THE SOCIAL SIGNIFICANCE OF INCOME DISTRIBUTION STATISTICS

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In both political discussions and scientific literature the income distribution has come to occupy a central position for the consideration of social welfare and economic equalization. It has been assumed that an individual's income reflects his consumption opportunities and therefore his standard of living or economic welfare. The thesis of this paper is, however, that there are reasons for being quite pessimistic about drawing meaningful conclusions from income distribution data. As illustrated by the use of Swedish data, the distribution of income gives an extremely incomplete picture of the distribution of consumption for a wide variety of definitional and statistical reasons. The distribution of consumption, furthermore, cannot be transformed into a corresponding distribution of welfare, since there is no well defined concept of welfare. The treatment of public consumption in empirical analysis of the distribution of welfare also raises problems. The paper closes with the presentation of the conceptual basis for an alternative to the traditional method of analyzing the distribution of income.

INTRODUCTION

If we economists are to be able to make any contribution to actual political discussions on problems related to the distribution of economic resources among different groups in society we must, of course, take care to use the same concepts as those which occur in this discussion. For example, if we are to illustrate statistically the economic equalization with which the political debate is concerned, we must try to find data showing this kind of equalization—and not something quite different. On this point we have committed a good many sins. Our empirical analyses of distribution are usually performed rather mechanically. The analyses have been concentrated on *income* distributions without much discussion of the fundamental questions as to (1) what type of distribution we should, in fact, be looking for in order to get results which are relevant to the problem at hand, and (2) what type of relationship we may expect to find between this "relevant" distribution and the corresponding distribution of incomes. Criticism may also be levelled at the methods used for measuring the degree of inequality, since these methods have been designed without due regard to economic theory. Their significance is therefore dubious.

The purpose of this paper is to discuss to what extent it is possible to draw conclusions from available income distribution statistics regarding the distribution which is truly relevant in the general debate on social welfare and economic equalization. As a starting point for my argument I shall discuss briefly the relationships among three different types of distribution which are of basic importance in this context, namely (1) the income distribution, (2) the consumption distribution and (3) the welfare distribution.

The central position which the income distribution has come to occupy both in the political discussion and in the scientific literature is due, of course, to the social importance which has been attached to it. It has been assumed that the individual's income reflects his consumption opportunities and therefore his standard of living. Since the distribution of the standard of living or, more generally, economic welfare, has been the subject of political evaluations, the income distribution has therefore also, indirectly, been the subject of such evaluations. It is obviously this income—consumption—welfare nexus which is the reason for the great political interest in the income distribution. Without this supposed nexus this distribution would be uninteresting, at least from the social point of view.

We may now ask how much justification there is in reality for this supposed connection between income, consumption and welfare. To what extent do the income distribution and its variations reflect the corresponding consumption distribution and its variations and to what extent do the consumption distribution and its changes reflect the corresponding welfare distribution and its changes? The answers to these questions are obviously of fundamental importance for all empirical research on income distributions. For if it is, as I have argued, the distribution of *welfare* that is the relevant concept in political discussion, the economists' empirical analyses of income distributions will be of interest only on the assumption that there is a fairly close connection between this distribution and the corresponding welfare distribution.

It is quite well known that the income distribution is not *identical* with the consumption distribution and it is equally well known that the consumption distribution does not *perfectly* reflect the welfare distribution—however we define welfare. This has been pointed out many times and much discussion has been devoted to the problem of how to define income and income-receiver in order to get a closer connection between income and standard of living. The statistical material at hand, however, gives us a limited freedom in the choice of definitions. We have to work with statistical data which are in many respects inappropriate for our purposes. But *how* inappropriate are our income distribution data? To what extent can we use them for drawing conclusions about the corresponding consumption or welfare distributions?

