} = p_N^{(C)}$$

following convenient notations that appear in his comments [4]. The point had already been made in my work [5] which preceded my article under question. Accordingly, I have few points that disagree with his specific comments but rather would stress the emphasis of my paper.

The emphasis of my paper is placed not so much on the invention of a new formula that is applied to the deflation of P_r and N as on the exploration of what is implied by the application of such a formula. It is claimed in my paper that the terms of trade effect resulting from the application of $p_N^{(K)}$ is expressed by a weighted average of $M(1 - p_2/p_1)$ and $-X(1 - p_1/p_2)$, which are terms of trade effects under the conditions of export surplus and deficit respectively formulated by Geary [6]. It should be noted that the application of $p_N^{(C)}$ in place of $p_N^{(K)}$ fails to reach the same formula. It follows from the application of $p_N^{(C)}$ that he is unable to discover a simple form of synthesis of Geary's terms of trade effects. Similar findings can be observed in the derivation of the terms of trade effect arising from the relative changes in prices between outputs and intermediate inputs which is discussed in the section 4 of my paper.

2. Although he does not comment on the relation between the terms of trade effect arising from relative changes in prices between outputs and intermediate inputs on the one hand and changes in factor productivity on the other hand, I have considered the point in section 4 of my paper. It is demonstrated in that section that the terms of trade between outputs and intermediate inputs are firmly associated with productivity changes of factors, as the relation (4.11) of my paper indicates. It is hinted in the last footnote that the term expressing productivity changes of factors, i.e., G of my notation, may be transformed into the

Divisia form of index numbers. The point can be further extended. Suppose that factor inputs of a production sector are divided into three categories, i.e., intermediate inputs, labour inputs and other primary inputs. Accordingly, factor payments are composed of the payments for intermediate inputs, compensation for employees and operating surplus. The total factor payments of a production sector in current prices are described by the following relation:

$$pX = p_u U + p_w W + p_u V, \tag{2}$$

in which the following notations are employed:

U, the volume of intermediate inputs,
W, the volume of labour inputs,
V, the volume of other primary inputs,
p_u, the price of intermediate inputs,
p_w, the price of làbour inputs,
p_v, the price of other primary inputs,
X, the volume of total factor inputs,
p, the price of total factor inputs.

The introduction of other primary inputs into the category of primary factors creates a new difficulty of measurement, in which the quantity component must be distinguished from the price component. Meanwhile, the problem of measurement is ignored in this discussion. It is required by the property of the Divisia index number formula that the following relation hold for total factor payments:

$$\left(\frac{\dot{X}}{X} + \frac{\dot{p}}{p}\right) = w_1\left(\frac{\dot{U}}{U} + \frac{\dot{p}_u}{p_u}\right) + w_2\left(\frac{\dot{W}}{W} + \frac{\dot{p}_w}{p_w}\right) + w_3\left(\frac{\dot{V}}{V} + \frac{\dot{p}_v}{p_v}\right) \tag{3}$$

where w_i denotes the share of ith factor payments in the total factor payments as indicated below:

$$w_1 = \frac{p_u U}{pX}, \qquad w_2 = \frac{p_w W}{pX}, \qquad w_3 = \frac{p_v V}{pX}. \tag{4}$$

It is interesting to note that a dual property is observed between the price components and the quantity components in (3). Indeed, if the Divisia quantity index of total factor inputs is defined by

$$\frac{\dot{X}}{X} = w_1\frac{\dot{U}}{U} + w_2\frac{\dot{W}}{W} + w_3\frac{\dot{V}}{V} \tag{5}$$

then we must have a Divisia price index of total factor inputs as a counterpart:

$$\frac{\dot{p}}{p} = w_1\frac{\dot{p}_u}{p_u} + w_2\frac{\dot{p}_w}{p_w} + w_3\frac{\dot{p}_v}{p_v}\,. \tag{6}$$

In the neighbourhood of the base year, it is readily seen that the balance between gross output and total factor outlays in constant prices for a production sector cannot be maintained unless the Divisia index of the volume of gross output is equated to the Divisia index of the volume of total factor inputs.[1]

[1] The term "gross" is used here for including intermediate inputs, and is not for including capital consumption allowances.

Where Y and q are the volume of gross output and its price respectively, the difference between gross output in constant prices and total factor outlays in constant prices is expressed by

$$q_0 Y - p_0 X = \left(\frac{\dot{Y}}{Y_0} - \frac{\dot{X}}{X_0}\right) p_0 X_0. \tag{7}$$

The right-hand side of (7) may be regarded as a correction term to maintain the balancing relation of a sector production account in constant prices. It is also noted that the term in parentheses stands for the effect of productivity change. Equation (5), the effect of productivity change, which is represented in a form of Divisia index, may be now transformed into the following relation:[2]

$$\left(\frac{\dot{Y}}{Y} - \frac{\dot{X}}{X}\right) = \left[\left(1 - \bar{w}_1 \frac{Q_u}{Q}\right) + \left(1 - \bar{w}_2 \frac{Q_w}{Q}\right) - \left(1 + \bar{w}_3 \frac{Q_v}{Q}\right)\right] \tag{8}$$

where Q, Q_u, Q_w and Q_v stand for (Laspeyres) quantity indexes for outputs, intermediate inputs, labour inputs and other primary inputs respectively.

