COMBINING INPUTS TO SECURE A MEASURE OF TOTAL FACTOR INPUT*

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An underlying theme in this paper is that the differences in approach in this area arise partly from the complexity of the phenomena in the real world being studied, the implications of this interrelated complexity for unbiassed and efficient estimates of the structural relations, and the problems of getting an adequate number of observations of the required form.

Two distinct approaches have been used in the study of marginal and total factor productivity. One approach is to use the factor shares in national income as weights to combine the individual factor inputs to make an index of total factor input, and to use a factor's relative share to measure its marginal contribution. The second approach is to estimate the production relation from the data being used, and derive the marginal contribution of the productive factors to output from the estimated relation. The longest parts of the paper review the procedures followed to cope with the main problems in the real world, and the strengths and limitations in the two approaches. The discussion emphasizes the issues for the economy as a whole, and touches only briefly on the issues in disaggregation.

Three major themes are emphasized in the conclusions to the paper. One is that many of the problems, the differences in view, and the controversies grow out of the range of interrelated issues in practical applications. A second major theme is that most of the attempts to solve particular issues by those using the factor shares approach are rather similar to those followed by researchers estimating the production relations directly. A third theme is to encourage more studies that will look at the interconnections between production relations and income distribution, from the points of view of both economic theory (and its predictions about the relevance to concrete applications) and statistical estimation.

I. Introduction

For several decades after Keynes's *General Theory*, macroeconomics emphasized the demand side and problems of economic stabilization. During the last decade, renewed concern and interest with the supply side of the economy has begun to emerge. This shift in economic theory, empirical work, and the concerns of government economic policy is a welcome step. The shift reflects, to an important extent, the fact that during the postwar experience of most countries, the fluctuations in demand have been mild and departures from full employment and potential output have been short and moderate. However, if one looks at the experience of a variety of countries, there are large differences in the levels of real output per person employed (or in relation to total factor input) at a point in time, and large differences in the rates of growth of output in relation to both labor and total factor inputs. The reasons for these differences and their

*This paper builds on the earlier work on Canada–United States income differences initiated in cooperation with Dorothy Walters, using the framework developed in Edward F. Denison, assisted by Jean-Pierre Poullier, *Why Growth Rates Differ: Postwar Experience in Nine Western Countries* (Washington: The Brookings Institution, 1967). This paper goes further in trying to be explicit on the main assumptions involved in the estimation of the contribution of factor inputs, and the problems in attaining an ideal solution.

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implications for economic policy have become of increased interest to economists, business organizations, and governments.

The purposes of this paper are to review the main problems in making empirical estimates of the relative contribution of the individual factor inputs as part of estimating total factor inputs, to speculate on an "ideal" solution, and to explore the main alternative pragmatic solutions. The main aspects of economic theory, statistical estimation, and data sources relevant to the central theme will be introduced, as appropriate. To limit the problem to more manageable dimensions, the paper will concentrate on the issues that arise at the aggregative level, recognizing that the total economy is a sum of the individual parts, and that the solutions to some of the questions may be found at a finer level of disaggregation (if they are to be found at all in the next decade or so).

The relative contribution of factor inputs to growth and income differences has reflected a fairly wide difference of view within the profession during the 1960's. The differences of view have usually been related to differences about other areas more at the center of the various controversies about economic growth—such as the form of the production function including factor substitution (the Cobb–Douglas and the C.E.S. production function), the division of output determinants between input and productivity, and capital theory (and the degree to which new technology is imbedded in new machinery and equipment). Furthermore, there has been little discussion of the question of weighting factor inputs in the main earlier studies and surveys.[1] Where it has been discussed as one part of a larger study, the nature of the problems and how, or the degree to which, they can be met in other approaches are rarely explored.

II. The Nature and Origins of the Problems

An underlying theme in this paper is that the differences in approach in this area partly arise from the complexity in the real world being studied, the implications of this interrelated complexity for unbiassed and efficient estimates of the structural relations, and the problems of getting an adequate number of observations of the required form. Individual contributors have selected subtopics of this larger whole, and used diverse data from a number of countries (with varying emphasis on disaggregation) with various statistical approaches. Without attempting to be either exhaustive or intensive, the following themes seem to appear in the literature.

The following comments are expressed in terms of changes over time, but essentially all the ideas could also be expressed in terms of differences between countries. No attempt is made in Section II to footnote all the points from the literature, although the selected references at the end of the paper and the footnotes in later parts of this paper discuss most of the points covered here. The emphasis will be on points relevant to the distribution of income to productive factors, and the contribution of those factors to physical output.

[1] F. H. Hahn and R. C. O. Matthews, "The Theory of Economic Growth: A Survey," *Economic Journal*, 74 (1964), pp. 779–902, reprinted in *Surveys of Economic Theory*, Vol. II (New York: St. Martin's Press, 1965), pp. 1–124; Murray Brown, editor, *The Theory and Empirical Analysis of Production*, Studies in Income and Wealth, Vol. 31, New York, NBER, 1967; and M. Ishaq Nadiri, "Some Approaches to the Theory and Measurement of Total Factor Productivity: A Survey," *Journal of Economic Literature*, December 1970, pp. 1137–77.

A. *The Interrelations in the Real World*

It is important to recognize that many factors in the real world can affect the contribution of the individual factors of production to real output, both in theory and in practice, and these changes in production conditions can have related influences on other parts of the economy. More complex statistical methods are appropriate to estimate the underlying structural relations in such a complex system of simultaneous economic relations, if the results are to conform to the basic relevant economic theory, and if the resulting estimates are to be unbiassed. The problems become more acute if it is recognized that the speeds of reaction differ in various parts of the system, with long lags sometimes being present, and that individual observations are not likely to correspond exactly to the estimated relation, but can diverge from it to varying degrees. The following remarks provide a framework or perspective on these problems, setting out initially the main interrelations present in practical applications, and, subsequently, the implications for estimation method and data.

1. *Final demand:* Changes in final demand for the goods and services produced in the economy can affect the distribution of income and the contribution of factors of production in at least two ways. First, it is well established that the extent of pressure of demand relative to supply, and changes in the extent of that pressure, can affect the output performance of the economy, and the distribution of income. A relatively strong (but not necessarily excessive) demand pressure encourages higher operating rates and higher levels of output in relation to measured inputs, while slack demand has the opposite effect. If there are differences in the degree of demand pressure over the period of estimation (or between the country comparisons at a point in time), this can affect the rate of growth in output in relation to inputs (or the differences in levels of output in relation to factor inputs). These variations in demand pressure and rates of productivity change are reflected in much more marked variations in corporation profits than in other income flows, with subsequent effects on decisions on investment in inventories and fixed capital formation. Second, long-term growth in final demand is reflected in long-term shifts in resources between industries, partly reflecting differences in the long-term income elasticities of demand for different types of commodities. The long-term shift out of agriculture, and the low income elasticity of demand for basic food products at the farm level illustrate one extreme. The long-term increase in the service industries in highly industrialized countries illustrates the other extreme. If those industries use different proportions of the basic productive factors, or the rates of return to labor and capital are different in the different industries, the long-term changes can affect both the distribution of income to factors and the output derived from the various factor inputs. However, this issue of disaggregation and inter-industry shifts will be largely ignored in the balance of this paper.

2. *Factor supply:* The scope for changes in the relative supply of the factors of production can be illustrated for the classical threefold distinction of productive factors into labor, capital, and land. Changes in the supply of labor can occur from changes in the basic population, reflecting changes in birth and death rates. Changes in the number in the labor force can occur from

changing participation rates of women, or from changes in school-leaving age and age of retirement, and changes in hours worked of both a longer-term trend and shorter-term cyclical variety can occur. The quality of the labor force can also change with rising levels of education and changes in the extent of on-the-job training. There have also been marked increases in the supply of physical capital in the form of buildings, machinery and equipment, and inventories. The supply of land, and forest and mineral resources, on the other hand, has changed very little. These changes in relative supplies can affect the distribution of income, and the relative contribution of these factors to output. Very little has been done, however, to study the long-term changes in the supplies of the factors and such effects, if any, on their prices and rates of return.