It is, I suppose, not possible to give a general and precise answer to these questions. The situation may vary from country to country. My thesis in this paper is, however, that there are reasons for being quite pessimistic about drawing meaningful conclusions from income distribution data—at least in cases where tax assessment data are used. As far as I can see the available data on income distributions yield little or no information about *real* distribution problems. In the following sections I shall try to corroborate this thesis with some statistical data and some simple argumentation. My discussion will be related to the Swedish situation, but I do not think that Sweden is an exception in this respect, so I am convinced that my views will be valid for other countries as well.

My discussion will proceed as follows: In the next section I shall consider the connection between the distribution of income and the corresponding consumption distribution. In the succeeding section I shall consider the connection between the consumption distribution and the corresponding welfare

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distribution, and in the last section I shall give some views on measuring the degree of inequality in welfare distributions.

INCOME DISTRIBUTION AND CONSUMPTION DISTRIBUTION

In discussing how the income distribution is related to the corresponding consumption distribution, I shall completely disregard the purely statistical sources of error which occur in the empirical material. That these sources of error are large and that they involve great problems for empirical analysis is so well known that I need not dwell on them. However, I cannot also brush aside the question of defining the concept of "income-receiver". It is of importance for the subsequent development of my argument and I must therefore say a few words about it.

All who have worked on empirical analyses of distributions of personal incomes know that the very definition of the concept of income-receiver contains a great many problems. There are alternative definitions and the choice among them is by no means easy to make. I shall not consider these problems in this paper but take as my starting point the definition used in Sweden's—and in also most other countries'—tax-assessment statistics. An income-receiver is any person with an income above a certain limit. In order not to run into immediate problems however, this definition must be modified somewhat. The requirement of a certain minimum level of income cannot be justified theoretically and we know from experience that it leads to unreasonable results insofar as the changes in the distribution from one period to another are strongly dominated by the "migratory movements" over this minimum limit. In order to avoid this absurdity, I shall consider as income receivers all *potential* income earners, and in this group include all persons over the age of 17 years. Further, as there is little sense in talking about the income-consumption relationship of a married woman, married couples will be considered in this paper as one income receiver. Where nothing else is said I will use income as a synonym for *disposable* income, i.e. income after taxes and transfers.

In order to understand how the income distribution—with the above definition of the concept of "income receiver"—is related to the corresponding consumption distribution, it may be well to recall why it is that the two distributions are not identical. There are three reasons for this—if we define consumption as *real* consumption valued at market prices. (1) There is a definitional difference between income and consumption—saving and dis-saving. (2) The consumption of certain individuals is paid for in whole or in part by other individuals or by institutions. (3) The real purchasing power which an income yields is not identical for all, since regional, individual or other price differences can occur. In order to be able to transform a certain given income distribution into the corresponding consumption distribution, we must accordingly know the simultaneous distribution of incomes, savings, various supports and price differences. Moreover, in order to be able to transform a given change in the income distributions into the corresponding change in the consumption distribution, we further need to know the development in time of the simultaneous distribution of the above-mentioned factors. In this case, there is another factor which complicates the picture, *viz.*

changes in the price relations. Insofar as the price development appears to be more favourable to some individuals than to others, a discrepancy arises between the changes in the income distribution and those in the consumption distribution.

Since we do not have access to all the data which are required to transform a given income distribution into the corresponding consumption distribution, we cannot obtain much information on the discrepancies which occur between these two distributions. We are not entirely ignorant on this point, however. It is well known, for example, that the consumption distribution normally displays a considerably smaller degree of inequality than the income distribution. That this is the case may be inferred, *inter alia*, from the fact that a large number of income-receivers have incomes far below the subsistence level, without, for that reason, being destitute.

The connection between income distribution and consumption distribution is, of course, based on the idea that, for the majority of income-receivers, the amount of income is the principal determinant of the amount of consumption. As has just been remarked, this condition obviously does not apply to all people. There are large groups for whom the difference between income and consumption is so great that income cannot be regarded as in any way an indicator of the level of consumption. One way of trying to find out how well the income distribution reflects the corresponding consumption distribution is to look for such groups and try to estimate how many income receivers they include. I shall follow this method in the present section.

