ESTIMATION OF CES PRODUCTION FUNCTIONS WITH NEUTRAL TECHNICAL CHANGE FOR INDUSTRIAL SECTORS IN THE FEDERAL REPUBLIC OF GERMANY 1958–1968

BY JOACHIM FROHN

Freie Universität Berlin, associated with the German Institute of Economic Research, Berlin

This paper discusses the estimation of the CES Production Function with Hicks-neutral technical change, and gives the results of an empirical study based on time series data (quarterly values) for sixteen industrial sectors in the Federal Republic of Germany (including West-Berlin), 1958–1968. The validity of the basic assumptions of the production model used in this investigation—neutrality of technical change and perfect competition—is tested by estimation of alternative specifications of the equations of this model. For this purpose two different methods of estimation were used.

I. INTRODUCTION

This paper is concerned with the estimation of CES Production Functions with neutral technical change. The empirical study is based on time series data for 16 industrial sectors in the Federal Republic of Germany (including West-Berlin). These data are the same as were used in the preceding article by Rolf Krengel, with the exception of utilized capital stock.[1]

In the next section the estimation procedures used in this study will be introduced. We shall formulate a simple production model which will serve as the basis for our empirical study, and we shall propose some tests for examination of the assumptions underlying this model. This is followed by a discussion of the data used in this investigation. The fourth section will give the empirical results which are summarized in the concluding paragraph.

2. ESTIMATION OF THE CES PRODUCTION FUNCTION

This study is based on the well known CES Production Function with Hicks-neutral technical change:

$$Y_t = \gamma e^{\lambda t}[\delta K_t^{-\rho} + (1-\delta)L_t^{-\rho}]^{-r/\rho} \tag{2.1}$$

where Y, K and L denote output, capital input and labor input. Estimation of the parameters γ, λ, δ, ρ and r can be done by

(1) using (2.1) in a single equation approach,

(2) using a production model consisting of (2.1) and one or more other equations.

Estimation problems caused by the nonlinearity of the function can be solved by using iterative procedures like LTR as described in the appendix.

[1]All these data are published in "Empirische Messung des technischen Fortschritts in der Verarbeitenden Industrie in der BRD", research project of Deutsches Institut für Wirtschaftsforschung, Berlin, Ifo-Institut für Wirtschaftsforschung, Munich, 1972.

Let us first give a short evaluation of the practicability of the first approach. Single equation estimation has the advantage that we do not need any additional assumptions like, for instance, perfect competition. But experience has shown that this method is only of limited practical significance if time series data are used: in this case the capital-labor-ratio usually does not vary very much over time. Variation of this coefficient, however, is necessary for estimation of δ and ρ. In case where K_t/L_t is constant over time, which is equal to $K_t = L_t$ if one uses appropriate scales, (2.1) becomes

$$Y_t = \gamma \, e^{\lambda t} K_t^r \tag{2.2}$$

i.e. a function without δ and ρ. If there is only slight variation of K_t/L_t it may be possible to estimate these parameters but the results cannot be regarded as very reliable.[2]

A second difficulty often comes from high correlation between t, $\ln K_t$, and $\ln (K_t/L_t)$. That this can cause estimation problems can be seen by using logarithms in (2.1), carrying out a Taylor series approximation of the term

$$\ln [\delta K_t^{-\rho} + (1-\delta) L_t^{-\rho}]$$

about $\rho = 0$ and curtailing this expansion after the second derivative:[3]

$$\begin{aligned} \ln Y_t = \ln \gamma + r\delta \ln K_t + r(1-\delta) \ln L_t \\ - \tfrac{1}{2}\rho r\delta(1-\delta)(\ln K_t - \ln L_t)^2 + \lambda t \end{aligned} \tag{2.3}$$

In our case these two difficulties made it impossible to estimate the parameters of (2.1) by single equation methods. Therefore we had to base the estimation on a production model. The disadvantage of this approach is the necessity of formulating assumptions concerning the situation in product and factor markets, thus making the model not as generally valid as a single equation model consisting only of the production function. On the other hand, if tests show that the assumptions of the model are valid, the additional equations enable us to make use of further information.

Production models are usually constructed by assuming:

(1) perfect competition in both factor and product markets,
(2) profit maximization or cost minimization.

If we assume (2.1) to be valid and make use of cost minimization and of perfect competition with the further assumption that the income shares of labor and capital add up to 1 we get the following equations:

$$\frac{w_t}{z_t} = \frac{1-\delta}{\delta}\left(\frac{K_t}{L_t}\right)^{1+\rho} \tag{2.4}$$

and

$$\frac{\alpha_t}{1-\alpha_t} = \frac{1-\delta}{\delta}\left(\frac{K_t}{L_t}\right)^{\rho} = a_0\left(\frac{K_t}{L_t}\right)^{a_1} \tag{2.5}$$

[2]For some sensitivity-tests see J. Frohn, "Untersuchungen zur CES Produktionsfunktion", Würzburg, 1970, pp. 97–107.