3. *Factor substitution:* The effects of the changes in factor supplies in the previous paragraph depend on the nature of factor substitution which occurs. This effect is reflected in the elasticity of substitution between factors. Technical change is also relevant, if it encourages the substitution of capital for labor or the reverse. Such changes can affect the distribution of income between factors, the relative demand for the factors, and the relative contribution of the factors to output. Attention should also be drawn to the capital embodiment thesis, which emphasizes the role that new machinery and equipment, based on later technology, plays. This view would argue that later machinery would be more productive than older facilities, and would force a lower rate of return on existing capital facilities. This view of the world and its implications for technology and income distribution would overlap some of the distinctions in earlier and later paragraphs.

4. *Shifts in production conditions:* If one looks at the available data on output and most measures of input, one is impressed by the indications of changes in production conditions in the major industrialized countries. This is the conclusion that I draw, whether I look at the evidence for a twenty, a forty, or less complete and less satisfactory data for a hundred-year period. Such changes could develop from a variety of influences. Innovation and technical change can be important, especially if it includes changes in organization, changes in management, and the degree to which best practice is actually implemented in organizations. Economies of scale can occur if, in addition to considerations of plant size and firm size, the effects of improved transportation (through lower rates, faster service, and more flexible traffic routes) on size of market are considered. Product diversity and length of run can also be important, and this can be affected by the extent of tariffs and non-tariff barriers to trade, especially for smaller countries. The evidence on many of these changes relates primarily to the extent of change in output in relation to labor (and other factor inputs). The evidence on the effects of such changes on the marginal contribution of the various factors of production, and the incomes they receive, is much less complete, but some effects are likely to have taken place.

5. *Market imperfections, equilibrium, and other unexplained deviations:* Under the assumptions of perfect competition (large numbers of buyers and sellers, a homogeneous product, and perfect knowledge), it is easy to relate the marginal contribution of the various factors of production to the distribution

of income, via the marginal productivity theory of distribution. A number of the extremely simplifying assumptions can be relaxed and one can still use the marginal productivity theory to relate income distribution and the marginal contribution of the factors to output. Much of that discussion has taken place in a relatively static setting. However, the greater the degree of market imperfections and the greater the change that may have occurred in the extent of such imperfections over the relevant time period, the more unsatisfactory does that approach become. Although some work has been done on the extent of market imperfections in product markets, and the effects of unions on the relative wages of union workers, this has not been drawn into the discussion of the weights for combining factor inputs as far as I know. If the marginal productivity theory of distribution is dismissed, either another approach to integrate production and distribution has to be provided, or the connection between the factor contribution to output and the distribution of income to productive factors has to be dropped.

One further problem is that the individual observations should lie on, or close to, the underlying relation that one is trying to measure. Only if that condition is approximately met can one estimate the true structural relationship. In economic terms, the relationships should be close to equilibrium, in terms of the individual time periods of observations. The unexplained deviations of the individual observations from that underlying relationship should be relatively small.

B. *The Conditions for Effective Estimation*

If one accepts some, but not necessarily all, the aspects of the interrelations in the real world, there are important implications for the estimation procedures that one should use, if one wants unbiassed estimates of the underlying structural relations. The statistical procedure should be a simultaneous estimation of all the parameters in the complete system (or in the subsystems if some parts of the system have a relatively unrelated connection to the balance of the economy). This type of estimation procedure assumes certain conditions on the feasibility of estimation (the identification questions), and certain conditions about the random disturbances in the individual equations (e.g., a normal distribution and independence of the disturbances in successive time periods). The conditions and procedures for estimation have been most fully developed for systems of linear equations with quick responses from the various exogenous variables on the system of endogenous relations. However, the empirical evidence tends to indicate that the complete economic system consists of important non-linearities and many long lags in response. The frequent presence of multicollinearity in the exogenous and endogenous variables is a further problem, especially over long time periods.[2]

A similar concern for the problems in the disentangling of these issues is reflected in part of the concluding remarks by Nadiri in his recent survey.

[2]For fuller discussion, see such typical econometric discussions as A. A. Walters, *An Introduction to Econometrics*, New York, W. W. Norton and Company, 1968, Parts III and IV; L. R. Klein, *Textbook of Econometrics*, Evanston, Row, Peterson and Company, 1953; and T. Koopmans, ed., *Statistical Inference in Dynamic Econometric Models*, New York, Wiley, 1950. These are in increasing order of technicality.

> The identification of the separate contribution of disembodied, embodied and biased technical change, of economies of scale, and of changes in the elasticity of substitution to the growth of productivity is still not achieved. The core of the problem is the inherent dynamic interactions of these technical aspects with the factor-price relations on the one hand, and among themselves on the other.[3]

One other problem area should be mentioned. Even though this paper emphasizes the economy as a whole, it should be realized that a number of problems in index numbers are encountered. The interrelations between value, volume and price come up on the expenditure side of output, and analogous problems come up in the aggregation of factor inputs.

C. *Data Problems*

It is apparent that very real practical problems are encountered in obtaining data of the type one would want to estimate the range of inter-connected relationships that have been touched on in the previous pages. Even if one limits one's attention to a small part of these issues and to a particular country, one encounters problems!

D. *An "Ideal" Solution*

A basic theme of modern econometric estimation procedures is the need to ensure that all of the relationships can be properly identified and then to estimate all the relationships simultaneously to ensure unbiassed estimates. It requires that the underlying relationships persist over all the units of observation, and that the unexplained disturbances have certain specified properties (such as lack of serial correlation, independence of disturbances in separate equations) and that the exogenous variables have certain properties.

It is pretty clear that no study has yet attempted to do this in the area, and it is not likely that this can be fully achieved during the current decade. One outstanding problem is the evidence suggesting that significant shifts have been occurring in the production relations over time and that there are important differences in production conditions between plants and firms within a country and important differences between countries at a point in time. This is a basic problem that will occur in many studies. There are a range of options open. At one extreme, one can withdraw from an attempt to explore practical applications and concentrate on theoretically complete theorems about general equilibrium. What most researchers have done is choose a subset of relationships, accumulate the relevant data, and try to form some conclusions about that part of the real world of interest to them. If concrete and important applications are being explored, such an approach can narrow the range of uncertainty about the parameter values, and suggest new questions for further study. In the light of the increased interest in the policy implications in this area, it is likely that the extent and depth of such work will continue to grow.

I am not aware of comprehensive studies of the magnitudes of possible bias in using partial systems of least squares estimates of the production relations.

[3]Nadiri, *Op. cit.*, p. 1170.

Such studies of aggregate demand relations indicate important biasses in large samples, but the differences between single equation least squares and unbiassed estimation methods are sometimes less in small samples, and many macro model builders still use least squares estimates in their initial work. It is probable that many researchers will continue to use less expensive estimation methods, even though they recognize the risks of bias that can occur under such conditions.

In the literature reviewed for this paper the various authors followed a wide range of routes and approaches, with varying emphasis and concern for economic theory, comprehensiveness, and statistical approach. These differences can arise for a variety of reasons, of which the following seem relevant, but not necessarily complete. There are differences in priorities with respect to an emphasis on relevance to the real world, and a desire to influence public policy. The authors can have a narrow or a wide range of interests in their studies in terms of comprehensiveness of questions studied. There are differences of emphasis and view on what relations are regarded as stable and quantitatively important. Other differences reflect data availability. Later comments on particular studies and authors will be selective rather than exhaustive in light of the range of possible topics and studies.