I begin by giving some data which emerged in a Swedish investigation in 1959 of the economic circumstances of persons with incomes (before taxes and transfers) below or in the neighbourhood of what was then regarded as the subsistence level. This investigation concerned (1) unmarried persons who had stated in their income-tax returns that they had incomes of less than 6,000 kronor (1,200 U.S. dollars), (2) married couples who had stated that they had incomes of less than 10,000 kronor (2,000 U.S. dollars), and (3) persons who had not sent in any income-tax returns at all. It was limited to unmarried persons aged 17–66 and to married couples in which the husband belonged to this age range. This limitation obviously excluded the majority of old-age pensioners.

The number of unmarried persons and married couples—hereinafter called "low-income-earners"—included in the investigation amounted to 35 per cent of all the income-earners in Sweden. It seems clear that this group included many persons living in difficult economic circumstances; about a quarter of the whole group stated that they had been unable to work all the year round on account of illness or unemployment. Though most persons in this category received considerable transfer incomes from health insurance and unemployment insurance, a large number of them were probably reduced to a low standard of living. For the remaining 75 per cent of the low-income-earners, however, the situation was quite different. The majority of them belonged to groups for whom the amount of income cannot normally be assumed to determine the standard of consumption. They were mainly students, apprentices, conscripts doing their military service, persons who had entered the labour market during the year, and young people under 20 years of age. Altogether these groups formed a good 50 per cent of the whole low-income-earner category. Concerning the remaining 25 per cent,

it is more difficult to draw any definite conclusions. Since this group consisted largely of self-employed persons (especially farmers) and persons who may be assumed to have been employed in private families or small family firms, however, there is reason to believe that their consumption level was substantially higher than their income level.

Among the low-income-earners, *at least* 60 per cent—probably considerably more—had *disposable* incomes less than what was then regarded as the subsistence level. However only 20 per cent were receiving public assistance. This indicates that at least 40 per cent, and probably considerably more, of all low-income-earners had a consumption level substantially in excess of the amount of income. To a certain extent this would seem to have been made possible by supports of different kinds. For instance, nearly one-third were living with their parents, with other relatives or in institutions.

These figures show that the large bottom layer of the income distribution consisted only to a small extent of hard-core poverty. It mainly consisted of persons for whom the amount of income does not normally determine the level of consumption. The low-income-earners assignable to this latter category constituted altogether 20–25 per cent of all the income receivers in Sweden.

This shows that the amount of income is not the principal determinant of the consumption level, at least for the majority of people belonging to the bottom layer of the income distribution. Since this is so we may ask ourselves whether the connection between income and consumption is equally diffuse in all income brackets. This is probably not the case, but there are obviously large groups among those not assignable to the low-income-earner category for whom there is, *normally*, a considerable difference between income and consumption. If this is so, it is natural to ask: Which are these groups? Without trying to give a complete answer to that question, it may be said that the following groups of individuals are in such a situation that as a rule their income level is not the determining factor of their consumption level: (1) persons aged 17–20 years, (2) students over 20 years of age, (3) conscripts doing their military service, (4) other persons living with their parents or with other relatives or in institutions, (5) self-employed farmers, and (6) persons who have died, emigrated or immigrated during the year. The inclusion of group (6) in the list is, of course, due to the fact that the length of the consumption period is not a whole year for those who die during the year and that the available data tell us nothing about *annual* incomes of those who have emigrated or immigrated.

A calculation of the number of Swedish income receivers belonging to any of the above-mentioned groups indicates that in 1967 the total number amounted to about 40 per cent of all income receivers in the country. This figure is, of course not a maximum figure since the list above is not exhaustive. It is, for instance, very probable that a considerable number of old age pensioners by spending their accumulated capital maintain a much higher consumption than that which corresponds to their income.

