AN INDEX NUMBER APPROACH TO THE INTERNATIONAL COMPARISON OF CONSUMPTION: WESTERN EUROPE AND THE UNITED STATES*

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The quantity index comparing the per capita consumption of one country vis-à-vis the other often gives widely different figures, depending on which country's prices are used as "weights". In this paper, this gap between two quantity indexes is divided into a substitution effect and an income effect by assuming common tastes between nations. For this division, we estimate the points of over-compensated variation and under-compensated variation in income from Gilbert and Kravis' data.

The results of our estimation show that the income effect is smaller than the substitution effect. But the sign of the income effect indicates that this effect is generally in the same direction as the difference between two quantity indexes. Translated into the Bortkiewicz covariance, this means that the income elasticities are inversely related to the relative prices; the higher the income elasticity of a good, the lower is its price in the high income country relative to the low income country.

Since we only approximate the points of exactly compensated variation in income, we cannot estimate "true" quantity indexes. However, our result implies that the two indexes in Gilbert and Kravis' data do form the upper and lower boundaries to the true index-numbers.

The difference in living standard between countries is usually quantified by "quantity" index numbers in "constant prices". To take the U.S. (denoted with a superscript A) and a country in Europe (denoted with E) as an example, quantities of individual goods and services consumed per capita in two countries (x_i^A's and x_i^E's) are first valued in U.S. prices. Then the ratio of total consumption expenditure of one country to the other gives an index number, $\bar{Q}^A = \Sigma_i p_i^A x_i^E / \Sigma_i p_i^A x_i^A$. Similarly, one may construct another index number, $\bar{Q}^E = \Sigma p^E x^E / \Sigma p^E x^A$, by expressing two countries' expenditures in the prices of a European country. Empirical results sometimes show a wide gap between the estimated $\bar{Q}^A$ and $\bar{Q}^E$.

This essay is an attempt to investigate the meaning of this gap. First, it is often presumed that some "true" index to the international difference in consumption is found within this gap with $\bar{Q}^A$ and $\bar{Q}^E$ as the upper and lower boundaries. Once one defines what is meant by the true index, the guess becomes an empirical question whose validity can be tested. Second, one must explain the gap between $\bar{Q}^A$ and $\bar{Q}^E$. To discuss this in a simplified analytical framework, we make the following assumptions. We set aside the problem of aggregation of individual preferences and interpret the data on national consumption per capita as if they reveal the preference of a "representative" individual. In addition, common tastes between nations are assumed. Furthermore, we consider only a

*This research was supported in part by a Canada Council Grant. The author is much indebted to Professors Abram Bergson and H. S. Houthakker for helpful comments. Needless to say, the author retains the sole responsibility for remaining errors.

static model in which a consumer uses up his income for spending on perishable goods and services. After these simplifications are made, we focus our attention on the international difference in income level and price structure. The gap between the two indexes is thus divided into the part attributed to the difference in per capita income and the part attributed to the difference in prices. As will be clear later, this division of the gap into an income effect and a substitution effect is closely related to our first question.

Income Effects in Index Numbers

We first define a pair of "true indexes." The true index in U.S. prices, Q^A, is the ratio $\Sigma p^A x^{*A} / \Sigma p^A x^A$, where x^{*A}'s are quantities of goods consumed per capita at the point of exactly compensated change in income from a European observation. Similarly, the true index in the prices of a European country is defined as $\Sigma p^E x^E / \Sigma p^E x^{*E}$, where x^{*E}'s represent the point of exact compensation from the U.S. observation.

In addition, we introduce the third pair of indexes that are related to the points of under-compensated change in income. Let $\underline{x}_i^A$'s and $\underline{x}_i^E$'s represent the point of under-compensated change in income from a European observation and that from the U.S. observation, respectively.[1] We estimate the expenditures at these points in equilibrium prices and take ratios of them to the observed expenditures. The two indexes, $\underline{Q}^A = \Sigma p^A \underline{x}^A / \Sigma p^A x^A$ and $\underline{Q}^E = \Sigma p^E x^E / \Sigma p^E \underline{x}^E$, are thus obtained.