III. The Two Main Alternative Approaches

In the study of factor productivity (including both total factor productivity and the marginal contribution of the individual factors of production) two distinct approaches have been used. First, the contribution of the individual factor inputs have been estimated by using the income share of that factor as a weight to combine with the growth rate in the quantity index of that factor input to estimate its marginal contribution to ouptut. The weighted changes in the individual factor inputs when aggregated provide a measure of total factor input. The difference between the change in total factor input and the change in total real output gives a measure of total factor productivity. Some authors go on to try to disentangle some of the individual component items that contribute to this change in total factor productivity, and it would be generally agreed that Ed Denison has gone further in attempting to isolate these individual contributions to growth than any other author.[4] A number of assumptions are usually involved, either explicitly or implicitly, about the real world in using this approach, such as the applicability of the marginal productivity theory of distribution, the nature of technological change, the existence of economies of scale, the conditions of substitution between factors of production, etc.

The second general approach is to estimate the contribution of the individual factors of production to output by using multiple regression methods.[5]

[4]For a partial list of other authors who have sometimes used this approach, see the selected references at the end of this paper, especially those by M. Abramovitz, E. F. Denison, Paul H. Douglas, Solomon Fabricant, Zvi Griliches, Dale W. Jorgenson, John W. Kendrick, Robert Solow and Dorothy Walters. See also, further references in the studies by Hahn and Matthews, Murray Brown and Ishaq Nadiri, referred to in Footnote 1.

[5]For a partial list of authors who have sometimes used this approach, see the references at the end of this paper to studies by Arrow, Chenery, Minhas and Solow; Beckman and Sato; Murray Brown; Lester Thurow; and further references in the studies by Murray Brown, Ishaq Nadiri and Marc Nerlove.

This approach usually specifies certain conditions about the real world such as the nature of technological change, the nature of substitution between factors of production, etc. This approach need not necessarily draw any conclusions about the implications of the results for the distribution of income to productive factors. However, several important studies have explored the interrelations between productivity change using regression methods and income distribution.[6]

In the following two sections, the two main approaches to estimating the marginal contribution of the individual factors to real output and income distribution will be discussed. With each approach, the key points in procedure will be reviewed, followed by a discussion of the number of factor inputs analyzed, the treatment of shorter-term demand fluctuations, the implications of technological change and other factors on income distribution, and the treatment, if any, for market imperfections. A final section will summarize the conclusions. The treatment of depreciation, taxation and capital gains in factor share weights will be considered in an Appendix.

IV. Weighting Inputs by Factor Shares

The recent use of national income shares as weights for the analysis of the contribution of factor inputs to economic growth was apparently first introduced by Schmookler in his 1952 article, and this approach has been used subsequently by such researchers as Abramovitz, Denison, Fabricant, Griliches, Jorgenson, Kendrick, Lithwick, Solow, and Walters. There has been more empirical work using this approach in North America than in other countries. However, the origins of some of the ideas go back to the interests of the classical economists in economic growth and the distribution of income, and later developments in the marginal productivity theory of distribution, Euler's theorem, and the Cobb–Douglas contributions on production and income distribution.

The most comprehensive application of this approach has been by Ed Denison, and the balance of this section will emphasize his approach (as being the fullest and most complete), with some attention to the modifications and comments of others, including both the users and critics of this approach. Some selections from *Why Growth Rates Differ* will provide some perspective on his reasons for using income shares.

> What fraction of the increase in real national income that would result from a 1 per cent increase in all factors of production is obtained from a 1 per cent increase in only one factor or group of factors? The question refers to conditions in which available resources are utilized at the same

[6]Murray Brown and John S. de Cani, "Technological Change and the Distribution of Income," *International Economic Review*, September 1963, pp. 289–309; Murray Brown, *On the Theory and Measurement of Technological Change*, pp. 180–91; Lester Thurow, "Disequilibrium and the Marginal Productivity of Capital and Labor," *RE Stat*, February 1968, pp. 23–31; Lester Thurow, "Disequilibrium under Alternative Production Conditions" in Paul Streeten, ed., *Unfashionable Economics: Essays in Honour of Lord Balogh* (London: Weidenfeld and Nicolson, 1970), pp. 325–47, and Michael Bruno, "Estimation of Factor Contribution to Growth under Structural Disequilibrium", *International Economic Review*, February 1968, pp. 49–62.

rate and rather fully. It does not refer to short-term "cyclical" changes in the position of the economy caused by fluctuations in aggregate demand.

Given the complexities of economies and the limitations of data, this question cannot be answered with absolute precision, but an approximation that is sufficiently accurate for the purposes of this study can be obtained: The fraction is the same as the fraction of total national income that is earned by the factor or group of factors that increases. Suppose, for example, that the factor is labor and that labor earns 80 per cent of the national income (my estimate for the United States in the 1960–62 period). A 1 percent increase in the quantity of every type of labor in use will then be equivalent to an increase of 0.80 per cent in all types of input. If a 1 percent increase in the quantity of all the factors would yield an increase in total product of 1 percent, as would be the case in an economy operating under constant returns to scale, than a 1 percent increase in labor input will yield a 0.80 per cent increase in output . . . .

This factor share approach, which has been widely used in economic literature, derives from marginal productivity analysis. It provides an accurate estimate if the earnings (prices) of the various factors of production are proportional to the value of their marginal products . . . .

The proportionality of earnings and marginal products must be present *if economic units combine factors in such a way as to minimize costs.* Production at minimum cost to an enterprise implies that, given the price at which each factor can be obtained, factors are combined in that proportion which makes the marginal product of each factor proportional to the marginal cost of obtaining it. Enterprises can always reduce costs by changing factor proportions unless the marginal costs of obtaining various types of labor, capital, and land are proportional to their marginal products. This is so even if the price of a factor is set monopolistically, or by law, or in some other fashion, above the "competitive" price at which its entire potential supply can be utilized. The result of such price setting will not be to destroy this proportionality but to prevent use of the full potential supply of the factor. Hence a tendency toward proportionality must be present provided only that economic units seek to minimize costs and are free to select the combination of resources that best achieves this purpose.

Of course, the tendency toward proportionality is only a tendency toward an equilibrium position that is itself constantly changing. The most efficient combination of factors in any process may be altered by the discovery or dissemination of new techniques of production and distribution or by changes in the relative prices of the factors resulting from changes in their supply, in the pattern of final demand, or other causes. Moreover, proprietors and managers do not have exact knowledge of least-cost combinations, and competitive pressure may not suffice to eliminate promptly enterprises that do not arrive at the best combination. Furthermore, managers (particularly in some of the European countries) may not be wholly free to arrive at the best combinations because of legal and

institutional restrictions, the most important of which are those controlling new investment and those imposing special costs for dismissing labor.

Despite all such qualifications, the reasons for believing that, in periods of fairly full utilization of available resources, relative marginal value products fairly closely approximate proportionality to relative earnings are compelling. The incentive (and in most situations the pressure) for enterprises or other producing units to combine factors in a way that will minimize costs is pervasive and strong, and in the long run the opportunity to do so is generally present. One chief task of any entrepreneur or manager in a private, free enterprise sector of an economy is to maximize profit by finding the combination that yields the desired result at the lowest cost. His enterprise will be at a competitive disadvantage if he is less successful than his competitors in doing so. Although ignorance, poor foresight, and time lags cause departures from proportionality in particular situations, these may in general be expected to be random, rather than biased in such a way as to raise or lower the relative returns to labor, capital, and land, and to particular types of these factors, in economies as a whole.

The working hypothesis of this study is that, on the average for all producing units, the tendency toward proportionality of factor prices and marginal products under conditions of reasonably high employment is sufficiently strong in the United States and, though perhaps weaker, in Western Europe for distributive shares to provide an adequate basis for analysis of the relative contributions of the various factors to growth. The general similarity of income distributions for different time periods and for the various countries that are derived in this chapter somewhat strengthens its acceptability.

It is important to note that nowhere in this study do I assume that the allocation of employed resources among alternative uses, or among industries and firms, is optimal.

