THE IMPACT OF CHANGES IN TERMS OF TRADE ON A SYSTEM OF NATIONAL ACCOUNTS: AN ATTEMPTED SYNTHESIS*

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The objective of this paper is to reformulate the terms of trade effect within a framework of national accounts in constant prices. The issue has been discussed by Professor R. C. Geary, Dr. G. Stuvel and others. In what follows the author proposes a new formula for deflating the net factor income from abroad and the net lending to the rest of the world. It is shown that the terms of trade effect which follows from the formula be expressed as a synthesis of Geary's and Stuvel's approaches.

The author also shows that a similar approach be applied to the construction of the sector production account in constant prices. By formulating an appropriate deflator which is right for deflating factor incomes he concludes that the terms of trade arising from changes in inputs prices relative to output price be closely associated with the term which expresses the effect of productivity changes.

1. INTRODUCTION

In this paper the author will be concerned with the presentation of the terms of trade effect within a framework of national accounts. The issue has already been taken up by Professor R. C. Geary, Dr. G. Stuvel and other authors. In the next section, the author presents a system of national accounts in a matrix form which constitutes a conceptual framework for further discussion. He offers a brief summary of the arguments advanced by Geary and Stuvel. It will be pointed out that the nature of the terms of trade effects which are offered by them is primarily dependent on the rule for selecting the deflators for those items which express the non-commodity flow, such as the net factor income from abroad and the net lending to the rest of the world. Since it becomes apparent that the terms of trade effects introduced by them have further disadvantages, the author proposes a new approach for formulating the deflators of the net factor income from abroad and the net lending to the rest of the world. A generalized form which expresses the terms of trade effect readily follows from this new formulation. It is interesting to see that a synthesis between Geary's and Stuvel's approaches is attained by the generalized expression of terms of trade effect.

In section 3 the author discusses the feasibility of fitting the generalized expression of the terms of trade effect to the system of national accounts expressed in constant prices. Three sorts of gains may be distinguished in our system of national accounts in constant prices if the deflator for saving is explicitly defined. They may be termed the expenditure gains, the external trade gains and internal trade gains following the terminology adopted by Mr. Broderick. It is illuminating

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to demonstrate that the external trade gains are distributed into the expenditure gains and internal trade gains respectively if the deflator for saving is appropriately chosen. It is noted that the fact may be considered as an extension of Broderick's argument.

In the last section, the author attempts to link changes in terms of trade with productivity changes. In doing this, the author begins with the formulation of the sector production account in constant prices. Subject to changes in volumes of inputs, both of intermediate products and of labour, relative to outputs, it is maintained that the terms which express productivity changes be introduced for formulating the sector production account in constant prices. It is also pointed out that the term which may be regarded as terms of trade arising from changes in inputs prices relative to output price be introduced for balancing the sector production account in constant prices if the deflator which is right for deflating factor incomes is reasonably defined. The author proposes a new formula for the deflator. The article concludes by showing that a meaningful relationship which connects the terms of trade with the productivity changes results from this formula.

2. Terms of Trade Effect and the Rest of the World Account in Constant Prices

For the convenience of subsequent discussion the present analysis opens with the presentation of national accounts in a matrix form. The matrix is presented in Table 1, in which the following notations are used:

V, gross domestic capital formation
C, consumers' expenditure on goods and services
X, sales of goods and services to the rest of the world
I, net domestic capital formation
Q, gross domestic products
D, consumption of fixed capital
P_r, net factor income received from the rest of the world
S, saving
K_r, net capital transfer received from the rest of the world
M, purchases of goods and services from the rest of the world
T_r^f, net income transfer paid to the rest of the world
F, net increase of lending to the rest of the world

Regions are divided into two parts in this matrix. They are the domestic economy and the rest of the world which are shown in abbreviated form by the Roman characters I and II respectively. A sub-matrix is constituted by the transactions which emerge from the economic activities carried out within the domestic economy, indicated in the first four rows and columns. They are (1) production, (2) capital formation, (3) formation of income and its consumption and (4) capital financing respectively. It is evident from the construction of the sub-matrix that the idea of so-called "the real and financial dichotomy" is adopted, because the first two rows and columns refer to real flow of goods and services and the remaining two are concerned with the flow of financial claims as assets or liabilities.