The fact that nearly half—and perhaps even a much greater proportion—of all the income-receivers in the country are normally in such a situation that the amount of their income does not determine the amount of their consumption justifies us, in my opinion, in seriously asking what information our data on

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income distribution really gives us. Do they tell us anything at all about the distribution of consumption? We have even greater reason to ask what information the *changes* in the income distribution from one period to another give us, especially since the number of individuals belonging to the groups considered above may vary. Thus, for example, the great expansion of education in the last few years must have resulted in a tendency to increased inequality in the income distribution, but whether this expansion has also resulted in a similar tendency in the consumption distribution is extremely doubtful.

It can be argued, however that the result of the above calculations is greatly dependent on the fact that young people under the age of 20 have been included as income-receivers. By simply excluding this group of young people, we would get a result that showed the income and consumption levels to be more closely correlated. This is certainly true, but, by excluding the group of young people, we simultaneously invalidate the income distribution; in that case it will not include all the income-earners. In addition, there is the fact that the available statistical material is not always of such a nature as to permit a grouping by age. Finally, it may be observed that, *even if* the groups of young people are excluded, a calculation of the kind made above yields a fairly high figure. By excluding the age group 17–19 years from the above calculation, the proportion would fall from about 40 per cent to about 30 per cent. Even this figure seems to be high enough to arouse skepticism about the possibilities of drawing conclusions about the consumption distribution from the income distribution.

A further objection could be raised to the above calculation. The large disparity between consumption and income may be a result of choosing the individual and not the *household* as the income receiving unit. For households the agreement would have been substantially greater. This objection is quite correct. It would, from many point of view, be more appropriate to work with the household rather than with the individual income-receiver as the unit. It is unfortunate, however, that in most countries the statistical material does not permit such a procedure.

The lack of sufficient data on the economic situation of households destroys not only the possibility of finding relevant relationships between income and consumption but also the possibility of finding the consumption variable which is of relevance for the standard of living. It is self-evident that the consumption variable which appears in the individual welfare function must be related, in one way or another, to the number of persons belonging to the household. Therefore, even if we had perfect knowledge of the consumption expenditures of every single income receiver this knowledge would not give us much information about the standard of living, unless we knew individual differences in the burden of maintenance.

To summarize the data which we have on the distribution of incomes give an extremely incomplete picture of the corresponding distribution of consumption, and we cannot expect that the changes which may be observed in the income distribution from one period to another will be matched by similar changes in the consumption distribution. If we wish to chart the consumption distribution empirically, we shall probably have to do so by means of *consumption* investigations. It seems impossible to go via the income distribution.

While the trouble connected with the transformation of a given income distribution into the corresponding consumption distribution is mainly due to the lack of appropriate statistical data, the problem connected with the transformation of a given consumption distribution into the corresponding welfare distribution is due to our lack of knowledge concerning the very nature of welfare functions. The latter transformation cannot, obviously, be made without a well-defined concept of welfare. Unfortunately this does not exist. The welfare concepts which appear in the general political discussion lack a precise definition. Studying the arguments put forward in favour of income equalization, for instance, we can easily see that there is much confusion about how different factors such as age, working hours, working capacity, etc. fit into the picture.

It must be stressed, however, that the lack of a precise welfare definition is not a sufficient reason for complete cynicism in this field. The acceptance of such cynicism would deny the possibility of making empirical studies which could be of importance to the general distribution debate. If our empirical studies of income (or consumption) distributions are to be something more than playing with figures we cannot avoid making some welfare judgements. Without them we cannot even arrange our statistical data in a meaningful way. Let me illustrate this with some examples.