> It may be noted in passing that the presence of some degree of imperfection in product markets, and therefore monopoly profits, is implied insofar as increasing returns to scale in national economies as a whole arise because of economies of scale that are internal to firms in certain industries; increasing returns to scale internal to firms are incompatible with pure competition.
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> For any reader who may be puzzled by my combination of assumptions that (1) there are returns to scale, some of which may be internal to the firm, and (2) income shares can be used to measure relative marginal products, let me make clear that I do not assume pure competition in product markets. Its absence does not alter the combination of labor, capital, and land that produces a given amount of output at least cost, or factor prices at that level of output, and would cause no difficulty in the use of income shares as weights, which derives from the cost minimization principle, if returns to labor, capital, and land could actually be isolated. Because profits arising from imperfect

competition in product markets are counted statistically as returns to capital and land, I admit some probable overweighting of these factors, and corresponding underweighting of labor.[7]

In considering the marginal productivity theory of distribution (which underlies the rationale for the use of income shares), it should be noted that it is essentially a theory of the demand for the factors of production, derived from the demand for the output of goods and services. To have a complete theory of the prices of the factors of production, the supply of the factors of production should also be considered and discussed. If there are limitations on the supply of particular factors, or legal or customary limits on the prices of particular factors, these are a necessary part of a complete explanation of price determination in the factor markets.[8] The presence of monopoly elements is no basis of dismissing or disregarding the potential relevance or applicability of marginal productivity theory. It should also be noted that Denison's approach does *not* involve the assumptions of perfect competition and constant returns to scale sometimes attributed to it.[9]

If it is agreed that there are a number of productive factors contributing to output, and one wants to distinguish between the contribution of individual and total factor inputs to growth, it seems pretty clear that some method of combining factor inputs must be used. The critics of the marginal productivity theory and the use of factor shares have some responsibility to the profession to be explicit about an alternative. Clearly there are alternative theories of distribution that have arisen out of aggregative theories of demand, but they have not been related to supply conditions, the demand for individual factors of production, and the micro theory of the firm and industry.[10] If one really

[7]Edward F. Denison, assisted by Jean-Pierre Poullier, *Why Growth Rates Differ: Postwar Experience in Nine Western Countries* (Washington: The Brookings Institution, 1967), pp. 33–36, including footnote 5 on p. 36.

[8]Milton Friedman, *Price Theory: A Provisional Text*, Revised Edition, Chicago: Aldine, 1966, pp. 172–225, especially p. 172.

[9]See the summary of the discussion in OECD, *The Residual Factor and Economic Growth*, Paris, 1964, especially pp. 263 and 264 by Scitovsky and pp. 271, 273, and 275 by Nield, which incorrectly attribute the assumptions of perfect competition and constant returns to scale to Denison. The wording in the most recent volume is, "The working hypothesis of this study is that, on the average for all producing units, the tendency toward proportionality of factor prices and marginal products . . . is sufficiently strong . . . for distributive shares to provide an adequate basis for analysis of the relative contributions of the various factors to growth." (*Op. cit.*, p. 35.) The marginal productivity theory of distribution is applicable to monopolistic competition and other sectors in addition to perfect competition, and Denison makes an allowance for changes in degree of competition where appropriate and feasible. He also allows for increasing returns to scale, and this factor is important in the difference between growth in a number of European countries and the United States. Some of the continuing misunderstanding may emerge from an insufficient appreciation that some of the restrictive assumptions made in the initial use of factor shares to combine factor inputs is subsequently relaxed in the discussion of the changes in output in relation to total factor inputs. Furthermore, it is not sufficiently recognized that the marginal productivity theory of distribution is essentially a theory of the *demand* for factors of production and that the conditions of supply (including any allowance for restrictions on the supply and prices of those factors) are a necessary part of the determination of factor prices.

[10]See the discussion of the theories of Kalecki, Kaldor and Joan Robinson in C. E. Ferguson, *The Neoclassical Theory of Production and Distribution*, Cambridge, Cambridge University Press, 1969, pp. 308–33.

wants to work with a number of productive factors and an aggregate of them there does not really appear to be any alternative to the marginal productivity theory, in a broad sense, to relate the factor inputs, production conditions, output, and income distribution.

Several important advantages of this use of factor shares might be pointed up initially. First, it is relatively simple to understand and apply. This has advantages in its use with students in an early stage in their introduction to economics. It is also simple to explain and use as a method for deriving and explaining projections of medium and long-term economic growth, as a background to business and government planning.[11] Second, a wide range of factor inputs can be combined. *Why Growth Rates Differ*, for example, uses labor (with breakdowns for age and sex, education, and hours of work), capital (with breakdowns into non-residential structures and equipment, dwellings, international assets, and inventories), and land (with breakdowns into non-residential sites, agricultural land, and mineral resources). The use of additional factor inputs does not create new problems in estimation or interpretation. Third, by using changing time periods for weights, the approach can allow for long-term differential rates of change in the supply of the individual factors and in their relative prices. It is not invariant to such changes, but the changes have to be quite large and persistent to change the overall factor shares to any appreciable extent.

One of the recognized limitations in this approach is that it has not dealt with the simultaneous interrelations in the real world. This is a situation that this approach shares with most of the other empirical work in this field. However, there does not appear to be much discussion of the extent of possible bias in ignoring the interrelations when the primary interest is in estimating production relations from time series data.[12]

In the following paragraphs, the degree to which this approach has been able to cope with the problems outlined in Section II will be reviewed.

1. *Final demand variations:* It is widely recognized that the growth in output in relation to labor input and total factor inputs does not grow steadily, but undergoes marked alterations, reflecting changes in the degree of demand pressure in the economy.[13] In Denison's first study, *The Sources of Economic*

[11]Projections using this framework have been used to project potential output in several of the *Annual Reviews* of the Economic Council of Canada, while other methods were being developed and tested.

[12]A. Zellner, J. Kmenta and J. Drèze, "Specification and Estimation of Cobb-Douglas Production Function Models," *Econometrica*, October 1966, pp. 784–95, show that "From the classical sampling theory point of view, the new model thus vindicates the single-equation approach to estimation of the Cobb-Douglas function from cross-section data." This applies, of course, to cross-section data and does not relate to time series data, which has been the primary focus of most of the studies.

[13]This was recognized by W. C. Mitchell in his 1913 volume on business cycles. A considerable renewed interest in this question developed with the development of slack in the North American economy in the late 1950's and early 1960's (with a short and incomplete expansion from 1958 to 1960 and two recessions starting within three years). As examples of the discussion, see S. Fabricant, *Basic Facts on Productivity Change* (New York: Columbia University Press, NBER, 1959); T. Hultgren, *Costs, Prices and Profits: Their Cyclical Relations* (New York: Columbia University Press, NBER, 1959); E. Kuh, "Cyclical and Secular Labor

[*Continued on next page*

Growth, Denison's initial and terminal years were roughly comparable in their degree of resource utilization. In the volume, *Why Growth Rates Differ*, however, 1962 in the United States clearly reflected more slack than in 1950, and there were milder demand variations in some of the eight European countries there considered.[14] In *Why Gowth Rates Differ*, these irregular fluctuations in demand pressure were treated as one component in the growth of output per unit of input, but an alternative would have been to adjust the observed growth rates before allocating them to the various sources of growth.

It is pretty clear that an adjustment of this kind is necessary, and differences in the procedure to make such estimates do not seem to lead to large differences in results.

It should also be noted that these short-term variations in demand have a very marked impact on corporate profits, one of the most volatile income categories over the business cycle. The income shares used as weights usually omit years of weak demand because of the volatility in the corporate profits share used as a key part of the return to capital,[15] although they may sometimes average years of strong and weak demand.