TABLE 1

Regions		I				II
	No.	1	2	3	4	5
I	1		V	C		X
	2				I	
	3	Q	$-D$			P_r
	4			S		K_r
II	5	M		T_r^f	F	

It is well recognized that the rest of the world account is derived from the matrix in Table 1:

$$X + P_r + K_r = M + T_r^f + F \tag{2.1}$$

It is convenient for the subsequent discussion to restate the account by the following relation:

$$CS + P_r = N \tag{2.2}$$

$$CS = X - M, \qquad N = T_r^f + F - K_r$$

According to the idea of Professor R. Stone, CS represents the commodity flow and the remaining two variables in (2.2) fall into the category of the non-commodity flow.

The rest of the world account in constant prices will not necessarily be maintained if a rule for deflating the non-commodity flow items in (2.2) is formulated. In practice, as Geary has pointed out, the following rule for the deflation of P_r and N yields the imbalance of the rest of the world account in constant prices[1]: (rule 1) The deflator of X is used for the deflators of P_r and N if $CS > 0$. On the other hand, the deflator of M is used for the deflators of P_r and N if $CS < 0$.

Under such a circumstance, a correction term must be introduced into the rest of the world account in constant prices in order to maintain the balance of its receipt and expenditure. Let $\bar{T}$ stand for the correction term, the balance of the rest of the world account in constant prices is represented by (2.3).

$$\bar{X} + \bar{P}_r + \bar{T} = \bar{M} + \bar{N} \tag{2.3}$$

It is easily shown that the correction term $\bar{T}$ stands for the effect arising from changes in terms of trade. In fact, $\bar{T}$ is expressed by

$$\text{(2.4)} \quad \text{(i)} \quad \bar{T} = \frac{M}{p_2}\left(1 - \frac{p_2}{p_1}\right) = \bar{M}\left(1 - \frac{p_2}{p_1}\right) \quad \text{if} \quad CS > 0$$

$$\text{(ii)} \quad \bar{T} = -\frac{X}{p_1}\left(1 - \frac{p_1}{p_2}\right) = -\bar{X}\left(1 - \frac{p_1}{p_2}\right) \quad \text{if} \quad CS < 0$$

[1] R. C. Geary, "Problems in the Deflation of National Accounts: Introduction", *Income and Wealth*, Series IX, London 1961.

where p_1 and p_2 are deflators of X and M respectively. $(1 - p_2/p_1)$ or $(p_1/p_2 - 1)$ stands for the unit gain (or loss) due to changes in terms of trade.

Attention is particularly called to the fact that the symmetry is observed between the rest of the world account and the domestic economy whether viewed from the side of the domestic economy or from that of the rest of the world. Noting that the national accounts for the rest of the world can be written in the form of Table 2, its rest of the world account is expressed by (2.5).

$$X_2 + P_{r2} + K_{r2} = M_2 + T_{r2}{}^f + F_2 \tag{2.5}$$

TABLE 2

Regions		I	II			
	No.	1	2	3	4	5
I	1		M_2		$T_{r2}{}^f$	F_2
II	2	X_2		V_2	C_2	
	3					I_2
	4	P_{r2}	Q_2	$-D_2$		
	5	K_{r2}			S_2	

X_2 and M_2 in (2.5) stand for the sales of goods and services to the domestic economy and the purchases of goods and services from the domestic economy respectively. Suffix 2 is used for distinguishing the rest of the world from the domestic economy. Taking note of the fact that

$$\begin{aligned} CS_2 &= X_2 - M_2 = M_1 - X_1 = -CS_1 \\ P_{r2} &= -P_{r1} \end{aligned} \tag{2.6}$$