[3]See J. Kmenta, "On Estimation of the CES Production Function", *Intern. Ec. Review*, vol. 8, 1967, pp. 180–189.

with w = wage rate, z = rate of return on capital and α = income share of labor. The production model consisting of (2.1) and (2.5) will be the basis for the empirical study.[4]

For estimation of the parameters[5] we assume a multiplicative stochastic term v_t in (2.5) and use logarithms:

$$\text{(2.5a)} \qquad \ln\left(\frac{\alpha_t}{1-\alpha_t}\right) = \ln a_0 + a_1 \ln\left(\frac{K_t}{L_t}\right) + \ln v_t.$$

We make the usual assumptions about $\ln v_t$.[6] (2.5a) now allows estimation of $\ln a_0$ and a_1 by least squares. This gives $\hat{\delta} = 1/(1+\hat{a}_0)$ and $\hat{\rho} = \hat{a}_1$. With these estimates it is possible to compute

$$\text{(2.6)} \qquad \hat{V}_t = [\hat{\delta} K_t^{-\hat{\rho}} + (1-\hat{\delta}) L_t^{-\hat{\rho}}]^{-1/\hat{\rho}}.$$

Inserting this in (2.1), and assuming a multiplicative stochastic term u_t, we get the following logarithmic form of (2.1):

$$\text{(2.7)} \qquad \ln Y_t = \ln \gamma + \lambda t + r \ln \hat{V}_t + \ln u_t.$$

We make the same assumptions about $\ln u_t$ as $\ln v_t$; $\ln \gamma$, λ, and r are estimated by least squares again.[7]

If one starts from a CES Function with constant returns to scale (2.4) and (2.5) remain unchanged, but now

$$\text{(2.8)} \qquad \ln\left(\frac{Y_t}{\hat{V}_t}\right) = \ln \gamma + \lambda t + \ln u_t$$

allows estimation of $\ln \gamma$ and λ.

In the following we shall refer to this stepwise procedure for estimation as Approach I. It is obvious that if we use this method the difficulties mentioned in connection with single equation estimation will not occur.

It is evident that the reliability of the estimates resulting from this method depends on correct specification of (2.5). Let us consider the following two possibilities of errors:

(1) Some important explanatory variables in addition to K/L have been excluded from (2.5).

(2) It is not correct to identify a_0 and a_1 with $(1-\delta)/\delta$ and ρ respectively.

These two cases are of special importance in connection with the two assumptions "technical change is exclusively Hicks-neutral" and "perfect competition".

[4]Similar models have been used for estimation by R. K. Diwan, "An Empirical Estimate of the Elasticity of Substitution Production Function", *Indian Economic Journal*, vol. 12, 1964, pp. 347–366; K. J. Arrow, H. B. Chenery, B. S. Minhas, R. M. Solow, "Capital-Labour Substitution and Economic Efficiency", *Review of Economics and Statistics*, vol. 43, 1961, pp. 225–250; C. E. Ferguson, "Time Series Production Functions and Technological Progress in American Manufacturing Industry", *Journal of Political Economy*, 1965, pp. 135–147; and others.

[5]See R. K. Diwan, "An Empirical Estimate . . .", pp. 348, 349.

[6]$\ln v_t$ is assumed to have zero expectation and finite variance, and shall not be autocorrelated.

[7]For a discussion of simultaneous equation difficulties which may arise in connection with the estimation of this model see M. Nerlove, "Recent Empirical Studies of the CES and Related Production Functions", *Studies in Income amd Wealth*, vol. 31, 1967, pp. 100–119.

If one allows technical change to be non-neutral as P. A. David and T. van de Klundert[8] did by using the factor augmenting concept one gets instead of (2.1):

$$Y_t = [(A_0\, e^{\lambda_K t} K_t)^{-\rho} + (B_0\, e^{\lambda_L t} L_t)^{-\rho}]^{-r/\rho} \tag{2.9}$$

with

$$A_0 = \gamma^{1/r}\delta^{-1/\rho}; \qquad B_0 = \gamma^{1/r}(1-\delta)^{-1/\rho}.$$

And instead of (2.5) one now finds

$$\frac{\alpha_t}{1-\alpha_t} = \left(\frac{A_0}{B_0}\right)^{\rho} e^{\rho(\lambda_K-\lambda_L)t}\left(\frac{K_t}{L_t}\right)^{\rho} = \frac{1-\delta}{\delta}\, e^{\mu t}\left(\frac{K_t}{L_t}\right)^{\rho}. \tag{2.10}$$

Therefore it seems to be necessary to examine the assumption of Hicks-neutrality by using (2.10) for alternative estimation.

Of course, if (2.10) provides us with a significantly better fit this need not always be due to the fact that the model includes only Hicks-neutral technical change, for this is only one possibility of wrong specification. Other variables correlated with t might have been excluded from (2.5) too. But a much better fit of (2.10) compared with (2.5) certainly indicates that (2.5) cannot be regarded as a correct specification for the explanation of $\alpha_t/(1-\alpha_t)$. Furthermore, (2.5) and (2.10) provide only two alternative explanations of $\alpha_t/(1-\alpha_t)$. Obviously, there is quite a variety of further possible specifications. Therefore, after the decision whether or not (2.10) should be preferred to (2.5) is made, it must be decided whether the fit of the selected equation can be regarded as good enough to make use of this equation in the investigation.