2. *Factor supply and factor substitution:* Three points will be mentioned here. For one thing, Denison essentially takes the supplies of factors as given for his analysis of economic growth. There is not a complete analysis of the reasons for the longer-term changes in the supply of the various factors, although the main facts and some important influences have been explored for the United States.[16]

The elasticity of substitution has been an area of controversy in the study of production relations. The use of the Cobb–Douglas form of relation with no allowance for a change in income shares implies a unitary elasticity of substitution. However, Denison does test for changes in the income shares over time, and by occasional changes in weights does introduce some flexibility. The introduction of changing weights as appropriate is a desirable and widely used practice in index numbers, and this is pretty clearly recognized as desirable in both separating quantity and price changes on the expenditure side of the accounts and combining individual inputs to make a measure of total factor input on the input side. Denison does not limit the defence of his procedure to

Productivity in the United States Manufacturing," *RE Stat.*, February 1965, pp. 1–12; A. Okun, "Potential GNP: Its Measurement and Significance," in *ASA Proceedings of the Business and Economics Statistics Section*, 1963, pp. 98–104; and L. Thurow and L. Taylor, "Interaction Between the Actual and the Potential Rates of Growth," *RE Stat.*, November 1966, pp. 351–60. This point was also mentioned by Sol Fabricant in his survey given at the Ronneby Conference.

[14]Denison and Poullier, *Op. cit.*, pp. 273–77 and pp. 441–46. It should be noted that this adjustment is complex. See, for example, the comments by Denison on Dorothy Walter's estimates for Canada in D. Walters, *Canadian Growth Revisited*, 1950–1967, Staff Study No. 28 prepared for the Economic Council of Canada, Ottawa, The Queen's Printer, 1970, p. 64.

[15]Charles L. Schultze, "Short-Run Movements of Income Shares," in *The Behavior of Income Shares: Selected Theoretical and Empirical Issues*, Studies in Income and Wealth, Vol. 27, Conference on Research in Income and Wealth (Princeton: Princeton University Press, 1964), pp. 143–82; and T. Hultgren, *Op. cit.*

[16]Irving Kravis "Relative Income Shares in Fact and Theory," *AER*, December 1959, pp. 917–49; and J. Kendrick and R. Sato, "Factor Prices, Productivity and Economic Growth," *AER*, December 1963, pp. 974–1003.

the statistical argument from index numbers, but goes on to point out that the procedure used is relatively insensitive to reasonable alternative values for the elasticity of substitution, relying on results from R. Nelson.[17]

A third area to note is the concept of capital, and the role it plays in technological change.[18] Some authors have emphasized new investment as a key part of the diffusion and implementation of technical change. This area still seems to be under dispute and the paper by T. K. Rymes explores one point of view on this topic. A greater clarification of this issue might occur if more attention to the implications of the capital embodiment thesis were explored for the distribution of income in addition to its implications for the measurement of production relations.

3. *Shift in production conditions:* A key theme from the Denison studies is the degree of importance that changes in output in relation to total factor inputs plays in economic growth, both in terms of changes in total real output and in terms of real output per employed person. An important part of *Why Growth Rates Differ* is devoted to an examination of such changes for the United States and the eight European countries studied. The differences in output in relation to total factor input are also large in the comparisons of output levels on a per person employed basis.[19] These differences partially reflect differences in production conditions.

Within the Denison framework, these changes over time (and differences in level) are associated with changes in the excessive allocation of labor to farming and self-employment, gains from economies of scale, more efficient transportation and distribution systems, obstacles to international trade, issues in the

[17]R. Nelson, "Aggregate Production Functions and Medium Range Growth Projections," *AER*, September 1964, pp. 575–606; and Denison and Poullier, *Op. cit.*, footnote 6, p. 36, which concludes that "If the true elasticity in fact lies within, or even close to, the 0.5 to 1.0 range the error resulting from the assumption of unit elasticity of substitution cannot be large in the present study." The same point had been made earlier in M. Bronfenbrenner, "A Note on Relative Shares and the Elasticity of Substitution," *JPE*, 1960, pp. 284–87. Murray Brown dissents from this view that the elasticity of substitution is of negligible importance in his comment in Murray Brown, ed., *The Theory and Empirical Importance of Production*, Studies in Income and Wealth, Vol. 31, New York, Columbia University Press, 1967, pp. 133–35.

[18]See, for examples, studies listed at the end of this paper by Abramovitz; Arrow; Arrow, Chenery, Minhas and Solow; Jorgenson; Jorgenson and Griliches; and Solow. Denison, in "The Unimportance of the Embodied Question," *AER*, March 1964, pp. 90–94, argues that the issue has been exaggerated and suggests that the implications of this approach be tested for its implications to income distribution. Such a test has rarely been tried. An unpublished test by Leo Bakoney for Canadian data, while with the Economic Council of Canada, indicated that the introduction of an allowance for embodied capital reduced the marginal contribution of labor rather than the residual. The implications of the two concepts of capital for the distribution of income were very similar, once the return to capital included capital consumption allowances.

[19]For similar material on a Canada–United States basis, see D. J. Daly and D. Walters, "Factors in Canada–United States Real Income Differences," *The Review of Income and Wealth*, December 1967, pp. 285–309; Dorothy Walters, *Canadian Income Levels and Growth: An International Perspective*, Staff Study No. 23 for the Economic Council of Canada (Ottawa: The Queen's Printer, 1968), pp. 20–23 and 169–191. For a fuller discussion of some aspects of economic growth and international trade to intercountry comparisons, see D. J. Daly "Uses of International Price and Output Data," in D. J. Daly, ed., *International Comparisons of Prices and Real Income*, Studies in Income and Wealth, Vol. 37, New York, NBER, 1972.

mobility and use of labor, and competition. Advances in knowledge is a further item in the changes in output in relation to total factor input.

One question that does not seem to have had much discussion is whether these changes in output in relation to total factor input are neutral on the measurements of marginal factor contributions and neutral on the distribution of income. The development of the marginal productivity theory of distribution took place when the primary concern was with comparative statics. The question, however, is the degree to which a more dynamically oriented concern with growth affects the empirical applicability of the use of income shares for weights for factor inputs. If the assumption of approximate proportionality of marginal products and factor prices is useful and applicable in analysis, no bias in the measurement of marginal factor contributions need occur. It would take significant technical change that was non-neutral on factor use and very low elasticities of substitution for such changes to have important effects on the income share weights.[20]

4. *Market imperfections and equilibrium:* The use of factor shares to derive the marginal contribution of the individual factors of production was based on the marginal productivity theory of distribution. Although this approach has been criticized,[21] there is no alternative theory of income distribution that relates the distribution of income factor shares to the physical aspects of production. There has been some theoretical and empirical work on the effects of variations in demand on the distribution of income to labor and capital, which emphasizes the cyclical variability of profits, and the implications of this for consumption, savings and investment.[22] There have also been some interpretations that emphasize mark-up pricing.[23]. Neither of these approaches to the distribution of income provide links with the conditions of production at the level of the firm, the industry, or the economy as a whole.

It should be noted that the marginal productivity theory is primarily a theory of the demand for the individual factors of production, and that a complete theory requires a theory of both the demand for and the supply of factors of production.[24] Although it was developed historically for variable proportions of the factors of production, it can be applied to situations of fixed proportions (or constant coefficients of production).[25] It was developed initially in the context of perfect competition in both product markets and factor markets, but it can

[20]One of the few studies of this question for the United States suggests that over the period from 1890 to 1960, non-neutral technological change has increased relatively and that non-neutral technological change had shifted from labor-saving to labor-using over that period. See Murray Brown, *On the Theory and Measurement of Technological Change* (Cambridge: Cambridge University Press, 1966), pp. 155–64.

[21]OECD, *The Residual Factor and Economic Growth*, Paris, 1964, pp. 263, 264, 271, 273, 275, for example.