(2.7) easily follows.

$$N_2 = -(CS_1 + P_{r1}) = -N_1 \tag{2.7}$$

where the suffix 1 indicates the domestic economy. Owing to the symmetry observed in N and P_r; the selection of deflators for N_2 and P_{r2} directly follows from the rule 1 as indicated below:

(Rule 1′) p_2 is used for the deflators of N_2 and P_{r2} if $CS_2 > 0$. On the other hand, p_1 is used for the deflators of N_2 and P_{r2} if $CS_2 < 0$.

For the rest of the world economy, its rest of the world account in constant prices can be presented in a balancing form as (2.8), if a correction term is introduced:

$$\bar{X}_2 + \bar{P}_{r2} + \bar{T}_2 = \bar{M}_2 + \bar{N}_2 \tag{2.8}$$

where $\bar{T}_2$ is the correction term for the rest of the world. $\bar{T}_2$ is also considered as the gain or loss of the rest of the world due to changes in terms of trade. It is easily proved that the sum total of trade gains in the world as a whole is reduced to zero, i.e.

$$\bar{T}_1 + \bar{T}_2 = 0 \tag{2.9}$$

(2.9) is conveniently termed *the zero-sum condition of trade gains*. In fact, (i) if $CS_1 > 0$ and $CS_2 < 0$, then

$$\bar{T}_1 = \frac{M_1}{p_2}\left(1 - \frac{p_2}{p_1}\right) = \frac{X_2}{p_2}\left(1 - \frac{p_2}{p_1}\right)$$

$$\bar{T}_2 = -\frac{X_2}{p_2}\left(1 - \frac{p_2}{p_1}\right)$$

and we obtain

$$\bar{T}_1 + \bar{T}_2 = 0$$

(ii), conversely, if $CS_1 < 0$ and $CS_2 > 0$, then

$$\bar{T}_1 = -\frac{X_1}{p_1}\left(1 - \frac{p_1}{p_2}\right)$$

$$\bar{T}_2 = \frac{M_2}{p_1}\left(1 - \frac{p_1}{p_2}\right) = \frac{X_1}{p_1}\left(1 - \frac{p_1}{p_2}\right)$$

and we obtain

$$\bar{T}_1 + \bar{T}_2 = 0.$$

In the derivation of (2.9) it is taken for granted that the exports of the domestic economy are identical with the imports of the rest of the world and vice versa. On account of inconsistencies observed in statistical measurements among various countries, the condition may not necessarily be guaranteed for actual data. The existence of the discrepancy between the exports of the domestic economy and the imports of the rest of the world will create further complication. The point is ignored in this article so that we may not complicate the matter by unnecessary additions.

The disadvantage of this formulation of terms of trade effect is that the term is solely dependent on either $\bar{X}$ or $\bar{M}$, aside from the term expressing the unit gain (or loss) due to changes in terms of trade. Stuvel's rule for the selection of deflators of N and P_r aims to surmount the disadvantage.[2] His rule may be expressed as below:

> (Rule 2) All entries of national accounts are deflated by a single deflator, say GDP deflator, which reflects the change in general prices. Thus, GDP deflator is used for the common deflator of both P_r and N.

Although he does not indicate the specific deflator which reflects the change in general prices, it is worthwhile to note the fact that the terms of trade effect which is derived from his argument becomes valid if and only if GDP deflator is chosen as the common deflator for all entries of national accounts. According

[2]G. Stuvel, "Asset Revaluation and Terms of Trade Effects in the Framework of the National Accounts", *Economic Journal*, June 1959.

