THE SEERS MODIFIED INPUT-OUTPUT TABLE: SOME PROJECTION TECHNIQUES

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Dudley Seers and his colleagues in working with various less developed economies have proposed a modified version of an input-output table for making projections and tests of consistency in planning. The table includes only the important inter-sectoral flows. By making a simplifying assumption with regard to the non-included inter-sectoral flows, an algebraic formulation of the modified input-output table is possible. The resulting matrix of input-output coefficients and final demands can be transformed into a Leontief input-output matrix which is block triangular and composed of two-blocks, one of which is diagonal. Given a set of final demands it is very easy to solve for the total output of each of the sectors. The amount of computation involved is directly related to the number of inter-sectoral flows included in the original modified input-output table.

I. INTRODUCTION

In a less developed country, often the statistics necessary for a full-fledged input-output table are not available. Furthermore, the compilation of statistics for such a table would be expensive in terms of skilled manpower used. Since, typically, inter-industry flows are so limited in most less-developed countries, the information that a fully articulated set of inter-industry accounts reveals is very often not worth the time and effort necessary to compile them (see Peacock [3]).

An alternative procedure is offered in [6]. This approach, suggested by Mr. Dudley Seers in working with the Ghanaian economy [5], and developed in research by Seers, and Reddaway [4] on various Latin American economies and the Jamaican, Zambian and Indian economies, concentrates on inter-industry flows only from industries primarily producing intermediate goods. Table 1 is a schematic illustration of such a "modified" input-output table.

II. FEATURES OF THE MODIFIED INPUT-OUTPUT TABLE

The most notable feature of the table from a theoretical or conceptual point of view is the lack of a one-to-one correspondence between industry rows and industry columns. Only a subset of those industries which have been allocated rows are also allocated columns, i.e., there are n industry rows and only n^* industry columns ($n^* < n$). The industries which are given columns are those which are primarily producers of intermediate products or which supply intermediate products to several industries. The inputs of each of these "intermediate goods industries" into each of the other industries is included in the table in columns labelled *1* through n^*. The inputs of all other industries ($n^* + 1, \ldots, n$) into each of the n industries is aggregated in the column labelled *0*. Another interesting aspect of the modified table is that distribution margins and indirect taxes on gross output and imports of

TABLE 1

SCHEMATIC MODIFIED INPUT-OUTPUT TABLE

Basic Inputs						Composition of Supply					Composition of Demand	
	MI	0	1	2 . . . n*	VA	GO	MF	D	IT	S/D	FD	SOS
1	MI_1	X_{01}	X_{11}	X_{21} . . . X_{n^*1}	VA_1	GO_1	MF_1	D_1	IT_1	SD_1	FD_1	SOS_1
2	MI_2	X_{02}	X_{12}	X_{22} . . . X_{n^*2}	VA_2	GO_2	MF_2	D_2	IT_2	SD_2	FD_2	SOS_2
.	.	.	.	. .	.	.	.	.	.	.	.	.
.	.	.	.	. .	.	.	.	.	.	.	.	.
.	.	.	.	. .	.	.	.	.	.	.	.	.
n*	MI_{n^*}	X_{on^*}	X_{1n^*}	X_{2n^*} . . . $X_{n^*n^*}$	VA_{n^*}	GO_{n^*}	MF_{n^*}	D_{n^*}	IT_{n^*}	SD_{n^*}	FD_{n^*}	SOS_{n^*}
.	.	.	.	. .	.	.	.	.	.	.	.	.
.	.	.	.	. .	.	.	.	.	.	.	.	.
.	.	.	.	. .	.	.	.	.	.	.	.	.
n	MI_n	X_{on}	X_{1n}	X_{2n} . . . X_{n^*n}	VA_n	GO_n	MF_n	D_n	IT_n	SD_n	FD_n	SOS_n

KEY:

MI: Imports of Intermediate Products
VA: Value Added
GO: Gross Output
MF: Imports of Final Products