[22]See the discussion of theories based upon aggregate demand of Kaldor and Joan Robinson in C. E. Ferguson, *The Neoclassical Theory of Production and Distribution*, Cambridge, Cambridge University Press, 1969, pp. 314–33.

[23]*Ibid.*, pp. 309–14. See also Tibor Scitovsky, "A Survey of Some Theories of Income Distribution," in *The Behavior of Income Shares*, Studies in Income and Wealth, Vol. 27, Princeton, Princeton University Press, 1964, pp. 15–31.

[24]Milton Friedman, *Price Theory*, Revised Edition, Chicago, Aldine, 1966, pp. 172.

[25]*Ibid.*, Chapter 8, pp. 162–71.

also be applied to situations of monopoly and monopolistic competition in the product market[26] and monopsony in the factor market.[27] In the monopolistic product markets, the sloping demand curve for the individual firm (and the related declining marginal revenue curve) reduces the remuneration of a factor below the value of its marginal physical product. The presence of monopolistic competition need not lead to any higher rate of return to capital in such industries—non-price competition can lead to rates of return similar to those in more competitive industries. Product diversity and short length of runs and a larger number of selling outlets can occur, and this can be reflected in the levels of physical output in relation to total factor inputs.

There is some evidence on the extent of the monopolized industries in the United States. Stigler made an estimate of the extent of competitive and monopolized industries, and concluded that about 20 percent of the labor force was in monopolized industries in 1940, a moderate decline from the 1920's, while about 80 percent was in competitive industries.[28] Although some questions of treatment arise, and the estimate is not recent, a quick dismissal of the importance of competition seems inadvisable. Furthermore, the effect of monopolist product markets may be reflected in inefficient use of resources and may not have much effect on the distribution of income between factor shares (such as labor and capital). Arnold Harberger has estimated the welfare loss from enterprise monopoly at 0.1 percent of GNP.[29]

An estimate has also been made of the effects of unions on relative wages and employment. About 25 percent of the U.S. labor force is unionized, and the effects of unions on wages in the unionized compared to the non-unionized workers with similar occupations and skills do not appear to exceed 15 percent. After allowing for both the relative wage and relative employment effect, the welfare loss of unions in 1957 was estimated at 0.14 percent of national output.[30]

These estimates emphasize the effects of monopoly and unions on relative factor shares, which is appropriate for income shares being used as weights for factor inputs. The influence of monopoly and unions on the general level of wages and prices or on continuing inflation is not relevant to this question.[31]

Even if monopolies or unions had a larger influence on efficient resource use than suggested by the above evidence, it need not bias the analysis of economic growth made along these lines. If the degree of monopoly or the extent of unions was the same at the beginning and end of the period under study, the relative contribution of factors and output per unit of factor input would not be biassed in any way. If the degree of monopoly or unions had *changed* over time (or was different between countries), a direct estimate of any effects of such changes on

[26]Edward H. Chamberlin, "Monopolistic Competition and the Productivity Theory of Distribution", in William Fellner and Bernard F. Haley, *Readings in the Theory of Income Distribution*, Philadelphia, Blakiston, 1946, pp. 143–57.

[27]Friedman, *Op. cit.*, pp. 187–90.

[28]George J. Stigler, *Five Lectures on Economic Problems* (London: Longmans, Green and Co., 1949), p. 52.

[29]Arnold Harberger, "Monopoly and Resource Allocation", *AER*, May, 1954, p. 77.

[30]Albert Rees, "The Effects of Unions on Resource Allocation," *JPE*, October 1963, p. 71. The emphasis on *relative wages* is appropriate in the discussion of weights for factor inputs in the analysis of economic growth.

[31]For a discussion of this question, see Martin J. Bailey, *National Income and the Price Level*, Second Edition, New York, McGraw-Hill, 1971, pp. 48–51 and 82–85.

output in relation to total factor input must be attempted. Similar quantitative studies of the effects of monopoly and union elements on relative prices and effective use of resources would be more appropriate than the dismissal of the marginal productivity theory of distribution.

These pieces of evidence suggest that important competitive elements are present in the U.S. economy. The marginal productivity theory has continuing relevance, even if market imperfections are present in either product or factor markets or both. The use of factor shares as weights for factor inputs is an appropriate working hypothesis under such market conditions. The important empirical question is the effect of changes in the degree of monopoly or unionization on relative factor prices and the efficiency of resource use.

V. Direct Estimation of Factor Contributions

The number of studies of production relations using econometric methods to estimate the marginal contribution of factors has expanded tremendously during the 1960's. The authors and references touched on here are only a partial reflection of the amount of published work. Furthermore, there is no single author or study whose contribution dominates the work in this field. It is thus harder to generalize accurately and convey the flavor of the work. The following points seem to be some key advantages and limitations of this approach.

This approach seems to have three important advantages. First, there is more scope for alternative specifications of the relevant variables and their form which can influence the marginal productivities of the factors. The resulting relationships emerge empirically from the data, with less initial specification of their form. Second, this approach can go further towards a simultaneous estimation of the demand and supply relations than the factor shares approach (which is closer to the partial equilibrium approach of traditional economic theory). However, many authors have estimated such relationships without using simultaneous estimation methods and without discussing the questions of identification and possible bias in the methods they have used. A third advantage is to allow the possibility of *marginal* relationships for the recent past and future to depart from the *average* income distribution of a recent period. This could be important if the statistical methods were both sensitive enough to detect such differences but robust to alternative possible specifications.

There are important limitations in this procedure in practice, however. A key limitation is the marked instability of the results to minor modifications in specification, length of time period, and estimation method. This instability has been documented most thoroughly in several surveys by Marc Nerlove, but the same instability continues to appear in a number of studies published subsequently.[32] A second important problem is that problems in estimation have

[32]Marc Nerlove, "Recent Empirical Studies of the CES and Related Production Functions," in Murray Brown, ed., *The Theory and Empirical Analysis of Production* (New York: Columbia University Press, NBER, 1967), pp. 55–122, and the subsequent discussion of the reasons for the differences among the estimates; and "Notes on the Production and Derived Demand Relations Included in Macro-Econometric Models," *IER*, June 1967, pp. 223–43. Nadiri comments, "The empirical evidence seems to indicate that the parameters of the CES production function are highly sensitive to slight changes in the data, measurement of variables, and methods of estimation." Nadiri, *Op. cit.*, p. 1151.

arisen if more than two factor inputs are included in the relation being estimated.[33] This creates problems in trying to attain increased applicability to real world problems, where a variety of factor inputs are suggested both from theory and practical experience.

Before turning to specific comments on the approach followed to deal with the problems in the real world, it might be noted that essentially all the problems mentioned in Section II are encountered with direct estimates of factor contributions. Furthermore, the methods used to cope with them are frequently fairly similar to those followed with the income share method of estimating marginal productivities of factors discussed in the last section. As there have been many studies using this approach, the references will be illustrative rather than complete.

1. *Final demand:* In the light of the evidence on the interrelations between variations in demand and the shorter-term variations in production relations, almost all studies make explicit allowance for demand variations. The degree of slack in the 1930's was the most extreme illustration of demand variation, but the slack of the late 1950's and early 1960's and the persistence of mild business cycle fluctuations provide important illustrations of demand variations.

Murray Brown had to deal with such demand variations in his study of technological change in the United States from 1890 to 1960. He included a cyclical variable in addition to labor and capital inputs, even after allowing for variations in capacity utilization in the measure of utilized capital services.[34]

Thurow and Taylor also used a utilization adjustment, and developed series for potential man-hours which took account of cyclical variations in participation rate and hours of work per week.[35] Utilization adjustments were tested in both a linear and nonlinear form (U and U^2), but the linear form was adopted as the range of unemployment in the postwar period was not large enough to estimate nonlinear effects.[36] They concluded that reductions in unemployment from 4 to 3 percent would not be reflected in as large an increase in real output as in Arthur Okun's results (2.4 percent in Thurow and Taylor, compared to 3.5 percent by Okun).[37]

These studies illustrate a general recognition of the need to take explicit account of the effects of demand variations on the shorter-term variations in supply conditions. Other studies making direct measures of production conditions typically make explicit allowances for such demand variations also.