to his argument, the rest of the world account (for the domestic economy) in constant prices is written as

$$\bar{X}_1 + \bar{\bar{P}}_{r1} + \bar{\bar{T}}_1 = \bar{M}_1 + \bar{\bar{N}}_1 \tag{2.10}$$

where

$$\bar{\bar{P}}_{r1} = \frac{P_{r1}}{P}, \qquad \bar{\bar{N}}_1 = \frac{N_1}{P}$$

letting P stand for GDP deflator. $\bar{\bar{T}}_1$ stands for the correction term which expresses the terms of trade effect as indicated below:

$$\bar{\bar{T}}_1 = \bar{M}_1\left(1 - \frac{p_2}{P}\right) - \bar{X}_1\left(1 - \frac{p_1}{P}\right). \tag{2.11}$$

But, the formulation of the terms of trade effect in (2.11) creates another difficulty, because (2.11) no longer ensures the zero-sum condition of trade gains. Thus, it is required that a new rule for choosing the deflators of P_r and N which overcomes the disadvantages indicated before be sought.

What is proposed by the author to meet the requirement is the following rule.

(Rule 3) P_{r1} and N_1 is deflated by a new deflator p_N which is constructed as a weighted harmonic mean of p_1 and p_2:

$$p_N = \frac{1}{\alpha(1/p_1) + (1 - \alpha)(1/p_2)} \tag{2.12}$$

where $0 < \alpha < 1$ stands for the weight in (2.12) and is specified by

$$\alpha = \frac{X_1}{X_1 + M_1}. \tag{2.13}$$

A correction term $\tilde{T}_1$ must be added so that the rest of the world account in constant prices may be established in the following form,

$$\bar{X}_1 + \tilde{P}_{r1} + \tilde{T}_1 = \bar{M}_1 + \tilde{N}_1 \tag{2.14}$$

if p_N defined in (2.12) is applied to P_{r1} and N_1 as their deflators. It is readily shown that the correction term is considered as the effect due to changes in terms of trade and is expressed by[3]

$$\tilde{T}_1 = \bar{X}_1(1 - \alpha)\left(\frac{p_1}{p_2} - 1\right) - \bar{M}_1\alpha\left(\frac{p_2}{p_1} - 1\right) \tag{2.15}$$

[3] M. R. Courbis has proposed another rule for selecting the deflator of P_{r1} and N_1, in his elaborate article, "Comptes Économique Nationax A Prix Constants", *Etudes et conjoncture*, Juillet 1964. As the deflator he chooses what is defined by

$$p_N = \alpha p_1 + (1 - \alpha)p_2, \text{ where } \alpha = \frac{\bar{X}_1}{\bar{X}_1 + \bar{M}_1}.$$

His choice of p_N creates somewhat complicated expression of $\tilde{T}_1$, which has no longer close association with any form of terms of trade effects derived from either rule 1 or rule 2. His derivation of terms of trade effect is written by

$$\tilde{T}_1 = \bar{X}_1\left[\frac{1}{\alpha + (1 - \alpha)(p_2/p_1)} - 1\right] - \bar{M}_1\left[\frac{1}{(1 - \alpha) + \alpha(p_1/p_2)} - 1\right].$$

It is interesting to note that the terms of trade effect obtained in (2.15) is expressed by a weighted average of $\bar{M}_1(1 - p_2/p_1)$ and $-\bar{X}_1(1 - p_1/p_2)$, which are the terms of trade effects produced by rule 1. Furthermore, it is recalled that the expression of $\tilde{T}_1$ in (2.15) exhibits symmetry with respect to X_1 and M_1. Owing to this symmetry, the zero-sum condition of trade gains is ensured. Thus, letting $\tilde{T}_2$ stand for the terms of trade effect originating from the rest of the world economy, (2.16) holds:

$$\tilde{T}_1 + \tilde{T}_2 = 0. \tag{2.16}$$

Replacing P by p_1 or p_2 in (2.11), it may be also noted that the expression of $\tilde{T}_1$ in (2.15) assumes a generalized form of (2.11).