But even if (2.5) is preferred to (2.10) and can be fitted to the data very well it still may be wrong to use Approach I for estimation. If, for instance we replace the assumption of perfect competition by assuming a demand function for the product and supply functions for the factors, each of the type $x = bp^{\eta_x}$ (x = product or factor, p = price, η_x = elasticity), (2.5) changes to

$$\frac{\alpha_t}{1-\alpha_t} = \left(\frac{m_K}{m_L}\right)\frac{1-\delta}{\delta}\left(\frac{K_t}{L_t}\right)^{\rho} = a_0^*\left(\frac{K_t}{L_t}\right)^{a_1} \tag{2.11}$$

with

$$m_K = 1+1/\eta_K; \qquad m_L = 1+1/\eta_L.$$

(2.11) is of the same form as (2.5). It is obvious that without further examination a good fit of (2.5) will not justify taking $\hat{\delta}$ from (2.5) for computation of $\hat{V}_t$ according to (2.6). For if (2.11) is correct $\hat{\delta}^* = 1/(1+\hat{a}_0^*)$ will be a biased estimator of δ according to the ratio δ^*/δ which consequently can lead to an incorrect estimate of λ. This bias will of course vanish if $m_K = m_L$.

Thus, in such a case Approach I obviously cannot be regarded as a correct method of estimation. Therefore in addition to Approach I one should estimate the parameters according to the following procedure: (2.5a) is used only for estimation of ρ; all the other parameters, including δ, are estimated from (2.1) using LTR. In the following discussion this method is referred to as Approach II.

[8] P. A. David, T. van de Klundert, "Biased Efficiency Growth and Capital-Labor Substitution in the U.S., 1899–1960", *American Ec. Review*, vol. 55, 1965, pp. 357–399.

It should be mentioned that by using this approach we again may be confronted with some of the difficulties described in connection with single equation estimation.

After the parameters have been estimated according to the two approaches, the results can then be compared. If $\hat{\delta}$ estimated by Approach I is far from the corresponding estimate using Approach II, this indicates that correct specification of (2.5) must be doubted.

A final remark concerning $\hat{\rho}$ is needed. In Approach II we still use $\hat{\rho}$ estimated from (2.5a); but if the supply and demand functions for the inputs and the output are not of the type used for derivation of (2.11) it must be expected that the identity $a_1 = \rho$ too will not be correct. This is the reason why in 4. we add some sensitivity tests using alternative values of ρ in order to get some idea of the importance of this estimate.

3. The Data

The following empirical study is based on time series data for factor input, output and income shares covering the period I/1958 to IV/1968, i.e. 44 quarters. In Rolf Krengel's article one finds a description of the concepts and methods of computation of these data. In this paragraph we will discuss briefly the applicability of these time series for estimation according to Approach I and II. The description of the data shows that there are no original time series of Y_c, L_c and K_u. This is due to the fact that for 1958–1968 there is no original information on the degree of utilization for all sectors and groups. Only for 16 of the 29 sectors[9] the Ifo-Institut für Wirtschaftsforschung of Munich has provided a coefficient of utilization $({}_IC_t)$ based on interviews with several firms in each sector.

In order to estimate technical change in all 29 sectors, Krengel calculated Y_c and L_c by assuming the following relations:

$$L_{ct} = e^{\hat{c}_0 + \hat{c}_1 t} K_{ct} \tag{3.1}$$

$$Y_{ct} = e^{\hat{d}_0 + \hat{d}_1 t} K_{ct} \tag{3.2}$$

with $\hat{c}_0$, $\hat{c}_1$, and $\hat{d}_0$ and $\hat{d}_1$ being calculated by use of K_{ct}, Y_{ut} and L_{ut}.[10]

Computation of K_u is based on the assumption that the coefficient of utilization of Y equals that of K; using (3.2) we get

$$K_{ut} = {}_DC_t K_{ct} = \left(\frac{Y_{ut}}{Y_{ct}}\right) K_{ct} = Y_{ut}/(e^{\hat{d}_0 + \hat{d}_1 t}). \tag{3.3}$$

[9]The 16 sectors are: Building Materials; Rubber and Asbestos Manufactures; Sawmills and Timber Processing; Cellulose and Paper; Machinery; Vehicles; Electric Engineering and Electronics; Precision Engineering and Optics; Steel Forging, Hardware, Metal Goods; Textiles; Leather; Fine Ceramics; Timber Manufactures; Glass; Paper and Board Manufactures; Plastics Manufactures.

[10]For a description of this calculation, see A. Boness, "Vierteljährliche Indexziffern der Kapazitätsauslastung der Verarbeitenden Industrie in der Bundesrepublik Deutschland", *Vierteljahreshefte zur Wirtschaftsforschung*, Berlin, 1969, pp. 190–206. If one compares the resulting coefficient of utilization ${}_DC_t = Y_{ut}/Y_{ct}$, with ${}_IC_t$ one finds that in general the series are similar. It should be noted that in the 16 sectors mentioned above the estimates of the rate of neutral technical change based on ${}_DC_t$ and ${}_IC_t$ are almost equal (see 4.1).

(3.3) shows that there is the same relation between Y_u and K_u as between Y_c and K_c.

What are the consequences of these calculations for the intended estimation in this study? (3.1) to (3.3) make clear that it is impossible to estimate the CES Function (2.1) (or an adequately formulated Cobb Douglas Function) by single equation methods using either Y_c, K_c, L_c or Y_u, K_u, L_u because of the interdependence between the variables. For in this case the estimation obviously would result in a perfect fit with $\hat{\lambda}$ being equal to $\hat{d}_1$. The same is true for Approach II. In addition to that (3.1) proves that it also will be inappropriate to use Approach I with Y_{ct}, K_{ct} and L_{ct}. In this case (2.5) represents only a relation between $\alpha_t/(1-\alpha_t)$ and t so that testing of (2.5) by using (2.10) is impossible.