The three pairs of indexes so far defined are ordered

$$\underline{Q}^A \leq Q^A \leq \bar{Q}^A \tag{1}$$

and

$$\bar{Q}^E \leq Q^E \leq \underline{Q}^E. \tag{2}$$

In order for the limits to the two true indexes to be complete, four indexes have to be estimated. But the estimates of $\underline{Q}^A$ and $\underline{Q}^E$ are not readily available, because to estimate them the intersection of one country's expenditure path with the other's budget line has to be found. The question then arises as to when

$$\bar{Q}^E \leq Q^A \tag{3}$$

and

$$Q^E \leq \bar{Q}^A \tag{4}$$

so that in view of (1) and (2), $\bar{Q}^E$, one of the two available estimations, may be smaller than both Q^A and Q^E; and $\bar{Q}^A$, the other available estimation, may be greater than both Q^A and Q^E. If the international data satisfy the relationships (3) and (4), the two true indexes must be found somewhere in the range between two available estimations.

[1]Samuelson [7].

[2]Liviatan and Patinkin [5].

Using Hicks' index number formulation,[3] it can be shown that the inequality (3) is satisfied if Hicks' income effect on the U.S. expenditure path is positive.[4] The relationship (4) is also satisfied if the income effect on a European path is positive. Thus the positive income effects are sufficient conditions for (3) and (4).

It is also convenient to put the index number gap in relative terms, namely $(\bar{Q}^A - \bar{Q}^E)/\bar{Q}^A$, because this formulation can be expressed in a weighted covariance between quantity ratios x^E/x^A and price ratios p^E/p^A.[5] Thus the relative gap indicates the overall price-quantity interrelation between two observations. By inserting the over-compensation point $\bar{x}^A$, the relative gap is divided into two components: the approximate income effect from x^A to $\bar{x}^A$ on the U.S. expenditure path and the approximate substitution effect from $\bar{x}^A$ to x^E at the income level of a European country.

$$(\bar{Q}^A - \bar{Q}^E)/\bar{Q}^A = \bar{i}^A + \bar{s}^E. \tag{5}$$

Each effect can also be expressed in a covariance.[6]

We may also show the relative gap by a covariance between quantity ratios x^A/x^E and price ratios p^A/p^E with European expenditure shares as weights. Further, by inserting the over-compensation point $\bar{x}^E$, the gap may be broken down into the approximate income effect along a European path and the approximate substitution effect at the U.S. income level, that is,

$$(\bar{Q}^A - \bar{Q}^E)/\bar{Q}^A = \bar{i}^E + \bar{s}^A.[7] \tag{6}$$

The approximate income term in (5) is obtained by making an approximation to Hicks' income effect with $\bar{x}^A$ substituted for x^{*A} and dividing the effect by $\bar{Q}^A$. We recall that the positive sign of Hicks' income effect along the U.S. path is a sufficient condition for (3). Then

$$\bar{i}^A \geq 0 \tag{7}$$

may be regarded as a proxy for this sufficient condition. Similarly, we regard

$$\bar{i}^E \geq 0 \tag{8}$$

as a proxy for the sufficient condition for (4). The condition (7) requires a negative covariance between the price ratios of A to E relative to the purchasing power parity and the quantity ratios of the U.S. observation to the over-compensation point $\bar{x}^A$. And the quantity ratios are closely related to arc-expenditure-elasticities, $(\bar{x}^A/x^A - 1)/(\bar{Q}^A - 1)$. Hence, the negative covariance implies a general tendency that the lower the ratio of the prices of A to E the

[3][4] ch. 19.

[4]Toda [8].

[5]Bortkiewicz [1] and Marris [6, pp. 218–219].