In most countries there has been, during recent decades, a great increase in the proportion of old-age pensioners in the total population. This has in all probability resulted in a tendency to increased inequality both in the income distribution and in the consumption distribution. But has it resulted in a similar tendency in the welfare distribution? This is very doubtful. The answer depends, obviously, upon the nature of the welfare functions. If these functions do not vary with age we may say that the increase in the proportion of old-age pensioners has produced a tendency to increased inequality in the welfare distribution. If, however, the welfare function varies with age we cannot know how this distribution has been affected.

Similar interpretation difficulties arise because of the increasing number of married women gainfully employed. The fact that more and more married women receive incomes of their own has probably meant a tendency to an increase in the inequality of incomes. But to what extent has it meant a similar tendency in welfare? On the rather natural assumption that the gainfully employed devote less effort to household duties than other married women do we cannot answer that question without making a valuation of these duties. But how shall this valuation be done?

The examples above give rise to the more general question of how to arrange the statistical material in order to get meaningful figures. We may ask whether there is much sense in pooling together, within one population, individuals showing great differences in age, burden of maintenance, hours of work, marital status, wealth, etc. Perhaps it would be better to disaggregate and treat different groups separately. If the statistical material permits such a disaggregation, *some* of the problems discussed above could be avoided. This seems to be true, especially if we disaggregate according to age and burden of maintenance.

Another fundamental question for the empirical analysis of distribution problems is how to treat public consumption. This consumption has, obviously, important welfare effects and we can be sure that these effects are distributed among the citizens in a rather specific way. In some part, at least, the benefits of public consumption are negatively correlated with incomes. This ought to be so, since the very aim of some types of public consumption is to equalize welfare.

It may, perhaps, be argued that all types of public consumption have welfare effects and that therefore the whole body of this consumption is of importance to the welfare distribution. The goods and services included in public consumption are of so many different kinds, however, that it cannot be reasonable to treat all of them in the same way. It is, for instance, quite clear that goods and services which are consumed *collectively* by the citizens are not fully comparable to those goods and services which constitute private consumption. How this type of public consumption should come into the welfare picture is therefore a great problem.

The situation is less complicated when we consider goods and services which the authorities place at the citizens' disposal for *individual* use either quite freely or at subsidized prices. These goods and services are very much the same as those appearing in private consumption and, valued at costs of production, they may be assumed to have the same welfare effect as other goods and services.

It seems to be quite obvious that at least this latter type of public consumption has to be included in our analyses of distribution, if we want to obtain meaningful results. The neglect of this consumption is unreasonable, particularly since, in many countries, a substantial part of social policy has consisted in an expansion of this type of consumption. How could we defend ourselves in a discussion with politicians, if we have "forgotten" this public consumption?

A central question concerning public consumption is this: How is education to be treated? Is it, for instance, reasonable to regard all education above the primary school level as consumption assignable to those individuals who make use of the education facilities offered by the authorities? Or is education to be regarded as investment from the individual's point of view?

My purpose in the simple remarks above is only to point out that we run into fundamental difficulties as soon as we try to illustrate, in a sensible way, the welfare distribution by statistical data. The discouraging fact we have to face is this: Even if we had access to all relevant data concerning the separate individual's income and consumption we would not know how to arrange these data in order to show the welfare distribution meaningfully. As has been said above we do not know how to make the grouping of individuals and we do not know how to treat public consumption. These are, however, only two examples out of along list of problems. How shall we, for instance, treat differences in working hours, in the time structure of consumption, and in wealth. My conclusion is that there is much research still to be done in this field.

An Alternative to the Traditional Method of Analysis

In a paper presented to an earlier IARIW conference I criticized the traditional measures of inequality for their lack of connection to economic theory.[1] I argued that it was impossible to give these measures a meaningful interpretation and this was due to the fact that they were constructed without regard to the purpose they were expected to serve. In that paper I also sketched a new method of distribution analysis. Contrary to the traditional methods this one was designed on the basis of the economic problem that was supposed to be elucidated by the analysis. The starting point of my approach was, in fact, the supposition that the distribution analysis was motivated by an interest in the relationship between the income distribution and the consumption of different goods.