In consideration of these qualities of the data the empirical investigation is based on the time series of Y_{ut}, L_{ut} and ${}_IK_{ut} = {}_IC_tK_{ct}$; consequently the estimation is limited to the 16 sectors mentioned above. (Time series data for K_{ct}, L_{ct} and Y_{ct} have not been used at all in this study, while ${}_DK_{ut} = {}_DC_tK_{ct}$ have only been used once for the purpose of comparison with the results reported in Rolf Krengel's article.)

The study is organized as follows. We first use Approach I for estimation. This is followed by estimation of (2.10) in order to examine the specification of (2.5) and to decide in what cases estimation of δ and ρ from (2.5) can be accepted. Then Approach II is used to discover possible differences in the estimates. Finally, some alternative estimates are made with different values of ρ. A final remark is needed concerning the scale of the variables: in order to make sure that the estimates of γ will not be too close to zero the original values of K_{ut} and L_{ut} have been divided by 100.

4. The Results of the Empirical Investigation

4.1. *Approach I*

Table 1[11] shows the estimates resulting from Approach I by using Y_{ut}, L_{ut} and ${}_IK_{ut}$ data. There has been no assumption on the returns to scale. In this table $R^2(1)$ represents the coefficient of determination for (2.5a) and $R^2(2)$ the same coefficient for the CES Production Function, using the estimates from (2.5a) and (2.7). Estimated standard errors are put in brackets. $\hat{\sigma}$ represents the estimate of the elasticity of substitution calculated according to $\hat{\sigma} = 1/(1+\hat{\rho})$. $\hat{\sigma}$ varies from 0.59 (fine ceramics) to 1.16 (precision engineering and optics). If we assume $\ln v_t$ to be distributed normally and use a 1 per cent confidence interval for ρ we find that in four sectors (sawmills and timber processing; electric engineering and electronics; steel forging, hardware, metal goods; glass) ρ cannot be regarded as significantly different from zero which is the case of the Cobb Douglas production function. The estimated quarterly rate of technical change varies from -0.55 per cent (vehicles) up to $+1.14$ per cent (textiles).

[11]For lack of space some tables had to be suppressed. The results of these tables are summarized in the text.

TABLE 1

ESTIMATES OF THE PARAMETERS ACCORDING TO APPROACH I ($r \neq 1$)

Industry	$\hat{\gamma}$	$\hat{\delta}$	$\hat{r}$	$\hat{\rho}$	$\hat{\sigma}$	$\hat{\lambda}$	$R^2(1)$	$R^2(2)$
Building Materials	0.0289	0.5127	1.8298 (0.1178)	−0.0748 (0.0220)	1.0808	−0.0031 (0.0016)	0.2151	0.8968
Rubber and Asbestos Manufactures	0.3475	0.0685	1.2229 (0.1045)	+0.5927 (0.0596)	0.6278	+0.0033 (0.0010)	0.7016	0.9507
Sawmills and Timber Processing	3.4116	0.2341	0.7879 (0.0763)	+0.1880 (0.0779)	0.8418	+0.0087 (0.0004)	0.1219	0.9476
Cellulose and Paper	11.9206	0.2046	0.5863 (0.1837)	+0.3803 (0.0516)	0.7245	+0.0086 (0.0006)	0.5637	0.9404
Machinery	0.0121	0.0875	1.5649 (0.1159)	+0.3958 (0.0228)	0.7164	−0.00004 (0.0008)	0.8778	0.9203
Vehicles	0.1877	0.1682	1.3806 (0.0907)	+0.5302 (0.0309)	0.6535	−0.0055 (0.0016)	0.8754	0.9597
Electric Engineering and Electronics	0.2905	0.3917	1.2568 (0.0913)	−0.1206 (0.1290)	1.1371	+0.0074 (0.0008)	0.0204	0.9689
Precision Engineering and Optics	0.0127	0.3135	1.8143 (0.1513)	−0.1392 (0.0360)	1.1617	+0.0081 (0.0005)	0.2624	0.9518
Steel, Forging, Hardware, Metal Goods	0.0165	0.2227	1.6232 (0.0884)	+0.0542 (0.0247)	0.9486	+0.0037 (0.0005)	0.1028	0.9723
Textiles	0.0075	0.1272	1.6542 (0.0772)	+0.3591 (0.0294)	0.7358	+0.0114 (0.0003)	0.7806	0.9725
Leather	0.0072	0.0566	1.7170 (0.1220)	+0.4157 (0.0678)	0.7064	+0.0088 (0.0006)	0.4724	0.8558
Fine Ceramics	0.2117	0.0339	1.2082 (0.0976)	+0.6743 (0.0793)	0.5973	+0.0090 (0.0003)	0.6326	0.9402
Timber Manufactures	0.0001	0.1268	2.4346 (0.1925)	+0.2369 (0.0369)	0.8085	+0.0088 (0.0006)	0.4957	0.9440
Glass	0.1744	0.4319	1.6088 (0.1307)	−0.0221 (0.0229)	1.0226	+0.0013 (0.0012)	0.0217	0.9816
Paper and Board Manufactures	1.2029	0.0426	0.9288 (0.1258)	+0.6505 (0.0943)	0.6059	+0.0058 (0.0014)	0.5309	0.9679
Plastics Manufactures	0.1214	0.3765	1.5582 (0.0744)	−0.0856 (0.0248)	1.0936	+0.0032 (0.0017)	0.2206	0.9967

In most cases the estimates of r are extremely high (in 9 sectors above 1.5!). This can be explained by the following. If in (2.7) $\ln \hat{V}_t$ remains relatively constant over time it is impossible to separate γ and r, i.e., to distinguish between proportional input effects and economies of scale. If $\ln \hat{V}_t$ and t are correlated to a certain extent it is impossible to separate r and λ, i.e., to distinguish between economies of scale and technical progress. The data show that in all 16 sectors one or the other of these two conditions is valid. Therefore it seems to be reasonable to make use of the assumption of constant returns to scale, being aware of the fact that now $\hat{\lambda}$ not only reflects technical change but probably also some effects of economies of scale.