[6]The income effect equals the covariance (with negative sign) between $(\bar{x}^A/x^A)/(\Sigma p^A\bar{x}^A/\Sigma p^Ax^A)$ and $(p^E/p^A)/(\Sigma p^Ex^A/\Sigma p^Ax^A)$ with the expenditure shares at A observation as weights. The substitution effect which is positive from our assumptions may also be expressed in the weighted covariance (with negative sign) between $\{(x^E - \bar{x}^A)/x^A\}/(\Sigma p^A\bar{x}^A/\Sigma p^Ax^A)$ and the price ratios as shown above.

[7]$\bar{i}^E$ is equal (except for the sign) to the covariance between $(\bar{x}^E/x^E)/(\Sigma p^E\bar{x}^E/\Sigma p^Ex^E)$ and $(p^A/p^E)/(\Sigma p^Ax^E/\Sigma p^Ex^E)$ with the shares at E as weights. The sign of $\bar{s}^A$ is positive.

smaller are the quantity ratios of $\bar{x}^A$ to x^A and, therefore, the higher are the arc-expenditure-elasticities. The same relationship can also be found in (8).

In parallel with the relative gap between $\bar{Q}^A$ and $\bar{Q}^E$, we consider the relative gap between $\underline{Q}^A$ and $\underline{Q}^E$. This gap can also be viewed as consisting of the approximate income effect along the U.S. expenditure path and the approximate substitution effect on the U.S. budget line,

$$(\underline{Q}^E - \underline{Q}^A)/\underline{Q}^E = \underline{i}^A + \underline{s}^A. \tag{9}$$

[8]

The U.S. observation is inserted as an intermediate point to make this breakdown possible. Likewise, a division of the gap may also be made as

$$(\underline{Q}^E - \underline{Q}^A)/\underline{Q}^E = \underline{i}^E + \underline{s}^E \tag{10}$$

where $\underline{i}^E$ and $\underline{s}^E$ represent the approximate income effect along an European expenditure path and the approximate substitution effect on a European budget line.[9] Similar to (7) and (8) the conditions

$$\underline{i}^A \leq 0 \tag{11}$$

and

$$\underline{i}^E \leq 0 \tag{12}$$

can be regarded as proxies for the sufficient conditions for (3) and (4), respectively.[10] The conditions (11) and (12) imply the similar price-quantity interrelation to what was found in (7) and (8). All these proxies thus indicate a general tendency that the lower the price ratios of the U.S. to Europe the higher are the expenditure elasticities. In other words, they roughly require that the U.S. prices relative to Europe are low for luxuries and high for necessities.

Estimation on International Cross-section Data

With the use of international cross-sectional data, we attempt to measure expenditures at the points of over-compensation and under-compensation. We then estimate the indexes which, as shown in (1) and (2), form limits to the true indexes. Further, the relative gap between the estimated indexes is divided into two effects. One of the two, the approximate income effect, is examined to see whether or not its sign satisfies the proxy for the sufficient condition.

We rely on Gilbert and Kravis' study which estimates the consumption levels and the prices of eight West European countries in 1950 in comparison with the U.S.[11] We assume the expenditure functions which explain the per capita consumption on ith commodity group in kth country with per capita

[8] The overall effect may be expressed in a covariance (with negative sign) between $(\underline{x}^E/\underline{x}^A)/(\Sigma p^A\underline{x}^E/\Sigma p^A\underline{x}^A)$ and $(p^E/p^A)/(\Sigma p^E\underline{x}^A/\Sigma p^A\underline{x}^A)$ with the shares at $\underline{x}^A$ as weights. Substituting x^A for $\underline{x}^E$ in the above formulation, one obtains $\underline{i}^A$. From our assumption, $\underline{s}^A$ is positive.

[9] A European observation is inserted as the intermediate point.

[10] The reason for this can be seen by recalling the definition of Hicks' income effect, $(\Sigma p^A x^{*A}/\Sigma p^A x^A) - (\Sigma p^E x^{*A}/\Sigma p^E x^A)$, and comparing this with the approximate income effect $\underline{i}^A$ which equals $(\Sigma p^A x^A/\Sigma p^A\underline{x}^A - \Sigma p^E x^A/\Sigma p^E\underline{x}^A) \cdot \underline{Q}^A$.