3. Terms of Trade Effect and a System of National Accounts in Constant Prices

So far we have discussed the terms of trade effect only within the scope of the rest of the world account. But, the effect necessarily generates significant impact on other segments of the domestic economy. The issue becomes considerably important as we consider the effect within *a system of national accounts in constant prices* instead of one independent account in constant prices. Noting the relationship in (2.2), the conceptual framework shown in Table 1 is further simplified as Table 3.

TABLE 3

Regions		I				II
	No.	1	2	3	4	5
I	1		V	C		X
	2				I	
	3	Q	$-D$			P_r
	4			S^*		
II	5	M			N	

where $S^* = S + K_r - T_r^f$.

Suppose that GDP deflator and the deflator for net capital formation are implicitly determined by means of the production account and the capital formation account which follow from the matrix in Table 3. The deflators for remaining entries which express non-commodity flow, i.e. P_r, N and S^*, are left indeterminate, unless rules for determining these entries are properly furnished. The rule 4 which will be given below and the rule 3 already rendered make it possible to produce a system of national accounts in constant prices and to determine appropriate deflators for P_r, N and S.

(Rule 4) The deflator of S is formulated by

$$p_S = \frac{1}{\beta(1/p^*) + (1 - \beta)(1/p_C)} \tag{3.1}$$

where p^* and p_C stand for the implicit NDP deflator and the deflator for consumers' expenditure on goods and services respectively.[4] β stands for a weight and is presented by

$$\beta = \frac{P}{P + C} \qquad (0 < \beta < 1). \tag{3.2}$$

If these deflators are applied to the constituent entries in the consumption account, a correction term $\bar{\theta}_1$ must be introduced into the consumption account in constant prices so that it may maintain the balance. So we obtain

$$\bar{C} + \bar{S}^* + \bar{\theta}_1 = \bar{P} + \bar{P}_r \tag{3.3}$$

The correction term can be expressed as (3.4):

$$\bar{\theta}_1 = \left[\bar{P}(1 - \beta)\left(1 - \frac{p^*}{p_C}\right) - \bar{C}\beta\left(1 - \frac{p_C}{p^*}\right)\right] + P_r\left(\frac{1}{p_N} - \frac{1}{p_S}\right) \tag{3.4}$$

$\bar{\theta}_1$ may be termed the expenditure losses, because they account for the use of additional real flow of purchasing power which arises from changes in relative prices. Obviously, it is seen that

$$\bar{\theta}_1 > 0 \quad \text{if} \quad p^* < p_C \quad \text{and} \quad p_S > p_N$$

$$\bar{\theta}_1 < 0 \quad \text{if} \quad p^* > p_C \quad \text{and} \quad p_S < p_N.$$

It is also noted that the expenditure loss is caused not only by the relative prices between NDP deflator and the deflator for the consumers' expenditure but also the terms of trade between p_N and p_S.

Similarly, an additional correction term must be inserted in the capital finance account so that it may maintain the balance. Let $\bar{\theta}_2$ stand for the correction term. The capital finance account in constant prices is written by (3.5):

$$\bar{I} + \bar{N} + \bar{\theta}_2 = \bar{S}^* \tag{3.5}$$

It can be readily shown that the correction term is expressed by (3.6):

$$\bar{\theta}_2 = -(\bar{\theta}_1 + \tilde{T}) \tag{3.6}$$

$\bar{\theta}_2$, which is analogous to $\bar{\theta}_1$, may be termed the internal trade losses because the term originates from changes in relative prices between the domestic prices and the price for exports and amounts to the internal use of real purchasing power.

[4]The NDP deflator is implicitly defined by the following definitional relation;

$$P = Q - D$$

if the deflator for Q and D are explicitly defined.