With this assumption we get the results shown in Table 2. $\hat{\lambda}$ now varies from +0.07 per cent (vehicles) to +1.60 per cent (plastics manufactures). We notice that the quality of fit usually is not reduced very much by moving from $r \neq 1$ to $r = 1$ (the only exceptions are building materials and leather). In case of plastic manufactures, for instance, $R^2(2)$ remains almost the same while $\hat{\lambda}$ changes from 0.32 per cent to 1.60 per cent.

It seems to be worthwhile to compare these values of the quarterly rate of technical change with the corresponding estimates reported in the article by Rolf Krengel which are derived from a Cobb–Douglas Model with the same assumptions as used in this study. For this purpose we first must know about the sensitivity of the results in relation to the two different coefficients of utilization. Therefore we repeated the estimation of (2.5a) and (2.7), (2.8) respectively, now using ${}_DC_t$ instead of ${}_IC_t$ for computation of K_{ut}. The results of this calculation can be summarized as follows. There is almost no difference at all as far as $\hat{\delta}$, $\hat{\rho}$, $\hat{\lambda}$ and $R^2(1)$ are concerned; only the estimates of γ and r and $R^2(2)$ are subject to some moderate changes. The following list of the 16 sectors gives the estimates of the percentage quarterly rate of technical change from estimation of (2.5a), (2.8) using ${}_IK_{ut}$ (${}_DK_{ut}$) and the corresponding values derived in Rolf Krengel's study ($r = 1$):

Building Materials	0.46 (0.39)—0.4
Rubber and Asbestos Manufactures	0.50 (0.47)—0.5
Sawmills and Timber Processing	0.85 (0.89)—0.9
Cellulose and Paper	0.75 (0.76)—0.8
Machinery	0.31 (0.27)—0.3
Vehicles	0.07 (−0.03)—0.0
Electric Engineering and Electronics	0.93 (0.89)—0.9
Precision Engineering and Optics	0.95 (0.93)—0.9
Steel Forging, Hardware, Metal Goods	0.65 (0.66)—0.7
Textiles	1.01 (1.01)—1.0
Leather	0.62 (0.67)—0.7
Fine Ceramics	0.86 (0.84)—0.8
Timber Manufactures	1.14 (1.14)—1.1
Glass	0.65 (0.63)—0.6
Paper and Board Manufactures	0.51 (0.50)—0.5
Plastics Manufactures	1.60 (1.58)—1.6

We notice that in all 16 sectors the estimates of λ are very similar. This even applies in cases where $\hat{\sigma}$ is considerably lower than 1 (for instance, rubber and asbestos manufactures, paper and board manufactures). We may take this as an indication that in case of the data used in this study the estimated rate of

TABLE 2

ESTIMATES OF THE PARAMETERS ACCORDING TO APPROACH I (r = 1)

Industry	$\hat{\gamma}$	$\hat{\delta}$	r	$\hat{\rho}$	$\hat{\sigma}$	$\hat{\lambda}$	$R^2(2)$
Building Materials	3.0663	0.5127	1.0	−0.0748 (0.0220)	1.0808	+0.0046 (0.0017)	0.7990
Rubber and Asbestos Manufactures	1.1940	0.0685	1.0	+0.5927 (0.0596)	0.6278	+0.0050 (0.0005)	0.9458
Sawmills and Timber Processing	1.1492	0.2341	1.0	+0.1880 (0.0779)	0.8418	+0.0085 (0.0004)	0.9341
Cellulose and Paper	1.3191	0.2046	1.0	+0.3803 (0.0516)	0.7245	+0.0075 (0.0004)	0.9324
Machinery	1.0220	0.0875	1.0	+0.3958 (0.0228)	0.7164	+0.0031 (0.0005)	0.8791
Vehicles	2.2044	0.1682	1.0	+0.5302 (0.0309)	0.6535	+0.0007 (0.0006)	0.9476
Electric Engineering and Electronics	1.7389	0.3917	1.0	−0.1206 (0.1290)	1.1371	+0.0093 (0.0004)	0.9652
Precision Engineering and Optics	1.1814	0.3135	1.0	−0.1392 (0.0360)	1.1617	+0.0095 (0.0005)	0.9081
Steel, Forging, Hardware, Metal Goods	1.2188	0.2227	1.0	+0.0542 (0.0247)	0.9486	+0.0065 (0.0005)	0.9397
Textiles	0.8619	0.1272	1.0	+0.3591 (0.0294)	0.7358	+0.0101 (0.0004)	0.9268
Leather	0.6434	0.0566	1.0	+0.4157 (0.0678)	0.7064	+0.0062 (0.0005)	0.7299
Fine Ceramics	0.6863	0.0339	1.0	+0.6743 (0.0793)	0.5973	+0.0086 (0.0003)	0.9346
Timber Manufactures	0.8321	0.1268	1.0	+0.2369 (0.0369)	0.8085	+0.0114 (0.0008)	0.8700
Glass	2.6875	0.4319	1.0	−0.0221 (0.0229)	1.0226	+0.0065 (0.0004)	0.9707
Paper and Board Manufactures	0.8083	0.0426	1.0	+0.6505 (0.0943)	0.6059	+0.0051 (0.0005)	0.9677
Plastics Manufactures	1.4608	0.3765	1.0	−0.0856 (0.0248)	1.0936	+0.0160 (0.0004)	0.9941

technical change does not depend very much on the decision whether estimation is based on CES or Cobb–Douglas functions.