[11] [2] Tables 27–30 and [3] Tables 38–41.

total consumption expenditure of this country and its retail prices relative to the U.S. as independent variables, that is,

$$p_i x_i = f_i(\Sigma px, \cdots, p_h/p_h{}^A, p_i/p_i{}^A, \cdots). \tag{13}$$

First, eight $\bar{x}_i{}^A$'s are estimated. We substitute Gilbert's estimations of the European consumption in U.S. prices, $\Sigma p^A x^E = \Sigma p^A \bar{x}^A$, for the income variable of (13). The price variables are fixed at the U.S. structure, that is, $p_h/p_h{}^A = p_i/p_i{}^A = \cdots = 1$. The resulting values of the dependent variables, $p_i{}^A \bar{x}_i{}^A$, give the over-compensation point from a European income. Secondly, we estimate eight $\underline{x}^A$'s. The total consumption, $\Sigma p^A \underline{x}^A$, to be substituted for the income variable in (13) should satisfy the definition of an under-compensation point, $\Sigma_i (p^E/p^A)(p^A \underline{x}^A) = \Sigma p^E x^E$. With this substitution and the assumed unitary price ratios, the expenditures on individual commodity groups at the under-compensation point are obtained from (13).[12] In this way, the difference in per capita consumption between x^E and x^A is broken down into two parts: the substitution effect from x^E to $\bar{x}^A$ (or $\underline{x}^A$), and the income effect from $\bar{x}^A$ (or $\underline{x}^A$) to x^A.

Sixteen compensation points from the U.S. income ($\bar{x}^E$'s and $\underline{x}^E$'s) are obtained by a similar method. Thus the difference in consumption between two observations is divided into another pair: the substitution effect from x^A to $\bar{x}^E$ (or $\underline{x}^E$) and the income effect from $\bar{x}^E$ (or $\underline{x}^E$) to x^E.[13]

The results of our estimation are shown in three tables. Table 1 presents the indexes in four different forms. Those in Columns 1 and 6 are taken from Gilbert's data. Since the corresponding indexes are smaller in Column 1 than in Column 6, they conform with the widely found relationship that the consumption levels of two countries are closer when they are measured in the prices of a rich country than those of a poor country.

The indexes $\underline{Q}^A$ in Columns 2 and 3 are all smaller than the corresponding indexes in Column 6, as suggested in (1). Moreover, all of them are also greater than the corresponding $\bar{Q}^E$'s in Column 1. Therefore, the index $\bar{Q}^E$ not only forms the lower limit to a true index Q^E, but is also lower than even the lower limit to another true index Q^A. It is thus obvious that the relationship (3) is satisfied.

[12]This computation involves an iteration.

[13]Since the available data have several limitations, the estimation method as we explained it is not directly applicable. The limited scope and accuracy that ensued in our estimation are the following: (1) We only compare the U.S. with each of eight European countries. Comparisons between every pair of European countries are not possible. (2) The price ratio of each commodity group divided by the purchasing power parity, $(p^k/p^A)/(\Sigma p^k x^A/\Sigma p^A x^A)$, is actually used as the only price variable. (3) The total consumption is disaggregated either into 14 or 23 groups of goods and services. We thus make two separate estimations of compensation points. (4) The expenditures have to be expressed in "constant" (U.S.) prices and not in "current" prices (prices of each country) as shown in (13). (5) The log-log form is used to estimate (13). In this form, however, the sum of dependent variables differs from the total expenditure to be used as an independent variable. Although we use the sum of dependent variables to estimate the compensation point, much work remains to improve the estimation method. Furthermore, this form does not enable one to trace back to a utility function. Although the estimated price elasticities are all negative except one commodity group and the estimated expenditure elasticities are positive and assume reasonable values, the compensation points thus estimated must be regarded as crude approximations. The results of elasticity estimations in [8] are omitted.