2. *Factor supply:* No study has thus far been located that makes explicit empirical estimates of the conditions of supply for all factor inputs. The possible bias in estimating production relations without taking account of the conditions of simultaneity are well known, but rarely dealt with explicitly in the available studies.

[33]Nadiri, *Op. cit.*, pp. 1158–59.

[34]Murray Brown, *On The Theory and Measurement of Technological Change*, p. 144.

[35]Lester C. Thurow and L. D. Taylor, "The Interaction between the Actual and the Potential Rates of Growth," *RE Stat*, November 1966, pp. 352–53.

[36]*Ibid.*, p. 357.

[37]*Ibid.*, p. 359.

3. *Factor substitution:* The two-factor constant elasticity of substitution production function has been the most widely discussed and used function in the literature in recent years. This can be looked at as a more general function, which includes the Cobb–Douglas and Leontief production functions as special cases. The approach does differ from the Cobb–Douglas–Denison formulation in that the magnitude of the elasticity of substitution is determined from the data, rather than being constrained in value before estimation. However, the empirical results obtained thus far are very unstable, and a wide range of values can be found in the literature. This result emerges if a variety of assumptions about the production function form are explored with the same set of data. The unconstrained value for the elasticity of substitution has some appealing features in principle, but encounters a significant degree of instability in practice.

4. *Shifts in production conditions:* Most of the estimated production functions find shifts in the relations over time, even after allowing for embodied technical change in the factor inputs.[38] How consistent such changes might be over time, and what can be done by government and business to accelerate such changes, are essentially similar to those encountered in combining factor inputs.

5. *Production relations and income distribution:* Most of the studies of production relations concentrate on the measurement of the contribution of the major influences on the supply of output, and rarely explore the implications of their results to the distribution of income. The potential fruitfulness of such checking for consistency can be seen in the work by Murray Brown.

Further study of estimates of the marginal productivity of labor and capital have been made by Lester Thurow, and the results were compared with the data on the distribution of income to labor and capital. His studies are based on the estimation of a number of production functions (using alternative assumptions on embodied and disembodied technical change and returns to scale) for the period 1929 to 1965 in the United States.[39] He concludes

> The ramifications of extensive disequilibrium are many. Most theoretical models rest on the assumption of equilibrium and many empirical studies make use of the same assumption. The results of this paper indicate that there is a major clash between the analysis provided by constant parameter

[38]An exception was the article by Jorgenson and Griliches that argues that the growth in total input, as they measure it, largely explains the growth in total output. This would leave no room for increasing returns to scale, learning by doing with existing capital facilities, or more widespread adoption of best practice. See D. W. Jorgenson and Z. Griliches, "The Explanation of Productivity Change," *RE Studies*, July 1967, pp. 249–82. See the detailed criticism of this study by E. F. Denison, "Some Major Issues in Productivity Analysis: An Examination of Estimates by Jorgenson and Griliches," *Survey of Current Business*, May 1969, pp. 1–27. A later article written by Christensen and Jorgenson corrects the measure of relative utilization of capital input but does not deal with the other questions raised by Denison. See L. R. Christensen and D. W. Jorgenson, "U.S. Real Product and Real Factor Input, 1929–1967," *Review of Income and Wealth*, March 1970, pp. 19–50. See also Dale W. Jorgenson and Zvi Griliches, "Issues in Growth Accounting: A Reply to Edward F. Denison," forthcoming in *Survey of Current Business*, for a fuller reply.

[39]Lester C. Thurow, "Disequilibrium and the Marginal Productivity of Capital and Labor," *RE Stat.*, February 1968, pp. 23–31; and "Disequilibrium under Alternative Production Conditions" in Paul Streeten, ed., *Unfashionable Economics: Essays in Honour of Lord Balogh* (London: Weidenfeld and Nicolson, 1970), pp. 325–47.

aggregate production functions and the assumptions underlying general equilibrium theory. Given these results the two techniques do not seem to fit together in the private American economy. One or the other must be empirically incorrect.

Production functions analysis indicates the marginal product of labor is larger than the actual returns to labor and that the marginal product of capital is smaller than the actual returns to capital. The gaps are large and no set of constant production function parameters can eliminate them at every point between 1929 and 1965. Not only are the disequilibrium gaps large and varying over time, but the marginal products of capital and labour are very different in different sectors of the economy. Extensive disequilibrium exists between different sectors of the economy as well as between capital and labour.[40]

However, these results raise a number of problems in interpretation. In estimating such economic relationships as these, it is assumed that the individual observations are in, or close to, equilibrium (with certain assumptions about the independence and statistical properties of the disturbances in individual equations and the complete system). If it is argued that major types of disequlibrium can persist for extended periods, it does not seem valid to argue that the resulting regressions are applicable to the theoretical models being estimated and tested. If the marginal product of capital has been below the actual returns to capital for almost four decades, why has the increase in the stock of capital been so large relative to labor inputs over the postwar period? How does one reconcile the results for the capital embodiment variations with the expectation of those supporting the capital embodiment theory of technological change that the actual rate of return on existing capital facilities would be pushed down by more recent capital facilities?[41] These results are extremely difficult to interpret and relate to other empirical work on production relations, economic growth, and income distribution for the United States and other countries.

Michael Bruno has obtained results for Israel rather similar to Thurow's for the United States. He has used data for manufacturing and the total private economy from 1953 to 1964. The lack of competitive factor markets in Israel is attributed to the role of unions in manufacturing and urban areas and government development loans at low rates of interest. The special role of immigrants and capital inflows presumably permitted the disequilibrium to persist.[42] However, by 1964 the difference between the marginal productivity of capital and the actual rate of return had gradually declined to about half its original value.[43] It would seem that disequilibrium can be present in Israel due to the special factors emphasized by Bruno, but these influences are not applicable to the same degree in the United States.

[40]Thurow, "Disequilibrium under Alternative Production Conditions," pp. 341–42.

[41]R. M. Solow, *Growth Theory: An Exposition* (Oxford: Clarendon Press, 1970), pp. 46, 54, and 56.

[42]Michael Bruno, "Estimation of Factor Contribution to Growth Under Structural Disequilibrium," *International Economic Review*, February 1968, pp. 49–62, especially pp. 53, 54, and 59. Apparently the studies by Thurow and Bruno were done independently.

[43]*Ibid.*, p. 59.

It is to be hoped that more studies of the mutual interrelations between production relations and income distribution will be undertaken, as these interrelations are an important and interconnected part of the history of economic thought, and an explicit allowance for the interrelations is important on the basis of the modern theory of the simultaneous estimation of economic relations.

VI. Conclusions

A major theme of this paper has been that many of the problems, the differences in method, the differences in view, and the controversies grow out of the range of interrelated issues found in the real world. We are a long way from an ideal solution, but the degree of professional and public interest suggests that attempts will continue to find reasonably workable solutions to problems that are very intractable.

A second major theme is that some of the attempts to solve particular issues by those using the factor shares approach are rather similar to those followed by researchers estimating the production relationships directly. The areas of mutual problems and similarity of solutions are frequently overlooked and played down, while the differences and uniqueness of new results are emphasized.

A third theme is to encourage more studies that will look at the interconnections between production relations and income distribution, from the points of view of both economic theory (and its predictions about and relevance to the real world) and statistical estimation.