Turning now to the coefficients of determination which are reported in Tables 1 and 2, we notice that $R^2(2)$ usually is very high while $R^2(1)$ varies from 0.02 to 0.88. In the four sectors which may be described by Cobb–Douglas functions the coefficient of determination for (2.5a) is lower than 0.13. One reason for these low values of $R^2(1)$ in a great number of sectors is that in many cases α_t and therefore—even more—$\alpha_t/(1-\alpha_t)$ underlie distinct cyclical fluctuations while K_t/L_t—the only explanatory variable in (2.5)—is much more stable, usually showing a rather continuous increase over time. This too is the reason why the values of the Durbin–Watson statistic in almost all cases indicate serial correlation ($\alpha = 0.01$).

As mentioned in paragraph 2 the reliability of the results of estimation according to Approach I depends on correct specification of (2.5). Therefore we next estimated (2.10) to examine the assumption of neutrality of technical change. Unfortunately, in our case this estimation did not provide a safe basis for a decision in this respect. Due to an extremely high correlation between t and $\ln(K_t/L_t)$ (in all 16 sectors the coefficient of correlation is above 0.97!) the coefficient of determination usually rises only by a very small margin when (2.10) is estimated instead of (2.5); exceptions are building materials ($R^2\,(2.5) = 0.22$; $R^2\,(2.10) = 0.36$) and steel forging, hardware, metal goods ($R^2\,(2.5) = 0.10$; $R^2\,(2.10) = 0.18$). In almost all the other sectors the corrected coefficient of determination for (2.5) is higher than for (2.10). As a further consequence of this correlation high standard errors of $\hat{\rho}$ and $\hat{\mu}$ were estimated, so that in most cases neither of the two parameters can be regarded as significantly different from zero ($\alpha = 0.01$). Therefore, with the exception of the two mentioned sectors alternative estimation of (2.10) did not lead to a rejection of (2.5). So in almost all cases we had to base the decision whether or not (2.5) can be regarded as a correct specification solely on the results from the estimation of this equation. The decision was made by inspection of R^2 and the scatter diagrams. (2.5) was accepted only in case of a coefficient of determination of at least 0.50 with the further condition that the differences between the observed ratio of the income shares and the estimated values should cluster randomly around the time-axis, except for some minor cyclical fluctuations. Using these criteria, (2.5) was accepted for estimation only in the following 5 sectors: rubber and asbestos manufactures (0.50),[12] machinery (0.13), vehicles (0.07), fine ceramics (0.86), textiles (1.01). In all five sectors the fit of the CES production function can be regarded as good; the elasticity of substitution is in all cases considerably lower than unity. Calculation of the values of the Durbin–Watson statistic indicates the possibility of positive serial correlation in fine ceramics ($\alpha = 0.01$). This can possibly be explained by seasonal fluctuations of Y_{ut}.

4.2. *Approach II*

We proceeded next with the estimation of the parameters according to Approach II, i.e., estimation of ρ from (2.5a) and estimation of all the other parameters from (2.1) by using the LTR method. The results for the five selected

[12]Estimated quarterly rate of technical change, assuming $r = 1$.

TABLE 3

ESTIMATES OF THE PARAMETERS ACCORDING TO APPROACH II ($r \neq 1$)

Industry	$\hat{\gamma}$	$\hat{\delta}$	$\hat{r}$	$\hat{\rho}$	$\hat{\sigma}$	$\hat{\lambda}$	$R^2(1)$	$R^2(2)$
Rubber and Asbestos Manufactures	0.4714 (0.3424)	0.0873 (0.0472)	1.2007 (0.1036)	+0.5927 (0.0596)	0.6279	+0.0021 (0.0029)	0.7016	0.9510
Machinery	0.0017 (0.0021)	−0.0019 (0.0284)	1.6708 (0.1282)	+0.3958 (0.0228)	0.7164	+0.0064 (0.0027)	0.8778	0.9304
Vehicles	0.0460 (0.0380)	0.0340 (0.0300)	1.4292 (0.0953)	+0.5302 (0.0309)	0.6535	+0.0023 (0.0026)	0.8754	0.9672
Textiles	0.0076 (0.0046)	0.1238 (0.0255)	1.6484 (0.0816)	+0.3591 (0.0294)	0.7358	+0.0116 (0.0018)	0.7806	0.9725
Fine Ceramics	0.2125 (0.1411)	0.0319 (0.0117)	1.2016 (0.1040)	+0.6743 (0.0793)	0.5973	+0.0094 (0.0020)	0.6326	0.9404

sectors are contained in Table 3 ($r \neq 1$) and Table 4 ($r = 1$). Due to the interdependence of the explanatory variables, estimated standard errors are much higher now. In machinery, vehicles and textiles the estimate of r must again be regarded as very high.