TABLE 1

PER CAPITA CONSUMPTION EXPENDITURES OF EUROPEAN COUNTRIES RELATIVE TO THE UNITED STATES IN 1950 (IN PER CENT)

Country	Index Number in the prices of each European Country (related to the Over-Compensation Point)	Index Number in U.S. Prices related to the Under-Compensation Point		Index Number in the Prices of each European Country related to the Under-Compensation Point		Index Number in U.S. Prices (related to the Over-Compensation Point)
(E)	$(\overline{Q}^E)$	$(\underline{Q}^A)$		$(\underline{Q}^E)$		$(\overline{Q}^A)$
		Based on Classification into 14 Commodity Groups	Based on Classification into 23 Commodity Groups	Based on Classification into 14 Commodity Groups	Based on Classification into 23 Commodity Groups	
	(1)	(2)	(3)	(4)	(5)	(6)
U.K.	49.6	53.1	53.9	63.5	63.1	65.6
Denmark	51.0	52.0	52.6	66.6	66.4	64.5
Belgium	50.7	51.7	52.5	58.9	58.5	59.9
Norway	42.4	46.5	47.8	58.6	58.1	57.2
France	36.6	39.9	40.3	47.3	46.7	52.5
Netherlands	35.8	37.5	38.2	47.0	46.8	48.3
W. Germany	28.5	30.0	31.2	40.1	39.2	41.4
Italy	17.7	20.6	21.9	26.6	25.2	30.7

As indicated in (2), the Q^E's in Columns 4 and 5 are all greater than the $\bar{Q}^E$'s in Column 1. Further, they are also smaller than the $\bar{Q}^A$'s in Column 6, except for two countries, Denmark and Norway. Therefore, except for these two, the index $\bar{Q}^A$ not only forms the upper limit to a true index Q^A but is also higher than even the upper limit to another true index Q^E. The relationship (4) is satisfied in most cases, although this is unclear for the two countries.

Turning now to Table 2, Column 1 presents the relative gap between $\bar{Q}^A$ and $\bar{Q}^E$. According to (5) and (6), this gap is divided into two effects. Here as well as in Table 1, eight countries are ordered according to their per capita consumption level (in U.S. prices) in 1950.

TABLE 2

ESTIMATION OF THE INCOME AND SUBSTITUTION EFFECTS APPROXIMATED BY OVER-COMPENSATION POINTS: COMPARISON BETWEEN THE U.S. AND WESTERN EUROPE IN 1950 (IN PER CENT)

Country (E)	Overall Effect	$\bar{i}^E$	$\bar{s}^A$	$\bar{i}^A$	$\bar{s}^E$
	(1) = (2) + (3) = (4) + (5)	(2)	(3)	(4)	(5)
Based on the classification into 14 *Commodity Groups*					
U.K.	24.4	4.8	19.6	2.6	21.8
Denmark	20.9	−4.1	25.0	0.8	20.1
Belgium	15.4	1.7	13.7	0.0	15.4
Norway	25.9	−3.8	29.7	2.1	23.8
France	30.2	10.7	19.5	1.1	29.1
Netherlands	25.9	3.5	22.4	3.2	22.7
W. Germany	31.3	2.5	28.8	2.8	28.5
Italy	42.3	14.7	27.6	8.0	34.3
Based on the classification into 23 *Commodity Groups*					
U.K.	24.4	5.0	19.4	4.1	20.3
Denmark	20.9	−3.1	24.0	2.0	18.9
Belgium	15.4	2.7	12.7	1.4	14.0
Norway	25.9	−2.5	28.4	4.8	21.1
France	30.2	12.4	17.8	2.2	28.0
Netherlands	25.9	4.1	21.8	3.8	22.1
W. Germany	31.3	5.8	25.5	5.9	25.4
Italy	42.3	19.5	22.8	12.6	29.7

First, the overall effects are all positive. In general, the greater the difference in income between two countries, the greater is the absolute gap between $\bar{Q}^A$ and $\bar{Q}^E$. The relative gaps in Column 1 also exhibit a similar, though much milder, tendency.