With some modifications the method sketched in my earlier paper can be applied also to analyses connected to welfare aspects. In this section I shall show, briefly, how this could be done.

We consider a group of individuals, who are assumed to have identical, cardinally measurable, welfare functions $w(q)$ where q denotes the level of consumption. The frequency function of the individual consumption levels is denoted by $f(q)$. The aggregate welfare of the group, W, can then be written in the form

$$W = n \int_0^\infty w(q)f(q)\, dq,$$

where n is the number of individuals.

The welfare function $w(q)$ is assumed to have the following—traditional—properties

$$w(q) > 0$$
$$w'(q) > 0$$
$$w''(q) < 0$$

for all values of q. This assumption implies, obviously, that W is smaller than or equal to $nw(m)$, where m is the mean consumption level. The equality sign holds good only if all individual levels of consumption are equal. Consequently, the maximum aggregate welfare that could be obtained by a redistribution of consumption between the individuals is $nw(m)$. Let us denote this value by M.

The difference between M and W can be interpreted as the total welfare loss caused by the dispersion of the individual consumption levels.

The expression

$$\frac{M - W}{M} = 1 - \frac{W}{M}$$

can, consequently, be interpreted as the relative welfare loss due to the dispersion.

[1] Some Aspects of the Economic Interpretation of Changes in the Inequality of Income Distribution, *Income and Wealth*, Series VI, London 1957.

This being so it seems natural to regard the ratio

$$E = \frac{W}{M}$$

as an index of what could be called the "welfare efficiency of distribution".

When income or consumption distributions are studied from the welfare point of view, it is, of course, the welfare-creating properties of the distributions that are of importance. The statistics to be used as instruments of analysis should therefore be designed in such a way that they represent these properties. The considerations above show that this requirement is met by the E-measure. This measure is not, however, applicable to empirical analyses since we do not know the precise shape of the welfare function $w(q)$. Still, the above analysis gives rise to an important conclusion, which can be formulated in this way: Looking for an appropriate measure, applicable to empirical analyses, we should choose between measures of the type

$$P = \frac{\displaystyle\int_0^\infty h(x)f(x)\,dx}{h(m)}$$

where m is the mean value of x and $h(x)$ is a function possessing the above-mentioned properties of a welfare function. Let us call a measure of this type a P-measure. Obviously, the measure E above belongs to the class of P-measures.

A P-measure has in many respects the character of a measure of equality. Three properties could be mentioned: (1) It is equal to 1 for a perfectly even distribution. (2) The greater the distributional dispersion is the smaller is the P-measure. (3) A transfer of consumption from one individual to another with a lower level of consumption, i.e. a decrease in inequality, results in an increase in P, and vice versa.

It should be stressed that the traditional measures of inequality do not belong to the P-measure class. Let us, for instance, regard the Lorenz measure (the rate of concentration). This is defined as

$$L = \tfrac{1}{2} \int_0^\infty [F(x) - G(x)]f(x)\,dx$$

where $F(x)$ is the ratio between the number of individuals having incomes less than x and the number of all individuals in the population under consideration, while $G(x)$ is the ratio between the income sum going to individuals with incomes less than x and the total sum of incomes. It is easy to see that the function within the brackets does not fulfill the condition that it should increase as x increases for all values of x.

The P-measures and the Lorenz measure react differently to changes in the income distribution, Consider, for example, a transfer of income from one person to another with lower income. The Lorenz measure is symmetrical in the sense that it reacts to such a transfer in the same way irrespective of whether it occurs at the bottom or at the top of the distribution. A P-measure, however, is not symmetrical in this sense. Because of the assumption that marginal welfare

is a decreasing function of income (or consumption), a transfer in the bottom of the distribution has a greater effect on aggregate welfare and on the P-measure —than a corresponding transfer at the top of the distribution.