A comparison between the estimates resulting from Approach I and II shows that in case of textiles and fine ceramics there is almost no difference. The coefficients of determination and the estimates of the parameters are almost equal. Considering the high standard errors of the estimates using Approach II this may be said of rubber and asbestos manufactures too. Thus in these three

TABLE 4

ESTIMATES OF THE PARAMETERS ACCORDING TO APPROACH II ($r = 1$)

Industry	$\hat{\gamma}$	$\hat{\delta}$	r	$\hat{\rho}$	$\hat{\sigma}$	$\hat{\lambda}$	$R^2(2)$
Rubber and Asbestos Manufactures	1.4707 (0.6591)	0.0944 (0.0610)	1.0	+0.5927 (0.0596)	0.6279	+0.0035 (0.0029)	0.9464
Machinery	0.6587 (0.3706)	0.0253 (0.0717)	1.0	+0.3958 (0.0228)	0.7164	+0.0056 (0.0036)	0.8820
Vehicles	1.4692 (0.6059)	0.0871 (0.0705)	1.0	+0.5302 (0.0309)	0.6535	+0.0033 (0.0033)	0.9500
Textiles	0.7330 (0.2609)	0.0979 (0.0598)	1.0	+0.3591 (0.0294)	0.7358	+0.0112 (0.0028)	0.9279
Fine Ceramics	0.7329 (0.1581)	0.0387 (0.0162)	1.0	+0.6743 (0.0793)	0.5973	+0.0080 (0.0020)	0.9348

sectors we may say that the results of Approach II support the use of Approach I, i.e., the assumption of perfect competition (or $m_K = m_L$).

This is different in cases of machinery and vehicles; in both sectors we now get somewhat higher values of R^2 and considerably lower estimates of δ, especially if we look at the results not subject to the assumption of constant returns to scale. In this case the estimate of the rate of technical change is now positive in contrast to the values resulting from Approach I! These differences become smaller if we assume r to be equal to 1. Even if we take the high standard errors into consideration these differences in the coefficients of determination and in the estimates give reason for questioning the estimation of δ from (2.5a); the results indicate that in these two sectors we might have been wrong in assuming perfect competition.

Under the assumption of constant returns to scale, we get the following estimates for the quarterly rate of technical change (in %): rubber and asbestos manufactures 0.35, machinery 0.56, vehicles 0.33, textiles 1.12, fine ceramics 0.80.

We completed this study by using the LTR procedure for direct estimation of the parameters γ, δ, r and λ from (2.1) assuming ρ to be 1.0 ($\sigma = 0.5$), 0.1111 ($\sigma = 0.9$), -0.3333 ($\sigma = 1.5$) respectively. We were especially interested in changes of the rate of technical change. The results (Table 5) show that in case of rubber and asbestos manufactures, machinery and vehicles the value of ρ is of almost no importance as far as $\hat{\lambda}$ is concerned. In all three sectors we find that a change of ρ can be compensated almost completely by a change of δ; the data do not permit separation of these two parameters with any certainty. In case of machinery there is almost no change at all of $\hat{\gamma}$, $\hat{r}$ and $\hat{\lambda}$, and R^2 remains the same for all three values of ρ.

In the other two sectors a change of ρ does have an influence on the value of $\hat{\lambda}$. In both cases $\hat{\lambda}$ becomes considerably lower with decreasing ρ. But the results do not contradict the estimation of ρ from (2.5a): in case of textiles $\hat{\rho}$ estimated from (2.5a) is not far from $\rho = 0.1111$ which gives the best fit according to Table 5. In case of fine ceramics some further estimation showed that ρ must be lower than -1.0 (i.e. $\sigma < 0.0$) for the best fit; therefore in this case too it seems to be correct to estimate ρ from outside information.

4.4. *Conclusion*

It was the intention of this investigation to estimate the parameters of CES production functions with Hicks-neutral technical change for the industrial sectors of the Federal Republic of Germany. Due to lack of appropriate data the investigation had to be limited to 16 sectors. The estimation was based on a production model consisting of the two equations (2.1) and (2.5). In order to explore the reliability of the results the assumptions of neutrality of technical change and of perfect competition were tested by further estimations using alternative specifications of the model.

The main results of this study can be summarized in the following four statements:

(1) A comparison of the estimate of λ based on CES functions with those based on Cobb–Douglas functions shows that there is almost no difference, even though the estimate of the elasticity of substitution in a considerable number of