Second, Column 4 shows that the approximate income effects on the U.S. path are all non-negative. Thus the proxy condition (7) is satisfied. On the other hand, $\bar{i}^E$ in Column 2 presents two negative figures. For these two, the proxy condition (8) is not met and, accordingly, one cannot be sure if $\bar{Q}^A$ in Table 1 is greater than a true index Q^E. The predominance of positive $\bar{i}$'s indicates that

in general the arc-expenditure-elasticities from E to $\overline{E}$ and those from $\overline{A}$ to A are inversely related to price ratios of E to A.[14] In regard to the size of the income effect, on the other hand, we detect little correspondence of it with the income level of a country, although Italy, the country with the lowest income, presents the largest income effect.

Third, as a consequence of our assuming common tastes among nations, the substitution effects in Columns 3 and 5 all present positive signs.

Finally in Table 3, we take the relative disparity between $\underline{Q}^E$ and $\underline{Q}^A$ (Columns 2–5 of Table 1) and divide it into two effects according to (9) and (10).

As shown in Column 1 of Table 3, the relative gap representing the overall price-quantity interrelationship is positive. It does not appear, however, that its size is significantly related to the income level of a European country relative to the U.S.

As to the income effects, $\underline{i}^A$'s in Column 4 are all negative and thus indicate that the condition (11) is satisfied.[15] Since $\underline{i}^E$'s in Column 2 present two positive

TABLE 3

ESTIMATION OF THE INCOME AND SUBSTITUTION EFFECTS APPROXIMATED BY UNDER-COMPENSATION POINTS: COMPARISON BETWEEN THE U.S. AND WESTERN EUROPE IN 1950 (IN PER CENT)

Country (E)	Overall Effect	$\underline{i}^E$	$\underline{s}^E$	$\underline{i}^A$	$\underline{s}^A$
	(1) = (2) + (3) = (4) + (5)	(2)	(3)	(4)	(5)
Based on the Classification into 14 *Commodity Groups*					
U.K.	16.4	−3.4	19.7	−7.0	23.4
Denmark	21.9	3.2	18.7	−2.0	23.9
Belgium	12.2	−1.7	13.9	−2.0	14.2
Norway	20.7	2.4	18.3	−9.6	30.3
France	15.6	−11.0	26.6	−8.9	24.5
Netherlands	20.3	−2.9	23.2	−4.6	24.8
W. Germany	25.2	−3.1	28.3	−5.5	30.7
Italy	22.5	−15.6	38.1	−16.3	38.8
Based on the Classification into 23 *Commodity Groups*					
U.K.	14.6	−4.0	18.6	−8.5	23.0
Denmark	20.8	2.9	17.9	−3.1	23.6
Belgium	10.4	−2.3	12.7	−3.5	13.7
Norway	17.8	1.5	16.2	−12.3	29.6
France	13.7	−12.4	26.2	−9.8	23.4
Netherlands	18.2	−3.4	21.6	−6.6	24.5
W. Germany	20.4	−5.6	26.0	−9.3	28.9
Italy	13.3	−21.6	34.9	−22.2	34.7

[14]We do not calculate correlation coefficients to test whether the positive $\overline{i}$'s are statistically significant or not.

[15]The negative $\underline{i}^A$ is equivalent to $\overline{Q}^E < \underline{Q}^A$. Similarly, $\underline{i}^E < 0$ is equivalent to $\overline{Q}^A > \underline{Q}^E$. (The proof is deleted.)

figures, however, the condition (12) is not satisfied in the two cases. Here we again find the general, though not universal, correspondence between price ratios and expenditure elasticities. However, similar to the magnitude of $\bar{i}$, a significant correspondence does not seem to exist between the magnitude of and the income level, although the absolute value of $\underline{i}$ is again the largest for Italy.[16]

As a consequence of our assumptions, all the substitution effects in Columns 3 and 5 give positive figures.[17]

Summary and A Concluding Comment

Major findings are summarized as follows:

1. As can be inferred from Table 1, the consumption indexes in European prices in Gilbert's data ($\bar{Q}^E$'s) are smaller than both true indexes (Q^E's and Q^A's). Except for two unclear cases (Denmark and Norway), the indexes in U.S. prices ($\bar{Q}^A$'s) are higher than both true indexes. Hence, by and large both true indexes are located within the range between the two indexes in Gilbert's data.