Thus, the preceding arguments imply far-reaching consequences. Firstly, it is implied that if the rule 3 and rule 4 are formulated for selecting deflators P_r, N and S then a system of national accounts in constant prices can be constructed on the basis of the matrix in Table 3:

$$
\begin{aligned}
(3.7) \qquad \bar{Q} &= \bar{V} + \bar{C} + \bar{X} - \bar{M} \\
\bar{V} &= \bar{D} + \bar{I} \\
\bar{C} + \bar{S}^* + \bar{\theta}_1 &= \bar{P} + \bar{P}_r \\
\bar{I} + \bar{N} + \bar{\theta}_2 &= \bar{S}^* \\
\bar{X} + \bar{P}_r + \tilde{T} &= \bar{M} + \bar{N}
\end{aligned}
$$

Secondly, (3.6) implies that the external trade gains are expressed as the sum of the expenditure gains and the internal trade gains noting changes in sign:

$$
(3.8) \qquad \tilde{T} = -(\bar{\theta}_1 + \bar{\theta}_2)
$$

(3.8) amounts to saying that the gains caused by changes in terms of trade are necessarily distributed into the gains arising from changes in relative prices of the domestic economy.

The issue raised in this section has already been discussed by several authors as Geary, Stuvel and Broderick. In particular, Broderick maintains that a system of national accounts in constant prices can be compiled if a set of deflators for the elements of the system which express the non-commodity flow is properly provided. He also points out that the expenditure gains of households, corporations and public authorities adds up to zero. What I have concluded from (3.8) is that his argument can be extended so as to establish explicitly the relationship between the external trade gains, the expenditure gains and the internal trade gains if the rule for selecting the deflators for those transactions which express the non-commodity flow are properly formulated.[5]

4. Terms of Trade Effect between Inputs and Outputs and Productivity Changes

If we note the fact that the terms of trade effect so far discussed be regarded as the effect caused by changes in relative prices between exports and imports, the preceding argument can be further extended to deal with the terms of trade effect generated by changes in relative prices between inputs and outputs. For the subsequent analysis it is of great help to construct the following table.[6]

[5]See J. B. Broderick, "National Accounts at Constant Prices", *The Review of Income and Wealth*, September 1967. In fact, a similar conclusion to (3.8) is also drawn by Stuvel (G. Stuvel, *op. cit.*). But, in his derivation, the entries which stand for the non-commodity flow are uniformly deflated by GDP deflator.

[6]Virtually similar table is introduced by Courbis. See R. Courbis, "Compatibilité Nationale à Prix Constants et à Productivité Constante", *The Review of Income and Wealth*, March 1969.

TABLE 4

	1	2	3	4	5	6	7	8
Output	X_0	$\tilde{\bar{X}}$	q	$\tilde{\bar{X}}$	q	$\tilde{\bar{X}}$	q	X_1
Intermediate products	U_0	$\tilde{\bar{U}}$	q	$\tilde{\bar{U}}$	q_u	$\tilde{\bar{U}}$	q_u	U_1
Compensation for employees	$\tilde{W}_0$	$\tilde{W}$	q	$\tilde{\bar{W}}$	q_w	W^*	q_y	W_1
Operating surpluses	Y_0	$\tilde{Y}$	q	$\tilde{Y}$	q	Y^*	q_y	Y_1
Gains due to price changes						T		
Gains due to productivity changes				G				
Column total	X_0	$\tilde{X}$		$\tilde{\bar{X}} + G$		$\tilde{\bar{X}} + T$		X_1

Column 1 indicates the base year values in current prices.
Column 2 indicates the current year values in constant prices when the volumes of outputs and inputs are kept unchanged.
Column 3 indicates the Laspeyres volume indexes which correspond with entries in column 2.
Column 4 indicates the current year values in constant prices. Intermediate products and labour inputs are deflated by individual deflators.
Column 5 indicates the Laspeyres volume indexes which correspond with entries in column 4.
Column 6 indicates the current year values in constant prices.
Column 7 indicates the Laspeyres volume indexes which correspond with entries in column 6.
Column 8 indicates the current year values in current prices.