D: Distribution
IT: Indirect Taxes
S/D: Total Supply (market price)/Total Demand

FD: Final Demand
SOS: Sales to Other Sectors
X_{ij}: Output of Industry i Used as Input in Industry j

final goods are entered in a section of the table called *composition of supply*. Distribution margins on imports of intermediate products are placed in the *basic inputs* section, so that the distribution industry has in effect been allocated two separate columns. The *composition of supply* section of the table also includes imports of final goods allocated according to the industry in which presumably they would have been produced if produced domestically. This breakdown of imports is especially useful in devising policies of import substitution. Total supply of each industry (at market price) is the sum of gross output, imports of final products, distribution margins, and indirect taxes. The total supply equals total demand, and the *composition of demand* includes final demand (private consumption, government consumption, investment, exports, and changes in stocks) and sales to other sectors.

The table developed by Seers includes other columns and rows not shown schematically in Table 1. It is not the purpose of this paper to discuss at length the format of Seers' modified input-output table (the reader is referred to [7]), but some features might be mentioned. In particular, the Seers table divides imports of intermediate goods into c.i.f. value and import duties. Value added is divided into its component parts, including, in the case of Zambia, value added by subsistence production and the breakdown of income by type of recipient (whether African or European). Final demand is also broken down into its component parts. Finally, the Seers table includes a set of rows to indicate the type of cash flow originating from the transactions indicated by the entries in the rest of the table. The extra rows include an industry row, a rest-of-the-world row, a government current account row and a savings and investment row. For example, the total of indirect taxes (the total of the entries in the first n rows of the IT column) is entered in the government current account row as a receipt (i.e., with a negative sign to indicate a receipt); the total of each of the import columns (MI and MF) is entered as a receipt in the rest-of-the-world row and total exports as a payment. This last set of rows incorporates into the table a set of appropriation accounts for government, households, the rest of the world, etc. A by-product is the balance of payments on current account and the government current account surplus.

The modified input-output table can be very useful in planning and in making planning projections. There are many different methods that one can use to make projections depending on which variables one chooses as target variables, which are policy variables, which are endogenously determined and which are residuals. The rest of this paper indicates the way in which one might make projections if the final demands for each of the n industries are target variables; imports of final products are policy variables depending on the assumed degree of import substitution; and all other entries in the modified table are determined by a set of structural equations.

III. Structural Coefficients

Table 2 summarizes the relevant structural coefficients of the model. The coefficients are derived from the base year entries in Table 1 as follows:

$m_i = \dfrac{MI_i}{GO_i} \equiv$ imports of intermediate products as a proportion of gross output of industry i (for $i = 1, \ldots, n$).

TABLE 2

STRUCTURAL COEFFICIENTS

	MI	0	1	2	... n^*	D	IT	SOS
1	m_1	a_{01}	a_{11}	a_{21}	... a_{n^*1}	d_1	t_1	–
2	m_2	a_{02}	a_{12}	a_{22}	... a_{n^*2}	d_2	t_2	–
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
n^*	m_{n^*}	a_{0n^*}	a_{1n^*}	a_{2n^*}	... $a_{n^*n^*}$	d_{n^*}	t_{n^*}	–
n^*+1	m_{n^*+1}	a_{0n^*+1}	a_{1n^*+1}	a_{2n^*+1}	... $a_{n^*n^*+1}$	d_{n^*+1}	t_{n^*+1}	η_{n^*1}
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
n	m_n	a_{0n}	a_{1n}	a_{2n}	... a_{n^*n}	d_n	t_n	η_n

$a_{ij} = \dfrac{X_{ij}}{GO_j} \equiv$ the input coefficient relating intermediate products from industry i used per unit of output of industry j (for $i = 1, \ldots, n^*$ and $j = 1, \ldots, n$).

$a_{0j} = \dfrac{X_{0j}}{GO_j} \equiv$ inputs of intermediate products, other than imports and other than those originating from industries *1* through n^*, per unit of output of industry j (for $j = 1, \ldots, n$).

$d_i = \dfrac{D_i}{GO_i + MF_i} \equiv$ distribution margin on gross output and imports of final products, of industry i (for $i = 1, \ldots, n$).

$t_i = \dfrac{IT_i}{GO_i + MF_i} \equiv$ rate of indirect taxation on gross output and imports of final products of industry i (for $i = 1, \ldots, n$).