If we want to apply a P-measure to empirical data, we obviously have to specify the function $h(x)$. But how should this be done? The answer to this question is, of course, that the choice of h-function must be done in accordance with our hypothesis about the character of the relationship between welfare and consumption. Of course a certain amount of arbitrariness must enter into such a choice. This seems, however, not to be too serious. The degree of arbitrariness involved in such a procedure seems to be much smaller than that involved in traditional measures of inequality, since these measures are defined without any regard to the theory behind the analysis. In addition these measures do not show the properties we are interested in.

Let me illustrate how a P-measure analysis could be performed. Assume, that the h-function is specified as Ax^e, where A and e are constants. In fact, the factor A is quite unimportant for the P-measure, since it appears as a factor both in the nominator and in the denominator and can be cancelled. The constant e is, however, of central importance and, if we choose not to specify its value, P will be a function of e. Then, we get the following expression

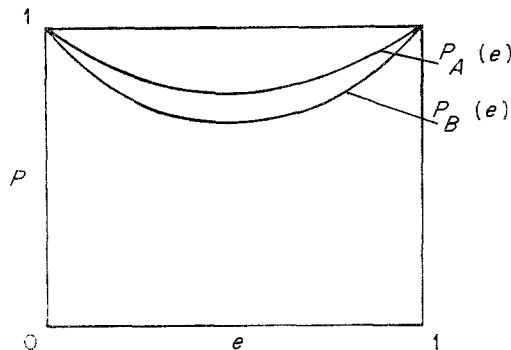$$P(e) = \frac{\int_0^\infty x^e f(x)\,dx}{m^e}.$$

This function $P(e)$ has the following general properties:

$$P = 1 \quad \text{for} \quad e = 0$$
$$P < 1 \quad \text{for} \quad 0 < e < 1$$
$$P = 1 \quad \text{for} \quad e = 1$$

Suppose now that we want to compare two or more distributions. For each one of these, we can calculate a number of P-values corresponding to different values of e. Such calculations allow us to draw a diagram like this:

where P_A and P_B represent the P-functions of two distributions A and B respectively.

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Assuming that the "real" welfare function is approximately reflected by the function Ax^e we can interpret the diagram in the following way. The distribution A possesses a greater "efficiency of distribution" than distribution B and the size of this difference in efficiency is represented, for alternative values of e, by the distance between the two curves. Using less technical terms we could express the same thing by saying that distribution A is less unequal than distribution B and that the distance between the two curves represents, for alternative values of e, those differences in aggregate welfare which are due to dissimilarities in dispersion.

Until now the analysis in this section has been based on the assumption that all individuals belonging to the population under consideration have identical welfare functions. The approach could, however, be applied also to more general situations. There are no theoretical difficulties involved in generalizing the analysis by considering two or more population groups with different welfare functions. If, for instance, we consider two such groups, denoted by A and B, the formula for the "welfare efficiency of distribution" would be

$$
E = \frac{n_A \int_0^\infty w_A(q) f_A(q)\,\mathrm{d}q + n_B \int_0^\infty w_B(q) f_B(q)\,\mathrm{d}q}{n_A w(q_A) + n_B w(q_B)},
$$

where q_A and q_B stand for those values of q which constitute equality between the marginal welfare of all individuals. This equality is, of course, the condition for a maximum of aggregate welfare under the constraint that aggregate consumption is given.

The generalization of the class of P-functions could be done in an analogous way. The necessary choice of function within this class is, however, more difficult than in the one-group case, since there are more than one h-functions to be specified. This double choice may, certainly involve great difficulties. These are, however, not due to the method but to difficulties connected to the very problem of finding a meaningful interpretation of the consumption (or income) distribution within a population consisting of individuals with different welfare functions. Usually we avoid this problem by pretending not to see it. This seems, however, not to be the very best procedure. It might be better to face the problem and to make serious efforts to solve it.