Recalling the fact that the ratio of the current year value to the base year value is expressed by the product of the corresponding Laspeyres quantity index and Paasche price index, the correction term for balancing a sector production account in constant prices in (2) is written by

$$G^* = \left(\frac{\dot{Y}}{Y} - \frac{\dot{X}}{X}\right) p_0 X_0 = \left(1 - \frac{\bar{w}_1 Q_u + \bar{w}_2 Q_w + \bar{w}_3 Q_v}{Q}\right) \bar{Y}. \tag{9}$$

Attention is also called to the fact that the effect of productivity change which is shown by the term in parentheses in (9) is identical with what is proposed by Jorgenson, Griliches and Christensen in [9], [10] and [11].[3]

The correction term for establishing a sector production account in constant prices which reflects the change in the productivity of factors is formulated in my paper by

$$G = \left[\left(1 - \frac{Q_u}{Q}\right)\bar{U} + \left(1 - \frac{Q_w}{Q}\right) \bar{W}\right]. \tag{10}$$

Noting the difficulty that an appropriate deflator for the inputs of other primary factors may not be easily determined, the formulation is derived from the assumption that the inputs of other primary factors change as much as outputs.

[2] It should be remembered that the transformation of the Divisia form into the Laspeyres or Paasche form is only possible in the neighbourhood of the base year. Because the index number that is derived from the Divisia form is expressed by a line integral, as is indicated by Richter [7] and Roy [8], it is necessary for discrete points of time that the continuous Divisia index be approximated. The transformation indicated in (8) may be regarded as one form of such approximation. If we want to compare the base year with a time far in the future, a form of chain index whose components are constituted by successive approximations may be applied.

[3] Investigation of tables that are reported in Christensen and Jorgenson [11] reveals a discrepancy between gross private domestic product in constant prices and gross private factor outlay in constant prices. The discrepancy is interpreted as a correction term for productivity change in factors which is necessary for maintaining the balance in the production account in *constant prices* for the domestic private company. It is readily seen that the correction term is expressed by a similar form as is shown in (9).

But, if the difficulty is neglected, as I suppose in this comment, the formula is readily generalized so that the productivity change in other primary factors may be additionally taken into account and is expressed by

$$(11) \qquad G^{**} = \left[\left(1 - \frac{Q_u}{Q}\right)\bar{U} + \left(1 - \frac{Q_w}{Q}\right)\bar{W} + \left(1 - \frac{Q_v}{Q}\right)\bar{V}\right].$$

Again (11) is written by

$$(12) \qquad G^{**} = \left[\left(1 - \frac{Q_u}{Q}\right) w_1{}^* + \left(1 - \frac{Q_w}{Q}\right) w_2{}^* + \left(1 - \frac{Q_v}{Q}\right) w_3{}^*\right] \bar{Y}(1 + t)$$

where

$$(13) \qquad w_1{}^* = \frac{\bar{U}}{\bar{X}}, \qquad w_2{}^* = \frac{\bar{W}}{\bar{X}}, \qquad w_3{}^* = \frac{\bar{V}}{\bar{X}} \quad \text{and} \quad t = \frac{\bar{T}}{\bar{Y}}.$$

It is interesting to note that a sector production account in constant prices as indicated below is implied in the derivation of (12):

$$(14) \qquad \bar{Y} + \bar{T} = \bar{X}, \qquad \text{where } \bar{X} = \bar{U} + \bar{W} + \bar{V}$$

The term $\bar{T}$ in (14) stands for the terms of trade effect arising from changes in relative prices between outputs and factor inputs, whose properties are the focus of studies by Courbis [3] and me [1].

The comparison of (9) with (12) reveals different characteristics that exist between Jorgenson, Griliches and Christensen's approach and the author's approach:

(1) In Jorgenson, Griliches and Christensen's approach the change in total factor productivity appears as a relative difference between the volume of output and the weighted average of the volume of inputs. Fixed weights at the base year are used.

(2) In the author's approach the change in total factor productivity is formulated as the weighted average of relative differences between the volume of output and the volume of inputs for individual factors, i.e., partial productivity of factors, after the adjustment for the terms of trade effect. It is also noted that the weights are variable, because they are defined as the shares of individual factor outlays in constant prices.

References

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