In Table 4 the following notations are also employed:

X, outputs
U, the inputs of intermediate products
W, compensation for employees
Y, operating surpluses
T, gains or losses due to changes in relative prices
G, gains or losses due to changes in productivity
q, (Laspeyres) volume index of X
q_u, (Laspeyres) volume index of U
q_w, (Laspeyres) volume index of W
q_y, (Laspeyres) volume index of Y. q_y is derived from W^*/W_0 which is defined below.

Noting the fact that the right hand side of the following production account (in current price),

$$Y + W = X - U \tag{4.1}$$

consists of the variables which express the commodity flow, it is reasonable to formulate the deflators for Y and W, which express the non-commodity flow, by the following rule.

(Rule 5) The deflator p_Y defined by

$$p_Y = \frac{1}{\eta(1/p) + (1 - \eta)(1/p_u)} \tag{4.2}$$

where η stands for a weight and is expressed by

$$\eta = \frac{X_1}{X_1 + U_1} \qquad (0 < \eta < 1) \tag{4.3}$$

is universally used as the deflators for Y_1 and W_1. Here, p and p_u are the deflators of outputs and the inputs of intermediate products respectively.

A correction term T for maintaining the balance of production account in constant prices must be introduced, if the rule 5 is applied to column 6 in Table 4 for obtaining W^* and Y^*. Thus,

$$\bar{X} + T = \bar{U} + W^* + Y^* \tag{4.4}$$

where

$$W^* = \frac{W_1}{p_Y}, \qquad Y^* = \frac{Y_1}{p_Y}.$$

It is easy to see that the correction term in (4.4) is expressed by

$$T = (1 - \eta)\bar{X}\left(\frac{p}{p_u} - 1\right) - \eta\bar{U}\left(\frac{p_u}{p} - 1\right). \tag{4.5}$$

Obviously,

$$T > 0 \quad \text{if} \quad p > p_r,$$
$$T < 0 \quad \text{if} \quad p < p_r.$$

As $(1 - p_u/p)$ indicates the unit gain (or loss) due to changes in relative prices between outputs and intermediate inputs, the correction term in (4.5) indicates the terms of trade effect arising from changes in relative prices. T expresses the gain arising from the terms of trade between outputs and intermediate products if it is positive, and T expresses the loss suffered from corresponding changes in relative prices if it is negative.

It should also be remembered that the following relations are established between the variables in column 2 of Table 4 and those in column 4:

$$\tilde{U} = \frac{q_u}{q}\bar{U}, \qquad \tilde{W} = \frac{q_w}{q}\bar{W}. \tag{4.6}$$

The balancing relation constituted by the variables listed in column 4 will not be maintained unless it is supplemented by a correction term. Let G stand for the correction term, we obtain

$$\bar{X} + G = \tilde{U} + \tilde{W} + \tilde{Y}. \tag{4.7}$$

Using the relationships in (4.6), the balance for column 2 may be written by

$$\tilde{X} = \tilde{U} + \tilde{W} + \tilde{Y} \tag{4.8}$$
$$= \left(\frac{q_u}{q}\tilde{U} + \frac{q_w}{q}\tilde{W} + \tilde{Y}\right).$$

It is demonstrated by (4.7) and (4.8) that the correction term appeared in (4.7) is broken up into the terms which express the effect of productivity changes in factors of production:

$$G = \left(1 - \frac{q_u}{q}\right)\tilde{U} + \left(1 - \frac{q_w}{q}\right)\tilde{W}. \tag{4.9}$$

In fact, the first term appeared in the right-hand side of (4.9) stands for a unit change in productivity arising out of the input of intermediate products. Similarly, the second term in the right-hand side of (4.9) indicates a unit productivity change arising out of labour inputs. Obviously,

$$G > 0 \quad \text{if} \quad q > q_u \quad \text{and} \quad q > q_w$$
$$G < 0 \quad \text{if} \quad q < q_u \quad \text{and} \quad q < q_w.$$

If $G > 0$ is the case, it is called that the effect of productivity changes takes place the increase of productivity.