2. Overall effects between observation points and those between under-compensation points are both positive. The former is greater than the latter in most cases.

3. Income effects approximated by over-compensation points are predominantly positive. Those approximated by under-compensation points are negative in most cases. This suggests a general, though not universal, tendency that the relative prices are high for necessities and low for luxuries in a country with high income level.

[16]We examine which overall effect and which approximation to the income and substitution effects gives larger estimations.

1. Comparison of two overall effects: From the results in Footnote 15, if $\underline{i}^A$ and $\underline{i}^E$ are both negative, then the overall effect between observations is greater than that between under-compensation points. But, if two $\underline{i}$'s are of different sign, one cannot definitely say which effect is greater.

2. Comparison of $\bar{i}$ and $\underline{i}$: One finds $\bar{i}^A + \underline{i}^A = -\mathrm{Cov}_A\{(\bar{x}^A/x^A)\bar{Q}^A, (p^E/p^A)/(\Sigma p^E x^A/\Sigma p^A x^A)\} + (\underline{Q}^A/\bar{Q}^E) \cdot \mathrm{Cov}_A\{(\underline{x}^A/x^A)/\underline{Q}^A, (p^E/p^A)/(\Sigma p^E \underline{x}^A/\Sigma p^A x^A)\}$. If the condition (7) is met, it is likely that both covariances are negative. The latter covariance is greater in absolute value than the former, because $\underline{x}^A < \bar{x}^A < x^A$ for most goods. Further, $\underline{Q}^A/\bar{Q}^E > 1$ from Footnote 15. Hence, $\bar{i}^A + \underline{i}^A < 0$, or a negative $\underline{i}$ is greater in absolute value than a positive $\bar{i}$. Between $\bar{i}^E$ and $\underline{i}^E$, however, a clear relationship cannot be found.

3. Comparison of $\bar{s}$ and $\underline{s}$: If the sign conditions are met, it is likely that the relationship $s^A > \bar{s}^A$ is found. (The proof is omitted.)

[17]As seen in two tables, the magnitudes of the estimated effects depend on the degree of disaggregation. When individual commodities are aggregated into several groups, the covariance concerning all individual commodities may also be shown as the between-group covariance *plus* the weighted-mean of within-group covariances (with the expenditure share of each group as a weight). For $\bar{i}$, it is likely that both between-group covariance and within-group covariance are negative. Therefore, $\bar{i}$ becomes greater with further disaggregation of total consumption. A similar argument can be made for $\underline{i}$. But the difference may taper off as the disaggregation proceeds. Indeed, with a proportionate change either in all prices (as in Hicks' definition of a commodity group) or in all quantities within a group, the within-group covariance vanishes. We omit an explanation for the reason that $\bar{s}$ and $\underline{s}$ become smaller with further disaggregation.

4. The size of an income effect is in general so small that it is outweighed by a positive substitution effect. In fact, all overall effects between under-compensation points in Table 3 turn out to be positive.

The positive overall effects between observations confirm a relationship widely found in both time-series and international cross-sectional data, namely, the inverse relationship between price ratios and quantity ratios of two observations. The small size of estimated income effects, on the other hand, suggests that the hypothesis of the inverse relationship between price ratios and income elasticities is restrictive. We may not expect that this hypothesis will be universally supported by empirical evidence, because this is a net effect of two diverging trends: low prices of income-elastic consumer durables and high prices of income-elastic, labor-intensive services in a country with high income level.

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