APPENDIX

Specific Issues in Factor Share Weights

The body of this paper has dealt with the similarities and differences in the two main approaches to the measurement of factor productivities. There are, however, significant differences in emphasis, methods and results between studies within the two approaches. For example, there have been large differences in the results and implications between Denison's work and the 1967 Jorgenson–Griliches study.[44] The latter study argued that there is almost no increase in output in relation to total factor inputs if the factor inputs are correctly measured. A major critique of this was published in the *Survey of Current Business*.[45] This conclusion was modified by a reworking of the measurement of factor inputs (particularly the modifications of the utilization adjustment of capital inputs).[46] A more complete restatement and modification of the 1967 paper has

[44]D. W. Jorgenson and Z. Griliches, "The Explanation of Productivity Change," *Review of Economic Studies*, Vol. 34, July 1967, pp. 249–83.

[45]Edward F. Denison, "Some Major Issues in Productivity Analysis: An Examination of Estimates by Jorgenson and Griliches," *Survey of Current Business*, May 1969, Part II, pp.1–27.

[46]Laurits R. Christensen and Dale W. Jorgenson, "U.S. Real Product and Real Factor Input, 1929–1967," *Review of Income and Wealth*, March 1970, pp. 19–50.

been prepared and presented at the Ronneby Conference.[47] This accepts and incorporates a number of the criticisms and suggestions made by Denison of the 1967 paper and shifts the area of their discussion in some important respects. The central issues were initially about measuring inputs, but the most recent Jorgenson–Griliches paper emphasizes consistency of measurement of national income, factor inputs (both capital and labor) and index numbers. The recent modification in the Jorgenson–Griliches estimates of total factor productivity raise their estimate from 0.30 to 1.03, which brings their estimate much closer to the Denison results of 1.37 (unadjusted, Denison's data).[48] However, only the issues associated with deriving weights for factor shares will be touched on here.

Many of the main assumptions about the need for weights for inputs, the acceptance of national income shares as weights, and the acceptance of the applicability of marginal productivity distribution theory are now clearly accepted by both Denison and Jorgenson–Griliches. Three areas still reflect differences of opinion, and these will be reviewed briefly here, recognizing that a further response from Denison is still to be expected. The three areas of differences in treatment relate to the treatment of depreciation in the weights for labor compared to the capital–land total; the treatment of taxes; and the treatment of capital gains. The latter two points affect the allocation of the total capital–land weight among components.

Depreciation: In the 1967 Jorgenson–Griliches paper, the estimate of the property share is 29.2 percent, while Denison uses 21.4 percent. Denison uses *net* national income for the total economy, while the Jorgenson–Griliches study concentrates on *gross* national product at factor cost for the private domestic economy.[49] When Denison adjusts his material to a comparable concept, it raises the property share from 21.4 to 32.8 percent, slightly higher than the Jorgenson–Griliches 1967 estimate of 29.2 percent. However, this division of input weights between labor and capital–land only explains .08 percentage points of an initial difference of 1.27 percentage points in their results in output per unit of input.

It is not clear how depreciation is handled in their reply to Denison in their recent paper. Table 4 provides the results on the relative proportions of capital stock by sector, but the paper does not seem to be explicit on whether these weights are gross or net of depreciation–replacement. They comment that "Denison is in error in asserting that we recommend the inclusion of depreciation in weights for the analysis of net product."[50] This is puzzling as it seems clear that depreciation has been included in their weights for capital–land in the 1967 paper.

[47]Dale W. Jorgenson and Zvi Griliches, "Issues in Growth Accounting: A Reply to Edward F. Denison," to be published in the *Survey of Current Business*.

[48]*Ibid.*, Table 25.

[49]Jorgenson and Griliches, "The Explanation of Productivity Change," p. 278, which states that "Total income from property is gross private domestic product in current prices less private domestic labour income."

[50]Jorgenson and Griliches, "Issues in Growth Accounting: A Reply to Edward F. Denison," mimeo, p. 94.

Treatment of Taxes: The treatment of indirect taxes, property taxes, and corporate profits taxes can affect the income share of the capital–land category, and also the distribution to assets within that category. The choices of national income at market prices or factor costs for weights is influenced by this question. This question had come up earlier in a review of Solow's book, *Capital Theory and the Rate of Return*, as Solow had included both indirect taxes and corporate profits taxes in estimating the share of income to property. Denison commented:

> But surely, market prices are irrelevant; there is no reason to count indirect taxes as capital income and little to think that they even fall disproportionately on property income. On the other hand, the corporate profits tax, treated in national accounts as a tax on profits, must in some degree (if only in regulated industries) be shifted. Hence the share of gross property income in corporate GNP at factor cost provides a maximum, not a minimum, estimate.[51]

The Jorgenson–Griliches 1967 paper included all indirect taxes and corporate profits taxes in the income of property, and this was criticized by Denison. The latest paper accepts the view that the 1967 treatment of sales and excise taxes and customs duties in the earnings of capital was an error and their revised estimates include only taxes levied on income from property.[52] Corporation profits taxes continue to be included in the weights for income from property, apparently.

Capital Gains in Relation to Property Share Weights: Once the distribution of national income between labor and capital has been done, the distribution of weights for individual property and land inputs must still be determined. The use of asset values is a necessary procedure to determine the distribution of earnings between different types of capital and land, as explained by Denison.

> The earnings of an enterprise can be measured, but most enterprises use more than one type of capital and land and there is no way to observe directly the earnings of each type. The analyst has no alternative but to assume that the individual enterprise earns the same rate of return on each. Given this assumption, the total net earnings of capital and land in each enterprise can be distributed among different types of assets in proportion to their value to obtain the earnings of each type.[53]

This general procedure is accepted by those using this method, but some differences in treatment of particular items still occur. In their 1967 paper, Jorgenson and Griliches make an adjustment for capital gains and losses on groups of assets as part of obtaining the rate of return on such assets by legal form of organization. Denison criticizes their procedure in his 1969 *Survey of Current Business* article.

[51]Edward F. Denison, "Capital Theory and the Rate of Return: A Review Article," *AER*, September 1964, p. 723.

[52]Jorgenson and Griliches, "Issues in Growth Accounting: A Reply to Edward F. Denison," p. 89.

[53]Edward F. Denison, "Some Major Issues in Productivity Analysis: An Examination of Estimates by Jorgenson and Griliches," p. 6.

> . . . The bias that Jorgenson and Griliches seek to eliminate is not present in the original data. Their capital gains adjustment thus introduces a bias in the opposite direction—that is, it overweights capital assets in which capital gains are small . . .
>
> The Jorgenson–Griliches formula may have theoretical interest. But as they have applied it, it is hardly to be taken seriously as a tool for statistical analysis. The alterations in weights, away from assets with large capital gains, that would be introduced by their simple "tax-absent" formula are untenable . . .
>
> (Their procedure) differs from proper procedure in two respects. First, they measure changes in prices from the average of one year to the average of the next, instead of from the beginning to the end of the year . . . Second, and more important, they use the implicit deflator for investment instead of the implicit deflator for the capital stock.[54]

In their latest paper they continue to include capital gains on assets by legal form of organization, and expand the number of asset groups on the basis of further work. They argue that it is desirable to measure real rates of return in measuring income from the supply of capital services, and by real rates of return they mean adjusting income as measured by accountants and taxation authorities for differential price changes for different classes of capital assets and land. These revaluations of asset values are on assets that continue to be held by the owner, and are thus unrealized capital gains. However, existing official estimates of national output, investment and saving only adjust for inventory valuation (to go from the business evaluation of book value of inventories to the current valuation of the physical change in inventories). For the private investigator interested in economic growth, a further adjustment of property income is made to adjust for the depreciation or replacement of capital stock to be consistent with the measurement of capital stock and use of capital services they prefer for analysis of economic growth. A further adjustment to take account of capital gains and losses on capital assets (arising from differential rates of price change of such capital assets) is a new step in national income accounting, and would involve revised definitions of national saving. The degree to which their procedure would depart from existing practices and introduce a degree of inconsistency in measuring property income relative to the measurement of output and savings does not seem to be fully appreciated by Jorgenson and Griliches.

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[54] *Ibid.*, pp. 6 and 12.

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