Substituting $\tilde{X}$ in (4.4) for what is obtained from (4.8), the terms of trade effect originated from changes in relative prices between outputs and the inputs of intermediate products may be expressed by means of the effect of productivity changes:

$$T = \left(1 - \frac{q_u}{q}\right)\tilde{U} + \left(1 - \frac{q_w}{q}\right)\tilde{W} + (Y^* - \tilde{Y}). \tag{4.10}$$

Noting the relation (4.9), it is clear that the effect of productivity changes is associated with the terms of trade effect arising from changes in relative prices which has already been introduced:[7]

$$T = G + (1 - \eta)\left(\frac{p}{p_u} - 1\right)\tilde{Y}. \tag{4.11}$$

[7]By definition, it is readily seen that

$$Y^* p_Y = Y_1 \quad \text{and} \quad \tilde{Y} p = Y_1$$

Thus, we obtain

$$(Y^* - \tilde{Y}) = \left(\frac{p}{p_Y}\tilde{Y} - \tilde{Y}\right) = \left(\frac{p}{p_Y} - 1\right)\tilde{Y}.$$

Recalling the definition of p_Y in (4.2), $[(p/p_Y) - 1]$ is written by

$$\left(\frac{p}{p_Y} - 1\right) = (1 - \eta)\left(\frac{p}{p_u} - 1\right).$$

Accordingly,

$$(Y^* - \tilde{Y}) = (1 - \eta)\left(\frac{p}{p_u} - 1\right)\tilde{Y}.$$

Noting that G is further restated by[8]

$$G = \left(\frac{1}{p_u} - \frac{1}{p}\right) [(1 - \eta)\, W_1 + U_1] \tag{4.12}$$

it is readily demonstrated from (4.12) that the effect of productivity changes is always positive if and only if the outputs prices are greater than the the prices of intermediate products. Also, (4.11) implies that the terms of trade effect arising from changes in relative prices between inputs and outputs always exceeds the effect of productivity changes if and only if the outputs prices are greater than the inputs prices. It should be recognized that the terms of trade between inputs and outputs are firmly associated with productivity changes as I have already shown.[9]

[8]Replacing T by (4.5), we obtain

$$\begin{aligned} G &= (1 - \eta)\left(\frac{p}{p_u} - 1\right)(\tilde{X} - \tilde{Y}) - \eta\left(\frac{p_u}{p} - 1\right)\bar{U} \\ &= (1 - \eta)\left(\frac{p}{p_u} - 1\right)(\tilde{U} + \tilde{W}) - \eta\left(\frac{p_u}{p} - 1\right)\bar{U} \end{aligned}$$

which results from (4.7). As we can see that

$$p\tilde{W} = W_1, \quad p\tilde{U} = U_1 \quad \text{and} \quad p_u\bar{U} = U_1$$

by definition, G is further written by

$$G = \left(\frac{1}{p_u} - \frac{1}{p}\right) [(1 - \eta)W_1 + U_1].$$

[9]It may be of some interest to remark that the effect of productivity changes indicated above can be written in terms of Divisia indexes which have been widely used by Jorgenson and others for the measurement of the total factor productivity. Let q^D, $q_u{}^D$ and $q_w{}^D$ stand for the Divisia quantity indexes of outputs, intermediates inputs and labour inputs respectively. The effect of productivity changes is expressed by

$$G = \frac{1}{1 + q^D}[(q^D - q_u{}^D)\bar{U} + (q^D - q_w{}^D)\tilde{W}].$$

In this expression, the productivity changes reveal themselves as either the difference between outputs and intermediate inputs or the difference between outputs and labour inputs. In connection with the application of Divisia indexes, reference may be made to D. W. Jorgenson and Z. Griliches, "The Explanation of Productivity Change", *Review of Economic Studies*, July 1967; L. R. Christensen and D. W. Jorgenson, "U.S. Real Product and Real Factor Input, 1929–1967", *The Review of Income and Wealth*, March 1970.