$\eta_i = \dfrac{SOS_i}{\sum_{i=n^*+1}^{n} SOS_i} = \dfrac{SOS_i}{\sum_{i=1}^{n} X_{0i}} \equiv$ intermediate goods output of industry i as a proportion of total intermediate sales of industries $i = n^* + 1, \ldots, n$.

Throughout the analysis, we assume all of the above parameters are constant. The constancy of the η_i implies that the intermediate sales of industries $n^* + 1$ through n increase proportionately. Other assumptions might be made, but this particular one is unlikely to cause gross inaccuracies in the projections since by definition industries $n^* + 1$ through n are not primarily sellers of intermediate products.

IV. The Structural Equations

We shall use the following notation:

$FD_i^* \equiv$ target final demand for industry i for $i = 1, \ldots, n$.
$MF_i^* \equiv$ imports of final products of industry i for $i = 1, \ldots, n$.
$F_i^* \equiv$ final demand of industry i which is to be supplied domestically for $i = 1, \ldots, n$.
$y_i \equiv$ gross output of industry i for $i = 1, \ldots, n$.

Given FD_i^* and MF_i^* (and by implication $F_i^* = FD_i^* - MF_i^*$), one can determine the y_i by use of a set of structural equations. Once the y_i have been determined, the values of all other variables can be computed easily.

Before writing out the structural equations, let us specify that industry 1 is the distribution industry since that industry plays a special role in the modified table. In order that intermediate sales plus final sales equal gross output of the distribution industry, the following equation must hold

$$\sum_{j=1}^{n} a_{1j} \cdot y_j + \sum_{j=1}^{n} d_j \cdot (y_j + MF_j^*) + F_1^* = y_1. \tag{1}$$

For other "intermediate goods industries," the following equations must hold:

$$\sum_{j=1}^{n} a_{ij} \cdot y_j + F_i^* = y_i \text{ for } i = 2, 3, \ldots, n^*. \tag{2}$$

For all other industries, the following must hold:

$$\eta_i \sum_{j=n^*+1}^{n} (y_j - F_j^*) + F_i^* = y_i \tag{3}$$

for

$$i = n^* + 1, n^* + 2, \ldots, n.$$

Furthermore, the total intermediate sales of industries $n^* + 1$ through n must satisfy the following input constraint

$$\sum_{j=1}^{n} a_{0j} \cdot y_j = \sum_{j=n^*+1}^{n} (y_j - F_j^*). \tag{4}$$

Finally, there are non-negativity restrictions on the variables y_i and a constraint on the parameters η_i.

$$y_i \geqq 0 \text{ for } i = 1, \ldots, n \tag{5}$$

and

$$\sum_{i=n^*+1}^{n} \eta_i = 1. \tag{6}$$

These structural equations may be cast into a Leontief input-output format by substituting the right-hand side of (4) into (3) and by collecting some terms.

$$\sum_{j=1}^{n} a'_{ij} \cdot y_j + F_i' = y_i \text{ for } i = 1, \ldots, n^* + 1, \ldots, n. \tag{7}$$

where

$$\begin{aligned} a'_{1j} &= a_{1j} + d_j \text{ for } j = 1, \ldots, n; \\ a'_{ij} &= a_{ij} \qquad \text{for } i = 2, \ldots, n^*, \text{ and} \\ &\qquad\qquad j = 1, \ldots, n; \\ a'_{ij} &= \eta_i \cdot a_{0j} \quad \text{for } i = n^* + 1, \ldots, n, \text{ and} \\ &\qquad\qquad j = 1, \ldots, n. \end{aligned} \tag{8}$$

and where

$$\begin{aligned} F_1' &= F_1^* + \sum_{j=1}^{n} d_j \cdot MF_j^* \\ F_i' &= F_i^* \text{ for } i = 2, \ldots, n. \end{aligned} \tag{9}$$

The usual methods of input-output analysis may be applied to this transformed version of the Seers model.