TABLE 5

ESTIMATES OF THE PARAMETERS, USING ALTERNATIVE VALUES OF ρ

Industry	$\hat{\gamma}$	$\hat{\delta}$	$\hat{r}$	$\hat{\rho}$	$\hat{\sigma}$	$\hat{\lambda}$	R^2
Rubber and Asbestos Manufactures	0.4171	+0.0261	1.1810	1.0000	0.5	+0.0022	0.9507
	(0.2865)	(0.0155)	(0.1024)			(0.0030)	
	0.6748	+0.2972	1.2226	0.1111	0.9	+0.0022	0.9511
	(0.5335)	(0.1212)	(0.1054)			(0.0029)	
	1.1192	+0.6197	1.2408	−0.3333	1.5	+0.0024	0.9508
	(0.8843)	(0.1355)	(0.1074)			(0.0029)	
Machinery	0.0017	−0.0002	1.6722	1.0000	0.5	+0.0063	0.9304
	(0.0021)	(0.0030)	(0.1321)			(0.0023)	
	0.0017	−0.0059	1.6699	0.1111	0.9	+0.0064	0.9304
	(0.0021)	(0.0814)	(0.1274)			(0.0030)	
	0.0016	−0.0371	1.6680	−0.3333	1.5	+0.0065	0.9304
	(0.0027)	(0.4320)	(0.1291)			(0.0033)	
Vehicles	0.0456	+0.0092	1.4157	1.0000	0.5	+0.0026	0.9675
	(0.0366)	(0.0079)	(0.0966)			(0.0023)	
	0.0492	+0.1005	1.4429	0.1111	0.9	+0.0022	0.9670
	(0.0433)	(0.0875)	(0.0946)			(0.0029)	
	0.0586	+0.2622	1.4579	−0.3333	1.5	+0.0022	0.9667
	(0.0583)	(0.2072)	(0.0947)			(0.0032)	
Textiles	0.0077	+0.0126	1.5555	1.0000	0.5	+0.0138	0.9704
	(0.0048)	(0.0032)	(0.0797)			(0.0016)	
	0.0091	+0.2570	1.6892	0.1111	0.9	+0.0109	0.9727
	(0.0056)	(0.0438)	(0.0839)			(0.0018)	
	0.0175	+0.6190	1.7558	−0.3333	1.5	+0.0098	0.9707
	(0.0107)	(0.0544)	(0.0917)			(0.0020)	
Fine Ceramics	0.1976	+0.0079	1.1745	1.0000	0.5	+0.0105	0.9381
	(0.1322)	(0.0031)	(0.1055)			(0.0031)	
	0.3637	+0.2768	1.2786	0.1111	0.9	+0.0068	0.9458
	(0.2441)	(0.0678)	(0.1026)			(0.0022)	
	0.8708	+0.7125	1.3626	−0.3333	1.5	+0.0047	0.9504
	(0.5186)	(0.0614)	(0.1032)			(0.0022)	

sectors is significantly different from unity. Therefore in the 16 sectors analyzed in this study it does not seem to be of great importance whether estimation is based on CES or Cobb–Douglas functions as far as the rate of technical change is concerned.

(2) In five of the 16 sectors (rubber and asbestos manufactures, machinery, vehicles, fine ceramics, textiles) equation (2.5) (or (2.11)!) can be accepted as a correct specification for explanation of the ratio of income shares. In the other sectors the validity of such an equation must be doubted.

(3) Estimation according to Approach II showed that in cases of machinery and vehicles it might be wrong to assume perfect competition; but high standard errors of the estimates make it difficult to come to definite conclusions.

(4) Alternative estimations with different values of ρ for the purpose of discovering the sensitivity of λ showed that in three sectors (rubber and asbestos manufactures, machinery, vehicles) variation of ρ does not have any influence on

this estimate while in case of textiles and fine ceramics it has. But in these two sectors the outcome of the sensitivity test did not contradict the use of the production model described.

This study can only be regarded as a first step. Since estimates of the rate of technical change could only be derived for five of the 16 sectors, further investigations using different production models must follow to provide estimates of the contribution of technical change to production for the remaining sectors.

APPENDIX

The LTR Method[1]

The elements δ_j ($j = 1, 2, \ldots, k$) of the parameter vector δ of the nonlinear function

$$y_i = f(x_i, \delta) + u_i \tag{1}$$

shall be estimated.

In (1): y_i = ith observation of the dependent variable Y ($i = 1, 2, \ldots, n$).
x_i = vector of the ith observations of P independent variables X_p ($p = 1, 2, \ldots, P$).
u_i = ith error term.

For estimation of δ preliminary estimates δ_j^0 of δ_j are determined (in case of the CES function the estimates resulting from Approach I were used). Then a Taylor series expansion about δ_0 is carried out and is curtailed at the first derivatives:

$$y_i = f(x_i, \delta^0) + \sum_{j=1}^{k} \left[\frac{\partial f(x_i, \delta)}{\partial \delta_j}\right]_{\delta=\delta^0} (\delta_j - \delta_j^0) + \nu_i^0, \tag{2}$$

where $\nu_i^0 = u_i$ + error of approximation.

We rewrite (2) as

$$y_i^0 = \sum_{j=1}^{k} Z_{ji}^0 \beta_j^0 + \nu_i^0 \tag{3}$$

with

$$y_i^0 = y_i - f(x_i, \delta^0); \qquad \beta_j^0 = (\delta_j - \delta_j^0); \qquad Z_{ji}^0 = \left[\frac{\partial f(x_i, \delta)}{\partial \delta_j}\right]_{\delta=\delta^0}.$$

In obvious matrix notation:

$$y^0 = Z^0 \beta^0 + \nu^0. \tag{4}$$

Applying least squares we get

$$\hat{\beta}^0 = (Z^{0\prime} Z^0)^{-1} Z^{0\prime} y^0. \tag{5}$$

$\hat{\beta}^0$ is an estimator of $\delta - \delta^0$. Therefore we may use the revised estimates $\delta^1 = \delta^0 + \hat{\beta}^0$ for a new Taylor series expansion and start from the beginning.

[1]For a detailed description including possible drawbacks of this method, see N. R. Draper, H. Smith, *Applied Regression Analysis*, New York, 1966, p. 263–284. (LTR = Linear Transformation.)

These successive revisions of the estimates are continued until the solution converges, i.e., until the differences $\delta_j^t - \delta_j^{t-1}$ become lower than a prespecified small amount.

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