Another approach, however, enables one to take computational advantage of the simple nature of the Seers analysis by a simple triangulation of the matrix of input-output coefficients. Letting $i = n^* + 1$, from (3) we have

$$\sum_{j=M^*+1}^{n} (y_j - F_j^*) = \frac{y_{n^*+1} - F^*_{n^*+1}}{\eta_{n^*+1}}. \tag{10}$$

Substituting (10) into (3), we have

$$(11) \qquad y_i = \frac{\eta_i}{\eta_{n^*+1}} (y_{n^*+1} - F^*_{n^*+1}) + F_i^* \text{ for } i = n^* + 2, \ldots, n.$$

Next substitute these expressions for y_i ($i = n^* + 2, \ldots, n$) back into (1) and (2) to obtain

$$(12) \qquad \sum_{j=1}^{n^*+1} a^o_{ij} \cdot y_j + F^o_i = y_i \text{ for } i = 1, \ldots, n^* + 1$$

where

$$a^o_{1j} = a_{1j} + d_j \quad \text{for } j = 1, \ldots, n^*$$

$$a^o_{1,n^*+1} = \frac{1}{\eta_{n^*+1}} \sum_{j=n^*+1}^{n} \eta_j(a_{1j} + d_j);$$

$$a^o_{ij} = a_{ij} \quad \text{for } j = 1, \ldots, n^*, \text{ and}$$

$$i = 2, \ldots, n^*;$$

$$(13) \qquad a^o_{i,n^*+1} = \frac{1}{\eta_{n^*+1}} \sum_{j=n^*+1}^{n} \eta_j \cdot a_{ij} \quad \text{for } i = 2, \ldots, n^*;$$

$$a^o_{n^*+1,j} = \eta_{n^*+1} \cdot a_{oj} \quad \text{for } j = 1, \ldots, n^*;$$

$$a^o_{n^*+1,n^*+1} = \sum_{j=n^*+1}^{n} \eta_j a_{oj}.$$

and where

$$(14) \qquad F^o_1 = F_1^* + \sum_{j=1}^{n} d_j \cdot MF_j^* + \frac{1}{\eta_{n^*+1}} \sum_{j=n^*+2}^{n} (a_{1j} + d_j)(\eta_{n^*+1} \cdot F_j^* - \eta_j \cdot F^*_{n^*+1})$$

$$F^o_i = F_i^* + \frac{1}{\eta_{n^*+1}} \sum_{j=n^*+2}^{n^*} a_{ij}(\eta_{n^*+1} \cdot F_j^* - \eta_j \cdot F^*_{n^*+1})$$

$$F^o_{n^*+1} = F^*_{n^*+1} + \sum_{j=n^*+2}^{n} a_{oj}(\eta_{n^*+1} \cdot F_j^* - \eta_j F^*_{n^*+1})$$

Equation (12) gives a reduced Leontief system which has $n^* + 1$ dimensions which may be considerably less than the n dimensions of the original problem. Equations (11) permit one to solve for $y_{n^*+2}, \ldots, y_n$ once the value of y_{n^*+1} is obtained from solving (12). For example, in the ECA[6] version of the Seers model for Zambia, there are only three intermediate industries ($n^* + 1 = 4$) while there are thirteen sectors altogether. Projections can be made by inverting a matrix of order four (quite simple using the power series approximation) and then using equations (11) to solve directly for the gross outputs of the other sectors.

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Dudley Seers et ses collègues dans leurs travaux sur les économies sous-developpées ont proposé une version modifiée d'un tableau d'échanges interindustriels qui permettrait de faire des projections et de vérifier la cohérence des plans. Le tableau comporte seulement les plus importants des flux intersectoraux. Si l'on fait une hypothèse de simplification au sujet des flux intersectoraux non-compris, une formule algébrique d'un tableau d'échanges interindustriels modifié devient possible. La matrice de coefficients entrée-sortie et de demandes finales qui en resulte peut être transformée en une matrice entrée-sortie du type Leontief d'aspect bloc-triangulaire et composée de deux blocs, un desquels est diagonal. Étant donné un ensemble de demandes finales, il est très facile de resoudre le système et de calculer la production brute de chacun des secteurs. Le volume des calculs à faire est directement lié au nombre de flux intersectoraux compris dans le tableau original modifié d'échanges interindustriels.